LINGO

The Modeling Language and Optimizer

LINDO SYSTEMS INC.

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Preface

LINGO is a comprehensive tool designed to make building and solving mathematical optimization models easier and more efficient. LINGO provides a completely integrated package that includes a powerful language for expressing optimization models, a full-featured environment for building and editing problems, and a set of fast built-in solvers capable of efficiently solving most classes of optimization models. LINGO's primary features include:

- ***Algebraic Modeling Language***
  LINGO supports a powerful, set-based modeling language that allows users to express math programming models efficiently and compactly. Multiple models may be solved iteratively using LINGO's internal scripting capabilities.

- ***Convenient Data Options***
  LINGO takes the time and hassle out of managing your data. It allows you to build models that pull information directly from databases and spreadsheets. Similarly, LINGO can output solution information right into a database or spreadsheet making it easier for you to generate reports in the application of your choice. Complete separation of model and data enhance model maintenance and scalability.

- ***Model Interactively or Create Turnkey Applications***
  You can build and solve models within LINGO, or you can call LINGO directly from an application you have written. For developing models interactively, LINGO provides a complete modeling environment to build, solve, and analyze your models. For building turn-key solutions, LINGO comes with callable DLL and OLE interfaces that can be called from user written applications. LINGO can also be called directly from an Excel macro or database application. LINGO currently includes programming examples for C/C++, FORTRAN, Java, C#.NET, VB.NET, ASP.NET, Visual Basic, Delphi, and Excel.

- ***Extensive Documentation and Help***
  LINGO provides all of the tools you will need to get up and running quickly. You get the LINGO Users Manual (in printed form and available via the online Help), which fully describes the commands and features of the program. Also included with Super versions and larger is a copy of Optimization Modeling with LINGO, a comprehensive modeling text discussing all major classes of linear, integer and nonlinear optimization problems. LINGO also comes with dozens of real-world based examples for you to modify and expand.

- ***Powerful Solvers and Tools***
  LINGO is available with a comprehensive set of fast, built-in solvers for linear, nonlinear (convex & nonconvex), quadratic, quadratically constrained, and integer optimization. You never have to specify or load a separate solver, because LINGO reads your formulation and automatically selects the appropriate one. A general description of the solvers and tools available in LINGO follows:

General Nonlinear Solver
LINGO provides both general nonlinear and nonlinear/integer capabilities. The nonlinear license option is required in order to use the nonlinear capabilities with LINDO API.

Global Solver
The global solver combines a series of range bounding (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branch-and-bound framework to find proven global solutions to nonconvex nonlinear programs. Traditional nonlinear solvers can get stuck at suboptimal, local solutions. This is no longer the case when using the global solver.

Multistart Solver
The multistart solver intelligently generates a sequence of candidate starting points in the solution space of NLP and mixed integer NLPs. A traditional NLP solver is called with each starting point to find a local optimum. For non-convex NLP models, the quality of the best solution found by the multistart solver tends to be superior to that of a single solution from a traditional nonlinear solver. A user adjustable parameter controls the maximum number of multistarts to be performed.

Barrier Solver
The barrier solver is an alternative way for solving linear, quadratic and conic problems. LINGO's state-of-the-art implementation of the barrier method offers great speed advantages for large-scale, sparse models.

Simplex Solvers
LINGO offers two advanced implementations of the primal and dual simplex methods as the primary means for solving linear programming problems. Its flexible design allows the users to fine tune each method by altering several of the algorithmic parameters.

Mixed Integer Solver
The mixed integer solver's capabilities of LINGO extend to linear, quadratic, and general nonlinear integer models. It contains several advanced solution techniques such as cut generation, tree reordering to reduce tree growth dynamically, and advanced heuristic and presolve strategies.

Stochastic Solver
The stochastic programming solver supports decision making under uncertainty through multistage stochastic models with recourse. The user describes the uncertainty by identifying the distribution functions, either built-in or user-defined, describing each random variable. The stochastic solver will optimize the model to minimize the cost of the initial stage plus the expected cost of future recourse actions over the planning horizon. Advanced sampling modes are also available for approximating continuous distributions. LINGO's stochastic solver also supports chance-constrained models, where one or more sets of constraints are allowed to be violated according to a specified probability.

Model and Solution Analysis Tools
LINGO includes a comprehensive set of analysis tools for debugging infeasible linear, integer and nonlinear programs, using advanced techniques to isolate the source of infeasibilities to the smallest subset of the original constraints. It also has tools to perform sensitivity analysis to determine the sensitivity of the optimal basis to changes in certain data components (e.g. objective vector and right-hand-size values).

Quadratic Recognition Tools
The QP recognition tool is a useful algebraic pre-processor that automatically determines if an arbitrary NLP is actually a convex, quadratic model. QP models may then be passed to the faster quadratic solver, which is available as part of the barrier solver option. When the barrier solver option is combined with the global option, LINGO will automatically recognize conic models, in addition to convex quadratic models.

Linearization Tools
Linearization is a comprehensive reformulation tool that automatically converts many non-smooth functions and operators (e.g., max and absolute value) to a series of linear, mathematically equivalent expressions. Many non-smooth models may be entirely linearized. This allows the linear solver to quickly find a global solution to what would have otherwise been an intractable nonlinear problem.

New Features for LINGO 19.0

LINDO Systems is proud to introduce LINGO 19.0. The new features added to LINGO include the following:

- *Enhancements to the simplex solvers, such that the average performance on large linear programs (where the number of variables is several times more than the number of constraints) has increased by several factors compared to previous releases.*

- *Improved performance on LP's when using multiple cores with concurrent execution of Primal, Dual, and Barrier solvers.*

- *Improved selection of defaults for cut generation and heuristics in MIP solver.*

- *Support for exponential and power cones in the quadratic and conic solvers, with faster solution time for models with* e^x *and* x^k *terms.*

- *Much improved ability to recognize convexity and various composite functions in global solver, especially involving logs and sums, with order-of-magnitude improvement in some cases.*

- *Large performance gains in global solver for models with objectives involving ratios.*

- *Improved performance on non-convex QCQP models in the global solver.*

- *More efficient storage of stochastic programming solutions allowing for considerably larger stochastic models.*

- *All-different capability extended to allow specification of a range of values allowed for the set of all-different integer variables.*

- *Addition of the @NULLSET function for deleting all members of a derived set.*

- *Ability to prompt user for input at runtime with the @INPUT function, and retrieve Lingo's version number with the @VERSION function.*

- *Significantly improved use of starting points to speed runtimes on integer programming models.*

- *Commands to write models to and read models from Excel, to facilitate the use of embedded LINGO models in Excel.*

We hope you enjoy this new release of LINGO. Many of the new features in this release are due to suggestions from our users. In particular, we'd like to thank both Robert Coughlan and Wu Jian (Jack) for their many useful suggestions for improving LINGO. If there are any features you'd like to see in the next release of LINGO, please let us know. Feel free to reach us at:

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1 Getting Started with LINGO

LINGO is a simple tool for utilizing the power of linear and nonlinear optimization to formulate large problems concisely, solve them, and analyze the solution. Optimization helps you find the answer that yields the best result; attains the highest profit, output, or happiness; or the one that achieves the lowest cost, waste, or discomfort. Often these problems involve making the most efficient use of your resources-including money, time, machinery, staff, inventory, and more. Optimization problems are often classified as linear or nonlinear, depending on whether the relationships in the problem are linear with respect to the variables.

If you are a new user, it is recommended you go through the first seven chapters of this document to familiarize yourself with LINGO. Then you may want to see *On Mathematical Modeling* for more information on the difference between linear and nonlinear models and how to develop large models. It may also be helpful to view some sample models in *Developing More Advanced Models* or *Additional Examples of LINGO Modeling* to see if a particular template example is similar to a problem you have. For users of previous versions of LINGO, the new features are summarized in the *Preface* at the beginning of the manual.

Getting Started on Windows

Installing LINGO on Windows

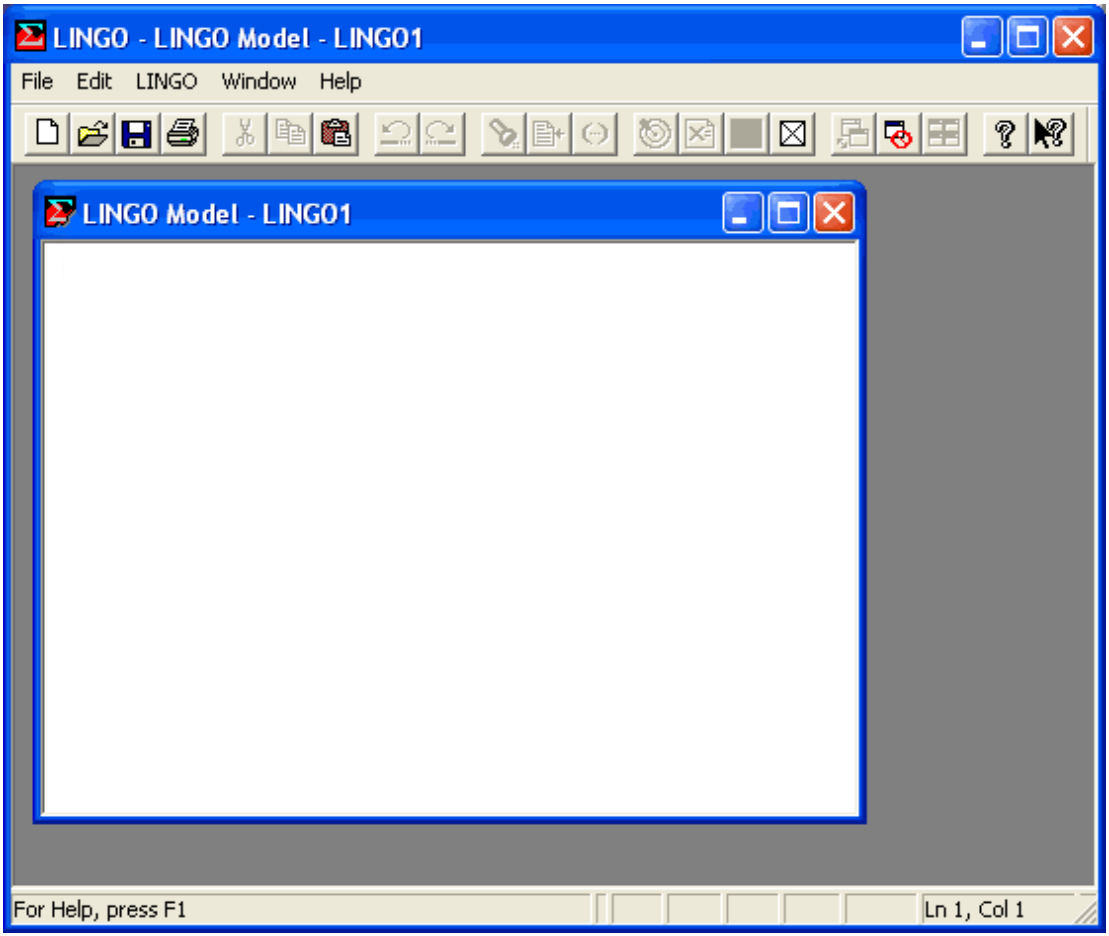
Installing the LINGO software is straightforward. To setup LINGO for Windows, place your CD in the appropriate drive and run the installation program *SETUP* contained in the LINGO folder. Alternatively, if you downloaded LINGO from the LINDO website, locate the download installation program and double-click it to run the installation. The LINGO installation program will open and guide you through the steps required to install LINGO on your hard drive.

Note:	If there is a previous version of LINGO installed on your machine, then you may need to uninstall it before you can install the new copy of LINGO. To uninstall the existing copy of LINGO, click on the *Windows Start* button, select the *Settings* command, select *Control Panel*, then double-click on the *Add or Remove Programs* icon. You should then be able to select LINGO and have the old version removed from your system.

Starting LINGO on Windows

Once LINGO is installed, you will find a new Lingo icon on your desktop:

You may double-click on the icon to start LINGO. Once LINGO is running, your screen will resemble the following:

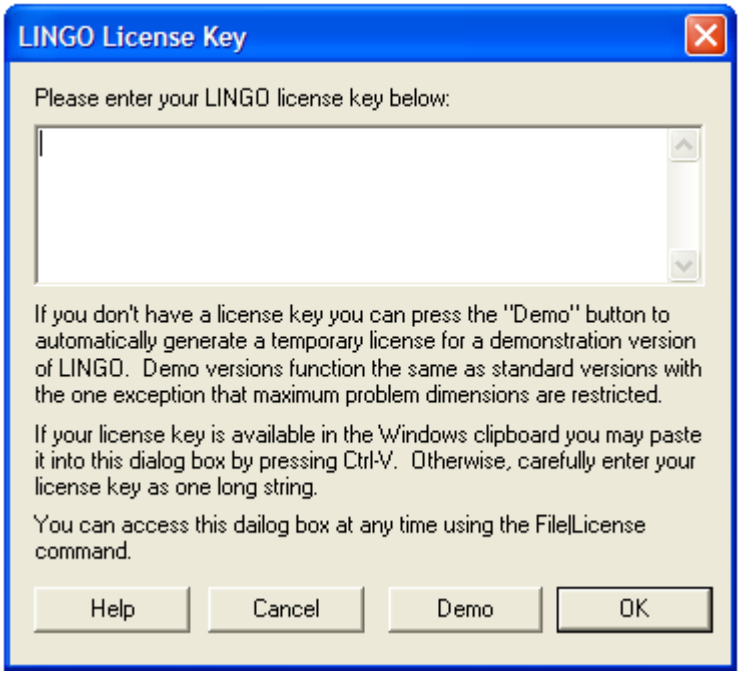


The outer window labeled LINGO is the main frame window. All other windows will be contained within this window. The top of the frame window also contains all the command menus and the command toolbar. See *Menu Commands* for details on the toolbar and menu commands. The lower edge of the main frame window contains a status bar that provides various pieces of information

regarding LINGO's current state. Both the toolbar and the status bar can be suppressed through the use of the *Options* command.

The smaller child window labeled *LINGO Model - LINGO1* is a new, blank model window. In the next section, we will be entering a sample model directly into this window.

Many copies of LINGO come with their licenses pre-installed. However, some versions of LINGO require you to input a license key. If your version of LINGO requires a license key, you will be presented with the following dialog box when you start LINGO:



Your license key may have been included in an email sent to you when you ordered your software. The license key is a string of letters, symbols and numbers, separated into groups of four by hyphens (e.g., `r82m-XCW2-dZu?-%72S-fD?S-Wp@`). Carefully enter the license into the edit field, including hyphens. License keys are case sensitive, so you must be sure to preserve the case of the individual letters when entering your key. Click the *OK* button and, assuming the key was entered correctly, LINGO will then start. In the future, you will be able to run LINGO directly without entering the key.

Note:	If you received your license key by email, then you have the option of cutting-and-pasting it into the license key dialog box. Cut the key from the email that contains it with the Ctrl+C key, then select the key field in LINGO dialog box and paste the key with the Ctrl+V key.

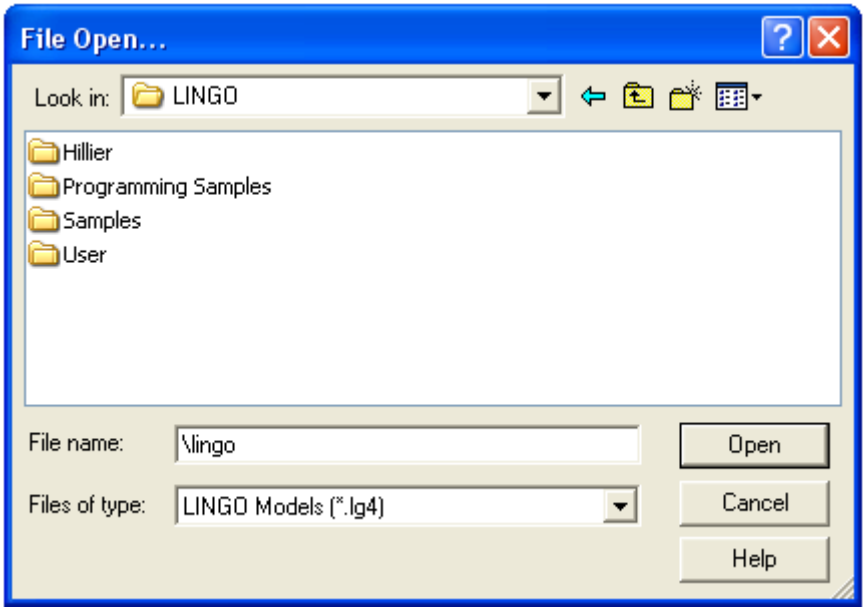
If you don't have a key, you can choose to run LINGO in demo mode by clicking the Demo button. In demo mode, LINGO has all the functionality of a standard version of LINGO with the one exception that the maximum problem size is restricted. Demo licenses expire after 180 days.

Opening a Sample Model on Windows

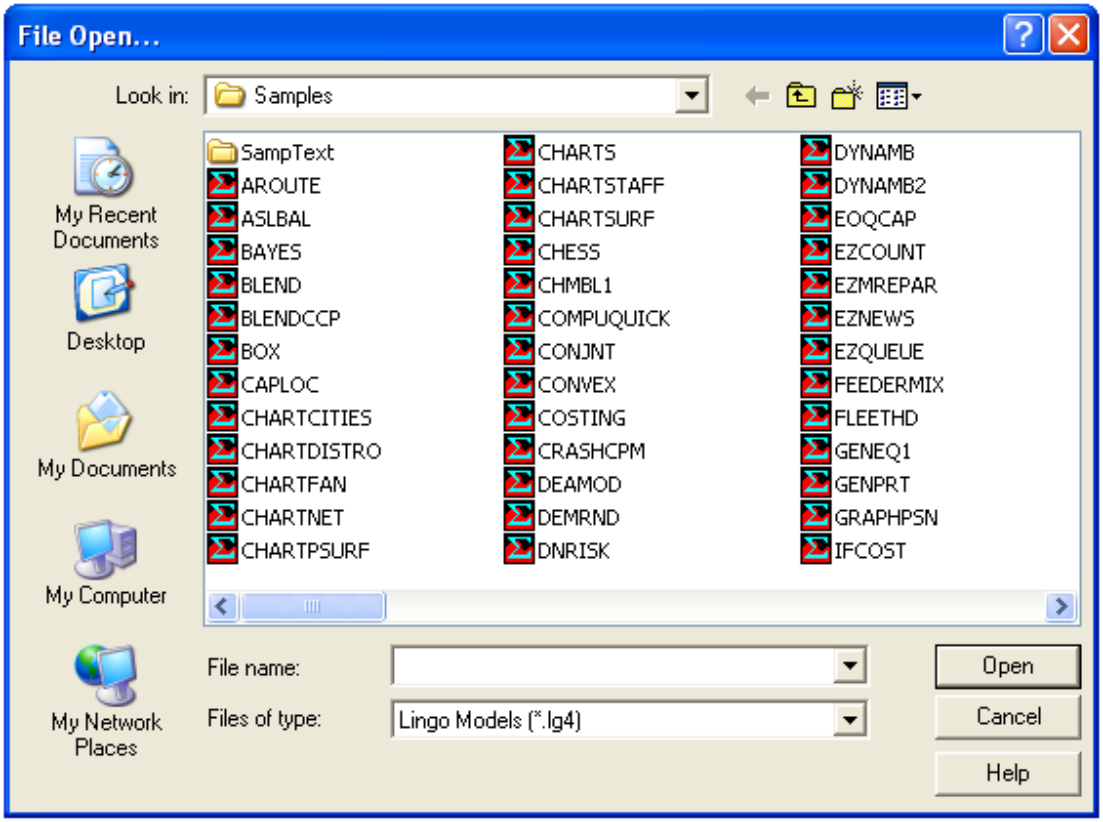
LINGO is shipped with a directory containing many sample models. These models are drawn from a wide array of application areas. For a complete listing of these models, see *Additional Examples of LINGO Modeling*. The sample model directory is titled *Samples* and is stored directly off the many LINGO directory.

To open a sample model in LINGO, follow these steps:

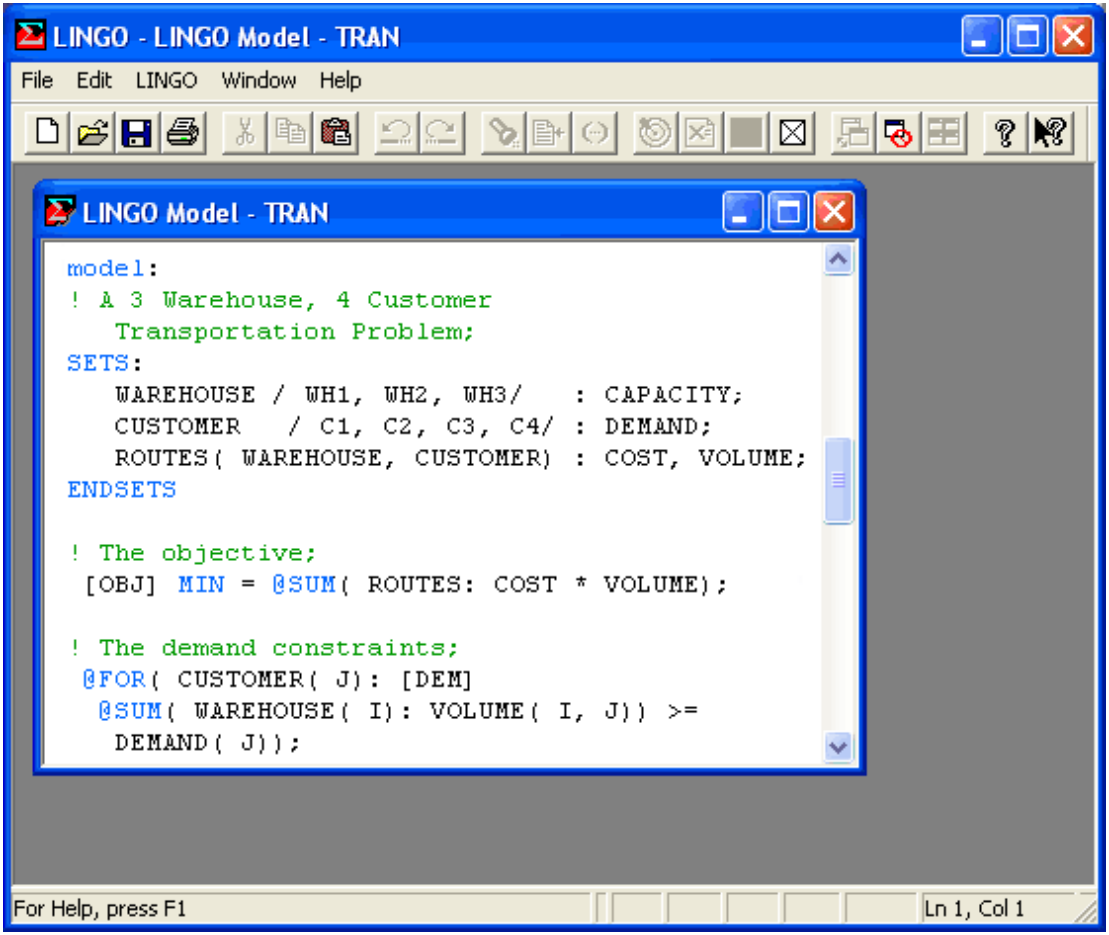
1. Pull down the *File* menu and select the *Open* command. You should see the following dialog box:



2. Double-click on the folder titled *Samples* contained in the main LINGO folder installed off your root folder, at which point you should see:

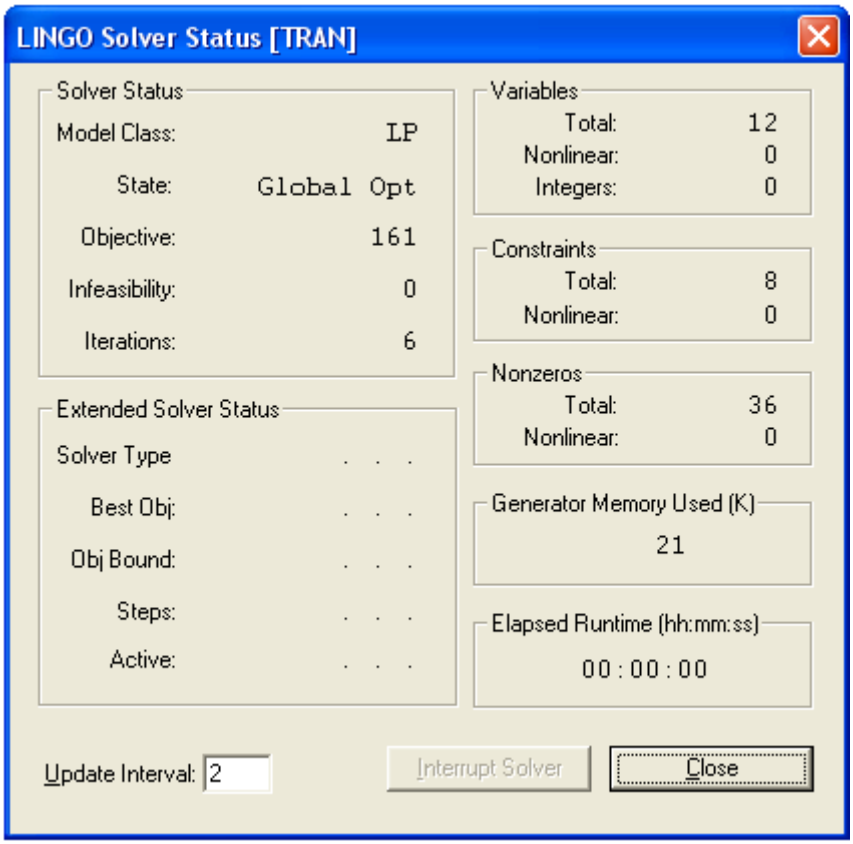


3. To read in a small transportation model, type `Tran` in the *File Name* field in the above dialog box and press the *Open* button. You should now have the model in an open window in LINGO as follows:



For details on developing a transportation model in LINGO see *A Sample Transportation Model* in this chapter.

You may now solve the model using the Solver|Solve command or by pressing the button on the toolbar. The optimal objective value for this model is 161. When solved, you should see the following solver status window:



Note the objective field has a value of 161 as expected. For an interpretation of the other fields in this window, see *Solver Status Window* in Getting Started with LINGO.

Behind the solver status window, you will find the solution report for the model. This report contains summary information about the model as well as values for all the variables. This report's header is reproduced below:

Solution Report - tran

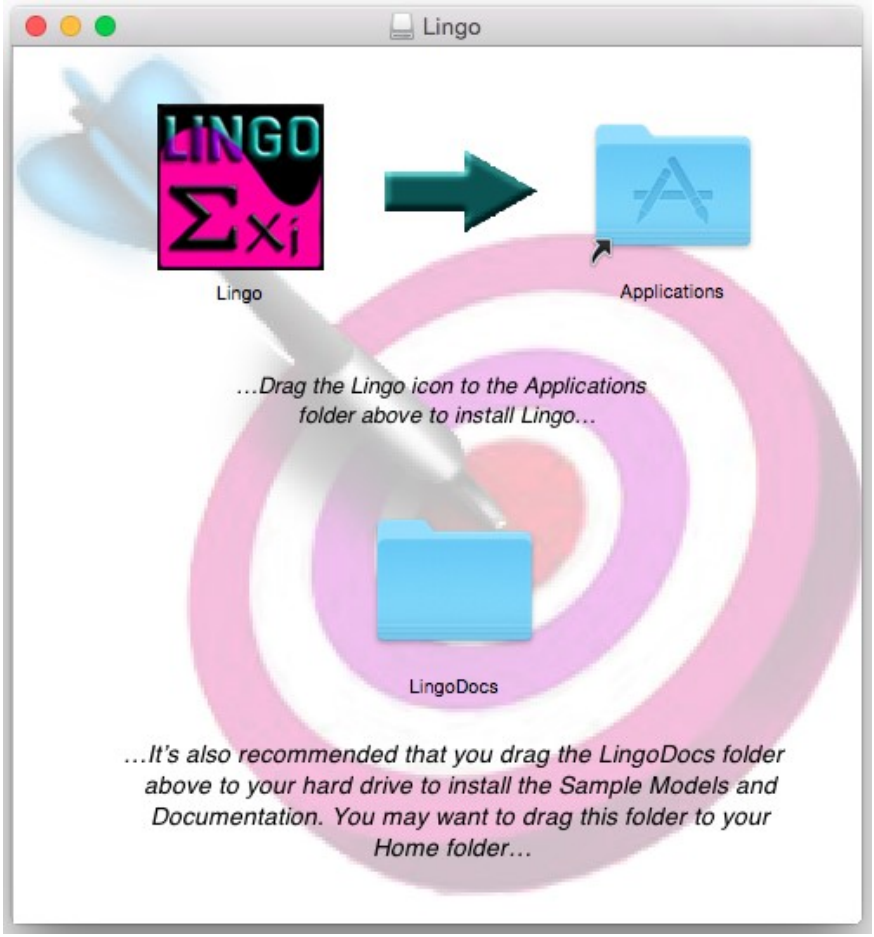
```
Global optimal solution found.
Objective value:                              161.0000
Infeasibilities:                              0.000000
Total solver iterations:                             6
Elapsed runtime seconds:                          0.03

Model Class:                                        LP

Total variables:                      12
Nonlinear variables:                   0
Integer variables:                     0

Total constraints:                     8
Nonlinear constraints:                 0

Total nonzeros:                       36
Nonlinear nonzeros:                    0

      Variable           Value        Reduced Cost
CAPACITY( WH1)        30.00000            0.000000
CAPACITY( WH2)        25.00000            0.000000
CAPACITY( WH3)        21.00000            0.000000
   DEMAND( C1)        15.00000            0.000000
   DEMAND( C2)        17.00000            0.000000
   DEMAND( C3)        22.00000            0.000000
   DEMAND( C4)        12.00000            0.000000
```

For information on interpreting the fields in the solution report, see *Sample Solution Report* in Getting Started with LINGO.

Getting Started on a Mac

Installing LINGO on a Mac

LINGO for the Mac is distributed as a .DMG file, or disk image file, titled *LINGO-OSX-64x86-19.0.dmg*. If you double-click on this file you should see a window similar to the following:

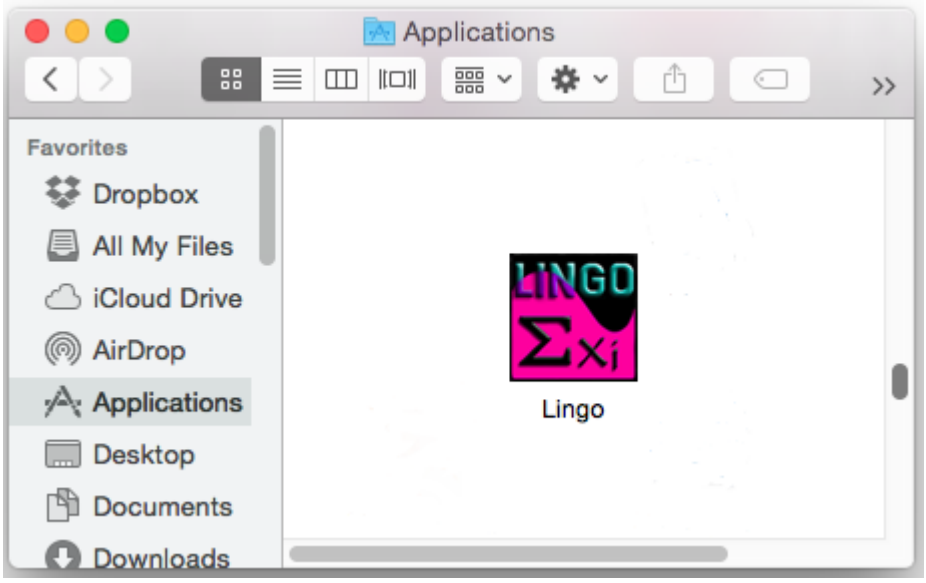


To install LINGO onto your Mac, drag the LINGO icon in the window to the *Applications* folder icon. This will place a copy of LINGO in the /*Applications* folder of your hard drive. The Mac version of LINGO requires that you have installed OS X 10.7, or later, on your system.

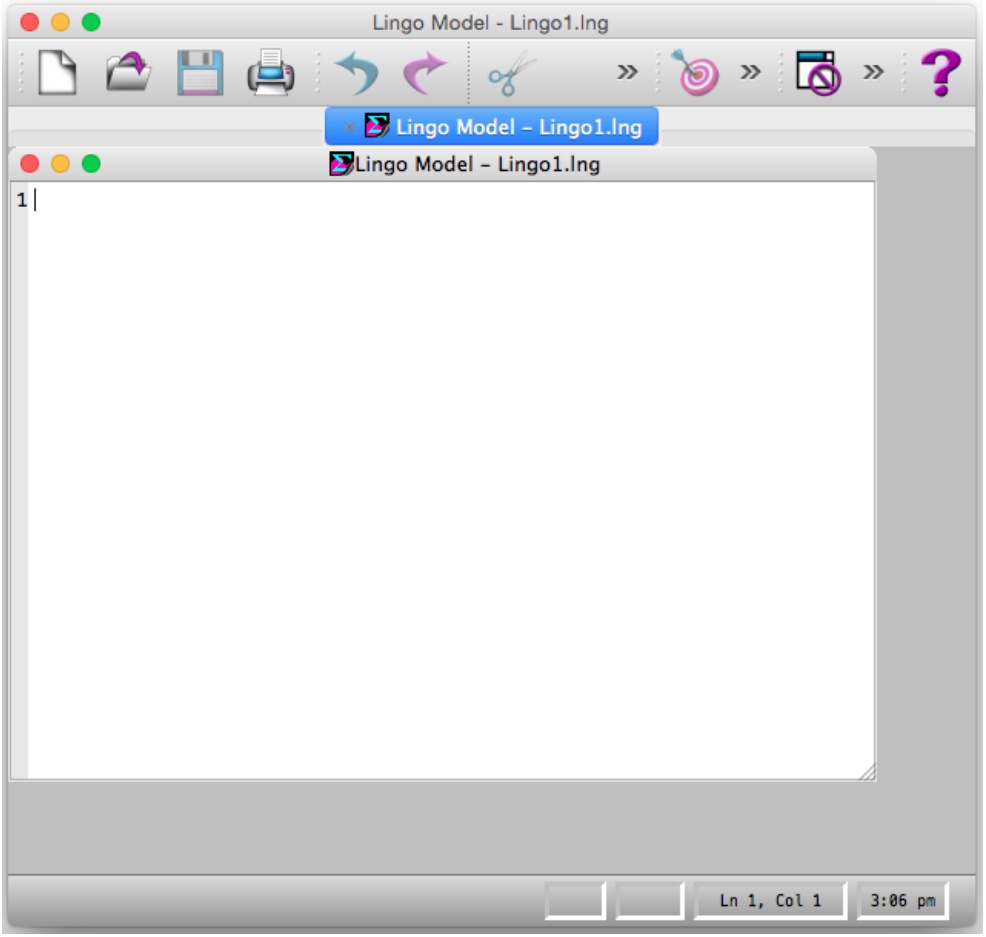
We also recommend that you drag a copy of the *LingoDocs* folder in the above window to your hard drive. The *LingoDocs* folder contains copies of LINGO's sample models and documentation. You may want to place this folder in your *$HOME* folder (subsequent documentation will assume this is the case).

Starting LINGO on a Mac

Once LINGO is installed, you will find a new LINGO icon in the */Applications* folder on your Mac's hard drive. The icon should appear as follows:



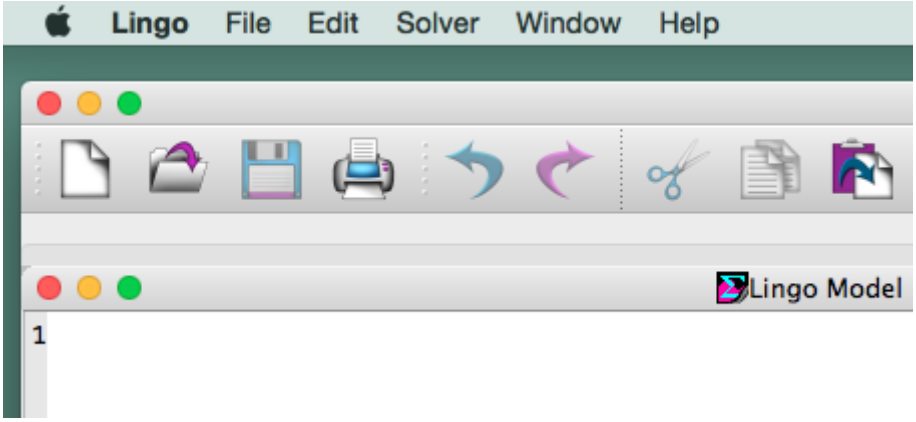
You may double-click on the icon to start LINGO. Once LINGO is running, your screen will resemble the following:



The outer window is the main frame window, and all other windows will be contained within this main frame window. The top of the main frame window also contains a toolbar for executing various LINGO commands.

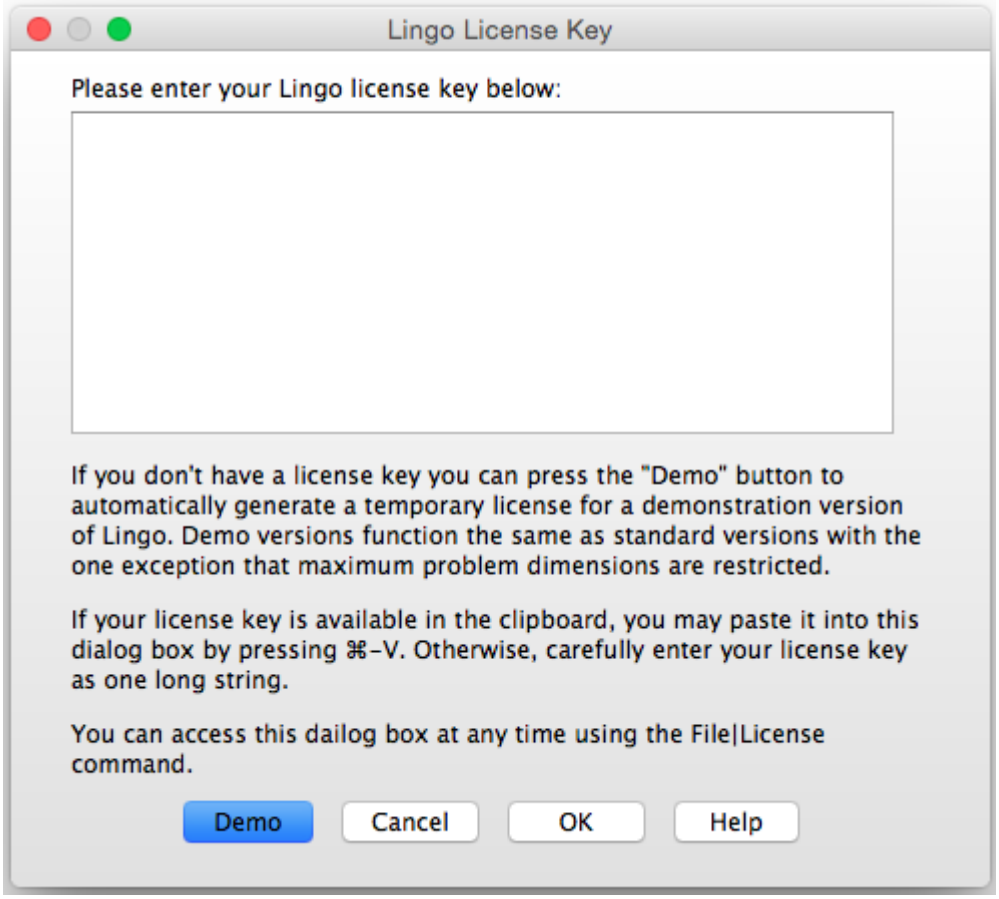
The smaller child window labeled *Lingo Model - Lingo1.lng* is a new, blank model window. In the *Developing a LINGO Model* section, we will be entering a sample model directly into this window.

Unlike the Windows version of LINGO, the command menus do not appear at the top of the main frame window, but appear in the Finder's menu bar whenever LINGO is the active application. Below, we see the six LINGO menus — *Lingo*, *File*, *Edit*, *Solver*, *Window* and *Help* — in the Finder's menu bar at the top of the screen:



See the *LINGO Commands* section for details on the toolbar and menu commands. The lower edge of the main frame window contains a status bar that provides various pieces of information regarding LINGO's current state. Both the toolbar and the status bar can be suppressed through the *LINGO|Preferences* command.

Many copies of LINGO come with their licenses pre-installed. However, some versions of LINGO require you to input a license key. If your version of LINGO requires a license key, you will be presented with the following dialog box when you start LINGO:



Your license key may have been included in an email sent to you when you ordered your software. The license key is a string of letters, symbols and numbers, separated into groups of four by hyphens (e.g., r82m-XCW2-dZu?-%72S-fD?S-Wp@). Carefully enter the license into the edit field, including hyphens. License keys are case sensitive, so you must be sure to preserve the case of the individual letters when entering your key. Click the OK button and, assuming the key was entered correctly, LINGO will then start. In the future, you will be able to run LINGO directly without entering the key.

Note:	If you received your license key by email, then you have the option of copying-and-pasting it into the license key dialog box. Copy the key from the email with the Command+C key, then select the key field in LINGO dialog box and paste the key with the Command+V key.

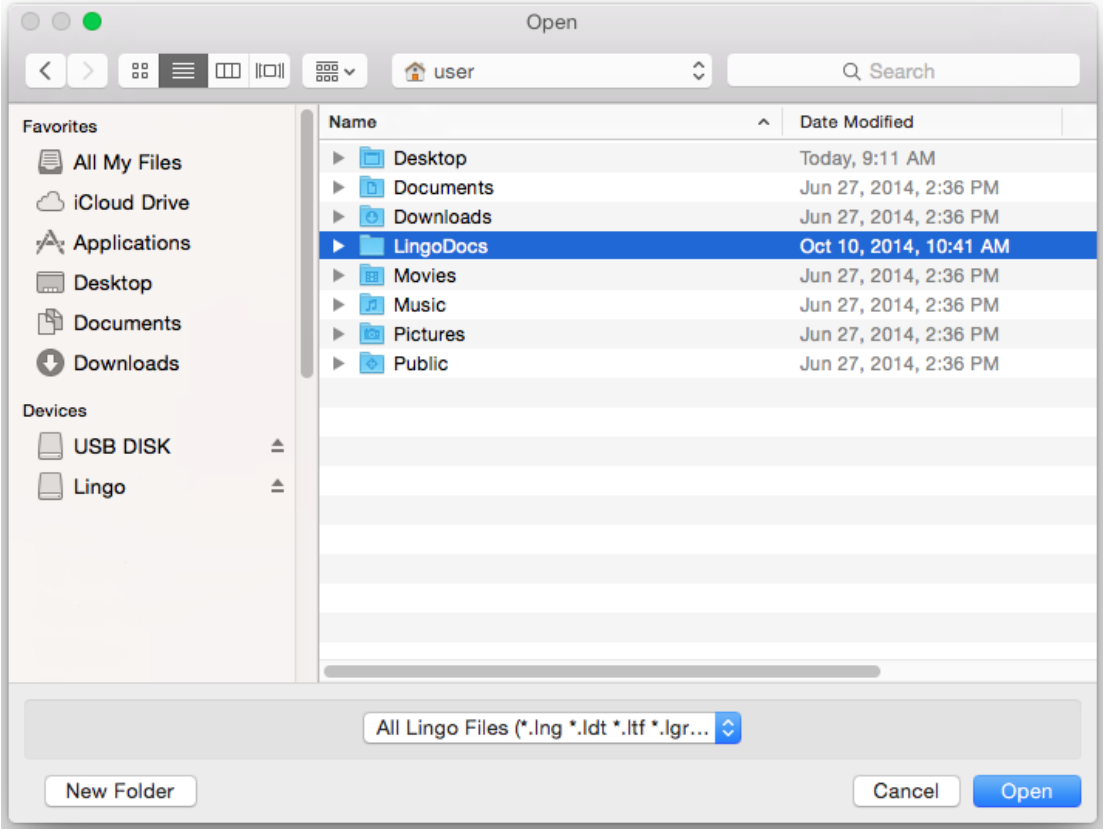
If you don't have a key, you can choose to run LINGO in demo mode by clicking the Demo button. In demo mode, LINGO has all the functionality of a standard version of LINGO with the one exception that the maximum problem size is restricted. Demo licenses expire after 180 days.

Opening a Sample Model on a Mac

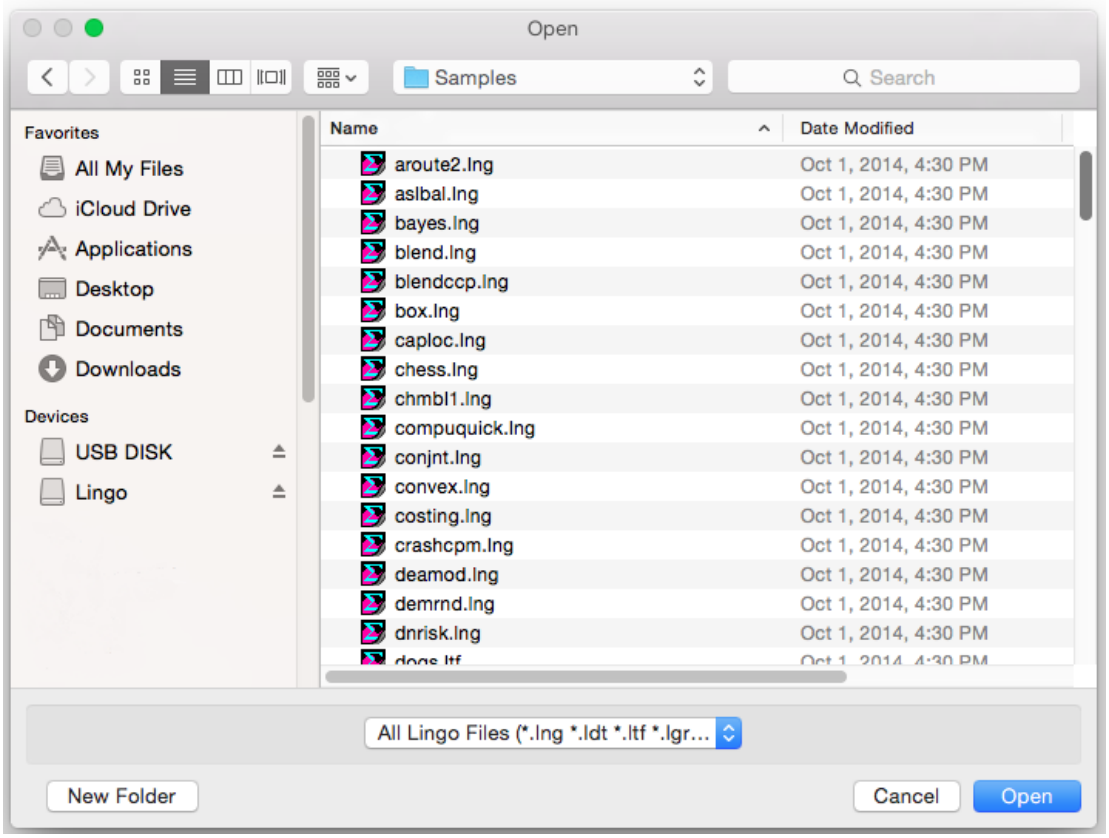
In the *Installing LINGO on a Mac* section above, we suggested that you install the *LingoDocs* folder in your *$HOME* directory. The *LingoDocs* folder contains a folder called *Samples* with numerous sample model. These models are drawn from a wide array of application areas. For a complete listing of these models, see *Additional Examples of LINGO Modeling*. The sample model directory is titled *Samples* and is stored directly off the main LINGO directory.

To open a sample model in LINGO, follow these steps:

1. Pull down the *File* menu and select the *Open* command. Migrate to where you installed your copy of the *LingoDocs* folder:



2. Double-click on the *LingoDocs* folder then double-click on the *Samples folder* contained in the *LingoDocs* folder, at which point you should see:



3. To read in a small transportation model, select the *tran.lng* model from the *Sample* folder and press the *Open* button. You should now have the model in an open window in LINGO as follows:

Lingo Model - tran.lng

Lingo Model – tran.lng

Lingo Model – tran.lng

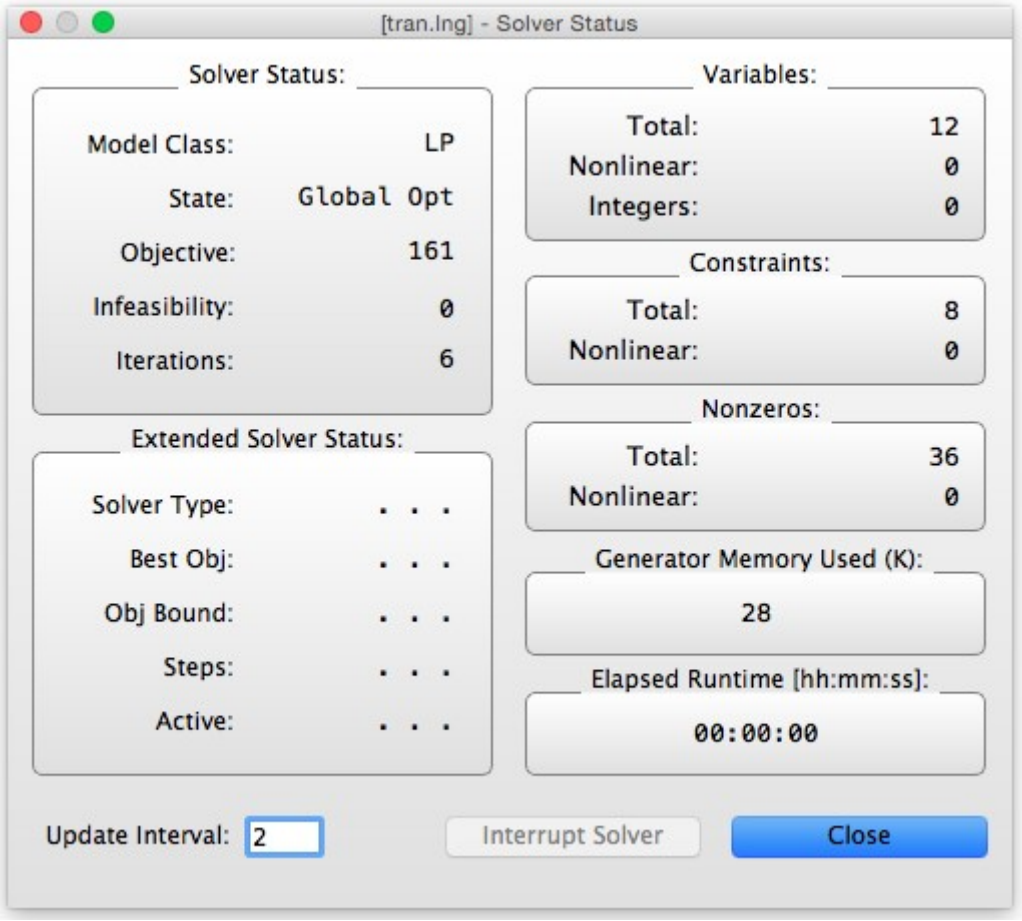
```
MODEL:
! A 3 Warehouse, 4 Customer
   Transportation Problem;
SETS:
   WAREHOUSE / WH1, WH2, WH3/    : CAPACITY;
   CUSTOMER   / C1, C2, C3, C4/ : DEMAND;
   ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The objective;
 [OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
 @FOR( CUSTOMER( J): [DEM]
  @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
   DEMAND( J));

! The supply constraints;
 @FOR( WAREHOUSE( I): [SUP]
  @SUM( CUSTOMER( J): VOLUME( I, J)) <=
   CAPACITY( I));

! Here are the parameters;
DATA:
   CAPACITY =   30, 25, 21 ;
   DEMAND =   15, 17, 22, 12;
   COST =      6,  2,  6,  7,
               4,  9,  5,  3,
               8,  8,  1,  5;
ENDDATA
END
```

Ln 1, Col 1 9:30 am

For details on developing a transportation model in LINGO see *A Sample Transportation Model* in this chapter.

You may now solve the model using the *Solver|Solve* command, or by pressing the Solve button on the toolbar:

The optimal objective value for this model is 161. When solved, you should see the following solver status window:



Note the objective field has a value of 161 as expected. For an interpretation of the other fields in this window, see *Solver Status Window* in Getting Started with LINGO.

Behind the solver status window, you will find the solution report for the model. This report contains summary information about the model as well as values for all the variables. This report's header is reproduced below:

Solution Report – tran.lng

```
Global optimal solution found.
Objective value:                              161.0000
Infeasibilities:                              0.000000
Total solver iterations:                             6
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                      12
Nonlinear variables:                   0
Integer variables:                     0

Total constraints:                     8
Nonlinear constraints:                 0

Total nonzeros:                       36
Nonlinear nonzeros:                    0


      Variable           Value        Reduced Cost
CAPACITY( WH1)        30.00000            0.000000
CAPACITY( WH2)        25.00000            0.000000
CAPACITY( WH3)        21.00000            0.000000
   DEMAND( C1)        15.00000            0.000000
   DEMAND( C2)        17.00000            0.000000
   DEMAND( C3)        22.00000            0.000000
   DEMAND( C4)        12.00000            0.000000
```

For information on interpreting the fields in the solution report, see *Sample Solution Report* in *Getting Started with LINGO*.

Features Not Currently Supported on the Mac

The Mac version of LINGO is a new addition and currently doesn't support the entire feature set found under Windows. The currently unsupported features are listed below:

- Excel interface
- ODBC database connectivity
- Callable LINGO API
- *@USER* user supplied function

We encourage you to check back at www.lindo.com frequently to see if any of these features have become available on Mac LINGO.

Getting Started on Linux

Installing LINGO on Linux

Linux versions of LINGO are distributed as a *Run* file, titled *LINGO-LINUX-64x86-19.0.run*, which is an automated installation program that will do the work required to set up LINGO on your Linux machine. Once you've copied this file onto your machine, you should first set the file's protections so that it is executable. You may do this by opening a Linux terminal window, changing to the directory with the Run file, and entering the command:

```
chmod 755 ./LINGO-LINUX-64x86-19.0.run -v
```

You may then start the install program by entering:

```
./LINGO-LINUX-64x86-19.0.run
```

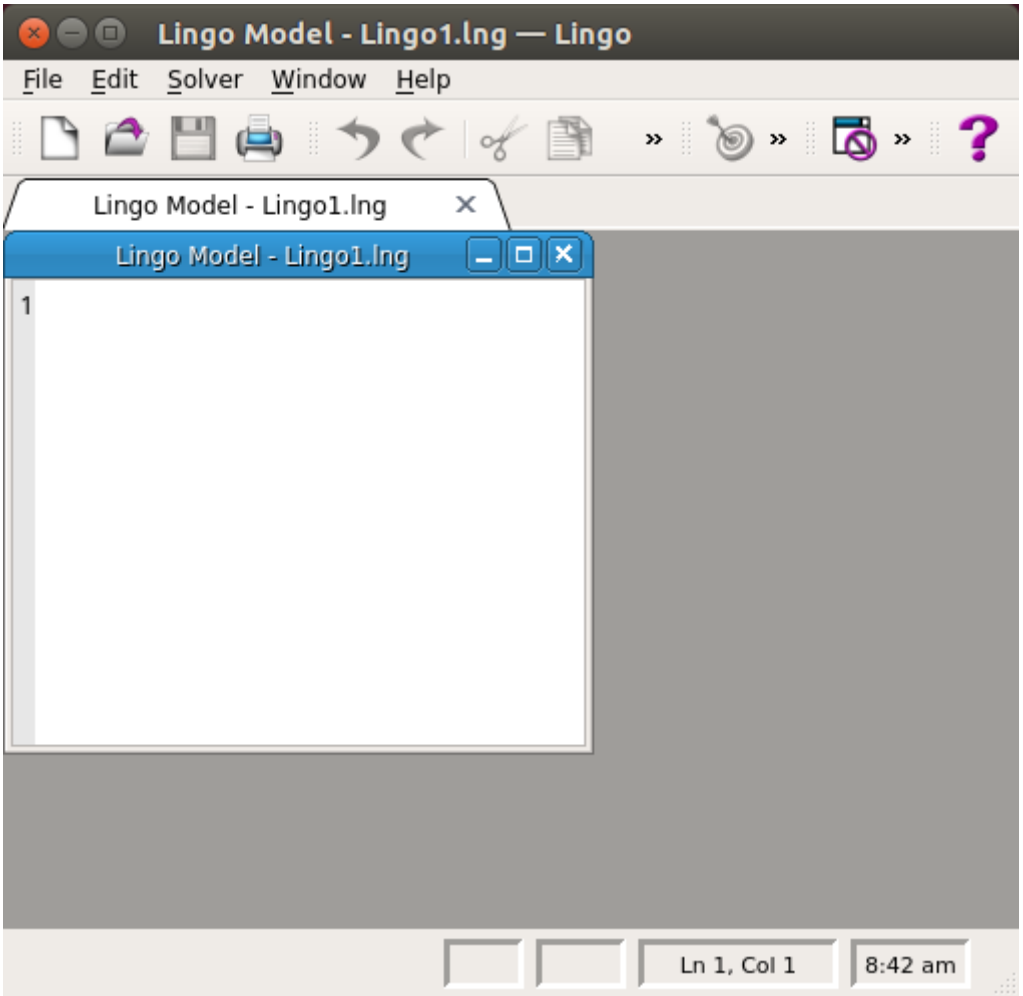
If you are installing LINGO with normal user rights, the install program defaults to putting LINGO in your *$HOME/lingo19* folder. If your LINGO license permits multiple users, then you can install LINGO as a super user, in which case, LINGO defaults to installing in the */opt/lingo19* folder, where it may be accessed by other users on the machine. In either case, the default location may be changed to suit your needs.

Starting LINGO on Linux

Once LINGO is installed, a new LINGO icon will appear on your desktop. The icon should appear as follows:

You may double-click on the icon to start LINGO.

Once LINGO is running, your screen will resemble the following:

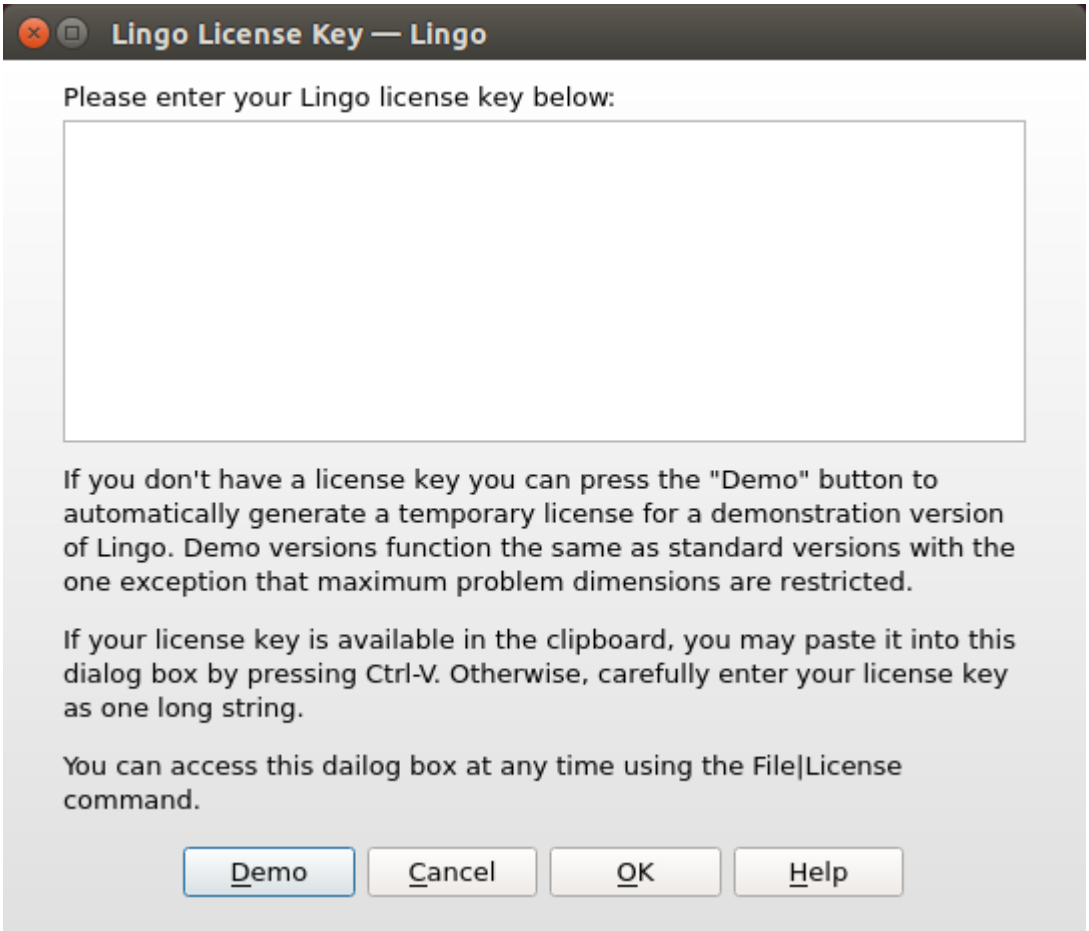


The outer window is the main frame window, and all other windows will be contained within this main frame window. The top of the main frame window also contains a toolbar for executing various LINGO commands.

The smaller child window labeled *Lingo Model - Lingo1.lng* is a new, blank model window. In the *Developing a LINGO Model* section, we will be entering a sample model directly into this window.

See the *Menu Commands* section for details on the toolbar and menu commands. The lower edge of the main frame window contains a status bar that provides various pieces of information regarding LINGO's current state. Both the toolbar and the status bar can be suppressed through the *Solver|Options* command.

Many copies of LINGO come with their licenses pre-installed. However, some versions of LINGO require you to input a license key. If your version of LINGO requires a license key, you will be presented with the following dialog box when you start LINGO:



Your license key may have been included in an email sent to you when you ordered your software. The license key is a string of letters, symbols and numbers, separated into groups of four by hyphens (e.g., `r82m-XCW2-dZu?-%72S-fD?S-Wp@`). Carefully enter the license into the edit field, including hyphens. License keys are case sensitive, so you must be sure to preserve the case of the individual letters when entering your key. Click the *OK* button and, assuming the key was entered correctly, LINGO will then start. In the future, you will be able to run LINGO directly without entering the key.

Note: If you received your license key by email, then you have the option of copying-and-pasting it into the license key dialog box. Copy the key from the email with the Ctrl+C key, then select the key field in LINGO dialog box and paste the key with the Ctrl+V key.

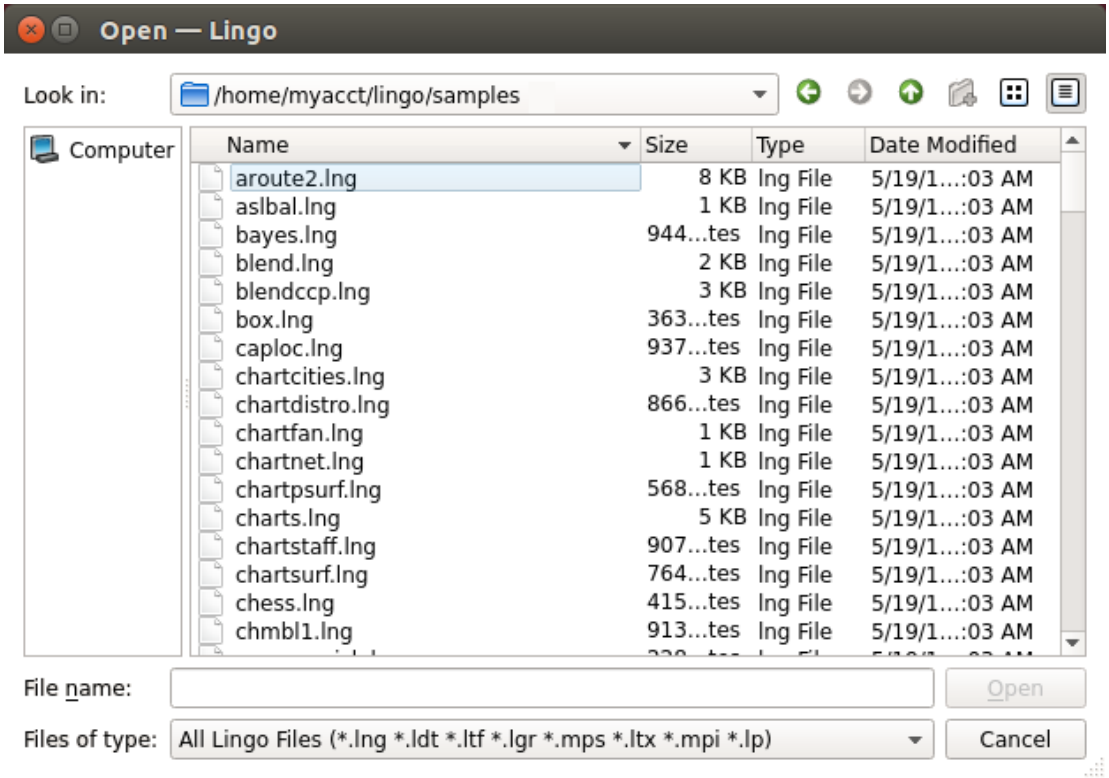
If you don't have a key, you can choose to run LINGO in demo mode by clicking the *Demo* button. In demo mode, LINGO has all the functionality of a standard version of LINGO with the one exception that the maximum problem size is restricted. Demo licenses expire after 180 days.

Opening a Sample Model on Linux

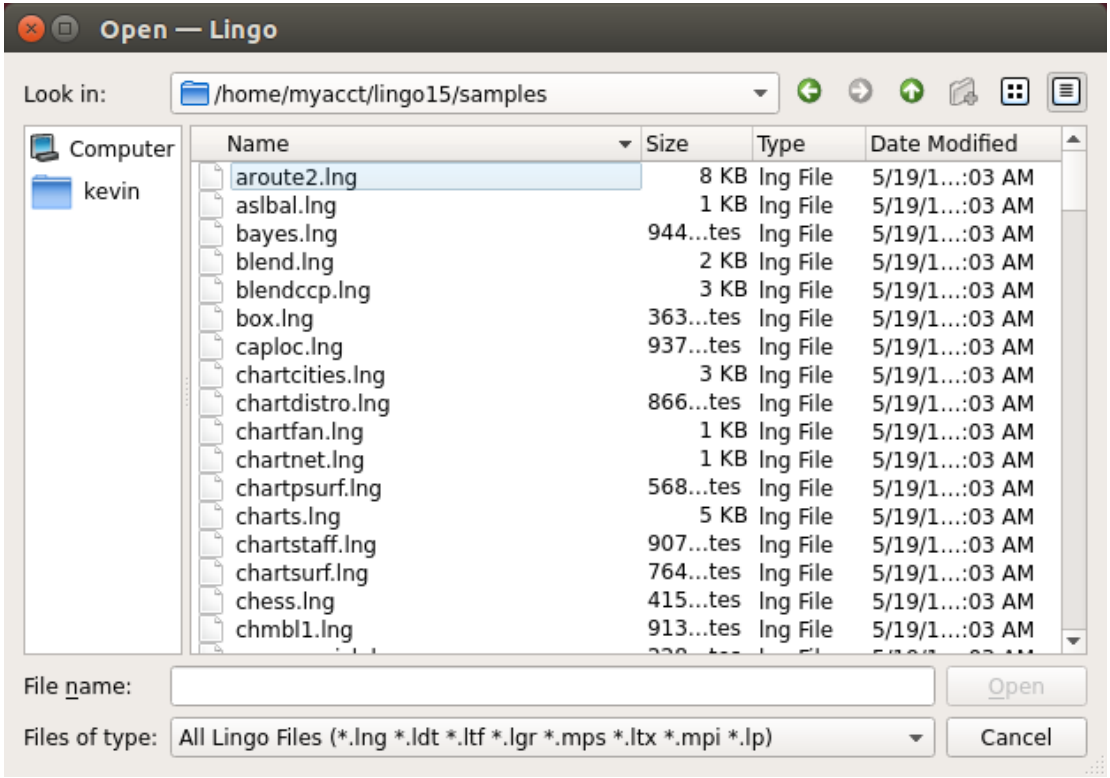
LINGO's installation program also copies a number of sample LINGO models onto your hard drive. These models can be found in the *samples* folder, immediately beneath the main *lingo19* folder. In most cases, the folder containing the samples will be *$HOME/lingo19/samples*. These models are drawn from a wide array of application areas. For more discussion of these models, see *Additional Examples of LINGO Modeling*.

To open a sample model in LINGO, follow these steps:

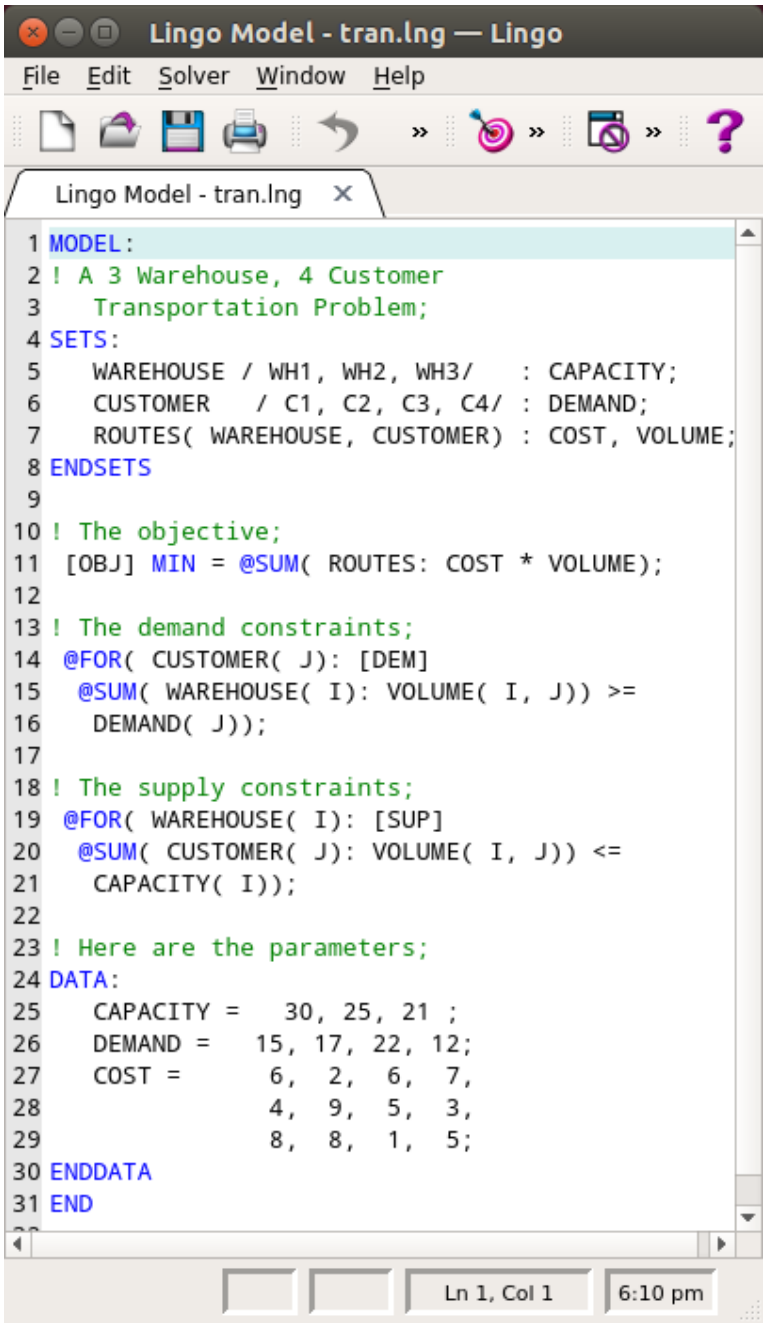
1. Pull down the *File* menu and select the *Open* command. You will be presented with the following file selection dialog, and it will be open to the *samples* folder:



2. Double-click on the *LingoDocs* folder then double-click on the *Samples folder* contained in the *LingoDocs* folder, at which point you should see:



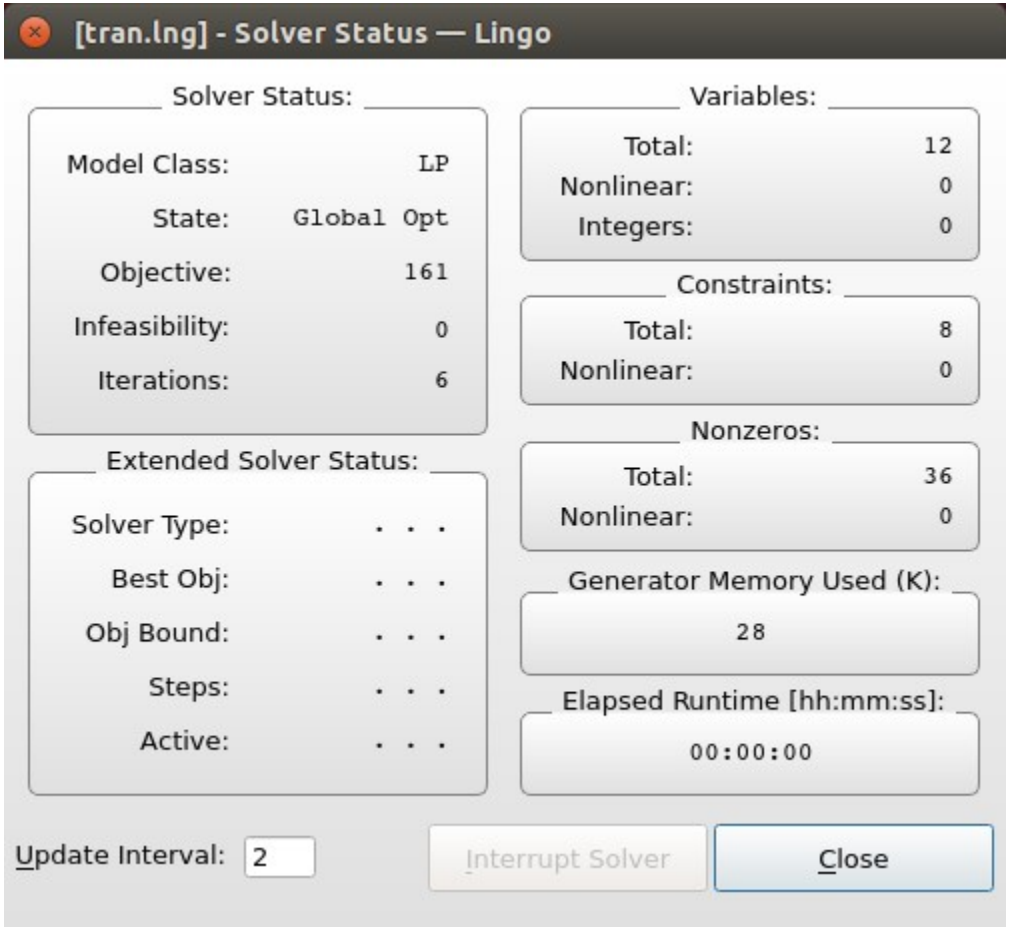
3. To read in a small transportation model, select the *tran.lng* model from the *samples* folder and press the *Open* button. You should now have the model in an open window in LINGO as follows:




```
MODEL:
! A 3 Warehouse, 4 Customer
   Transportation Problem;
SETS:
   WAREHOUSE / WH1, WH2, WH3/    : CAPACITY;
   CUSTOMER    / C1, C2, C3, C4/ : DEMAND;
   ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The objective;
 [OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
 @FOR( CUSTOMER( J): [DEM]
  @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
   DEMAND( J));

! The supply constraints;
 @FOR( WAREHOUSE( I): [SUP]
  @SUM( CUSTOMER( J): VOLUME( I, J)) <=
   CAPACITY( I));

! Here are the parameters;
DATA:
   CAPACITY =   30, 25, 21 ;
   DEMAND =   15, 17, 22, 12;
   COST =      6,  2,  6,  7,
               4,  9,  5,  3,
               8,  8,  1,  5;
ENDDATA
END
```

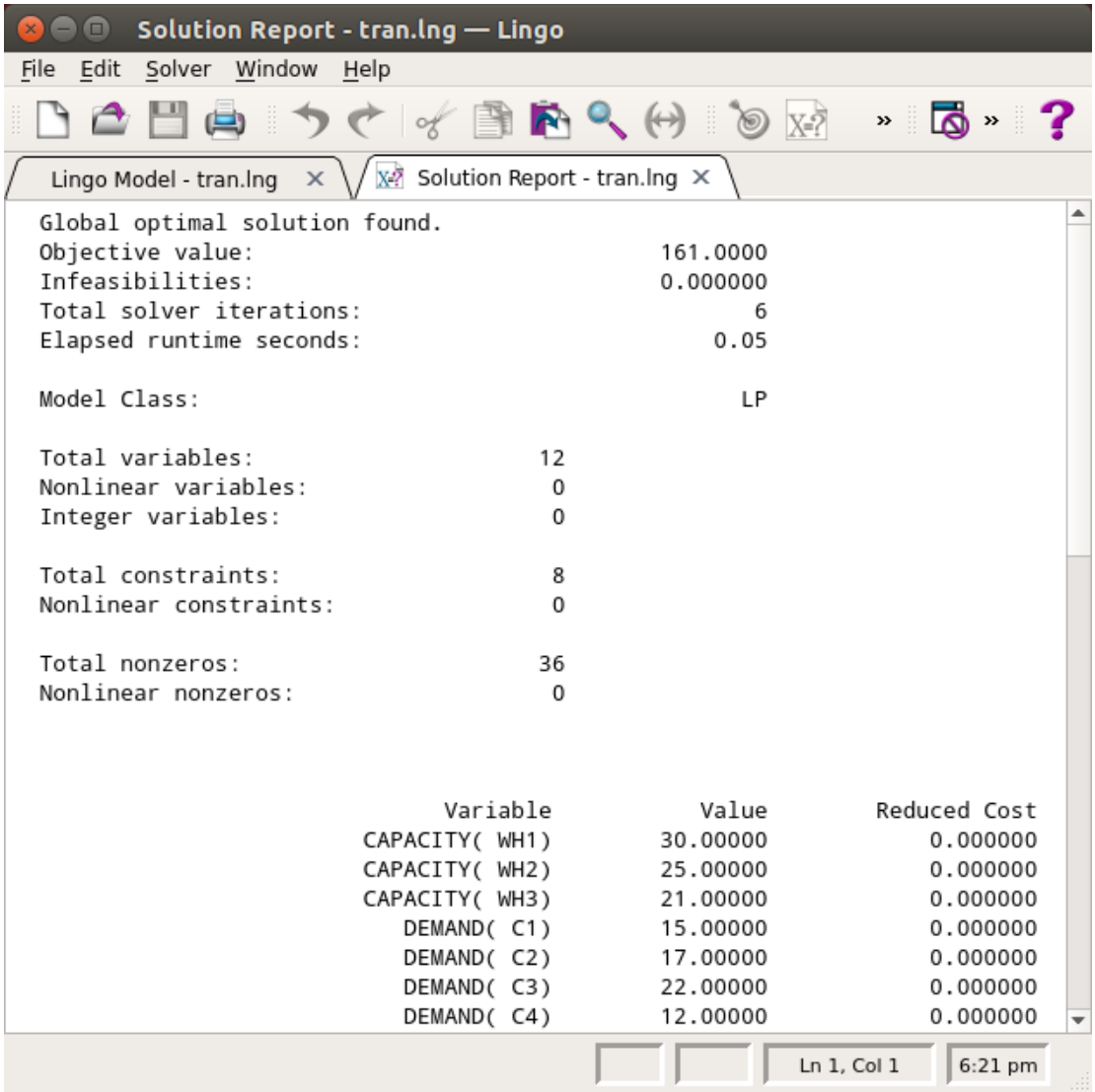
For details on developing a transportation model in LINGO see *A Sample Transportation Model* in this chapter.

You may now solve the model using the *Solver|Solve* command, or by pressing the *Solve* button on the toolbar: . The optimal objective value for this model is 161. When solved, you should see the following solver status window:



Note the objective field has a value of 161 as expected. For an interpretation of the other fields in this window, see *Solver Status Window* in Getting Started with LINGO.

Behind the solver status window, you will find the solution report for the model. This report contains summary information about the model as well as values for all the variables. This report's header is reproduced below:




```
Global optimal solution found.
Objective value:                              161.0000
Infeasibilities:                              0.000000
Total solver iterations:                             6
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                     12
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                0

Total nonzeros:                      36
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                         CAPACITY( WH1)        30.00000            0.000000
                         CAPACITY( WH2)        25.00000            0.000000
                         CAPACITY( WH3)        21.00000            0.000000
                            DEMAND( C1)        15.00000            0.000000
                            DEMAND( C2)        17.00000            0.000000
                            DEMAND( C3)        22.00000            0.000000
                            DEMAND( C4)        12.00000            0.000000
```

For information on interpreting the fields in the solution report, see *Sample Solution Report* in *Getting Started with LINGO*.

Features Not Currently Supported on Linux

The Linux version of LINGO is a new addition and currently doesn't support the entire feature set found under Windows. The currently unsupported features are listed below:

- *Excel interface*
- ODBC database connectivity
- @USER user supplied function

We encourage you to check back at www.lindo.com frequently to see if any of these features have become available on the Linux version of LINGO.

Command-Line Prompt

On machines other than Windows, Mac and Linux, you may have to interface with LINGO through the means of a command-line prompt. All instructions are issued to LINGO in the form of text command strings.

When you start a command-line version of LINGO, you will see a colon command prompt as follows:

```
LINGO


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rights reserved. Copying except as authorized in license
agreement is prohibited.

:
```

The colon character (:) at the bottom of the screen in LINGO's prompt for input. When you see the colon prompt, LINGO is expecting a command. When you see the question mark prompt, you have already initiated a command and LINGO is asking you to supply additional information related to this command such as a number or a name. If you wish to "back out" of a command you have already started, you may enter a blank line in response to the question mark prompt and LINGO will return you to the command level colon prompt. All available commands are listed in *Command-line Commands*.

Entering the Model from the Command-Line

When you enter a model in the command-line interface, you must first specify to LINGO that you are ready to begin entering the LINGO statements. This is done by entering the *MODEL:* command at the colon prompt. LINGO will then give you a question mark prompt and you begin entering the model line by line.

As an example, we will use the CompuQuick model discussed in the previous section. After entering the CompuQuick model, your screen should resemble the following (note that user input is in bold):

```
: MODEL:
? MAX = 100 * STANDARD + 150 * TURBO;
? STANDARD <= 100;
? TURBO <= 120;
? STANDARD + 2 * TURBO <= 160;
? END
:
```

The *END* command tells LINGO you are finished inputting the model. Once you enter the *END* command and return to the colon prompt, the model is in memory and ready to be solved.

Solving the Model from the Command-Line

To begin solving the model, type the *GO* command at the colon prompt and press the enter key. LINGO will begin compiling the model. This means LINGO will determine whether the model conforms to all syntax requirements. If the LINGO model doesn't pass these tests, you will be informed by an error message. For more information on error codes, see *Error Messages.*

If there are no formulation errors during the compilation phase, LINGO will invoke the appropriate internal solver to begin searching for the optimal solution to your model. When LINGO is done solving the CompuQuick model, it will send the following solution report to your screen:

```
: GO
  Global optimal solution found.
  Objective value:                        14500.00
  Infeasibilities:                        0.000000
  Total solver iterations:                       0
  Elapsed runtime seconds:                    0.03

  Model Class:                                  LP

  Total variables:                        2
  Nonlinear variables:                    0
  Integer variables:                      0

  Total constraints:                      4
  Nonlinear constraints:                  0

  Total nonzeros:                         6
  Nonlinear nonzeros:                     0

    Variable           Value        Reduced Cost
    STANDARD        100.0000            0.000000
       TURBO        30.00000            0.000000

          Row    Slack or Surplus      Dual Price
            1        14500.00            1.000000
            2        0.000000            25.00000
            3        90.00000            0.000000
            4        0.000000            75.00000
```

This solution tells us that CompuQuick should build 100 Standards and 30 Turbos each day to give them a total daily profit of $14,500. Click here for additional details on the various fields in this report.

Printing and Saving Your Work from the Command-Line

For command-line versions of LINGO, the *DIVERT* file command may be used to send all LINGO reports to a file rather than to the screen. You may then route this file to a printer or load it into a word processing program for printing.

For example, to create a text file for printing that contains a copy of your model and solution, issue the commands:

```
DIVERT MYFILE       !Opens an output file called MYFILE;
LOOK ALL            !Sends formulation to the file;
GO                  !Sends solution to the file;
RVRT                !Closes down output file;
```

To save your model to disk, issue the *SAVE* command followed by the name of a file to store your model under. For example, the command:

```
SAVE MYFILE.LNG
```

saves a copy of the current model to the file titled *MYFILE.LNG*. The model may be retrieved for use later with the *TAKE* command.

Please refer to *Command-line Commands* for more detailed information on these and other commands.

Creating and Solving a Model in LINGO

Developing a LINGO Model

For our sample model, we will create a small, product mix example. Let's imagine that the CompuQuick Corporation produces two models of computers - Standard and Turbo. CompuQuick can sell every Standard unit it produces for a profit contribution of $100, and each Turbo unit for a contribution of $150. At the CompuQuick factory, the Standard computer production line can produce, at most, 100 computers per day. At the same time, the Turbo computer production line can turn out 120 computers per day. Furthermore, CompuQuick has a limited supply of daily labor. In particular, there is a total of 160 hours of labor available each day. Standard computers require 1 hour of labor, while Turbo computers are relatively more labor intense requiring 2 hours of labor. The problem for CompuQuick is to determine the mix of Standard and Turbo computers to produce each day to maximize total profit without exceeding line and labor capacity limits.

In general, an optimization model will consist of the following three items:

- *Objective Function* -- The objective function is a formula that expresses exactly what it is you want to optimize. In business oriented models, this will usually be a profit function you wish to maximize, or a cost function you want to minimize. Models may have, at most, one objective function. In the case of our CompuQuick example, the objective function will compute the company's profit as a function of the output of Standards and Turbos.

- *Variables* -- Variables are the quantities you have under your control. You must decide what the best values of the variables are. For this reason, variables are sometimes also called decision variables. The goal of optimization is to find the values of a model's variables that generate the best value for the objective function, subject to any limiting conditions placed on the variables. We will have two variables in our example--one corresponding to the number of Standards to produce and the other corresponding to the number of Turbos to produce.

- *Constraints* -- Almost without exception there will be some limit on the values the variables in a model can assume--at least one resource will be limited (e.g., time, raw materials, your department's budget, etc.). These limits are expressed in terms of formulas that are a function of the model's variables. These formulas are referred to as constraints because they constrain the values the variables can take. In our CompuQuick example, we will have one constraint for each of our production lines and one constraint on the total labor used.

We will now construct the objective function for our example. We will let the variables STANDARD and TURBO denote the number of Standard and Turbo computers to produce, respectively. CompuQuick's objective is to maximize total profit. Total profit is calculated as the sum of the profit contribution of the Standard computer ($100) multiplied by the total Standard computers produced (STANDARD) and the profit contribution of the Turbo computer ($150) multiplied by the total Turbo computers produced (TURBO). Finally, we tell LINGO we want to maximize an objective function by preceding it with "MAX =". Therefore, our objective function is written on the first line of our model window as:

```
MAX = 100 * STANDARD + 150 * TURBO;
```

Note: Each mathematical expression in LINGO is terminated with a semicolon. These semicolons are required. Your model will not solve without them. For more information on the syntax of LINGO, see below.

Next, we must input our constraints on line capacity and labor supply. The number of Standard and Turbo computers produced must be constrained to the production line limits of 100 and 120, respectively. Do this by entering the following two constraints just below the objective function:

```
STANDARD <= 100;
TURBO <= 120;
```

In words, the first constraint says the number of Standard computers produced daily (STANDARD) must be less-than-or-equal-to (<=) the production line capacity of 100. Likewise, the second

constraint says the number of Turbo computers produced daily (TURBO) must be less-than-or-equal-to (<=) its line capacity of 120.

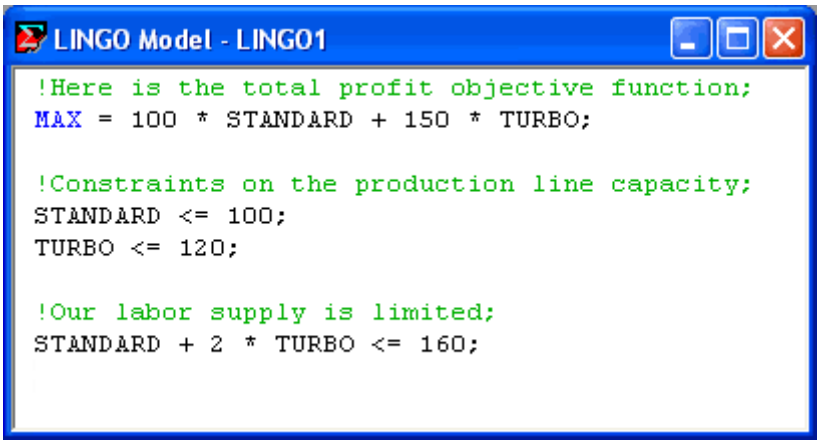
Note:	Since most computers do not have less-than-or-equal-to keys (≤), LINGO has adopted the convention of using the two character symbol <= to denote ≤. As an alternative, you may simply enter < to signify less-than-or-equal-to. In a similar manner, >= or > are used to signify greater-than-or-equal-to (≥).

The final constraint on the amount of labor used can be expressed as:

```
STANDARD + 2 * TURBO <= 160;
```

Specifically, the total number of labor hours used (STANDARD + 2 * TURBO) must be less-than-or-equal-to (<=) the amount of labor hours available of 160.

After entering the above and entering comments to improve the readability of the model, your model window should look like the following:



An expression may be broken up into as many lines as you want, but the expression must be terminated with a semicolon. As an example, we could have used two lines rather than just one to contain the objective function:

```
MAX = 100 * STANDARD
+ 150 * TURBO;
```

We have also entered some comments to improve the readability of our model. Comments begin with an exclamation point (!) and end with a semicolon (;). All text between an exclamation point and terminating semicolon is ignored by LINGO. Comments can occupy more than one line and can share lines with other LINGO expressions. For example:

```
X = 1.5 * Y + Z / 2 * Y; !This is a comment;
X = 1.5 * !This is a comment in the middle
of a constraint; Y + Z / 2 * Y;
```

You may have noticed we used all uppercase letters for our variable names. This is not a requirement. LINGO does not distinguish between uppercase and lowercase in variable names. Thus, the following variable names would all be considered equivalent:

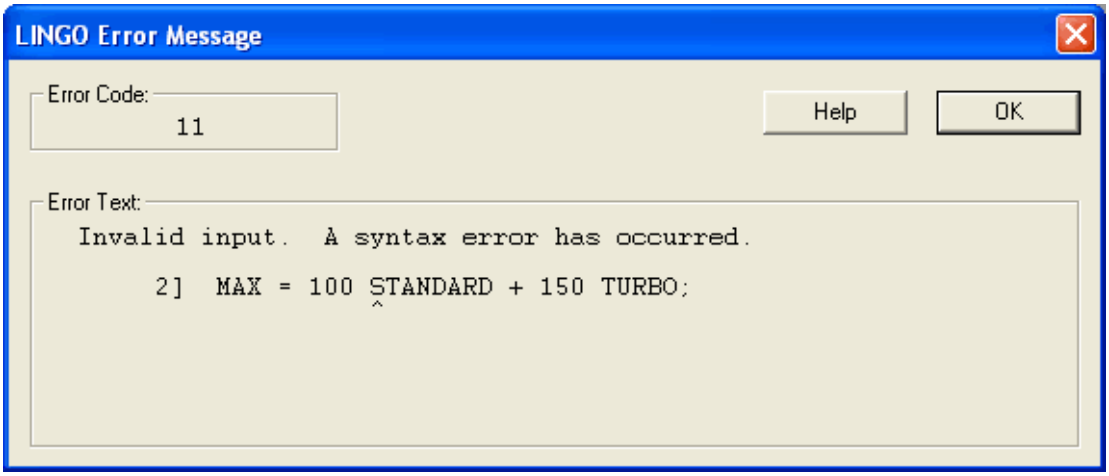
```
TURBO
Turbo
turbo
```

When constructing variable names in LINGO, all names must begin with an alphabetic character (A-Z). Subsequent characters may be either alphabetic, numeric (0-9), or the underscore (_). Names may be up to 64 characters in length.

A final feature you will notice is that LINGO's editor is "syntax aware." In other words, when it encounters LINGO keywords it displays them in blue, comments are displayed in green, and all remaining text is displayed in black. Matching parentheses are also highlighted in red when you place the cursor immediately following a parenthesis. You should find this feature useful in tracking down syntax errors in your models.

Solving the Model

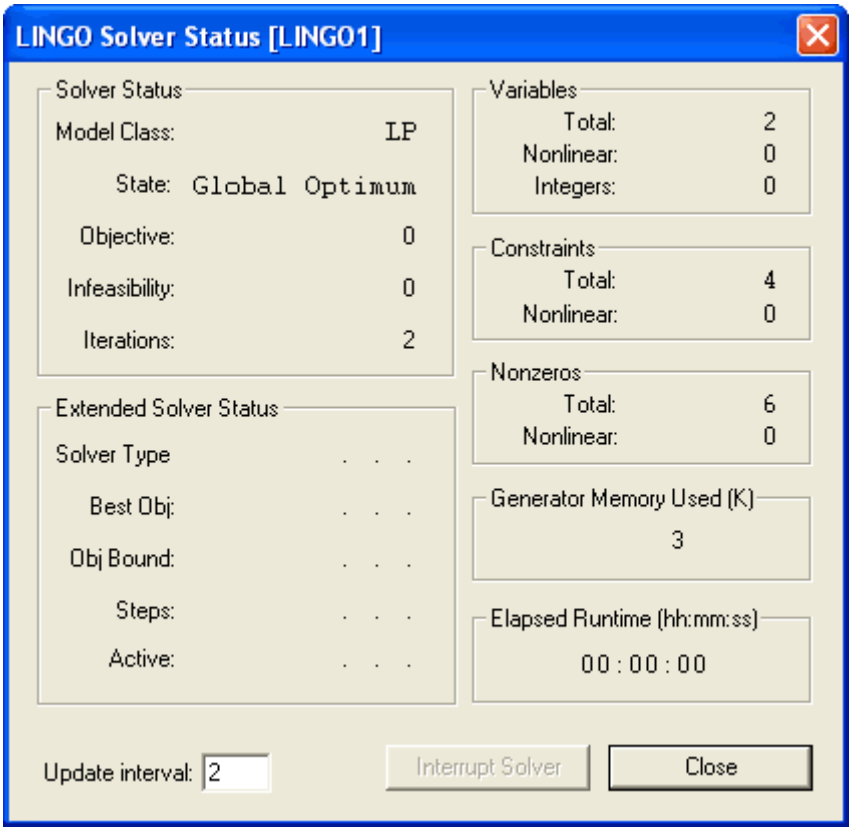
Your model has now been entered and it is ready to be solved. To begin solving the model, select the *Solve* command from the *LINGO* menu, or press the *Solve* button ( ) on the toolbar at the top of the main frame window. LINGO will begin compiling the model. During this step, LINGO will determine whether the model conforms to all syntax requirements. If the LINGO model doesn't pass these tests, you will be informed by an error message. In this model, for instance, if you forget to use the multiplication sign, you will get an error like the following:



LINGO lets you know there is a syntax error in your model, lists the line of the model it is in, and points to the place in the line where it occurred. For more information on error codes, see *Error Messages*.

Solver Status Window

If there are no formulation errors during the compilation phase, LINGO will invoke the appropriate internal solver to begin searching for the optimal solution to your model. When the solver starts, it displays a *solver status* window on your screen resembling the following:



The solver status window is useful for monitoring the progress of the solver and the dimensions of your model. The various fields are described in more detail below.

The solver status window also provides you with an Interrupt Solver button. Interrupting the solver causes LINGO to halt the solver on the next iteration. In most cases, LINGO will be able to restore and report the best solution found so far. The one exception is in the case of linear programming models (i.e., linear models without integer variables). If a linear programming model is interrupted, the solution returned will be meaningless and should be ignored. This should not be a problem because linear programs generally solve quickly, thus minimizing the need to interrupt.

Note: You must be careful how you interpret solutions after interrupting the solver. These solutions 1) will definitely not be optimal, 2) may not be feasible to all the constraints, and 3) are worthless if the model is a linear program.

Next to the Interrupt Solver button is another button labeled Close. Hitting the Close button will close the solver status window. This window can be reopened at any time by selecting the Window|Status Window command.

At the bottom of the solver status window, you will find a field titled: Update Interval. LINGO will update the solver status window every n seconds, where n is the value contained in the Update Interval field. You may set this interval to any value you desire. However, setting it to 0 will result in longer solution times—LINGO will spend more time updating the solver status window than solving your model. On larger models, LINGO may not always be able to update the solver status window on a regular interval. So, don't be concerned if you sometimes must wait longer than the indicated interval.

Variables Box

The *Variables* box shows the total number of variables in the model. The *Variables* box also displays the number of the total variables that are *nonlinear*. A variable is considered to be nonlinear if it enters into any nonlinear relationship in any constraint in the model. For instance, the constraint:

```
X + Y = 100;
```

would be considered linear because the graph of this function would be a straight line. On the other hand, the nonlinear function:

```
X * Y = 100;
```

is quadratic and has a curved line as its graph. If we were to solve a model containing this particular nonlinear constraint, the nonlinear variable count would be at least 2 to represent the fact that the two variables *X* and *Y* appear nonlinearly in this constraint.

As another example, consider the constraint:

```
X * X + Y = 100;
```

In this case, *X* appears nonlinearly while *Y* appears as a linear variable. This constraint would not cause *Y* to be counted as one of the nonlinear variables. See *On Mathematical Modeling* for more information on the difference between linear and nonlinear equations.

The *Variables* box in the solver status window also gives you a count of the total number of integer variables in the model. In general, the more nonlinear and integer variables your model has, the more difficult it will be to solve to optimality in a reasonable amount of time. Pure linear models without integer variables will tend to solve the fastest. For more details on the use of integer variables, refer to *Using Variable Domain Functions*.

The variable counts do not include any variables LINGO determines are fixed in value. For instance, consider the following constraints:

```
X = 1;
X + Y = 3;
```

From the first constraint, LINGO determines *X* is fixed at the value of 1. Using this information in constraint 2, LINGO determines *Y* is fixed at a value of 2. *X* and *Y* will then be substituted out of the model and they will not contribute to the total variable count.

Constraints Box

The *Constraints* box shows the total constraints in the expanded model and the number of these constraints that are *nonlinear*. A constraint is considered nonlinear if one or more variables appear nonlinearly in the constraint.

LINGO searches your model for *fixed* constraints. A constraint is considered fixed if all the variables in the constraint are fixed. Fixed constraints are substituted out of the model and do not add to the total constraint count.

Nonzeroes Box

The *Nonzeros* box shows the total *nonzero coefficients* in the model and the number of these that appear on nonlinear variables. In a given constraint, only a small subset of the total variables typically appears. The implied coefficient on all the non-appearing variables is zero, while the coefficients on the variables that do appear will be nonzero. Thus, you can view the total nonzero coefficient count as a tally of the total number of times variables appear in all the constraints. The nonlinear nonzero coefficient count can be viewed as the number of times variables appear nonlinearly in all the constraints.

Generator Memory Used Box

The *Generator Memory Used* box lists the amount of memory LINGO's model generator is currently using from its memory allotment. You may change the size of the generator's memory allotment using the *Solver|Options* command.

Elapsed Runtime Box

The *Elapsed Runtime* box shows the total time used so far to generate and solve the model. This is an elapsed time figure and may be affected by the number of other applications running on your system.

Solver Status Box

The *Solver Status* box shows the current status of the solver. A description of the fields appear in the table below followed by a more in depth explanation:

Field	Description
Model Class	Displays the model's classification. Possible classes are "LP", "QP", "CONE", "NLP", "MILP", "MIQP", "MICONE", "MINLP" "PILP", "PIQP", "PICONE", and "PINLP".
State	Gives the Status of the current solution. Possible states are "Global Optimum", "Local Optimum", "Feasible", "Infeasible", "Unbounded", "Interrupted", and "Undetermined".
Objective	Current value of the objective function.
Infeasibility	Amount constraints are violated by.
Iterations	Number of solver iterations.

Model Class Field

The *Model Class* field summarizes the properties of your model. The various classes you will encounter are listed below:

Abbreviation	Class	Description
LP	Linear Program	All expressions are linear and the model contains no integer restrictions on the variables.
QP	Quadratic Program	All expressions are linear or quadratic, and there are no integer restrictions.
CONE	Conic Program	The model is a conic (second-order cone) program and all variables are continuous.
NLP	Nonlinear Program	At least one of the relationships in the model is nonlinear with respect to the variables.

MILP	Mixed Integer Linear Program	All expressions are linear, and a subset of the variables is restricted to integer values.
MIQP	Mixed Integer Quadratic Program	All expressions are either linear or quadratic, and a subset of the variables has integer restrictions.
MICONE	Mixed Integer Conic Program	The model is a conic (second-order cone) program, and a subset of the variables is restricted to integer values.
MINLP	Integer Nonlinear Program	At least one of the expressions in the model is nonlinear, and a subset of the variables has integer restrictions. ***In general, this class of model will be very difficult to solve for all but the smallest cases.***
PILP	Pure Integer Linear Program	All expressions are linear, and all variables are restricted to integer values.
PIQP	Pure Integer Quadratic Program	All expressions are linear or quadratic, and all variables are restricted to integer values.
PICONE	Pure Integer Conic (Second-Order Cone) Program	The model is a conic (second-order cone) program, and all the variables are restricted to integer values.
PINLP	Pure Integer Nonlinear Program	At least one of the expressions in the model is nonlinear, and all variables have integer restrictions. ***In general, this class of model will be very difficult to solve for all but the smallest cases.***

Note: Certain model classes may not be available on your installation of LINGO: nonlinear models require the nonlinear option, quadratic models require the barrier option and conic models require the barrier, global and conic options.

State Field

When LINGO begins solving your model, the initial state of the current solution will be "Undetermined". This is because the solver has not yet had a chance to generate a solution to your model.

Once the solver begins iterating, the state will progress to "Infeasible". In the infeasible state, LINGO has generated tentative solutions, but none that satisfy all the constraints in the model.

Assuming a feasible solution exists, the solver will then progress to the "Feasible" state. In the feasible state, LINGO has found a solution that satisfies all the constraints in your model, but the solver is not yet satisfied it has found the best solution to your model.

Once the solver can no longer find better solutions to your model, it will terminate in either the "Global Optimum" or "Local Optimum" state. If your model does not have any nonlinear constraints, then any locally optimal solution will also be a global optimum. Thus, all optimized linear models will terminate in the global optimum state. If, on the other hand, your model has one or more nonlinear constraints, then any locally optimal solution may not be the best solution to your model. There may be another "peak" that is better than the current one, but the solver's local search procedure is unable to "see" the better peak. Thus, on nonlinear models, LINGO can terminate only in the local optimum state. LINGO may, in fact, have a globally optimal solution, but, given the nature of nonlinear problems, LINGO is unable to claim it as such. Given this fact, it is always preferred to formulate a model using only linear constraints whenever possible. For more details on the concept of global vs. local optimal points, refer to On Mathematical Modeling.

Note:	LINGO's optional global solver may be used to find globally optimal solutions to nonlinear models. For more information on the global solver, refer to the Nonlinear Solver Tab help topic.

Note:	If a model terminates in the "Unbounded" state, it means LINGO can improve the objective function without bound. In real life, this would correspond to a situation where you can generate infinite profits. Because such a situation is rare, if not impossible, you have most likely omitted or misspecified some constraints in your model.

Finally, the "Interrupted" state will occur when you prematurely interrupt LINGO's solver before it has found the final solution to your model. The mechanics of interrupting the solver are discussed in more detail above.

Objective Field

The Objective field gives the objective value for the current solution. If your model does not have an objective function, then "N/A" will appear in this field.

Infeasibility Field

The Infeasibility field lists the amount that all the constraints in the model are violated by. Keep in mind that this figure does not track the amount of any violations on variable bounds. Thus, it is possible for the Infeasibility field to be zero while the current solution is infeasible due to violated variable bounds. The LINGO solver may also internally scale a model such that the units of the Infeasibility field no longer correspond to the unscaled version of the model. To determine whether LINGO has found a feasible solution, you should refer to the State field discussed above.

Iterations Field

The Iterations field displays a count of the number of iterations completed thus far by LINGO's solver. The fundamental operation performed by LINGO's solver is called an iteration. An iteration involves finding a variable, currently at a zero value, which would be attractive to introduce into the solution at a nonzero value. This variable is then introduced into the solution at successively larger values until either a constraint is about to be driven infeasible or another variable is driven to zero. At this point, the iteration process begins anew. In general, as a model becomes larger, it will require more iterations to solve and each iteration will require more time to complete.

Extended Solver Status Box

The Extended Solver Status box shows status information pertaining to several of the specialized solvers in LINGO. These solvers are:

- BNP Solver
- Branch-and-Bound Solver
- Global Solver, and
- Multistart Solver.

The fields in this box will be updated only when one of these three specialized solvers is running. The fields appearing in the Extended Solver Status box are:

Field	Description
Solver Type	The type of specialized solver in use, and will be either "B-and-B", "Global", "Multistart", or "BNP".
Best Obj	The objective value of the best solution found so far.
Obj Bound	The theoretical bound on the objective.
Steps	The number of steps taken by the extended solver.
Active	The number of active subproblems remaining to be analyzed.

Solver Type Field
This field displays either "BNP", "B-and-B", "Global", or "Multistart", depending on the specialized solver in use.

LINGO employs a strategy called *branch-and-bound* to solve models with integer restrictions. Branch-and-bound is a systematic method for implicitly enumerating all possible combinations of the integer variables. Refer to Hillier and Lieberman (1995) for more information on the branch-and-bound algorithm.

In addition to the branch-and-bound solver, there are three other specialized solvers that may be invoked, which are: the *global solver*, the *multistart solver*, and the *BNP Solver*.

Many nonlinear models are non-convex and/or non-smooth. For more information see the Chapter 15, On Mathematical Modeling. Nonlinear solvers that rely on local search procedures (as does LINGO's default nonlinear solver) will tend to do poorly on these types of models. Typically, they will converge to a local, sub-optimal point that may be quite distant from the true, globally optimal point. The multistart solver and the global solver are specialized solvers that attempt to find the globally optimal solution to non-convex models. You can read more about these solvers in the *Nonlinear Solver Tab* section in Chapter 5.

The BNP solver is a mixed integer programming solver for solving linear models with block structure. Based on the decomposition structure, the solver divides the original problem into several subproblems and solves them (almost) independently, exploiting parallel processing if multiple cores are available. You can read more about the BNP solver in the *BNP Solver* section in Chapter 5.

Best Obj and Obj Bound Fields
The Best Obj field displays the best feasible objective value found so far. Obj Bound displays the bound on the objective. This bound is a limit on how far the solver will be able to improve the objective. At some point, these two values may become very close. Given that the best objective value can never exceed the bound, the fact that these two values are close indicates that LINGO's current best solution is either the optimal solution, or very close to it. At such a point, the user may choose to interrupt the solver and go with the current best solution in the interest of saving on additional computation time.

Steps Field
The information displayed in the Steps field depends on the particular solver that is running. The table below explains:

Solver	Steps Field Interpretation
BNP	Number of branches in the branch-and-bound tree.
Branch-and-Bound	Number of branches in the branch-and-bound tree.
Global	Number of subproblem boxes generated.
Multistart	Number of solver restarts.

Active Field
This field pertains to the BNP, branch–and–bound and global solvers. It lists the number of open subproblems remaining to be evaluated. The solver must run until this valve goes to zero.

Sample Solution Report

When LINGO is done solving the CompuQuick model, there will be a new window created on your screen titled *Solution Report* containing the details of the solution to your model. The solution report should appear as follows:

Solution Report - COMPUQUICK

```
Global optimal solution found.
Objective value:                              14500.00
Infeasibilities:                              0.000000
Total solver iterations:                             0
Elapsed runtime seconds:                          0.02

Model Class:                                        LP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    0

Total constraints:                    4
Nonlinear constraints:                0

Total nonzeros:                       6
Nonlinear nonzeros:                   0


      Variable           Value        Reduced Cost
      STANDARD        100.0000            0.00000
         TURBO        30.00000            0.000000

           Row    Slack or Surplus      Dual Price
             1        14500.00            1.000000
             2        0.000000            25.00000
             3        90.00000            0.000000
             4        0.000000            75.00000
```

This solution tells us that CompuQuick should build 100 Standards and 30 Turbos each day for a total daily profit of $14,500. Refer to the *Examining the Solution* section for additional details on the various fields in this report.

Printing Your Work

In Windows versions of LINGO, use the Print command in the File menu to print the active (frontmost) window, or click on the Print button (). You may print any window, including both model and report windows. If you wish to print just a portion of a window, use the Cut and Paste commands in the Edit menu to put the desired text in a new window before printing. You can also access the Cut command by clicking on the Cut button (). Likewise, the Paste command can be accessed through the Paste button (). To create a new window, use the File|New command, or click the New button ().

Saving Your Work

To save your model to a disk file, use the *File|Save* command, or press the *Save* button () in the toolbar. Unless you specify otherwise, LINGO will automatically append a *.LG4* extension to your file name on Windows. The *.LG4* file format is a binary RTF file format. On Mac and Linux, LINGO automatically saves model in the *.LNG* file format, which is a plain text file format, generally readable by any other programs that can read text files.

Examining the Solution Report

First, the solution report us that LINGO took 0 iterations to solve the model (the preprocessor was able to deduce the optimal solution without having to iterate). Second, the maximum profit attainable is $14,500. Third, the quantities of each computer to produce, *STANDARD* and *TURBO*, are 100 and 30, respectively. What's interesting to note is we make less of the relatively more "profitable" Turbo computer due to its more intensive use of our limited supply of labor. The *Reduced Costs*, *Slack or Surplus*, and *Dual Price* columns are explained in other sections.

Reduced Cost

In a LINGO solution report, you'll find a *reduced cost* figure for each variable. There are two valid, equivalent interpretations of a reduced cost.

First, you may interpret a variable's reduced cost as the amount that the objective coefficient of the variable would have to improve before it would become profitable to give the variable in question a positive value in the optimal solution. For example, if a variable had a reduced cost of 10, the objective coefficient of that variable would have to increase by 10 units in a maximization problem and/or decrease by 10 units in a minimization problem for the variable to become an attractive alternative to enter into the solution. A variable in the optimal solution, as in the case of *STANDARD* or *TURBO*, automatically has a reduced cost of zero.

Second, the reduced cost of a variable may be interpreted as the amount of penalty you would have to pay to introduce one unit of that variable into the solution. Again, if you have a variable with a reduced cost of 10, you would have to pay a penalty of 10 units to introduce the variable into the solution. In other words, the objective value would fall by 10 units in a maximization model or increase by 10 units in a minimization model.

Reduced costs are valid only over a range of values for the variable in questions. For more information on determining the valid range of a reduced cost, see the *Solver|Range* command in Chapter 5, *Windows Commands*.

Slack or Surplus

The *Slack or Surplus* column in a LINGO solution report tells you how close you are to satisfying a constraint as an equality. This quantity, on less-than-or-equal-to (≤) constraints, is generally referred to as *slack*. On greater-than-or-equal-to (≥) constraints, this quantity is called a *surplus*.

If a constraint is exactly satisfied as an equality, the slack or surplus value will be zero. If a constraint is violated, as in an infeasible solution, the slack or surplus value will be negative. Knowing this can help you find the violated constraints in an infeasible model—a model for which there doesn't exist a set of variable values that simultaneously satisfies all constraints. Nonbinding constraints, will have positive, nonzero values in this column.

In our CompuQuick example, note that row 3 (*TURBO <= 120*) has a slack of 90. Because the optimal value of *TURBO* is 30, this row is 90 units from being satisfied as an equality.

Dual Price

The LINGO solution report also gives a *dual price* figure for each constraint. You can interpret the dual price as the amount that the objective would improve as the right-hand side, or constant term, of the constraint is increased by one unit. For example, in the CompuQuick solution, the dual price of 75 on row 4 means adding one more unit of labor would cause the objective to improve by 75, to a value of 14,575.

Notice that “improve” is a relative term. In a maximization problem, improve means the objective value would increase. However, in a minimization problem, the objective value would *decrease* if you were to *increase* the right-hand side of a constraint with a *positive* dual price.

Dual prices are sometimes called *shadow prices*, because they tell you how much you should be willing to pay for additional units of a resource. Based on our analysis, CompuQuick should be willing to pay up to 75 dollars for each additional unit of labor.

As with reduced costs, dual prices are valid only over a range of values. Refer to the *Solver|Range* command in Chapter 5, *Windows Commands*, for more information on determining the valid range of a dual price.

Intro to LINGO's Modeling Language

One of LINGO's most powerful features is its mathematical modeling language. LINGO's modeling language lets you express your problem in a natural manner that is very similar to standard mathematical notation. Rather than entering each term of each constraint explicitly, you can express a whole series of similar constraints in a single compact statement. This leads to models that are much easier to maintain and scale up.

Another convenient feature of LINGO's modeling language is the *data section*. The data section allows you to isolate your model's data from the formulation. In fact, LINGO can even read data from a separate spreadsheet, database, or text file. With data independent of the model, it's much easier to make changes, and there's less chance of error when you do.

The simple CompuQuick model discussed above uses scalar variables. Each variable is explicitly listed by name (e.g., *STANDARD* and *TURBO*) and each constraint is explicitly stated (e.g., *TURBO <=120*). In larger models, you'll encounter the need to work with a group of several very similar constraints and variables. Using the scalar modeling approach we have illustrated to this point, you would need to undertake the repetitive task of typing in each term of each constraint. Fortunately, LINGO's ability to handle sets of objects allows you to perform such operations much more efficiently.

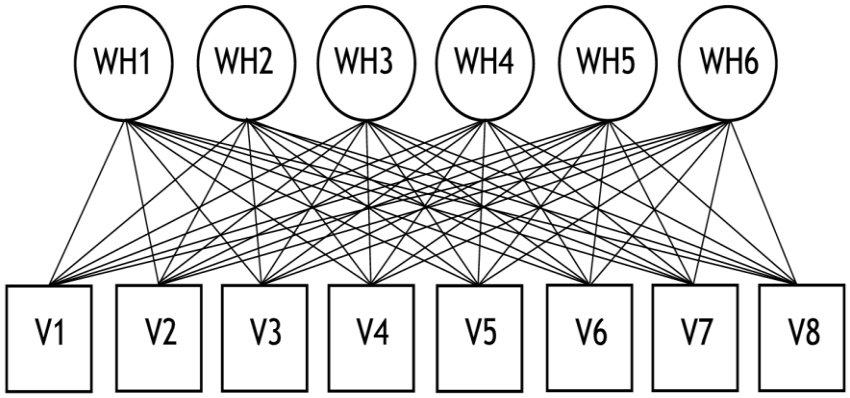
The section below is an example of how to use sets to solve a shipping problem. After reviewing this example, it should become clear that coupling the power of sets with LINGO's modeling language allows you to build large models in a fraction of the time required in a scalar oriented approach to modeling (See Chapter 2, *Using Sets*, for a detailed description of sets).

Developing a Set-Based Transportation Model

The Problem

For our example, suppose that the Wireless Widget (WW) Company has six warehouses supplying eight vendors with their widgets. Each warehouse has a supply of widgets that cannot be exceeded, and each vendor has a demand for widgets that must be satisfied. WW wants to determine how many widgets to ship from each warehouse to each vendor so as to minimize the total shipping cost. This is a classic optimization problem referred to as the *transportation* problem.

The following diagram illustrates the problem:



Wireless Widget's Shipping Network

Since each warehouse can ship to each vendor, there are a total of 48 possible shipping paths, or arcs. We will need a variable for each arc to represent the amount shipped on the arc.

The following data is available:

Warehouse	Widgets On Hand
1	60
2	55
3	51
4	43
5	41
6	52

Widget Capacity Data

Vendor	Widget Demand
1	35
2	37
3	22
4	32
5	41
6	32
7	43
8	38

Vendor Widget Demand

	V1	V2	V3	V4	V5	V6	V7	V8
Wh1	6	2	6	7	4	2	5	9
Wh2	4	9	5	3	8	5	8	2
Wh3	5	2	1	9	7	4	3	3
Wh4	7	6	7	3	9	2	7	1
Wh5	2	3	9	5	7	2	6	5
Wh6	5	5	2	2	8	1	4	3

Shipping Cost per Widget ($)

The Objective Function

Our first pass at formulating the model will be to construct the objective function. As mentioned, WW wants to minimize total shipping costs. We will let the *VOLUME_I_J* variable denote the number of widgets shipped from warehouse *I* to vendor *J*. Then, if we were to explicitly write out our objective function using scalar variables, we would have:

```
MIN = 6 * VOLUME_1_1 + 2 * VOLUME_1_2 +
      6 * VOLUME_1_3 + 7 * VOLUME_1_4 +
      4 * VOLUME_1_5 +
            .
            .
            .
      8 * VOLUME_6_5 + VOLUME_6_6 + 4 * VOLUME_6_7 +
      3 * VOLUME_6_8;
```

For brevity, we included only 9 of the 48 terms in the objective. As one can see, entering such a lengthy formula would be tedious and prone to errors. Extrapolate to the more realistic case where vendors could number in the thousands, and it becomes apparent that scalar based modeling is problematic at best.

If you are familiar with mathematical notation, you could express this long equation in a much more compact manner as follows:

$$Minimize\ \Sigma_{ij} COST_{ij} \bullet VOLUME_{ij}$$

In a similar manner, LINGO's modeling language allows you to express the objective function in a form that is short, easy to type, and easy to understand. The equivalent LINGO statement is:

```
MIN = @SUM(LINKS(I,J): COST(I,J) * VOLUME(I,J));
```

In words, this says to minimize the sum of the shipping *COST* per widget times the *VOLUME* of widgets shipped for all *LINKS* between the warehouses and vendors. The following table compares the mathematical notation to the LINGO syntax for our objective function:

Math Notation	LINGO Syntax
Minimize	`MIN =`
Σ_{ij}	`@SUM(LINKS(I, J):`
$COST_{ij}$	`COST(I, J)`
$\bullet$	`*`
$VOLUME_{ij}$	`VOLUME(I,J));`

The Constraints

With the objective function in place, the next step is to formulate the constraints. There are two sets of constraints in this model. The first set guarantees that each vendor receives the number of widgets required. We will refer to this first set of constraints as being the demand constraints. The second set of constraints, called the capacity constraints, ensures no warehouse ships out more widgets than it has on hand.

Starting with the demand constraint for Vendor 1, we need to sum up the shipments from all the warehouses to Vendor 1 and set them equal to Vendor 1's demand of 35 widgets. Thus, if we were using scalar-based notation, we would need to construct the following:

$$VOLUME_1_1 + VOLUME_2_1 + VOLUME_3_1 + VOLUME_4_1 + VOLUME_5_1 + VOLUME_6_1 = 35;$$

You would then need to type seven additional demand constraints, in a similar form, to cover all eight vendors. Again, as one can see, this would be a tedious and error prone process. However, as with our objective function, we can use LINGO's set-based modeling language to simplify our task.

Using mathematical notation, all eight demand constraints can be expressed in the single statement:

$$\Sigma_i VOLUME_{ij} = DEMAND_j, \text{ for all } j \text{ in } VENDORS$$

The corresponding LINGO modeling statement appears as follows:

```
@FOR(VENDORS(J):
 @SUM(WAREHOUSES(I): VOLUME(I, J)) =
  DEMAND(J));
```

This LINGO statement replaces all eight demand constraints. In words, this says for all *VENDORS*, the sum of the *VOLUME* shipped from each of the *WAREHOUSES* to that vendor must equal the corresponding *DEMAND* of the vendor. Notice how closely this statement resembles the mathematical notation above as the following table shows:

Math Notation	LINGO Syntax
For all j in VENDORS	`@FOR(VENDORS(J):`
Σ_i	`@SUM(WAREHOUSES(I):`
$VOLUME_{ij}$	`VOLUME(I, J))`
=	=
$DEMAND_j$	`DEMAND(J));`

Now, we will move on to constructing the capacity constraints. In standard mathematical notation, the six capacity constraints would be expressed as:

$$\Sigma_j VOLUME_{ij} <= CAP_i, \text{ for all } i \text{ in } WAREHOUSES$$

The equivalent LINGO statement for all capacity constraints would be:

```
@FOR(WAREHOUSES(I):
 @SUM(VENDORS(J): VOLUME(I, J))<=
  CAPACITY(I));
```

In words, this says, for each member of the set *WAREHOUSES*, the sum of the *VOLUME* shipped to each of the *VENDORS* from that warehouse must be less-than-or-equal-to the *CAPACITY* of the warehouse.

Putting together everything we've done so far yields the following complete LINGO model:

```
MODEL:
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I): VOLUME(I, J)) =
     DEMAND(J));
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J)) <=
     CAPACITY(I));
END
```

Model: WIDGETS

However, we still need to define sets of objects used in the model (vendors, warehouses and shipping arcs) as well as the data. We will do this in two additional model sections called the *sets section* and the *data section*.

Defining the Sets

Whenever you are modeling some situation in real life, you will typically find there are one or more sets of related objects. Examples would be such things as factories, customers, vehicles, and employees. Usually, if a constraint applies to one member of a set, then it will apply equally to each other member of the set. This simple concept is at the core of the LINGO modeling language. LINGO allows you to define the sets of related objects in the *sets section*. The sets section begins with the keyword *SETS:* on a line by itself and ends with *ENDSETS* on a line by itself. Once your set members are defined, LINGO has a group of set looping functions (e.g., *@FOR*), which apply operations to all members of a set using a single statement. See Chapter 2, *Using Sets* for more information.

In the case of our Wireless Widget model, we have constructed the following three sets:

- warehouses,
- vendors, and
- shipping arcs from each warehouse to customer.

The three sets are defined in the model's sets section as follows:

```
SETS:
   WAREHOUSES: CAPACITY;
   VENDORS: DEMAND;
   LINKS( WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS
```

The second line says that the set *WAREHOUSES* has an attribute called *CAPACITY*. The following line declares the vendor set and that it has an attribute called *DEMAND*.

The final set, titled *LINKS*, represents the links in the shipping network. Each link has a *COST* and a *VOLUME* attribute associated with it. The syntax used to define this set differs from the previous two. By specifying:

```
LINKS( WAREHOUSES, VENDORS)
```

we are telling LINGO that the *LINKS* set is *derived* from the *WAREHOUSES* and *VENDORS* sets. In this case, LINGO generates each ordered (warehouse, vendor) pair. Each of these 48 ordered pairs becomes a member in the *LINKS* set. To help clarify this, we list selected members from the *LINKS* set in the following table.

Member Index	Shipping Arc
1	*WH1→V1*
2	*WH1→V2*
3	*WH1→V3*
…	...
47	*WH6→V7*
48	*WH6→V8*

A nice feature of LINGO is that it will automatically generate the members of the *LINKS* set-based on the members of the *WAREHOUSES* and *VENDORS* sets, thereby saving us considerable work.

Inputting the Data

LINGO allows the user to isolate data within the data section of the model. In our Wireless Widget example, we have the following data section:

```
DATA:
   !set members;
   WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
   VENDORS = V1 V2 V3 V4 V5 V6 V7 V8;

   !attribute values;
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
```

The *data section* begins with the keyword *DATA:* on a line by itself and ends with *ENDDATA* on a line by itself.

Next, we input the list of warehouses and vendors. Had we preferred, we could have also used the following shorthand notation to the same end:

```
!set members;
   WAREHOUSES = WH1..WH6;
   VENDORS = V1..V8;
```

LINGO interprets the double-dots to mean that it should internally generate the six warehouses and eight vendors.

Both the *CAPACITY* attribute of the set *WAREHOUSES* and *DEMAND* attribute of the set *VENDORS* are initialized in a straightforward manner. The *COST* attribute of the two-dimensional set *LINKS* is a little bit trickier, however. When LINGO is initializing a multidimensional array in a data section, it increments the outer index the fastest. Thus, in this particular example, *COST(WH1, V1)* is initialized first, followed by *COST(WH1, V2)* through *COST (WH1, V8)*. Then, the next one to be initialized with be *COST(WH2, V1)*, and so on.

In this particular example, we have isolated all the model's data within a single data section. Given that the data is the most likely feature to change from one run of a model to the next, isolating data, as we have done here, makes modifications considerably easier. Contrast this to how difficult it would be to track down and change the data in a large, scalar model where data is spread throughout all the constraints of the model.

In order to facilitate data management further, LINGO has the ability to import data from external sources. More specifically, a LINGO model can import data from external text files, establish real-time OLE links to Excel, and/or create ODBC links to databases.

Putting together the data section, the sets section, the objective, and the constraints, the completed model is as follows:

```
MODEL:
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES: CAPACITY;
   VENDORS: DEMAND;
   LINKS( WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS
! Here is the data;
DATA:
   !set members;
   WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
   VENDORS = V1 V2 V3 V4 V5 V6 V7 V8;

   !attribute values;
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
! The objective;
   MIN = @SUM( LINKS( I, J):
    COST( I, J) * VOLUME( I, J));
! The demand constraints;
   @FOR( VENDORS( J):
    @SUM( WAREHOUSES( I): VOLUME( I, J)) =
     DEMAND( J));
! The capacity constraints;
   @FOR( WAREHOUSES( I):
    @SUM( VENDORS( J): VOLUME( I, J)) <=
     CAPACITY( I));
END
```

Model: WIDGETS

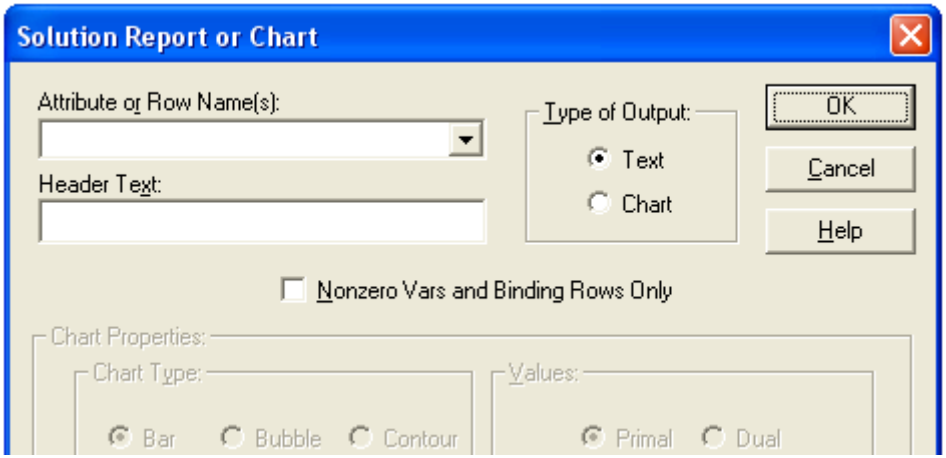
Note that we have again added comments to improve the readability of the model. The model is named *WIDGETS*, and can be found in the *SAMPLES* subdirectory off the main LINGO directory.

Solving the Transportation Model

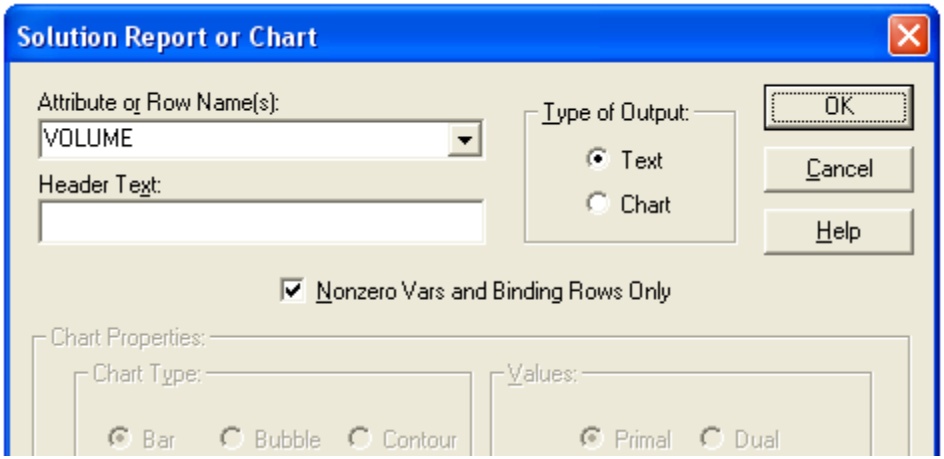
Now, let's solve the model to determine the optimal shipping volume for each warehouse to vendor link. In LINGO for Windows, choose *Solve* from the *LINGO* menu or press the *Solve* button ( ). On other platforms, enter the *GO* command at the command-line prompt. LINGO will respond by solving the model and returning a somewhat lengthy solution report containing the values for all the variables, constraints, and data in the model. Most of this information is not of immediate interest. What we would really like to know is the amount of widgets being shipped from the warehouses to the vendors.

Note: Anytime you find the amount of LINGO's output overwhelming, you can choose *Options…* from the LINGO menu, select the *Interface* tab, and set the Output Level option to *Terse*. LINGO will then display only the solutions status, objective value and number of iterations in the solution window. In non-Windows versions of LINGO, enter the ***TERSE*** command before giving the ***GO*** command.

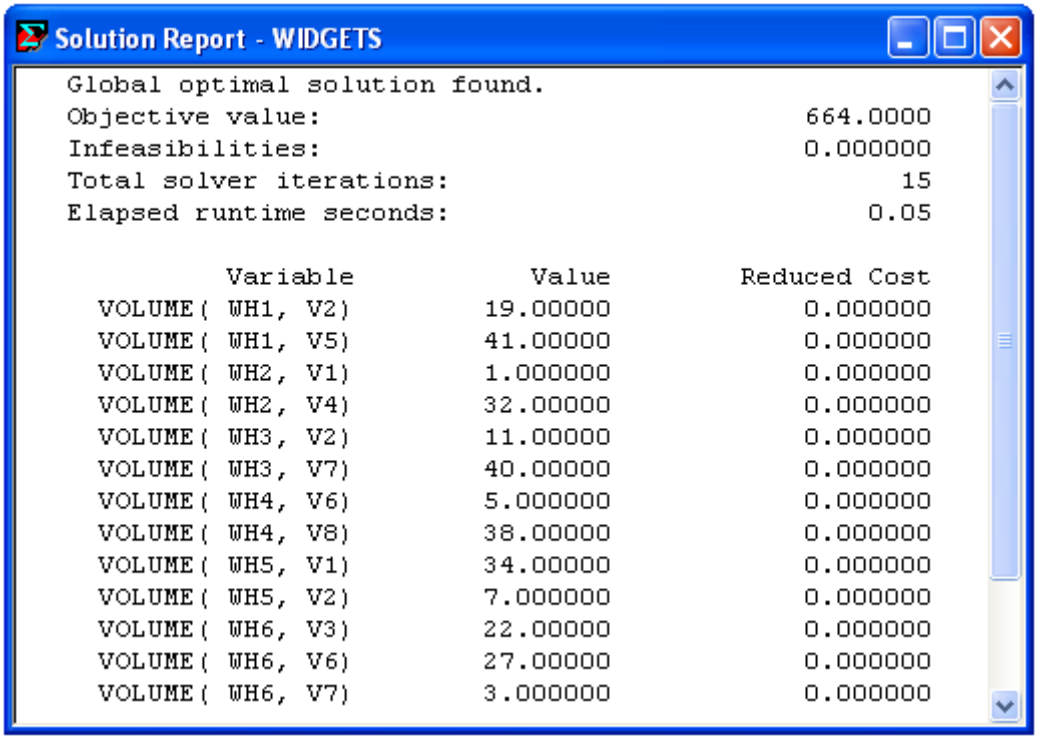
Note: To obtain a report containing only the nonzero values for *VOLUME*, we select the ***Solution*** command from the *LINGO* menu. We are then presented with the following dialog box:



Press down on the arrow button in the *Attribute or Row Name* field and select *VOLUME* from the list of names in the drop-down box. To suppress the printing of variables with zero value, click on the *Nonzero Vars and Binding Rows Only* checkbox. Once you have done this, the dialog box should resemble:



Now, click the *OK* button and you will be presented with the following report that contains the nonzero *VOLUME* variables:

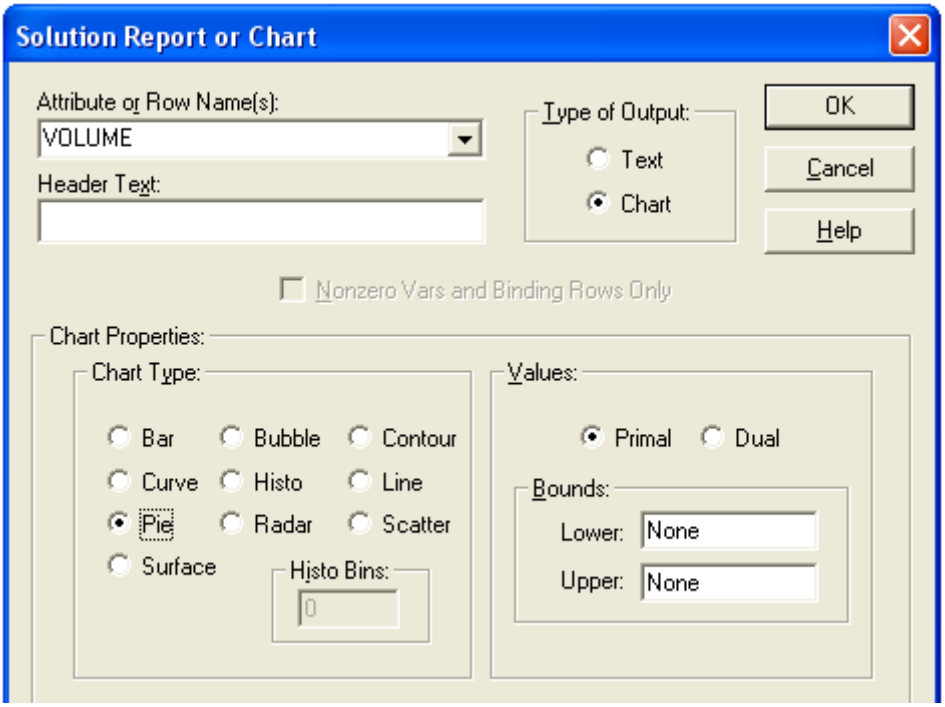


Solution Report - WIDGETS

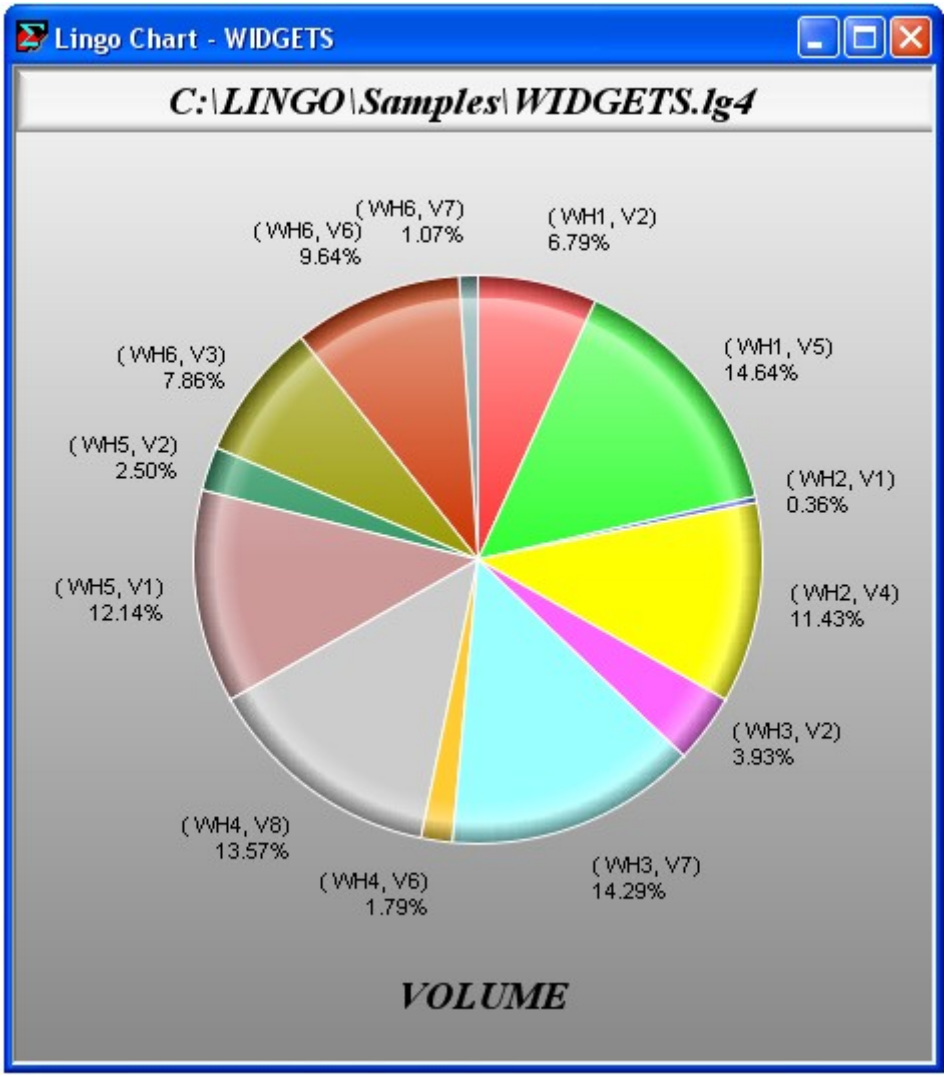
```
Global optimal solution found.
Objective value:                              664.0000
Infeasibilities:                              0.000000
Total solver iterations:                            15
Elapsed runtime seconds:                          0.05

          Variable           Value        Reduced Cost
  VOLUME( WH1, V2)        19.00000            0.000000
  VOLUME( WH1, V5)        41.00000            0.000000
  VOLUME( WH2, V1)        1.000000            0.000000
  VOLUME( WH2, V4)        32.00000            0.000000
  VOLUME( WH3, V2)        11.00000            0.000000
  VOLUME( WH3, V7)        40.00000            0.000000
  VOLUME( WH4, V6)        5.000000            0.000000
  VOLUME( WH4, V8)        38.00000            0.000000
  VOLUME( WH5, V1)        34.00000            0.000000
  VOLUME( WH5, V2)        7.000000            0.000000
  VOLUME( WH6, V3)        22.00000            0.000000
  VOLUME( WH6, V6)        27.00000            0.000000
  VOLUME( WH6, V7)        3.000000            0.000000
```

If you are running LINGO on a platform other than Windows, you can generate the same report by issuing the *NONZERO VOLUME* command.

LINGO can also display various forms of charts depicting your solution. Suppose you would like to display a pie chart of *VOLUME*. You may do this by first clicking on the *Chart* button in the *Type of Output* box, which will enable the *Chart Properties* box, where you select a chart type of *Pie*:



Clicking *OK* will cause LINGO to open a new window with the requested pie chart:



Summary

This section has begun to demonstrate the virtues of LINGO's set-based modeling language. By moving to a set-based approach to modeling, you will find that your models become easier to build, easier to understand, and easier to maintain. Set-based modeling takes a little more work to become comfortable with, but the benefits should substantially outweigh the extra effort involved in the learning process. We will delve further into the concepts of set-based modeling in the following chapter, *Using Sets*.

Additional Modeling Language Features

Constraint Names

LINGO gives you the ability to name the constraints in your model. This is a good practice for two reasons. First, the constraint names are used in solution reports making them easier to interpret. Secondly, many of LINGO's error messages refer to a given constraint by name. If you don't name your constraints, tracking down the source of these errors may, at best, be difficult.

Note: LINGO does not require you to name your constraints. However, if you do not name your constraints, LINGO defaults to using a name that corresponds to the internal index of the constraint. This internal index may have little to do with the order in which you defined the constraint, thus making the job of interpreting solution reports and error messages difficult. Therefore, it is strongly recommended that you always use constraint names in your models.

Naming a constraint is quite simple. All you need do is insert a name in square brackets at the very start of the constraint. The name must obey the standard requirements for a LINGO name. More specifically, all names must begin with an alphabetic character (A-Z). Subsequent characters may be either alphabetic, numeric (0-9), or the underscore (_). Names may be up to 64 characters in length. Some examples of constraint names follow:

Example 1:

```
[OBJECTIVE] MIN = X;
```

assigns the name *OBJECTIVE* to the model's objective row,

Example 2:

```
@FOR(CUSTOMERS( J): [DEMAND_ROW]
 @SUM(SOURCES( I): SHIP(I, J)) >=
  DEMAND( J));
```

assigns the name *DEMAND_ROW* to the demand constraints in a transportation model.

To further illustrate the use of row names, we have updated the *WIDGETS* model from the previous section to include constraint names (shown in bold):

```
MODEL:
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES: CAPACITY;
   VENDORS: DEMAND;
   LINKS( WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS
DATA:
   !set members;
   WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
   VENDORS = V1 V2 V3 V4 V5 V6 V7 V8;

   !attribute values;
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
! The objective;
   [OBJECTIVE] MIN = @SUM( LINKS( I, J):
    COST( I, J) * VOLUME( I, J));
! The demand constraints;
   @FOR( VENDORS( J): [DEMAND_ROW]
    @SUM( WAREHOUSES( I): VOLUME( I, J)) =
     DEMAND( J));
! The capacity constraints;
   @FOR( WAREHOUSES( I): [CAPACITY_ROW]
    @SUM( VENDORS( J): VOLUME( I, J)) <=
     CAPACITY( I));
END
```

WIDGETS with Constraint Names

The row section of the solution report is now considerably easier to interpret:

Row	Slack or Surplus	Dual Price
OBJECTIVE	664.0000	1.000000
DEMAND_ROW(V1)	0.0000000	-4.000000
DEMAND_ROW(V2)	0.0000000	-5.000000
DEMAND_ROW(V3)	0.0000000	-4.000000
DEMAND_ROW(V4)	0.0000000	-3.000000
DEMAND_ROW(V5)	0.0000000	-7.000000
DEMAND_ROW(V6)	0.0000000	-3.000000
DEMAND_ROW(V7)	0.0000000	-6.000000
DEMAND_ROW(V8)	0.0000000	-2.000000
CAPACITY_ROW(WH1)	0.0000000	3.000000
CAPACITY_ROW(WH2)	22.00000	0.000000
CAPACITY_ROW(WH3)	0.0000000	3.000000
CAPACITY_ROW(WH4)	0.0000000	1.000000
CAPACITY_ROW(WH5)	0.0000000	2.000000
CAPACITY_ROW(WH6)	0.0000000	2.000000

Row Report for WIDGETS with Constraint Names

Note that each row now has a name rather than a simple index number. Furthermore, if the constraint is generated over a set using the *@FOR* function, LINGO qualifies the constraint name by appending the corresponding set member name in parentheses.

Model Title

You can insert a title for a model anywhere you would normally enter a constraint. If a title is included, it will be printed at the top of solution reports. The title is also used as a default argument in the *@ODBC* function (see Chapter 10, *Interfacing with Databases*).

The model's title must begin with the keyword *TITLE* and end with a semicolon. All text between *TITLE* and the semicolon will be taken as the title of the model.

In the following, we have added a title to the beginning of the *WIDGETS* model:

```
MODEL:
TITLE Widgets;
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES: CAPACITY;
                        .
                        .
                        .
```

Excerpt from WIDGETS Model with a Title

Note that when we display the solution report, the title is now displayed along the top:

```
 Model Title: Widgets

       Variable           Value        Reduced Cost
 CAPACITY( WH1)        60.00000           0.0000000
 CAPACITY( WH2)        55.00000           0.0000000
 CAPACITY( WH3)        51.00000           0.0000000
 CAPACITY( WH4)        43.00000           0.0000000
                        .
                        .
                        .
```

Excerpt from Solution Report to WIDGETS Model with a Title

Indicating Convexity and Concavity

LINGO's global solver can exploit the fact that a constraint is either convex or concave to improve performance. In many cases, the global solver can automatically identify a constraint as either convex or concave. In other cases, though, the global solver can't make this determination. If you know a constraint is convex or concave then you can flag it as such. In order to do this, replace the standard equality and inequality operators (=, <= and >=) with either: =C=, <C= and >C=. The interpretations of these operators are:

Operator	*Interpretation*
<C=	The constraint *F(X)* <= *b* is convex.
>C=	The constraint *F(X)* >= *b* is concave.
=C=	The function *F(X)* of the constraint *F(X)* = *b* is a convex function.

As an example, consider the nonlinear model:

```
MODEL:
  MAX= 5 * W1 - 3 * X1 - 4 * Y1
               + 5 * W2 - 3 * X2 - 4 * Y2
               + 5 * W3 - 3 * X3 - 4 * Y3;
  - W1 <= 1;
  W1 <= 1;
  X1 + Y1 >= 1;
  [NLROW1] W1 ^ 3 - X1 * Y1 <C= 0;

  - W2 <= 1;
  W2 <= 1;
  X2 + Y2 >= 1;
  [NLROW2] W2 ^ 3 - X2 * Y2 <C= 0;

  - W3 <= 1;
  W3 <= 1;
  X3 + Y3 >= 1;
  [NLROW3] W3 ^ 3 - X3 * Y3 <C= 0;
```

Model: CONVEX

The three nonlinear rows in this model: NLROW1, NLROW2 and NLROW3, are convex, and we have marked them as such using the convexity inequality operator (<C=). This will allow the global solver to exploit the convexity of these rows and solve the model almost instantly. The global solver would run considerably longer on this model if we were to remove the convexity operators, reverting back to the standard inequality operator <=.

Note: If you have a constraint for which F(x) is only quasi-convex or quasi-concave, you may still apply the convexity markers to the same advantage. A convex objective, Min = F(x), can be identified by rewriting it: Min = z; z>C = F(x);

Maximum Problem Dimensions

Some versions of LINGO limit one or more of the following model properties: *total variables*, *integer variables*, *nonlinear variables*, *global variables,* and *constraints*. The total variable limit is on the total number of optimizable variables in your model (i.e., variables LINGO was unable to determine as being fixed at a particular value). The integer variable limit applies to the total number of optimizable variables restricted to being integers with either the *@BIN* or *@GIN* functions. The nonlinear variable limit applies to the number of optimizable variables that appear nonlinearly in the model's constraints. As an example, in the expression: *X + Y,* both *X* and *Y* appear linearly. However, in the expression: *X ^ 2 + Y, X* appears nonlinearly while *Y* appears linearly. Thus, *X* would count against the nonlinear variable limit. In some cases, nonlinear variables are allowed only if you have purchased the nonlinear option for your LINGO software. The global variable limit applies to the total number of nonlinear variables when using the global solver. The constraint limit refers to the number of formulas in the model that contain one or more optimizable variables. Keep in mind that a single *@FOR* function may generate many constraints.

The maximum sized problem your LINGO can handle depends on the version you have. The current limits for the various versions are:

Version	Total Variables	Integer Variables	Nonlinear Variables	Global Variables	Constraints
Demo/Web	300	30	30	5	150
Solver Suite	500	50	50	5	250
Super	2,000	200	200	10	1,000
Hyper	8,000	800	800	20	4,000
Industrial	32,000	3,200	3,200	50	16,000
Extended	*Unlimited*	*Unlimited*	*Unlimited*	*Unlimited*	*Unlimited*

You can also determine the limits of your version by selecting the *About LINGO* command from the *Help* menu in Windows, or by typing *HELP* at the command-line prompt on other platforms. If you determine you need a larger version of LINGO, upgrades are available from LINDO Systems. Please feel free to contact us for pricing and availability.

Note 1: The limits of different LINGO versions are subject to change. Check our website, http://www.lindo.com, for the most current sizes.

Note 2: In some versions of LINGO, the *Nonlinear Variable* limit will be 0 if you have not purchased the nonlinear option for your copy of LINGO. Similarly, the global variable limit will be o if the global solver option is not enabled.

Note 3: LINGO has two other implicit limits not given by the table above—*memory* and *time*. Large models may require more memory to solve than is available on your system, or they may require more time to solve than one would normally be willing to wait. So, when building large models, be aware that just because your model falls within LINGO's limits there is no guarantee it will be solvable in a reasonable amount of time on a particular machine.

How to Contact LINDO Systems

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For sales and product information, please contact us at:

Tel: **1-800-441-2378 or 312-988-7422**
e-mail: ***sales@lindo.com***

For technical support, we prefer you send your model and questions by email to *tech@lindo.com*. You may also speak to our technical support staff at 312-988-9421. Our technical support staff can help you with questions regarding the installation and operation of LINGO. If you have simple modeling questions, we can generally help get you pointed in the right direction. If you have extensive modeling questions, we can recommend third party consultants well versed in the specifics of LINGO and mathematical modeling in general, who can assist you in your modeling efforts.

2 Using Sets

As we mentioned in the previous chapter, whenever you are modeling situations in real life there will typically be one or more groups of related objects. Examples of such objects might include factories, customers, vehicles, or employees. LINGO allows you to group these related objects together into *sets*. Once the objects in your model are grouped into sets, you can make use of set-based functions to unleash the full power of the LINGO modeling language.

Having given you a brief introduction into the use of sets in Chapter 1, *Getting Started with LINGO*, we will now go into greater depth as to how you construct sets and initialize set attributes with data. This will then give us the ability to begin constructing some interesting and useful examples. Once you've read this chapter, you should have a basic understanding of how to go about applying set-based modeling techniques to your own models.

Why Use Sets?

Sets are the foundation of LINGO's modeling language—the fundamental building block of the program's most powerful capabilities. With an understanding of sets, you can write a series of similar constraints in a single statement and express long, complex formulas concisely. This allows you to express your largest models very quickly and easily. In larger models, you'll encounter the need to express a group of several very similar calculations or constraints. Fortunately, LINGO's ability to handle sets of information allows you to perform such operations efficiently.

For example, preparing a warehouse-shipping model for 100 warehouses would be tedious if you had to write each constraint explicitly (e.g., "Warehouse 1 must ship no more than its present inventory, Warehouse 2 must ship no more than its present inventory, Warehouse 3 must ship no more than its present inventory…", and so on). LINGO allows you to express formulas in the form easiest for you to read and understand (e.g., "Each warehouse must ship no more than its present inventory").

What Are Sets?

Sets are simply groups of related objects. A set might be a list of products, trucks, or employees. Each member in the set may have one or more characteristics associated with it. We call these characteristics *attributes*. Attribute values can be known in advance or unknowns that LINGO solves for. For example, each product in a set of products might have a price attribute; each truck in a set of trucks might have a hauling capacity attribute; and each employee in a set of employees might have a salary attribute, as well as a birth date attribute.

Types of Sets

LINGO recognizes two kinds of sets: *primitive* and *derived*.

A *primitive set* is a set composed only of objects that can't be further reduced. In the Wireless Widgets example (page 48), the *WAREHOUSES* set, which is composed of six warehouses, is a primitive set. Likewise, the set composed of eight vendors is a primitive set.

A *derived set* is defined using one or more other sets. In other words, a derived set *derives* its members from other preexisting sets. Again, using the Wireless Widgets example, the set composed of the links between the six warehouses and eight vendors (*LINKS*) is a derived set. It derives its members from the unique pairs of members of the *WAREHOUSES* and *VENDORS* sets. Although the *LINKS* set is derived solely from primitive sets, it is also possible to build derived sets from other derived sets as well. See the section below, *Defining Derived Sets*, for more information.

The Sets Section of a Model

Sets are defined in an optional section of a LINGO model called the *sets section*. Before you use sets in a LINGO model, you have to define them in the sets section of the model. The sets section begins with the keyword *SETS:* (including the colon), and ends with the keyword *ENDSETS*. A model may have no sets section, a single sets section, or multiple sets sections. A sets section may appear anywhere in a model. The only restriction is you must define a set and its attributes before they are referenced in the model's constraints.

Defining Primitive Sets

To define a primitive set in a sets section, you specify:

- the *name* of the set,
- optionally, its *members* (objects contained in the set), and
- optionally, any *attributes* the members of the set may have.

A primitive set definition has the following syntax:

setname [/ *member_list* /] [: *attribute_list*];

Note: The use of square brackets indicates an item is optional. In this particular case, a primitive set's *attribute_list* and *member_list* are both optional.

The *setname* is a name you choose to designate the set. It should be a descriptive name that is easy to remember. The set name must conform to standard LINGO naming conventions. In other words, the name must begin with an alphabetic character, which may be followed by up to 31 alphanumeric characters or the underscore (_). LINGO does not distinguish between upper and lowercase characters in names.

A *member_list* is a list of the members that constitute the set. If the set members are included in the set definition, they may be listed either *explicitly* or *implicitly*. If set members are not included in the set

definition, then they may be defined subsequently in a data section of the model. For details on defining set members in a data section, refer *to Introduction to the Data Section*.

When listing members explicitly, you enter a unique name for each member, optionally separated by commas. As with set names, member names must also conform to standard naming conventions. In the Wireless Widgets model, we could have used an explicit member list to define the set *WAREHOUSES* in the sets section as follows:

```
WAREHOUSES / WH1 WH2 WH3 WH4 WH5 WH6/: CAPACITY;
```

When using implicit set member lists, you do not have to list a name for each set member. Use the following syntax when using an implicit set member list:

setname / *member1*..*memberN* / [: *attribute_list*];

where *member1* is the name of the first member in the set and *memberN* is the name of the last member. LINGO automatically generates all the intermediate member names between *member1* and *memberN*. While this can be a very compact and convenient method for building a primitive set, there is one catch in that only certain formats of names are accepted for the initial and terminal member names. The following table details the available options:

Implicit Member List Format	Example	Set Members
1..*n*	`1..5`	`1, 2, 3, 4, 5`
stringM..stringN	`TRUCKS3..TRUCKS204`	`TRUCKS3, TRUCKS4, …, TRUCKS204`
dayM..dayN	`MON..FRI`	`MON, TUE, WED, THU, FRI`
monthM..monthN	`OCT..JAN`	`OCT, NOV, DEC, JAN`
monthYearM..monthYearN	`OCT2001..JAN2002`	`OCT2001, NOV2001, DEC2001, JAN2002`

When using the 1..*n* format, *n* may be any positive integer value, and the initial member must always be a 1.

The *stringM..stringN* format allows you to use any string to start both the initial and terminal member names as long as the string conforms to standard LINGO naming conventions. *M* and *N* must be nonnegative and integer, such that $M \leq N$.

The *dayM..dayN* format allows you to choose the initial and terminal member names for the names of the days of the week. All names are abbreviated to three characters. Thus, the available options are: Mon, Tue, Wed, Thu, Fri, Sat, and Sun.

The *monthM..monthN* format allows you to select from the months of the year, where all names are abbreviated to three characters. The available options are: Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, and Dec.

The *monthYearM..monthYearN* option allows you to specify a month and a four digit year.

As further illustration, in the *Wireless Widgets* example, we could have also defined the *WAREHOUSES* set as:

```
WAREHOUSES / 1..6/: CAPACITY;
```

As an alternative, when using this 1..*n* form of implicit definition, you may also place the length of the set in a data section, and then reference this length in a subsequent sets section as we do here:

```
DATA:
   NUMBER_OF_WH = 6;
ENDDATA

SETS:
   WAREHOUSES / 1..NUMBER_OF_WH/: CAPACITY;
ENDSETS
```

Set members may have one or more *attributes* specified in the *attribute_list* of the set definition. An attribute is simply some property each member of the set displays. For instance, in the *WAREHOUSES* set above, there is a single attribute titled *CAPACITY*, which is used to represent the shipping capacity of the *WAREHOUSES*. Attribute names must follow standard naming conventions and be separated by commas.

For illustration, suppose our warehouses had additional attributes related to their location and the number of loading docks. These additional attributes could be added to the attribute list of the set declaration as follows:

```
WAREHOUSES / 1..6/: CAPACITY, LOCATION, DOCKS;
```

In addition to listing a primitive set's members in a model's sets section, primitive set members may also be listed in a model's data section. Some users may prefer this alternative approach in that a set's members are actually input data for the model. Therefore, listing set members in a model's data section, along with all other data, is a more natural approach that makes a model more readable. All the various techniques listed above for enumerating a primitive set's members are also valid in a data section. Some examples of defining primitive set members in a data section follow:

```
SETS:
   WAREHOUSES: CAPACITY;
ENDSETS
DATA:
   WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
ENDDATA
```

Example 1: Listing a Primitive Set in a Data Section

```
SETS:
   WAREHOUSES: CAPACITY;
ENDSETS
DATA:
   NUMBER_OF_WH = 6;
   WAREHOUSES = 1..NUMBER_OF_WH;
ENDDATA
```

Example 2: Listing a Primitive Set in a Data Section

Defining Derived Sets

To define a derived set, you specify:

- the *name* of the set,
- its *parent sets*,
- optionally, its *members*, and
- optionally, any *attributes* the set members may have.

A derived set definition has the following syntax:

setname(*parent_set_list*) [/ *member_list* /] [: *attribute_list*];

The *setname* is a standard LINGO name you choose to name the set.

The *parent_set_list* is a list of previously defined sets, separated by commas. Without specifying a *member_list* element, LINGO constructs all combinations of members from each parent set to create the members of the new derived set. As an example, consider the following sets section:

```
SETS:
   PRODUCT / A B/;
   MACHINE / M N/;
   WEEK / 1..2/;
   ALLOWED(PRODUCT, MACHINE, WEEK);
ENDSETS
```

Sets *PRODUCT*, *MACHINE*, and *WEEK* are primitive sets, while *ALLOWED* is derived from parent sets, *PRODUCT*, *MACHINE*, and *WEEK*. Taking all the combinations of members from the three parent sets, we come up with the following members in the *ALLOWED* set:

Index	Member
1	*(A,M,1)*
2	*(A,M,2)*
3	*(A,N,1)*
4	*(A,N,2)*
5	*(B,M,1)*
6	*(B,M,2)*
7	*(B,N,1)*
8	*(B,N,2)*

ALLOWED Set Membership

The *member_list* is optional, and is used when you want to limit the set to being some subset of the full set of combinations derived from the parent sets. The *member_list* may alternatively be specified in a model's data section (for details on this see *Introduction to the Data Section* in Chapter 4, *Data and Init Sections*).

If the *member_list* is omitted, the derived set will consist of all combinations of the members from the parent sets. When a set does not have a *member_list* and, therefore, contains all possible combinations of members, it is referred to as being a *dense* set. When a set includes a *member_list* that limits it to being a subset of its dense form, we say the set is *sparse*.

A derived set's *member_list* may be constructed using either:

- an *explicit member list*, or
- a *membership filter*.

When using the explicit member list method to specify a derived set's *member_list*, you must explicitly list all the members you want to include in the set. Each listed member must be a member of the dense set formed from all possible combinations of the parent sets. Returning to our small example above, if we had used an explicit member list in the definition of the derived set, *ALLOWED*, as follows:

```
ALLOWED(PRODUCT, MACHINE, WEEK)
 / A M 1, A N 2, B N 1/;
```

then *ALLOWED* would not have had the full complement of eight members. Instead, *ALLOWED* would have consisted of the three member sparse set: *(A,M,1)*, *(A,N,2)*, and *(B,N,1)*. Note that the commas in the list of set members are optional and were added only for readability purposes.

If you have a large, sparse set, explicitly listing all members can become cumbersome. Fortunately, in many sparse sets, the members all satisfy some condition that differentiates them from the nonmembers. If you could just specify this condition, you could save yourself a lot of effort. This is exactly how the membership filter method works. Using the membership filter method of defining a derived set's *member_list* involves specifying a logical condition that each potential set member must satisfy for inclusion in the final set. You can look at the logical condition as a *filter* to keep out potential members that don't satisfy some criteria.

As an example of a membership filter, suppose you have already defined a set called *TRUCKS*, and each truck has an attribute called *CAPACITY*. You would like to derive a subset from *TRUCKS* that contains only those trucks capable of hauling big loads. You could use an explicit member list, and explicitly enter each truck that can carry heavy loads. However, why do all that work when you could use a membership filter as follows:

```
HEAVY_DUTY(TRUCKS)|CAPACITY(&1) #GT# 50000:
```

We have named the set *HEAVY_DUTY* and have derived it from the parent set, *TRUCKS*. The vertical bar character (|) is used to mark the beginning of a membership filter. The membership filter allows only those trucks that have a hauling capacity (*CAPACITY(&1)*) greater than (*#GT#*) 50,000 into the *HEAVY_DUTY* set. The *&1* symbol in the filter is known as a *set index placeholder.* When building a derived set that uses a membership filter, LINGO generates all the combinations of parent set members. Each combination is then "plugged" into the membership condition to see if it passes the test. The first primitive parent set's member is plugged into *&1*, the second into *&2*, and so on. In this example, we have only one parent set (*TRUCKS*), so *&2* would not have made sense. The symbol *#GT#* is a *logical operator* and means "greater than".

The logical operators recognized by LINGO are:

#EQ#	*equal*
#NE#	*not equal*
#GE#	*greater-than-or-equal-to*
#GT#	*greater than*
#LT#	*less than*
#LE#	*less-than-or-equal-to*

In addition to listing a derived set's members in a model's sets section, derived set members may also be listed in a model's data section. Some users may prefer this alternative approach in that a set's members are actually input data for the model. Therefore, listing set members in a model's data section, along with all other data, is a more natural approach that makes a model more readable. All the various techniques listed above for enumerating a primitive set's members are also valid in a data section, with the exception of the membership filter method. An example of defining derived set members in a data section follow:

```
SETS:
   PRODUCT;
   MACHINE;
   WEEK;
   ALLOWED( PRODUCT, MACHINE, WEEK);
ENDSETS
DATA:
   PRODUCT =  P1..P3;
   MACHINE =  M1..M3;
   WEEK    =  W1..W4;
   ALLOWED =
              P1  M1  W1
              P2  M1  W3
              P3  M1  W2
              P3  M3  W4
   ;
ENDDATA
```

Inputting a Derived Set in a Data Section

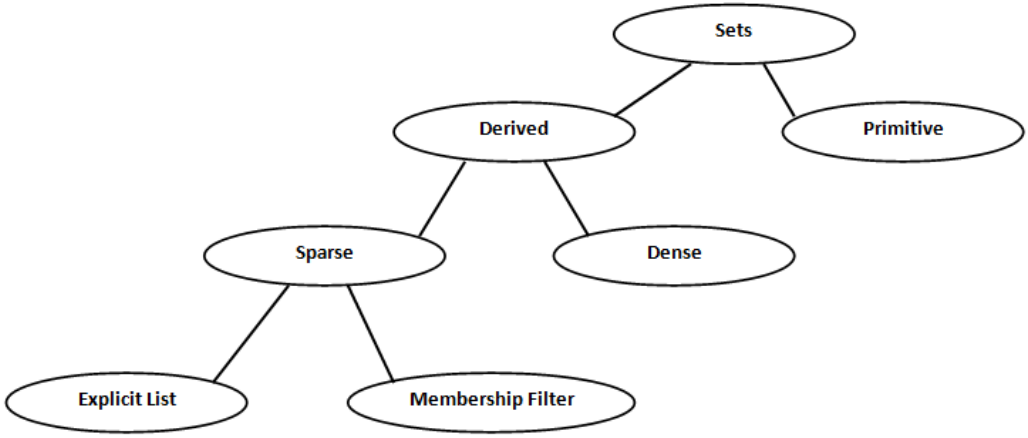
Summary

In summary, keep in mind that LINGO recognizes two types of sets—*primitive* and *derived*.

Primitive sets are the fundamental objects in a model and can't be broken down into smaller components. Primitive sets can be defined using either an *explicit* or *implicit* list. When using an explicit list, you enter each member individually in the set member list. With an implicit list, you enter the initial and terminal set members and LINGO generates all the intermediate members.

Derived sets, on the other hand, are created from other component sets. These component sets are referred to as the parents of the derived set, and may be either primitive or derived. A derived set can be either *sparse* or *dense*. Dense sets contain all combinations of the parent set members (sometimes this is also referred to as the *Cartesian product* or *cross* of the parent sets). Sparse sets contain only a subset of the cross of the parent sets, and may be defined by two methods——*explicit listing* or *membership filter*. The explicit listing method involves listing the members of the sparse set. The membership filter method allows you to specify the sparse set members compactly through the use of a logical condition that all members must satisfy. The relationships amongst the various set types are illustrated in the graph below.Derived sets, on the other hand, are created from other component sets. These component sets are referred to as the parents of the derived set, and may be either primitive or derived. A derived set can be either *sparse* or *dense*. Dense sets contain all combinations of the parent set members (sometimes this is also referred to as the *Cartesian product* or *cross* of the parent sets). Sparse sets contain only a subset of the cross of the parent sets and may be defined by two

methods—*explicit listing* or *membership filter*. The explicit listing method involves listing the members of the sparse set. The membership filter method allows you to specify the sparse set members compactly through the use of a logical condition all members must satisfy. The relationships amongst the various set types are illustrated in the graph below:



LINGO Set Types

At this point, you are probably thinking that set definition is, at best, somewhat complicated. We will be presenting you with plenty more examples in subsequent sections that should help to illustrate the concepts introduced in this section, and demonstrate that set definition is nowhere near as difficult as it may seem. For now, however, we will turn our attention to how data is input into a model. Then, we will examine a group of functions that are designed to operate on set members. Once we have accomplished this, we will be able to bring together all we have learned in order to begin building some interesting and relevant examples of set-based modeling.

The DATA Section

Typically, you will want to initialize the members of certain sets and assign values to certain set attributes. For this purpose, LINGO uses a second optional section called the *data section*. The *data section* allows you to isolate data from the equations of your model. This is a useful practice in that it leads to easier model maintenance and facilitates scaling up a model to larger dimensions.

Similar to the sets section, the data section begins with the keyword *DATA:* (including the colon) and ends with the keyword *ENDDATA*. In the data section, you can have statements to initialize the sets and/or attributes you defined in a sets section. These expressions have the syntax:

object_list = *value_list*;

The *object_list* contains the names of a set and/or attributes you want to initialize, optionally separated by commas. If there is more than one attribute name on in the object list, then all attributes must be defined on the same set. Furthermore, if a set name appears in the object list, then it must be the parent set of any attributes also in the object list. The *value_list* contains the values to assign to the objects in the object list, optionally separated by commas. For example, consider the following model:

```
MODEL:
SETS:
   SET1: X, Y;
ENDSETS
DATA:
   SET1 = A B C;
   X = 1 2 3;
   Y = 4 5 6;
ENDDATA
END
```

We have two attributes, *X* and *Y*, defined on the set *SET1*. The three values of *X* are set to 1, 2, and 3, while *Y* is set to 4, 5, and 6. We could have also used the following compound data statement to the same end:

```
MODEL:
SETS:
   SET1: X, Y;
ENDSETS
DATA:
   SET1 X Y = A 1 4
              B 2 5
              C 3 6;
ENDDATA
END
```

An important fact to remember is that when LINGO reads a compound data statement's value list, it assigns the first *n* values in the list to the first position of each of the *n* objects in the object list, the second *n* values to the second position of each of the *n* objects, and so on. In other words, LINGO is expecting the input data in column format rather than row format, which mirrors the flat file approach used in relational databases.

This section has served to give you a brief introduction into the use of the data section. In *Data and Init Sections*, you will learn more about the capabilities of the data section. You will learn data does not have to actually reside in the data section as shown in examples here. In fact, your data section can have OLE links to Excel, ODBC links to databases, and connections to text based data files.

Set Looping Functions

We have mentioned the power of set-based modeling comes from the ability to apply an operation to all members of a set using a single statement. The functions in LINGO that allow you to do this are called *set looping functions*. If your models don't make use of one or more set looping function, then you are missing out on the power of set-based modeling and, even worse, you're probably working too hard!

Set looping functions allow you to iterate through all the members of a set to perform some operation. There are currently four set looping functions in LINGO. The names of the functions and their uses are:

Function	Use
@FOR	The most powerful of the set looping functions, *@FOR* is primarily used to generate constraints over members of a set. *@FOR* may also be used in calc sections to assign values to attributes across the members of a set.
@SUM	Probably the most frequently used set looping function, *@SUM* computes the sum of an expression over all members of a set.
@MIN	Computes the minimum of an expression over all members of a set.
@MAX	Computes the maximum of an expression over all members of a set.
@PROD	Computes the product of an expression over all members of a set.

The syntax for a set looping function is:

@function(*setname* [(*set_index_list*) [|*conditional_qualifier*]] : *expression_list*);

where *@function* corresponds to one of the four set looping functions listed in the table above. *setname* is the name of the set you want to loop over.

set_index_list is optional. It is used to create a list of indices. Each index corresponds to one of the parent, primitive sets that form the set specified by *setname*. As LINGO loops through the members of the set *setname*, it will set the values of the indices in the *set_index_list* to correspond to the current member of the set *setname*.

The *conditional_qualifier* is optional, and may be used to limit the scope of the set looping function. When LINGO is looping over each member of *setname*, it evaluates the *conditional_qualifier*. If the *conditional_qualifier* evaluates to true, then the *@function* is performed for the set member. Otherwise, it is skipped.

The *expression_list* is a list of expressions that are to be applied to each member of the set *setname*. When using the *@FOR* function, the expression list may contain multiple expressions, separated by semicolons. These expressions will be added as constraints to the model. When using the remaining set looping functions (*@SUM*, *@MAX*, *@MIN* and *@PROD*), the expression list must contain one expression only. If the *set_index_list* is omitted, all attributes referenced in the *expression_list* must be defined on the set *setname*.

The following examples should help to illustrate the use of set looping functions.

@SUM Set Looping Function

In this example, we will construct several summation expressions using the *@SUM* function in order to illustrate the features of set looping functions in general, and the *@SUM* function in particular.

Consider the model:

```
MODEL:
SETS:
   VENDORS: DEMAND;
ENDSETS
DATA:
   VENDORS, DEMAND =  V1,5 V2,1 V3,3 V4,4 V5,6;
ENDDATA
END
```

Each vendor of the *VENDORS* set has a corresponding *DEMAND*. We could sum up the values of the *DEMAND* attribute by adding the following expression after the *ENDDATA* statement:

```
TOTAL_DEMAND = @SUM(VENDORS(J): DEMAND(J));
```

LINGO evaluates the *@SUM* function by first initializing an internal accumulator to zero. LINGO then begins looping over the members in the *VENDORS* set. The set index variable, *J*, is set to the first member of *VENDORS* (i.e., *V1*) and *DEMAND (V1)* is then added to the accumulator. This process continues until all *DEMAND* values have been added to the accumulator. The value of the sum is then stored in the *TOTAL_DEMAND* variable.

Since all the attributes in our expression list (in this case, only *DEMAND* appears in the expression list) are defined on the index set (*VENDORS*), we could have alternatively written our sum as:

```
TOTAL_DEMAND = @SUM(VENDORS: DEMAND);
```

In this case, we have dropped the superfluous index set list and the index on *DEMAND*. When an expression uses this shorthand, we say the index list is *implied*. Implied index lists are not allowed when attributes in the expression list have different parent sets.

Next, suppose we want to sum the first three elements of the attribute *DEMAND*. We can use a conditional qualifier on the set index to accomplish this as follows:

```
DEMAND_3 = @SUM(VENDORS(J)|J #LE# 3: DEMAND(J));
```

The *#LE#* symbol is called a *logical operator* (see p. 429 for more details). This operator compares the operand on the left (*J*) with the one on the right (3), and returns *true* if the left operand is less-than-or-equal-to the one on the right. Otherwise, it returns *false*. Therefore, when LINGO computes the sum this time, it plugs the set index variable, *J*, into the conditional qualifier *J #LE# 3*. If the conditional qualifier evaluates to true, *DEMAND(J)* will be added to the sum. The end result is LINGO sums up the first three terms in *DEMAND*, omitting the fourth and fifth terms, for a total sum of 9.

Note: Before leaving this example, one subtle aspect to note in this last sum expression is the value that the set index *J* is returning. Note, we are comparing the set index variable to the quantity 3 in the conditional qualifier *J #LE# 3*. In order for this to be meaningful, *J* must represent a numeric value. Because a set index is used to loop over set members, one might imagine a set index is merely a placeholder for the current set member. In a sense, this is true, but what set indices *really* return is the *index* of the current set member in its parent primitive set. The index returned is *one-based*. In other words, the value 1 is returned when indexing the first set member, 2 when indexing the second, and so on. Given that set indices return a numeric value, they may be used in arithmetic expressions along with other variables in your model.

@MIN and @MAX Set Looping Functions

The *@MIN* and *@MAX* functions are used to find the minimum and maximum of an expression over members of a set.

Again, consider the model:

```
MODEL:
SETS:
   VENDORS: DEMAND;
ENDSETS
DATA:
   VENDORS, DEMAND =  V1,5 V2,1 V3,3 V4,4 V5,6;
ENDDATA
END
```

To find the minimum and maximum *DEMAND*, all one need do is add the two expressions:

```
MIN_DEMAND = @MIN( VENDORS( J): DEMAND( J));
MAX_DEMAND = @MAX( VENDORS( J): DEMAND( J));
```

The resulting model with the new statements in bold would then be as follows:

```
MODEL:
SETS:
   VENDORS: DEMAND;
ENDSETS
DATA:
   VENDORS, DEMAND =  V1,5 V2,1 V3,3 V4,4 V5,6;
ENDDATA
   MIN_DEMAND = @MIN( VENDORS( J): DEMAND( J));
   MAX_DEMAND = @MAX( VENDORS( J): DEMAND( J));
END
```

As with the *@SUM* example, we can use an implied index list since the attributes are defined on the index set. Using implied indexing, we can recast our expressions as:

```
MIN_DEMAND = @MIN( VENDORS: DEMAND);
MAX_DEMAND = @MAX( VENDORS: DEMAND);
```

In either case, when we solve this model, LINGO returns the expected minimum and maximum *DEMAND* of:

```
Variable           Value
MIN_DEMAND        1.000000
MAX_DEMAND        6.000000
```

For illustration purposes, suppose we had just wanted to compute the minimum and maximum values of the first three elements of *DEMAND*. As with the @SUM example, all we need do is add the conditional qualifier *J #LE# 3*. We then have:

```
MIN_DEMAND3 =
   @MIN( VENDORS( J) | J #LE# 3: DEMAND( J));
MAX_DEMAND3 =
   @MAX( VENDORS( J) | J #LE# 3: DEMAND( J));
```

with solution:

```
Variable           Value
MIN_DEMAND3        1.000000
MAX_DEMAND3        5.000000
```

@FOR Set Looping Function

The *@FOR* function is used to generate constraints across members of a set. Whereas scalar based modeling languages require you to explicitly enter each constraint, the *@FOR* function allows you to enter a constraint just once, and LINGO does the work of generating an occurrence of the constraint for each set member. Thus, the *@FOR* statement provides the set-based modeler with a very powerful tool.

To illustrate the use of *@FOR*, consider the following set definition:

```
SETS:
   TRUCKS / MAC, PETERBILT, FORD, DODGE/: HAUL;
ENDSETS
```

Specifically, we have a primitive set of four trucks with a single *HAUL* attribute. If *HAUL* is used to denote the amount a truck hauls, then we can use the *@FOR* function to limit the amount hauled by each truck to 2,500 pounds with the following expression:

```
@FOR(TRUCKS(T): HAUL(T) <= 2500);
```

In this case, it might be instructive to view the constraints LINGO generates from our expression. You can do this by using the *Solver|Generate|Display model* command under Windows, or by using the *GENERATE* command on other platforms. Running this command, we find LINGO generates the following four constraints:

```
HAUL(MAC) <=   2500
HAUL(PETERBILT) <=   2500
HAUL(FORD) <=   2500
HAUL(DODGE) <=   2500
```

In other words, as we anticipated, LINGO generated one constraint for each truck in the set limiting it to a load of 2,500 pounds.

Here is a model that uses an *@FOR* statement (listed in bold) to compute the reciprocal of any five numbers placed into the *VALUE* attribute:

```
MODEL:
SETS:
   NUMBERS /1..5/: VALUE, RECIPROCAL;
ENDSETS
DATA:
   VALUE = 3 4 2 7 10;
ENDDATA
   @FOR( NUMBERS( I):
      RECIPROCAL( I) = 1 / VALUE( I)
   );
END
```

Solving this model gives the following values for the reciprocals:

```
     Variable           Value
RECIPROCAL(1)       0.3333333
RECIPROCAL(2)       0.2500000
RECIPROCAL(3)       0.5000000
RECIPROCAL(4)       0.1428571
RECIPROCAL(5)       0.1000000
```

Since the reciprocal of zero is not defined, we could put a conditional qualifier on our *@FOR* statement that causes us to skip the reciprocal computation whenever a zero is encountered. The following *@FOR* statement accomplishes this:

```
@FOR(NUMBERS(I)| VALUE(I) #NE# 0:
   RECIPROCAL(I) = 1 / VALUE(I)
);
```

The conditional qualifier (listed in bold) tests to determine if the value is not equal (*#NE#*) to zero. If so, the computation proceeds.

This was just a brief introduction to the use of the *@FOR* statement. There will be many additional examples in the sections to follow.

@PROD Set Looping Function

The *@PROD* function is used to find the product of an expression across members of a set. As an example, consider the model:

```
MODEL:
SETS:
  COMPONENTS: P;
ENDSETS
DATA:
  P = .95 .99 .98;
ENDDATA
  P_FAIL = 1 - @PROD( COMPONENTS( I): P( I));
END
```

Here we have a system of three components arranged in a series. The probability that each component functions successfully (.95, .99, and .98) is loaded into attribute *P* in the model's data section. We then compute the probability that the entire system will fail, *P_FAIL*, by taking the product of the component probabilities and subtracting it from 1:

```
P_FAIL = 1 - @PROD( COMPONENTS( I): P( I));
```

As an aside, an interesting feature to note about this model is that we never initialized the *COMPONENTS* set. When LINGO sees that an attribute of an undefined primitive set being initialized to n values in a data section, it automatically initializes the parent primitive set to contain the members: 1, 2, …, n. So, in this example, LINGO automatically assigned the member 1, 2 and 3 to the *COMPONENTS* set.

Nested Set Looping Functions

The simple models shown in the last section use *@FOR* to loop over a single set. In larger models, you'll encounter the need to loop over a set within another set looping function. When one set looping function is used within the scope of another, we call it *nesting*.

An example of a nested set looping function can be found in the Wireless Widgets shipping model (p.48). If you remember, WW's vendors had a demand for widgets that had to be met. The LINGO statement that enforces this condition is:

```
! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I): VOLUME(I, J)) =
     DEMAND(J));
```

Specifically, for each vendor, we sum up the shipments going from all the warehouses to that vendor and set the quantity equal to the vendor's demand. In this case, we have nested an *@SUM* function within an *@FOR* function.

@SUM, *@MAX*, and *@MIN* can be nested within any set looping function. *@FOR* functions, on the other hand, may only be nested within other *@FOR* functions.

Summary

This section demonstrated that set looping functions can be very powerful and can simplify the modeler's task. If you aren't making use of sets and set looping functions, you will have a considerably more difficult time building your models. Furthermore, the difficulty will grow dramatically as the sizes of your models grow.

We now know how to create sets, how to initialize sets and attributes using the data section, and how to work with sets using set looping functions. At this point, we now have the ability to start constructing some meaningful example models.

Set-Based Modeling Examples

Recall from the earlier discussion in this chapter, there are four types of sets that can be created in LINGO. These set types are:

1. primitive,
2. dense derived,
3. sparse derived - explicit list, and
4. sparse derived - membership filter.

If you would like to review the four set types, refer to the sections *What are Sets?* and *The Sets Section of a Model* at the beginning of this chapter. The remainder of this section will help develop your talents for set-based modeling by building and discussing four models, each introducing one of the set types listed above.

Primitive Set Example

The following staff scheduling model illustrates the use of a primitive set. In a staff scheduling model, there is demand for staffing over a time horizon. The goal is to come up with a work schedule that meets staffing demands at minimal cost.

The model used in this example may be found in the *SAMPLES* subdirectory off the main LINGO directory under the name *STAFFDEM*.

The Staff Scheduling Problem

Suppose you run the popular Pluto Dogs hot dog stand that is open seven days a week. You hire employees to work a five-day workweek with two consecutive days off. Each employee receives the same weekly salary. Some days of the week are busier than others and, based on past experience, you know how many workers are required on a given day of the week. In particular, your forecast calls for these staffing requirements:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Staff Req'd	20	16	13	16	19	14	12

You need to determine how many employees to start on each day of the week in order to minimize the total number of employees, while still meeting or exceeding staffing requirements each day of the week.

The Formulation

The first question to consider when building a set-based model is, "What are the relevant sets and their attributes?". In this model, we have a single primitive set, the days of the week. If we call this set *DAYS*, we can begin by writing our sets section as:

```
SETS:
   DAYS;
ENDSETS
```

Next, we can add a data section to initialize the set members of the *DAYS* set:

```
SETS:
   DAYS;
ENDSETS
DATA:
   DAYS = MON TUE WED THU FRI SAT SUN;
ENDDATA
```

Alternatively, we could use LINGO's implicit set definition capability and express this equivalently as:

```
SETS:
   DAYS;
ENDSETS
DATA:
   DAYS = MON..SUN;
ENDDATA
```

We will be concerned with two attributes of the *DAYS* set. The first is the number of staff required on each day, and the second is the number of staff to start on each day. If we call these attributes *REQUIRED* and *START*, then we may add them to the sets section to get:

```
SETS:
   DAYS: REQUIRED, START;
ENDSETS
```

After defining the sets and attributes, it is useful to determine which of the attributes are data, and which are decision variables. In this model, the *REQUIRED* attribute is given to us and is, therefore, data. The *START* attribute is something we need to determine and constitutes the decision variables. Once you've identified the data in the model, you may go ahead and initialize it. We can do this by extending the data section as follows:

```
DATA:
   DAYS =     MON TUE WED THU FRI SAT SUN;
   REQUIRED =  20  16  13  16  19  14  12;
ENDDATA
```

We are now at the point where we can begin entering the model's mathematical relations (i.e., the objective and constraints). Let's begin by writing out the mathematical notation for the objective. Our objective is to minimize the total number of employees we start during the week. Using standard mathematical notation, this objective may be expressed as:

Minimize: $\sum_i START_i$

The equivalent LINGO statement is very similar. Substitute "MIN=" for "Minimize:" and "@SUM(DAYS(I):" for $\sum_i$ and we have:

```
MIN = @SUM( DAYS( I): START( I));
```

Now, all that is left is to come up with our constraints. There is only one set of constraints in this model. Namely, we must have enough staff on duty each day to meet or exceed staffing requirements. In words, what we want is:

Staff on duty today $\geq$ *Staff required today, for each day of the week*

The right-hand side of this expression, *Staff required today,* is easy to calculate. It is simply the quantity *REQUIRED(I)*. The left-hand side, *Staff on duty today*, is a bit trickier to compute. Given that all employees are on a "five day on, two day off" schedule, the number of employees working today is:

Number working today = Number starting today +
Number starting 1 day ago + Number starting 2 days ago +
Number starting 3 days ago + Number starting 4 days ago.

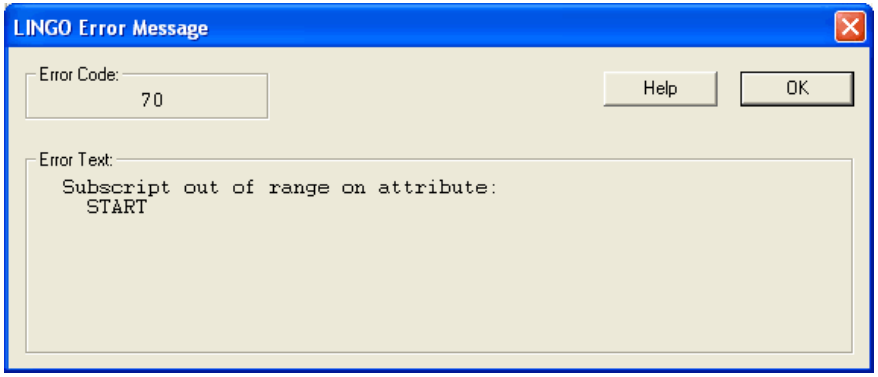
In other words, to compute the number of employees working today, we sum up the number of people starting today plus those starting over the previous four days. The number of employees starting five and six days back don't count because they are on their days off. So, using mathematical notation, what one might consider doing is adding the constraint:

$$\Sigma_{i = j-4, j}\, START_i \geq REQUIRED_j\,,\ for\ j \in DAYS$$

Translating into LINGO notation, we can write this as:

```
@FOR( DAYS( J):
   @SUM( DAYS( I) | I #LE# 5: START( J - I + 1))
      >= REQUIRED( J)
);
```

In words, the LINGO statement says, for each day of the week, the sum of the employees starting over the five day period beginning four days ago and ending today must be greater-than-or-equal-to the required number of staff for the day. This sounds correct, but there is a slight problem. If we try to solve our model with this constraint we get the error message:



To see why we get this error message, consider what happens on Thursday. Thursday has an index of 4 in our set *DAYS*. As written, the staffing constraint for Thursday will be:

```
START( 4 - 1 + 1) + START( 4 - 2 + 1) +
 START( 4 - 3 + 1) + START( 4 - 4 + 1) +
  START( 4 - 5 + 1) >= REQUIRED( 4);
```

Simplifying, we get:

```
START( 4) + START( 3) +
 START( 2) + START( 1) +
  START( 0) >= REQUIRED( 4);
```

The *START(0)* term is the root of our problem. *START* is defined for days 1 through 7. *START(0)* does not exist. An index of 0 on *START* is considered "out of range."

We would like to have any indices less-than-or-equal-to 0 *wrap around* to the end of the week. Specifically, 0 would correspond to Sunday (7), -1 to Saturday (6), and so on. LINGO has a function that does just this: *@WRAP*.

The *@WRAP* function takes two arguments—call them *INDEX* and *LIMIT*.
Formally, *J = @WRAP(INDEX, LIMIT)* returns *J* such that *J = INDEX - K * LIMIT*, where *K* is an integer such that 1 <= *J* < *LIMIT*+1. Informally, *@WRAP* will subtract or add *LIMIT* to *INDEX* until it falls in the range 1 to *LIMIT + 0.99999*. Therefore, this is just what we need to "wrap around" an index in multiperiod planning models.

Incorporating the *@WRAP* function, we get the corrected, final version of our staffing constraint:

```
@FOR( DAYS( J):
   @SUM( DAYS( I) | I #LE# 5:
      START( @WRAP( J - I + 1, 7)))
         >= REQUIRED( J)
);
```

The Solution

Below is our staffing model in its entirety:

```
MODEL:
SETS:
   DAYS: REQUIRED, START;
ENDSETS

DATA:
   DAYS =     MON TUE WED THU FRI SAT SUN;
   REQUIRED =  20  16  13  16  19  14  12;
ENDDATA

MIN = @SUM( DAYS( I): START( I));

@FOR( DAYS( J):
   @SUM( DAYS( I) | I #LE# 5:
      START( @WRAP( J - I + 1, 7)))
         >= REQUIRED( J)
);
END
```

Model: STAFFDEM

Solving the model, we get the solution report:

```
Global optimal solution found.
Objective value:                              22.00000
Infeasibilities:                              0.000000
Total solver iterations:                             5


     Variable           Value        Reduced Cost
REQUIRED( MON)        20.00000            0.000000
REQUIRED( TUE)        16.00000            0.000000
REQUIRED( WED)        13.00000            0.000000
REQUIRED( THU)        16.00000            0.000000
REQUIRED( FRI)        19.00000            0.000000
REQUIRED( SAT)        14.00000            0.000000
REQUIRED( SUN)        12.00000            0.000000
   START( MON)        8.000000            0.000000
   START( TUE)        2.000000            0.000000
   START( WED)        0.000000            0.000000
   START( THU)        6.000000            0.000000
   START( FRI)        3.000000            0.000000
   START( SAT)        3.000000            0.000000
   START( SUN)        0.000000           0.3333333

           Row    Slack or Surplus      Dual Price
             1        22.00000          -1.000000
             2        0.000000         -0.3333333
             3        0.000000           0.000000
             4        0.000000         -0.3333333
             5        0.000000           0.000000
             6        0.000000         -0.3333333
             7        0.000000         -0.3333333
             8        0.000000           0.000000
```

Solution to STAFFDEM

The objective value of 22 means we need to hire 22 workers.

We start our workers according to the schedule:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Start	8	2	0	6	3	3	0

If we look at the surpluses on our staffing requirement rows (rows 2 - 7), we see that the slack values are 0 on all of the days. This means there are no more workers than required and we just meet staffing requirements on every day. Even though this is a small model, trying to come up with a solution this efficient "by hand" would be a difficult task.

Dense Derived Set Example - Blending

This following model illustrates, among other things, the use of a dense derived set in a *blending* model. In a blending model, one is blending raw materials into a finished product that must meet minimal quality requirements on one or more dimensions. The goal is to come up with a blend of the raw materials to satisfy the quality requirements at minimal cost.

This model may be found in the *SAMPLES* subdirectory off the main LINGO directory under the name *CHESS*.

The Problem

The Chess Snackfoods Co. markets four brands of mixed nuts. The four brands of nuts are called Pawn, Knight, Bishop, and King. Each brand contains a specified ratio of peanuts and cashews. The table below lists the number of ounces of the two nuts contained in each pound of each brand and the price the company receives per pound of each brand.

	Pawn	Knight	Bishop	King
Peanuts (oz.)	15	10	6	2
Cashews (oz.)	1	6	10	14
Selling Price ($)	2	3	4	5

Chess has contracts with suppliers to receive 750 pounds of peanuts/day and 250 pounds of cashews/day. Our problem is to determine the number of pounds of each brand to produce each day to maximize total revenue without exceeding the available supply of nuts.

The Formulation

The primitive sets in this model are the nut types and the brands of mixed nuts. We can declare them in the sets section as follows:

```
SETS:
  NUTS: SUPPLY;
  BRANDS: PRICE, PRODUCE;
ENDSETS
```

The *NUTS* set has the single attribute *SUPPLY*, which we will use to store the daily supply of nuts in pounds. The *BRANDS* set has *PRICE* and *PRODUCE* attributes, where *PRICE* stores the selling price of the brands, and *PRODUCE* represents the decision variables of how many pounds of each brand to produce each day.

We need one more set, however, which is the dense derived set we have been promising. In order to input the brand formulas, we will need a two dimensional table defined on the nut types and the brands. To do this, we will generate a derived set named *NCROSSB* from the cross of the *NUTS* and *BRANDS* sets. Adding this derived set, we get the completed sets section:

```
SETS:
  NUTS: SUPPLY;
  BRANDS: PRICE, PRODUCE;
  NCROSSB( NUTS, BRANDS): FORMULA;
ENDSETS
```

We have titled the derived set *NCROSSB*. It has the single attribute *FORMULA*, which will be used to store the ounces of nuts used per pound of each brand. Since we have not specified the members of this derived set, LINGO assumes we want the complete, dense set that includes all pairs of nuts and brands, for a total of eight (nut,brand) pairs.

Now that our sets are declared, we can move on to building the data section. We initialize our three sets, *NUTS, BRANDS* and *NCROSSB,* as well as the two data attributes *SUPPLY and PRICE* as follows:

```
DATA:
  NUTS,     SUPPLY =
   PEANUTS   750
   CASHEWS   250;

  BRANDS,   PRICE =
   PAWN      2
   KNIGHT    3
   BISHOP    4
   KING      5;

  FORMULA = 15 10  6  2
             1  6 10 14;
ENDDATA
```

With the sets and data established, we can begin to enter our objective function and constraints. The objective function of maximizing total revenue is straightforward. We can express this as:

```
MAX = @SUM( BRANDS( I): PRICE( I) * PRODUCE( I));
```

Our model has only one class of constraints: We can't use more nuts than we are supplied with on a daily basis. In words, we would like to ensure:

> *For each nut i, the number of pounds of nut i used must be less-than-or-equal-to the supply of nut i.*

We can express this in LINGO as:

```
@FOR( NUTS( I):
@SUM( BRANDS( J):
 FORMULA( I, J) * PRODUCE( J) / 16) <= SUPPLY( I)
);
```

We divide the sum on the left-hand side by 16 to convert from ounces to pounds.

The Solution

Our completed blending model is:

```
MODEL:
SETS:
  NUTS: SUPPLY;
  BRANDS: PRICE, PRODUCE;
  NCROSSB( NUTS, BRANDS): FORMULA;
ENDSETS

DATA:
  NUTS,     SUPPLY =
   PEANUTS   750
   CASHEWS   250;

  BRANDS,   PRICE =
   PAWN      2
   KNIGHT    3
   BISHOP    4
   KING      5;

  FORMULA = 15 10  6  2
             1  6 10 14;
ENDDATA

MAX = @SUM( BRANDS( I): PRICE( I) * PRODUCE( I));

@FOR( NUTS( I):
@SUM( BRANDS( J):
 FORMULA( I, J) * PRODUCE( J) / 16) <= SUPPLY( I)
);
END
```

Model: CHESS

An abbreviated solution report for the model follows:

```
Global optimal solution found.
Objective value:                    2692.308
Infeasibilities:                    0.000000
Total solver iterations:                   2

       Variable     Value    Reduced Cost
 PRODUCE( PAWN)  769.2308       0.0000000
PRODUCE( KNIGHT)  0.000000       0.1538461
PRODUCE( BISHOP)  0.000000       0.7692297E-01
 PRODUCE( KING)  230.7692       0.0000000

        Row  Slack or Surplus  Dual Price
         1       2692.308        1.000000
         2       0.000000        1.769231
         3       0.000000        5.461538
```

Solution to CHESS

Sparse Derived Set Example - Explicit List

In this example, we will introduce the use of a sparse derived set with an explicit listing. As you recall, when we use this technique to define a sparse set, we must explicitly list all members belonging to the set. This will usually be some small subset of the dense set resulting from the full Cartesian product of the parent sets.

For our example, we will set up a PERT (Project Evaluation and Review Technique) model to determine the *critical path* of tasks in a project involving the roll out of a new product. PERT is a simple, but powerful, technique developed in the 1950s to assist managers in tracking the progress of large projects. PERT is particularly useful in identifying the critical activities within a project, which, if delayed, will delay the project as a whole. These time critical activities are referred to as the critical path of a project. Having such insight into the dynamics of a project goes a long way in guaranteeing it won't get sidetracked and become delayed. In fact, PERT proved so successful, the Polaris project that it was first used on was completed 18 months ahead of schedule. PERT continues to be used successfully on a wide range of projects. For more information on PERT, and a related technique called CPM (Critical Path Method), please refer to Schrage (2005) or Winston (1995).

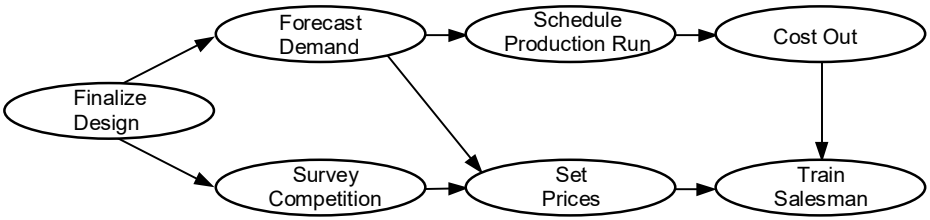
The formulation for this model is included in the *SAMPLES* subdirectory off the main LINGO directory under the name *PERT*.

A Project Scheduling Problem

Wireless Widgets is about to launch a new product: The Solar Widget. In order to guarantee the launch will occur on time, WW wants to perform a PERT analysis of the tasks leading up to the launch. Doing so will allow them to identify the critical path of tasks that must be completed on time in order to guarantee the Solar Widget's timely introduction. The tasks that must be accomplished before introduction and their anticipated times for completion are listed in the table below:

Task	Weeks
Finalize Design	10
Forecast Demand	14
Survey Competition	3
Set Prices	3
Schedule Production Run	7
Cost Out	4
Train Salesmen	10

Certain of the tasks must be completed before others can commence. These precedence relations are shown in the following graph:



Product Launch Precedence Relations

For instance, the two arrows originating from the Forecast Demand node indicate the task must be completed before the Schedule Production Run and the Set Prices tasks may be started.

Our goal is to construct a PERT model for the Solar Widget's introduction in order to identify the tasks on the critical path.

The Formulation

We will need a primitive set to represent the tasks of the project. We can add such a set to the model using the set definition:

```
SETS:
   TASKS: TIME, ES, LS, SLACK;
ENDSETS
```

We have associated four attributes with the *TASKS* set. The definitions of the attributes are:

TIME	Time to complete the task
ES	Earliest possible start time for the task
LS	Latest possible start time for the task
SLACK	Difference between LS and ES for the task

The *TIME* attribute is given to us as data. We will compute the values of the remaining three attributes. If a task has a 0 slack time, it means the task must start on time or the whole project will be delayed. The collection of tasks with 0 slack time constitute the critical path for the project.

In order to compute the start times for the tasks, we will need the precedence relations. The precedence relations can be viewed as a list of ordered pairs of tasks. For instance, the fact that the *DESIGN* task must be completed before the *FORECAST* task could be represented as the ordered pair (*DESIGN*, *FORECAST)*. Creating a two-dimensional derived set on the *TASKS* set will allow us to input the list of precedence relations. Specifically, we add the derived set definition *PRED*:

```
SETS:
   TASKS: TIME, ES, LS, SLACK;
   PRED( TASKS, TASKS);
ENDSETS
```

Next, we can input the *TASKS* set and task times in the data section by including:

```
DATA:
 TASKS,  TIME =
   DESIGN    10
   FORECAST  14
   SURVEY     3
   DUMMY      0
   PRICE      3
   SCHEDULE   7
   COSTOUT    4
   TRAIN     10
 ;
ENDDATA
```

The set *PRED* is the sparse derived set with an explicit listing that we want to highlight in this example. The set is a subset derived from the cross of the *TASKS* set upon itself. The set is sparse because it contains only 8 out of a possible 49 members found in the complete cross of *TASKS* on *TASKS*. The set is said to be an "explicit list" set, because we will explicitly list the members we want included in the set. Explicitly listing the members of a sparse set may not be convenient in cases where there are thousands of members to select from, but it does make sense whenever set membership conditions are not well defined and the sparse set size is small relative to the dense alternative. Adding the initialization of *PRED* to the data set give us:

```
DATA:
 TASKS,  TIME =
   DESIGN    10
   FORECAST  14
   SURVEY     3
   DUMMY      0
   PRICE      3
   SCHEDULE   7
   COSTOUT    4
   TRAIN     10
 ;
 PRED =
    DESIGN, FORECAST,
    DESIGN, SURVEY,
    FORECAST, DUMMY
    FORECAST, SCHEDULE,
    SURVEY, PRICE,
    SCHEDULE, COSTOUT,
    PRICE, TRAIN,
    COSTOUT, TRAIN,
    DUMMY, PRICE
 ;
ENDDATA
```

Keep in mind that the first member of this set is the ordered pair (*DESIGN*, *FORECAST*)—not just the single task *DESIGN*. Therefore, this set has a total of 9 members that all correspond to a directed arc in the precedence relations diagram. We added a dummy task to accommodate the fact that both *FORECAST* and *SURVEY* must precede *PRICE*. For more information on the use of dummy tasks, refer to: http://people.brunel.ac.uk/~mastjjb/jeb/or/netaoa.html.

Now, with our sets and data established, we can turn our attention to building the formulas of the model. We have three attributes to compute: earliest start (*ES*), latest start (*LS*), and slack time (*SLACK*). The trick is computing *ES* and *LS*. Once we have these times, *SLACK* is merely the difference of the two.

Lets start by coming up with a formula to compute *ES*. A task cannot begin until all its predecessor tasks are completed. Thus, if we find the latest finishing time of all predecessors to a task, then we have also found its earliest start time. Therefore, in words, the earliest start time for task *t* is equal to the maximum over all predecessors of task *t* of the sum of the earliest start time of the predecessor plus its completion time. The corresponding LINGO notation is:

```
@FOR( TASKS( J)| J #GT# 1:
 ES( J) = @MAX( PRED( I, J): ES( I) + TIME( I))
);
```

Note that we skip the computation for the first task by adding the conditional qualifier *J #GT# 1*. We do this because the first task has no predecessors. We will give the first task an arbitrary start time as shown below.

Computing *LS* is slightly trickier, but very similar to *ES*. In words, the latest time for task *t* to start is the minimum over all successor tasks of the sum of the successor's earliest start minus the time to perform task *t*. If task *t* starts any later than this, it will prohibit at least one successor from starting at its earliest start time. Converting into LINGO syntax gives:

```
@FOR( TASKS( I)| I #LT# LTASK:
 LS( I) = @MIN( PRED( I, J): ES( J) - TIME( I))
);
```

Here, we omit the computation for the last task since it has no successor tasks.

Computing slack time is just the difference between *LS* and *ES*, and may be written as:

```
@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));
```

We can set the start time of the first task to some arbitrary value. For our purposes, we will set it to 0 with the statement:

```
ES( 1) = 0;
```

We have now input formulas for computing the values of all the variables with the exception of the latest start time for the last task. It turns out, if the last project were started any later than its earliest start time, the entire project would be delayed. So, by definition, the latest start time for the last project is equal to its earliest start time. We can express this in LINGO using the equation:

```
LS( 8) = ES( 8);
```

This would work, but it's probably not the best way to express the relation. Suppose you were to add some tasks to your model. You'd have to change the 8 in this equation to the new number of tasks was. The whole idea behind LINGO's set-based modeling language is the equations in the model should be independent of the data. Expressing the equation in this form violates data independence. Here's a better way to do it:

```
LTASK = @SIZE( TASKS);
LS( LTASK) = ES( LTASK);
```

The *@SIZE* function returns the size of a set. In this case, it will return the value 8, as desired. However, if we changed the number of tasks, *@SIZE* would also return the new, correct value. Thus, we preserve the data independence of our model's equations.

The Solution

The entire *PERT* formulation and portions of its solution appear below.

```
MODEL:
SETS:
   TASKS: TIME, ES, LS, SLACK;
   PRED( TASKS, TASKS);
ENDSETS

DATA:
 TASKS,  TIME =
   DESIGN    10
   FORECAST  14
   SURVEY     3
   DUMMY      0
   PRICE      3
   SCHEDULE   7
   COSTOUT    4
   TRAIN     10
 ;
 PRED =
    DESIGN, FORECAST,
    DESIGN, SURVEY,
    FORECAST, DUMMY
    FORECAST, SCHEDULE,
    SURVEY, PRICE,
    SCHEDULE, COSTOUT,
    PRICE, TRAIN,
    COSTOUT, TRAIN,
    DUMMY, PRICE
 ;
ENDDATA

@FOR( TASKS( J)| J #GT# 1:
 ES( J) = @MAX( PRED( I, J): ES( I) + TIME( I))
);

@FOR( TASKS( I)| I #LT# LTASK:
 LS( I) = @MIN( PRED( I, J): ES( J) - TIME( I));
);

@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));

ES( 1) = 0;
LTASK = @SIZE( TASKS);
LS( LTASK) = ES( LTASK);

END
```

Model: PERT

```
Feasible solution found.
Total solver iterations:                  0

          Variable           Value
             LTASK        7.000000
       ES( DESIGN)       0.0000000
     ES( FORECAST)        10.00000
       ES( SURVEY)        10.00000
        ES( PRICE)        24.00000
     ES( SCHEDULE)        24.00000
      ES( COSTOUT)        31.00000
        ES( TRAIN)        35.00000
       LS( DESIGN)       0.0000000
     LS( FORECAST)        10.00000
       LS( SURVEY)        21.00000
        LS( PRICE)        32.00000
     LS( SCHEDULE)        24.00000
      LS( COSTOUT)        31.00000
        LS( TRAIN)        35.00000
    SLACK( DESIGN)       0.0000000
  SLACK( FORECAST)       0.0000000
    SLACK( SURVEY)        11.00000
     SLACK( PRICE)        8.000000
  SLACK( SCHEDULE)       0.0000000
   SLACK( COSTOUT)       0.0000000
     SLACK( TRAIN)       0.0000000
```

Solution to PERT

The interesting values are the slacks for the tasks. Both *SURVEY* and *PRICE* have slack in their start times of 11 weeks and 8 weeks, respectively. Their start times may be delayed by as much as these slack values without compromising the completion time of the entire project. The tasks *DESIGN*, *FORECAST*, *SCHEDULE*, *COSTOUT*, and *TRAIN*, on the other hand, have 0 slack times. These tasks constitute the critical path for the project and, if any of their start times are delayed, the entire project will be delayed. Management will want to pay close attention to these critical path projects to be sure they start on time and are completed within the allotted amount of time. Finally, the *ES(TRAIN)* value of 35 tells us the estimated time to the start of the roll out of the new Solar Widget will be 45 weeks _35 weeks to get to the start of training, plus 10 weeks to complete training.

A Sparse Derived Set Using a Membership Filter

In this example, we introduce the use of a sparse derived set with a membership filter. Using a membership filter is the third method for defining a derived set. When you define a set using this method, you specify a logical condition each member of the set must satisfy. LINGO then filters out potential set members that don't satisfy the membership condition.

For our example, we will formulate a *matching* problem. In a matching problem, there are *N* objects we want to match into pairs at minimum cost. The pair *(I,J)* is indistinguishable from the pair *(J,I)*. Therefore, we arbitrarily require *I* be less than *J* in the pair. Formally, we require *I* and *J* make a set of ordered pairs. In other words, we do not wish to generate redundant ordered pairs of *I* and *J,* but only those with *I* less than *J*. This requirement that *I* be less than *J* will form our membership filter.

The file containing this model may be found in the *SAMPLES* subdirectory off the main LINGO directory under the name *MATCHD*.

A Matching Problem

Suppose you manage your company's strategic planning department. You have a total of eight analysts in the department. Furthermore, your department is about to move into a new suite of offices. There are a total of four offices in the new suite and you need to match up your analysts into 4 pairs, so each pair can be assigned to one of the new offices. Based on past observations, you know some of the analysts work better together than they do with others. In the interest of departmental peace, you would like to come up with a pairing of analysts that results in minimal potential conflicts. To this goal, you have come up with a rating system for pairing your analysts. The scale runs from 1 to 10, with a 1 rating of a pair meaning the two get along fantastically. Whereas, a rating of 10 means all sharp objects should be removed from the pair's office in anticipation of mayhem. The ratings appear in the following table:

Analysts	1	2	3	4	5	6	7	8
1	-	9	3	4	2	1	5	6
2	-	-	1	7	3	5	2	1
3	-	-	-	4	4	2	9	2
4	-	-	-	-	1	5	5	2
5	-	-	-	-	-	8	7	6
6	-	-	-	-	-	-	2	3
7	-	-	-	-	-	-	-	4

Analysts' Incompatibility Ratings

Since the pairing of analyst *I* with analyst *J* is indistinguishable from the pairing of *J* with *I*, we have only included the above diagonal elements in the table. Our problem is to find the pairings of analysts that minimizes the sum of the incompatibility ratings of the paired analysts.

The Formulation

The first set of interest in this problem is the set of analysts. This is a primitive set that can be declared simply as:

```
SETS:
   ANALYSTS;
ENDSETS
```

The final set we want to construct is a set consisting of all the potential pairings. This will be a derived set that we will build by taking the cross of the *ANALYST* set on itself. As a first pass, we could build the dense derived set:

```
PAIRS( ANALYSTS, ANALYST);
```

This set, however, would include both *PAIRS(I, J)* and *PAIRS(J, I)*. Since only one of these pairs is required, the second is wasteful. Furthermore, this set will include "pairs" of the same analyst of the form *PAIRS(I, I)*. As much as each of the analysts might like an office of their own, such a solution is not feasible. The solution is to put a membership filter on our derived set requiring each pair *(I,J)* in the final set to obey the condition *J* be greater than *I*. We do this with the set definition:

```
PAIRS( ANALYSTS, ANALYSTS) | &2 #GT# &1;
```

The start of the membership filter is denoted with the vertical bar character (|). The *&1* and *&2* symbols in the filter are known as *set index placeholders*. Set index placeholders are valid only in membership filters. When LINGO constructs the *PAIRS* set, it generates all combinations in the cross of the *ANALYSTS* set on itself. Each combination is then "plugged" into the membership filter to see if it passes the test. Specifically, for each pair *(I,J)* in the cross of the *ANALYST* set on itself, *I* is substituted into the placeholder *&1* and *J* into *&2* and the filter is evaluated. If the filter evaluates to true, *(I,J)* is added to the pairs set. Viewed in tabular form, this leaves us with just the diagonal elements of the *(I,J)* pairing table.

We will also be concerned with two attributes of the *PAIRS* set. First, we will need an attribute that corresponds to the incompatibility rating of the pairings. Second, we will need an attribute to indicate if analyst *I* is paired with analyst *J*. We will call these attributes *RATING* and *MATCH*. We append them to the *PAIRS* set definition as follows:

```
PAIRS( ANALYSTS, ANALYSTS) | &2 #GT# &1:
 RATING, MATCH;
```

The completed sets section containing both set declarations is then:

```
SETS:
   ANALYSTS;
   PAIRS( ANALYSTS, ANALYSTS) | &2 #GT# &1:
    RATING, MATCH;
ENDSETS
```

Next, we initialize the *ANALYSTS* set and the *RATING* attribute to the incompatibility ratings in the data section:

```
DATA:
   ANALYSTS = 1..8;
   RATING =
      9  3  4  2  1  5  6
         1  7  3  5  2  1
            4  4  2  9  2
               1  5  5  2
                  8  7  6
                     2  3
                        4;
ENDDATA
```

We will use the convention of letting *MATCH(I, J)* be 1 if we pair analyst *I* with analyst *J*, otherwise 0. Given this, the *MATCH* attribute contains the decision variables for the model.

Our objective is to minimize the sum of the incompatibility ratings of all the final pairings. This is just the inner product on the *RATING* and *MATCH* attributes and is written as:

```
MIN = @SUM( PAIRS( I, J):
 RATING( I, J) * MATCH( I, J));
```

There is just one class of constraints in the model. In words, it is:

For each analyst, ensure that the analyst is paired with exactly one other analyst.

Putting the constraint into LINGO syntax, we get:

```
@FOR( ANALYSTS( I):
 @SUM( PAIRS( J, K) | J #EQ# I #OR# K #EQ# I:
  MATCH( J, K)) = 1
);
```

The feature of interest in this constraint is the conditional qualifier (*J #EQ# I #OR# K #EQ# I*) on the *@SUM* function. For each analyst *I*, we sum up all the *MATCH* variables that contain *I* and set them equal to 1. In so doing, we guarantee analyst *I* will be paired up with exactly one other analyst. The conditional qualifier guarantees we only sum up the *MATCH* variables that include *I* in its pairing.

One other feature is required in this model. We are letting *MATCH(I, J)* be 1 if we are pairing *I* with *J*. Otherwise, it will be 0. Unless specified otherwise, LINGO variables can assume any value from 0 to infinity. Because we want *MATCH* to be restricted to being only 0 or 1, we need to apply the *@BIN variable domain function* to the *MATCH* attribute. Variable domain functions are used to restrict the values a variable can assume. Unlike constraints, variable domain functions do not add equations to a model. The *@BIN* function restricts a variable to being *binary* (i.e., 0 or 1). When you have a model that contains binary variables, it is said to be an *integer programming* (IP) model. IP models are much more difficult to solve than models that contain only continuous variables. Carelessly formulated large IPs (with several hundred integer variables or more) can literally take *forever* to solve! Thus, you should limit the use of binary variables whenever possible. To apply *@BIN* to all the variables in the *MATCH* attribute, add the *@FOR* expression:

```
@FOR( PAIRS( I, J): @BIN( MATCH( I, J)));
```

The Solution

The entire formulation for our matching example and parts of its solution appear below.

```
MODEL:

SETS:
   ANALYSTS;
   PAIRS( ANALYSTS, ANALYSTS) | &2 #GT# &1:
    RATING, MATCH;
ENDSETS

DATA:
   ANALYSTS = 1..8;
   RATING =
      9  3  4  2  1  5  6
         1  7  3  5  2  1
            4  4  2  9  2
               1  5  5  2
                  8  7  6
                     2  3
                        4;
ENDDATA

MIN = @SUM( PAIRS( I, J):
 RATING( I, J) * MATCH( I, J));

@FOR( ANALYSTS( I):
 @SUM( PAIRS( J, K) | J #EQ# I #OR# K #EQ# I:
  MATCH( J, K)) = 1
);

@FOR( PAIRS( I, J): @BIN( MATCH( I, J)));

END
```

Model: MATCHD

```
Global optimal solution found.
Objective value:                              6.000000
Extended solver steps:                               0
Total solver iterations:                             0

    Variable           Value        Reduced Cost
MATCH( 1, 2)       0.0000000            9.000000
MATCH( 1, 3)       0.0000000            3.000000
MATCH( 1, 4)       0.0000000            4.000000
MATCH( 1, 5)       0.0000000            2.000000
MATCH( 1, 6)        1.000000            1.000000
MATCH( 1, 7)       0.0000000            5.000000
MATCH( 1, 8)       0.0000000            6.000000
MATCH( 2, 3)       0.0000000            1.000000
MATCH( 2, 4)       0.0000000            7.000000
MATCH( 2, 5)       0.0000000            3.000000
MATCH( 2, 6)       0.0000000            5.000000
MATCH( 2, 7)        1.000000            2.000000
MATCH( 2, 8)       0.0000000            1.000000
MATCH( 3, 4)       0.0000000            4.000000
MATCH( 3, 5)       0.0000000            4.000000
MATCH( 3, 6)       0.0000000            2.000000
MATCH( 3, 7)       0.0000000            9.000000
MATCH( 3, 8)        1.000000            2.000000
MATCH( 4, 5)        1.000000            1.000000
MATCH( 4, 6)       0.0000000            5.000000
MATCH( 4, 7)       0.0000000            5.000000
MATCH( 4, 8)       0.0000000            2.000000
MATCH( 5, 6)       0.0000000            8.000000
MATCH( 5, 7)       0.0000000            7.000000
MATCH( 5, 8)       0.0000000            6.000000
MATCH( 6, 7)       0.0000000            2.000000
MATCH( 6, 8)       0.0000000            3.000000
MATCH( 7, 8)       0.0000000            4.000000
```

Solution to MATCHD

From the objective value, we know the total sum of the incompatibility ratings for the optimal pairings is 6. Scanning the *Value* column for 1s, we find the optimal pairings: (1,6), (2,7), (3,8), and (4,5).

Summary

In this chapter, we've discussed the concept of sets, how to declare sets, and demonstrated the power and flexibility of set-based modeling. You should now have a foundation of knowledge in the definition and use of both primitive and derived sets. The next chapter will discuss the use of variable domain functions, which were briefly introduced in this chapter when we used *@BIN* in the previous matching model.

3 Using Variable Domain Functions

Unless specified otherwise, variables in a LINGO model default to being non-negative and continuous. More specifically, variables can assume any real value from zero to positive infinity. In many cases, this default domain for a variable may be inappropriate. For instance, you may want a variable to assume negative values, or you might want a variable restricted to purely integer values. LINGO provides nine *variable domain functions*, which allow you to override the default domain of a variable, and, in the case of *@POSD*, the domain of a matrix of variables. The names of these functions and a brief description of their usage are:

@GIN	restricts a variable to being an integer value
@BIN	makes a variable binary (i.e., 0 or 1)
@FREE	allows a variable to assume any real value, positive *or* negative
@BND	limits a variable to fall within a finite range
@SOS	defines a set of binary variables and places restrictions on their collective values
@CARD	defines a set of binary variables and places an upper limit on their sum
@SEMIC	restricts variables to being either zero or greater-than a specified constant
@PRIORITY	used to assign branching priorities to variables.
@POSD	restricts a square matrix to being symmetric matrix and positive semi-definite

In the remainder of this section, we'll investigate the mechanics of using these functions, and present a number of examples illustrating their usage.

Integer Variables

LINGO gives the user the ability to define two types of integer variables—*general* and *binary*. A general integer variable is required to be a whole number. A binary integer variable is further required to be either zero or one. Any model containing one or more integer variables is referred to as an *integer programming* (IP) model.

In many modeling projects, you will be faced with Yes/No types of decisions. Some examples would include Produce/Don't Produce, Open Plant/Close Plant, Supply Customer *I* from Plant *J*/Don't Supply Customer *I* from Plant *J*, and Incur a Fixed Cost/Don't Incur a Fixed Cost. Binary variables are the standard method used for modeling these Yes/No decisions.

General integer variables are useful where rounding of fractional solutions is problematic. For instance, suppose you have a model that dictates producing 5,121,787.5 blue crayons in your crayon factory. Whether you round the solution to 5,121,787 or 5,121,788 is inconsequential. On the other hand, suppose your planning model for NASA determines the optimal number of space stations to deploy is 1.5. Because building 0.5 space stations is impossible, you must *very* carefully consider how to round the results. When whole numbers are required and rounding can make a significant difference, general integer variables are appropriate.

LINGO does not simply round or truncate values to come up with an integer answer. Rounding of a solution will typically lead to either infeasible or suboptimal solutions. To illustrate this point, consider the small model:

```
MAX = X;
X + Y = 25.5;
X <= Y;
```

By examining this model, one can deduce the optimal solution is X=Y=12.75. Now, suppose we want an optimal solution with X being integer. Simply rounding X to 13 would make the model infeasible, because there would be no value for Y that would satisfy both the constraints. Clearly, the optimal solution is X=12 and Y=13.5. Unfortunately, "eyeballing" the optimal solution on larger models with many integer variables is virtually impossible.

To solve these problems, LINGO performs a complex algorithm called *branch-and-bound* that implicitly enumerates all combinations of the integer variables to determine the best feasible answer to an IP model. Because of the extra computation time required by this algorithm, formulating your problem to avoid the use of integer variables is advised whenever possible. Even so, although computation times may grow dramatically when you add integer variables, it often makes sense to ask LINGO for integer solutions when fractional values are of little or no use.

General Integer Variables

By default, LINGO assumes all variables in a model are continuous. In many applications, fractional values may be undesirable. You won't be able to hire two-thirds of a person, or sell half an automobile. In these instances, you will want to make use of the general integer variable domain function, *@GIN*.

The syntax of the *@GIN* function is:

@GIN(*variable_name*);

where *variable_name* is the name of the variable you wish to make general integer. The *@GIN* function may be used in a model anywhere you would normally enter a constraint. The *@GIN* function can be embedded in an *@FOR* statement to allow you to easily set all, or selected, variables of an attribute to be general integers. Some examples of *@GIN* are:

Example 1: `@GIN(X);`
makes the scalar variable *X* general integer,

Example 2: `@GIN(PRODUCE(5));`
makes the variable *PRODUCE(5)* general integer,

Example 3: `@FOR(DAYS(I): @GIN(START(I)));`
makes all the variables of the *START* attribute general integer.

General Integer Example - CompuQuick Product Mix

To illustrate the use of *@GIN* in a full model, we will consider a variation on the CompuQuick Corporation model in *Getting Started with LINGO*. CompuQuick has successfully rebalanced the Standard computer's assembly line. In so doing, they are now able to build an additional 3 Standard computers on the line each day, for a daily total of 103 computers. As a result, the constraint on the Standard's assembly line will now be:

```
STANDARD <= 103;
```

Incorporating this constraint into the original CompuQuick model, we have:

```
! Here is the total profit objective function;
MAX = 100 * STANDARD + 150 * TURBO;

! Constraints on the production line capacity;
STANDARD <= 103;
TURBO <= 120;

! Our labor supply is limited;
STANDARD + 2 * TURBO <= 160;
```

Solving this modified model, we get the solution:

```
Global optimal solution found.
Objective value:                          14575.00
Infeasibilities:                          0.000000
Total solver iterations:                         0
Elapsed runtime seconds:                      0.03

Model Class:                                    LP

Total variables:                       2
Nonlinear variables:                   0
Integer variables:                     0

Total constraints:                     4
Nonlinear constraints:                 0

Total nonzeros:                        6
Nonlinear nonzeros:                    0

   Variable           Value        Reduced Cost
   STANDARD        103.0000            0.000000
      TURBO        28.50000            0.000000

        Row    Slack or Surplus      Dual Price
          1        14575.00            1.000000
          2        0.000000            25.00000
          3        91.50000            0.000000
          4        0.000000            75.00000
```

Note that the new optimal number of Turbo computers, 28.5, is no longer an integer quantity. CompuQuick must produce whole numbers of computers each day. To guarantee this, we add *@GIN* statements to make both the *STANDARD* and *TURBO* variables general integer. The revised model follows:

```
! Here is the total profit objective function;
MAX = 100 * STANDARD + 150 * TURBO;

! Constraints on the production line capacity;
STANDARD <= 103;
TURBO <= 120;

! Our labor supply is limited;
STANDARD + 2 * TURBO <= 160;

! Integer values only;
@GIN( STANDARD); @GIN( TURBO);
```

Solving the modified model results in the integer solution we were hoping for:

```
Global optimal solution found.
Objective value:                          14550.00
Objective bound:                          14550.00
Infeasibilities:                          0.000000
Extended solver steps:                           0
Total solver iterations:                         0
Elapsed runtime seconds:                      0.06

Model Class:                                  PILP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    2

Total constraints:                    4
Nonlinear constraints:                0

Total nonzeros:                       6
Nonlinear nonzeros:                   0

   Variable           Value        Reduced Cost
   STANDARD        102.0000           -100.0000
      TURBO        29.00000           -150.0000

        Row    Slack or Surplus      Dual Price
          1        14550.00            1.000000
          2        1.000000            0.000000
          3        91.00000            0.000000
          4        0.000000            0.000000
```

Note that we now have a two new solution statistics: *Extended solver steps and Objective bound*.

For models with integer variables, such as this one, the extended solver steps statistic is a tally of the number of times integer variables had to be forced to an integer value during the branch-and-bound solution procedure. In general, this value is not of much practical use to the normal user, other than to give you a notion of how hard LINGO is working at finding an integer solution. If the number of steps gets quite large, LINGO is having a hard time finding good integer solutions to your model. In this case, given that the model is quite small, LINGO's preprocessor was able to find the optimal solution without having to resort to the branch-and-bound solver.

The objective bound statistic gives us a bound on the best possible solution for the model. In other words, there is no feasible solution to the model with an objective value better than the objective bound statistic. Here we see that the bound and the objective value are both equal, which is further evidence that we have a globally optimal solution. On larger models that take a while to run, you may decide to interrupt LINGO before a global solution is found. In this case, the objective bound and the objective value will probably not agree. In this case, the bound will let you know how far you are from the true optimal solution. You may also find it useful to interrupt the solver once the bound and

objective value get close to one another, with the idea being that any further potential gains in the best objective aren't worth the additional solve time.

Also of interest is the Model Class, which has changed from LP (Linear Program) to PILP (Pure Integer Linear Program).

General Integer Example - Staff-Scheduling

Recalling the staff-scheduling example in Chapter 2, *Using Sets*, for the Pluto hot dog stand, you will remember the solution told us how many employees to start on any given day of the week. You may also remember the optimal solution had us starting whole numbers of employees on every day even though we weren't using integer variables. It turns out this was just a happy coincidence. Let's return to the staffing model to demonstrate this.

In the original staffing model, we required the following number of people on duty for the seven days of the week: 20, 16, 13, 16, 19, 14, and 12. Let's change the second day requirement from 16 to 12 and the third day's requirement from 13 to 18. Incorporating this change into the model, we have:

```
MODEL:
SETS:
   DAYS: REQUIRED, START;
ENDSETS

DATA:
   DAYS =     MON TUE WED THU FRI SAT SUN;
   REQUIRED =  20  12  18  16  19  14  12;
ENDDATA

MIN = @SUM( DAYS( I): START( I));

@FOR( DAYS( J):
   @SUM( DAYS( I) | I #LE# 5:
      START( @WRAP( J - I + 1, 7)))
         >= REQUIRED( J)
);
END
```

After making this modest change and re-solving, we no longer have a pure integer solution. In fact, all the *START* variables are now fractional as the following, abbreviated solution report shows:

```
Global optimal solution found.
Objective value:                    23.66667
Total solver iterations:                   0

    Variable           Value        Reduced Cost
 START( MON)        9.666667            0.000000
 START( TUE)        2.000000            0.000000
 START( WED)        1.666667            0.000000
 START( THU)        5.666667            0.000000
 START( FRI)        0.000000            0.000000
 START( SAT)        4.666667            0.000000
 START( SUN)        0.000000            0.333333
```

In this particular model, we can always round the solution up and remain feasible. (In most models, we won't tend to be as lucky. Rounding the continuous solution in one direction or the other can lead to an infeasible solution.) There may be some extra staff on some of the days, but, by rounding up, we will never have a day without enough staff. Rounding the continuous solution up gives an objective of 10+2+2+6+5=25 employees.

Now, let's apply integer programming to the revised staffing model. First, we will need to use the *@GIN* function to make the *START* variables general integers. We could do this by adding the following to our model:

```
@GIN( START( MON));
@GIN( START( TUE));
@GIN( START( WED));
@GIN( START( THU));
@GIN( START( FRI));
@GIN( START( SAT));
@GIN( START( SUN));
```

However, an easier approach would be to embed the *@GIN* function in an *@FOR* function, so we can apply *@GIN* to each member of *START* using the single statement:

```
@FOR( DAYS( I): @GIN( START( I)));
```

This new statement says, for each day of the week, make the variable corresponding to the number of people to start on that day a general integer variable.

After inserting this *@FOR* statement at the end of our model and reoptimizing, we get the pure integer solution:

```
Global optimal solution found.
Objective value:                              24.00000
Extended solver steps:                               0
Total solver iterations:                             6


      Variable           Value        Reduced Cost
   START( MON)        10.00000            1.000000
   START( TUE)        2.000000            1.000000
   START( WED)        1.000000            1.000000
   START( THU)        6.000000            1.000000
   START( FRI)        0.000000            1.000000
   START( SAT)        5.000000            1.000000
   START( SUN)        0.000000            1.000000
```

Note that the objective of 24 beats the objective of 25 obtained by rounding. Thus, had we gone with the rounded solution, we would have hired one more employee than required.

Alldiff - All Different General Integer Variables

A common situation in integer programming is to have a set of general integer variables that are all required to take on different values. Examples would include the sequence of cities visited in a traveling salesman problem, or the 9 values in a square of a Sudoku puzzle.

The syntax of the *@ALLDIFF* function is:

@ALLDIFF('set_name', *variable_name*);

where 'set_name' is the name assigned to this set of general integer variables, and *variable_name* is the name of a variable belonging to the set. Note that the *@ALLDIFF* function can be used inside an *@FOR* looping function to place multiple variables into the same set. Some examples of *@ALLDIFF* are:

Example 1: `@ALLDIFF( 'SET1', X); @ALLDIFF( 'SET1', Y);`
makes the scalar variables *X* and *Y* members of the all-different set called 'SET1',

Example 2: `@FOR( CITIES( I): @ALLDIFF( 'SEQSET', Z( I)));`
assigns all variables of attribute *Z* to the all-different set 'SEQSET'.

By *all-different*, we mean that the variables in the set will all take on different integer values ranging from 1 to *N*, when *N* is the number of variables in the set.

Note: All-different variables must be general integer variables. However, you do not have to explicitly declare them as such, because LINGO will automatically mark them as general integer.

For an interesting application of *@ALLDIFF*, you can refer to the *SUDOKUALLDIFF.LG4* model in the LINGO *Samples* folder. This model solves the well-know Sudoku puzzle, where one fills a 9x9 grid with the digits 1,2,...9, so that each digit appears once in:

a) each column,
b) each row,
c) each of the nine 3x3 subsquares,
d) the main diagonal,
e) the reflected diagonal.

Some versions of the puzzle do not require (d) and (e).

Looking at the sample model *SUDOKUALLDIFF,* we find the following constraints that enforce properties a) through c) above:

```
! Each entry is in the interval [1, 9];
@for( sxs(i,j):
 @bnd( 1, y(i,j), 9);
   );

! For each row i, the entries must be all different integers;
@for( side( i):
  @for( side(j):
     @alldiff( 'row'+side(i), y(i,j));
      );
    );

! For each col j, the entries must be all different;
@for( side( j):
  @for( side(i):
     @alldiff( 'col'+side(j), y(i,j));
      );
    );

! For each subblock, the entries must be all different;
@for( sxs(i1,j1) | @mod(i1,3) #eq# 1 #and# @mod(j1,3) #eq# 1:
  @for( sxs(i,j) | i1 #le# i #and# i #le# i1+2
    #and# j1 #le# j #and# j #le# j1+2:
     @alldiff( 'blk'+side(i1)+side(j1), y(i,j));
      );
    );
```

Model: SUDUKOALLDIFF

Note that *Y(i,j)* represents the value stored in row *i* and column *j* of the puzzle.

An interesting feature of the calls to the *@ALLDIFF* function in this model are the references to the set *SIDE,* where *SIDE* is used to represent the 9 rows and 9 columns of the puzzle. Using set names allows us to construct unique names for the different sets of all-different variables. Generating the scalar version of this model, we find find the following *@ALLDIFF* references to enforce the all-different condition for row 1 of the puzzle:

```
@ALLDIFF( 'ROW1', Y_1_1); @ALLDIFF( 'ROW1', Y_1_2);
@ALLDIFF( 'ROW1', Y_1_3); @ALLDIFF( 'ROW1', Y_1_4);
@ALLDIFF( 'ROW1', Y_1_5); @ALLDIFF( 'ROW1', Y_1_6);
@ALLDIFF( 'ROW1', Y_1_7); @ALLDIFF( 'ROW1', Y_1_8);
@ALLDIFF( 'ROW1', Y_1_9);
```

In this case, we concatenated the set name of the first row (i.e., "1") to the word "ROW" to construct the all-different set name for the first row of the puzzle: "ROW1".

Solving this models gives us the following solution to the puzzle:

```
Sudoku Puzzle Solution:
.....................................
.  3  5  4  .  1  6  2  .  7  8  9  .
.  2  9  1  .  7  8  3  .  6  5  4  .
.  7  6  8  .  9  4  5  .  2  3  1  .
.....................................
.  6  1  2  .  8  3  9  .  4  7  5  .
.  8  4  5  .  6  7  1  .  9  2  3  .
.  9  3  7  .  2  5  4  .  1  6  8  .
.....................................
.  1  7  6  .  3  9  8  .  5  4  2  .
.  5  2  3  .  4  1  7  .  8  9  6  .
.  4  8  9  .  5  2  6  .  3  1  7  .
.....................................
```

Specifying Bounds on Variables on an Alldiff Set

You will note from above that the default values of the variables in an Alldiff set with *N* members will be contained in the set of positive integers {1..*N*}, with each variable having a unique value. For most applications this should be sufficient. However, there may be times when you wish to override these bounds of 1 to *N*, so that the variables can take on a different set of values. In the model fragment below, we create an Alldiff set called *MYALLDIFF* that contains the three variables *X, Y* and *Z*.

```
!First call passed the lower and upper bounds on
the Alldiff variables;
  @ALLDIFF( 'MYALLDIFF', 10, 20);

!Add X, Y amd Z variables to the set;
  @ALLDIFF( 'MYALLDIFF', X);
  @ALLDIFF( 'MYALLDIFF', Y);
  @ALLDIFF( 'MYALLDIFF', Z);
```

Note the additional *@ALLDIFF* call at the top to explicitly specify the variable bounds: *@ALLDIFF*('MYALLDIFF', 10, 20). In this case, instead of the variable values running from 1 to 3, they will now values contained in the interval of 10 to 20. Subsequent *@ALLDIFF* calls are as before, with two arguments consisting of the set name and a new variable name to add to the set.

Binary Integer Variables

A *binary* integer variable—also called a 0/1 variable—is a special case of an integer variable that is required to be either zero or one. It's often used as a switch to model Yes/No decisions.

The syntax of the *@BIN* function is:

@BIN(*variable_name*);

where *variable_name* is the name of the variable you wish to make binary. The *@BIN* function may be used in a model anywhere you would normally enter a constraint. The *@BIN* function can be embedded in an *@FOR* statement to allow you to easily set all, or selected, variables of an attribute to be binary integers. Some examples of *@BIN* are:

Example 1: `@BIN( X);`
makes the scalar variable, *X*, a binary integer,

Example 2: `@BIN( INCLUDE( 4));`
makes the variable *INCLUDE(4)* binary,

***Example 3*:** `@FOR( ITEMS: @BIN( INCLUDE));`
makes all variables in the *INCLUDE* attribute binary.

Binary Integer Example - The Knapsack Problem

The *knapsack* model is a classic problem that uses binary variables. In this problem, you have a group of items you want to pack into your knapsack. Unfortunately, the capacity of the knapsack is limited such that it is impossible to include all items. Each item has a certain value, or utility, associated with including it in the knapsack. The problem is to find the subset of items to include in the knapsack that maximizes the total value of the load without exceeding the capacity of the knapsack.

Of course, the knapsack euphemism shouldn't lead one to underestimate the importance of this class of problem. The "knapsack" problem can be applied to many situations. Some examples are vehicle loading, capital budgeting, and strategic planning.

The Problem

As an example, suppose you are planning a picnic. You've constructed a list of items you would like to carry with you on the picnic. Each item has a weight associated with it and your knapsack is limited to carrying no more than 15 pounds. You have also come up with a 1 to 10 rating for each item, which indicates how strongly you want to include the particular item in the knapsack for the picnic. This information is listed below:

Item	Weight	Rating
Ant Repellent	1	2
Beer	3	9
Blanket	4	3
Bratwurst	3	8
Brownies	3	10
Frisbee	1	6
Salad	5	4
Watermelon	10	10

The Formulation

We have only one set in this model—the set of items we are considering carrying in the knapsack. This is a primitive set, and we can define it in the sets section:

```
SETS:
  ITEMS: INCLUDE, WEIGHT, RATING;
ENDSETS
```

We have associated the three attributes *INCLUDE*, *WEIGHT*, and *RATING* with the set. *INCLUDE* will be the binary variables used to indicate if an item is to be included in the knapsack. *WEIGHT* is used to store the weight of each item, and *RATING* is used to store each item's rating.

Next, we will need to construct a data section to input the set members of set *ITEMS* and their associated weights and ratings. Here is a data section that accomplishes the task:

```
DATA:
 ITEMS          WEIGHT RATING =
   ANT_REPEL        1      2
   BEER             3      9
   BLANKET          4      3
   BRATWURST        3      8
   BROWNIES         3     10
   FRISBEE          1      6
   SALAD            5      4
   WATERMELON      10     10;

   KNAPSACK_CAPACITY = 15;
ENDDATA
```

Note that we have also included the knapsack's capacity in the data section. This is a good practice in that it isolates data from the constraints of the model.

Given that all the sets and data have been defined, we can turn to building our objective function. We want to maximize the sum of the ratings of the items included in our knapsack. Note that *INCLUDE(I)* will be 1 if item *I* is included. Otherwise, it will be 0. Therefore, if we take the inner product of *INCLUDE* with the *RATING* attribute, we will get the overall rating of a combination of included items. Putting this into LINGO syntax, we have:

```
MAX = @SUM( ITEMS: RATING * INCLUDE);
```

Note that we did not specify a set index variable in the *@SUM* function. Since all the attributes in the function (*RATING* and *INCLUDE*) are defined on the index set (*ITEMS*), we can drop the set index variable and use implicit indexing.

Our next step is to input our constraints. There is only one constraint in this model. Specifically, we must not exceed the capacity of the knapsack. In a similar manner as the objective, we compute the weight of a given combination of items by taking the inner product of the *INCLUDE* attribute with the *WEIGHT* attribute. This sum must be less-than-or-equal-to the capacity of the knapsack. In LINGO syntax, we express this as:

```
@SUM(ITEMS: WEIGHT * INCLUDE) <= KNAPSACK_CAPACITY;
```

Finally, we must make the *INCLUDE* variable binary. We could do this by adding:

```
@BIN( INCLUDE( @INDEX( ANT_REPEL)));
@BIN( INCLUDE( @INDEX( BEER)));
@BIN( INCLUDE( @INDEX( BLANKET)));
@BIN( INCLUDE( @INDEX( BRATWURST)));
@BIN( INCLUDE( @INDEX( BROWNIES)));
@BIN( INCLUDE( @INDEX( FRISBEE)));
@BIN( INCLUDE( @INDEX( SALAD)));
@BIN( INCLUDE( @INDEX( WATERMELON)));
```

(Note that the *@INDEX* function simply returns the index of a primitive set member in its set.) However, a more efficient and data independent way of doing this would be to embed an *@BIN* function in an *@FOR* function as follows:

```
@FOR( ITEMS: @BIN( INCLUDE));
```

The Solution

The entire model for our knapsack example and excerpts from its solution are listed below. The model formulation file may be found in your *SAMPLES* subdirectory off the main LINGO directory under the name *KNAPSACK*:

```
MODEL:

SETS:
  ITEMS: INCLUDE, WEIGHT, RATING;
ENDSETS

DATA:
  ITEMS          WEIGHT RATING =
   ANT_REPEL        1      2
   BEER             3      9
   BLANKET          4      3
   BRATWURST        3      8
   BROWNIES         3     10
   FRISBEE          1      6
   SALAD            5      4
   WATERMELON      10     10;

  KNAPSACK_CAPACITY = 15;
ENDDATA

MAX = @SUM( ITEMS: RATING * INCLUDE);

@SUM( ITEMS: WEIGHT * INCLUDE) <=
KNAPSACK_CAPACITY;

@FOR( ITEMS: @BIN( INCLUDE));

END
```

Model: KNAPSACK

```
Global optimal solution found.
Objective value:                              38.00000
Objective bound:                              38.00000
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0
Elapsed runtime seconds:                          0.03

 Model Class:                                      PILP

           Variable           Value        Reduced Cost
 INCLUDE( ANT_REPEL)        1.000000           -2.000000
      INCLUDE( BEER)        1.000000           -9.000000
   INCLUDE( BLANKET)        1.000000           -3.000000
 INCLUDE( BRATWURST)        1.000000           -8.000000
  INCLUDE( BROWNIES)        1.000000           -10.00000
   INCLUDE( FRISBEE)        1.000000           -6.000000
     INCLUDE( SALAD)        0.000000           -4.000000
INCLUDE( WATERMELON)        0.000000           -10.00000
```

Solution to KNAPSACK

Your knapsack is fully packed at 15 pounds, and we take along everything but the salad and watermelon. Your lunch of beer, bratwurst, and brownies may not be very healthy, but at least you will be happy!

An Extension - Modeling a Logical Or Condition

Binary variables are very useful for modeling logical conditions. For instance, suppose your physician reviews your picnic plans and, fearing for your health, insists you must take either the salad *or* the watermelon along on your picnic. You could add this condition to your model by simply appending the constraint:

```
INCLUDE( @INDEX( SALAD)) +
INCLUDE( @INDEX( WATERMELON)) >= 1;
```

In order to satisfy this constraint, either the salad, the watermelon, or both must be included in the knapsack. Unfortunately, constraints of this form are not good practice in that they are not data independent. Suppose that your list of picnic items changes. You may need to modify this new constraint to reflect those changes. A well formulated model should require no changes to the constraints as a result of changes to the data. The following model demonstrates a data independent way of incorporating your physician's request (additions to the original model are listed in bold).

```
MODEL:

SETS:

       ITEMS: INCLUDE, WEIGHT, RATING;
       MUST_EAT_ONE( ITEMS);
ENDSETS

DATA:
       ITEMS WEIGHT RATING =
       ANT_REPEL 1 2
       BEER 3 9
       BLANKET 4 3
       BRATWURST 3 8
       BROWNIES 3 10
       FRISBEE 1 6
       SALAD 5 4
       WATERMELON 10 10;

       MUST_EAT_ONE = SALAD WATERMELON;

       KNAPSACK_CAPACITY = 15;
ENDDATA

MAX = @SUM( ITEMS: RATING * INCLUDE);

@SUM( ITEMS: WEIGHT * INCLUDE) <=
KNAPSACK_CAPACITY;

@FOR( ITEMS: @BIN( INCLUDE));

@SUM( MUST_EAT_ONE( I): INCLUDE( I)) >= 1;

END
```

We have derived a set called *MUST_EAT_ONE* from the original picnic items, and used an explicit list to include the items we must carry as members. Then, at the end of the model, we added a constraint that forces at least one of the "must eat" items into the solution.

For those interested, the solution to the modified model is:

```
Global optimal solution found.
Objective value:                    37.00000
Objective bound:                    37.00000
Infeasibilities:                    0.000000
Extended solver steps:                     0
Total solver iterations:                   0
Elapsed runtime seconds:                0.03

Variable                          Value          Reduced Cost
INCLUDE( ANT_REPEL)            0.000000             -2.000000
INCLUDE( BEER)                 1.000000             -9.000000
INCLUDE( BLANKET)              0.000000             -3.000000
INCLUDE( BRATWURST)            1.000000             -8.000000
INCLUDE( BROWNIES)             1.000000             -10.00000
INCLUDE( FRISBEE)              1.000000             -6.000000
INCLUDE( SALAD)                1.000000             -4.000000
INCLUDE( WATERMELON)           0.000000             -10.00000
```

In short, we drop the ant repellent and blanket, and replace them with the salad.

Binary Integer Example – Product-Mix with Fixed Costs

In many situations, it is not unusual for a particular activity to incur a fixed cost. Examples where one might incur a fixed cost include opening a plant, producing a product, paying a commission on an order to buy stock, or retooling an assembly line.

In this next example, we will put together a product-mix model much like the CompuQuick example from Chapter 1, *Getting Started with LINGO*. In this case, however, there is a fixed setup charge associated with the production of an item. In other words, whenever we produce any amount of a product, we incur a fixed charge independent of the output level of the product.

The Problem

You're the manager of an airplane plant and you want to determine the best product-mix of your six models to produce. The six models currently under production are the Rocket, Meteor, Streak, Comet, Jet, and Biplane. Each plane has a known profit contribution. There is also a fixed cost associated with the production of any plane in a period. The profit and fixed costs are given in the following table:

Plane	Profit	Setup
Rocket	30	35
Meteor	45	20
Streak	24	60
Comet	26	70
Jet	24	75
Biplane	30	30

Each plane is produced using six raw materials—steel, copper, plastic, rubber, glass, and paint. The units of these raw materials required by the planes as well as the total availability of the raw materials are:

	Rocket	Meteor	Streak	Comet	Jet	Biplane	Available
Steel	1	4	0	4	2	1	800
Copper	4	5	3	0	1	0	1160
Plastic	0	3	8	0	1	0	1780
Rubber	2	0	1	2	1	5	1050
Glass	2	4	2	2	2	4	1360
Paint	1	4	1	4	3	4	1240

The problem is to determine the final mix of products that maximizes net profit (gross profit - setup costs) without exceeding the availability of any raw material. Your brand new Meteor model has the highest profit per unit of anything you've ever manufactured and the lowest setup cost. Maybe you should build nothing but Meteors? Then again, maybe not.

The Formulation

As you might guess, we will need two primitive sets in this model—one to represent the airplane models and one to represent the raw materials. We can construct these sets as follows:

```
PLANES:
  PROFIT, SETUP, QUANTITY, BUILD;
RESOURCES: AVAILABLE;
```

We added the following four attributes to the *PLANES* set:

- *PROFIT* stores profit contribution for the plane,
- *SETUP* stores setup cost to begin producing the plane,
- *QUANTITY* a variable for quantity of planes to produce, and
- *BUILD* a binary variable, 1 if we produce the plane, else 0.

The *AVAILABLE* attribute on the *RESOURCES* set will be used to store the availability of each resource.

We will also need to derive a dense set by taking the cross of the *RESOURCES* set with the *PLANES* set. We need this set in order to define a *USAGE* attribute to store the resource usage of each plane. We will call this derived set RXP, which, after inclusion into the sets section, gives us:

```
SETS:
   PLANES:
     PROFIT, SETUP, QUANTITY, BUILD;
   RESOURCES: AVAILABLE;
   RXP( RESOURCES, PLANES): USAGE;
ENDSETS
```

In our data section, we will initialize the set members: *PLANES* and *RESOURCES*, along with the data attributes: *PROFIT*, *SETUP*, *AVAILABLE*, and *USAGE*. Here is the data section we will use:

```
DATA:
   PLANES    PROFIT  SETUP =
     ROCKET     30     35
     METEOR     45     20
     STREAK     24     60
     COMET      26     70
     JET        24     75
     BIPLANE    30     30;
   RESOURCES AVAILABLE =
     STEEL,800 COPPER,1160 PLASTIC,1780
     RUBBER,1050 GLASS,1360 PAINT,1240;
   USAGE =  1 4 0 4 2 0
            4 5 3 0 1 0
            0 3 8 0 1 0
            2 0 1 2 1 5
            2 4 2 2 2 4
            1 4 1 4 3 4;
ENDDATA
```

With the sets and data sections complete, we can now turn our attention to the objective function. For our objective, we want to maximize total net profit. Specifically, this is computed as the sum of profit times quantity produced of each plane, minus its setup cost multiplied by the *BUILD* binary variable. In LINGO syntax, we express the objective as:

```
MAX = @SUM( PLANES:
 PROFIT * QUANTITY - SETUP * BUILD);
```

Since all attributes are defined on the index set, we can drop the set index variable and use implicit indexing.

For our first set of constraints, we want to be sure raw material supplies are not exceeded. In words, what we want is:

> *For each resource i, the sum over each plane j of the quantity of plane j built multiplied by the resource usage of resource i by plane j must be less-than-or-equal-to the availability of resource i.*

Given the vagaries of the English language, it's highly likely one would find the equivalent LINGO notation more concise and easier to understand:

```
@FOR( RESOURCES( I):
 @SUM( PLANES( J):
  USAGE( I, J) * QUANTITY( J)) <=
   AVAILABLE( I)
);
```

Our next set of constraints is not quite as intuitive. We are using the binary variable *BUILD* to represent if a plane is being built, so we can incorporate a fixed cost for the plane in the objective function. What we need is some constraint mechanism to force *BUILD(I)* to be 1 when we produce a nonzero quantity of plane *I*. The following constraint will do just that:

```
@FOR( PLANES:
 QUANTITY <= 400 * BUILD;
  @BIN( BUILD)
);
```

Given that *BUILD* is 0/1, as soon as *QUANTITY* goes nonzero the only feasible solution is for *BUILD* to go to 1. Constraints of this form used to force a binary variable to an appropriate value are sometimes referred to as *forcing constraints*.

The coefficient of 400 in our forcing constraints was chosen because we know from scanning our data that no more than 400 of any plane can be built. Can you verify this? Coefficients used in this manner are sometimes called *BigM* coefficients. For solver efficiency reasons, it's best to try to keep BigM values as small as reasonably possible.

Because the BigM coefficient of 400 is dependent upon the model's data, it is actually bad modeling practice to embed the coefficient in the model's constraints as we have done here. As we have discussed, it is best to try to keep the constraints of your model independent of the data to facilitate model maintenance. A more data independent formulation would actually involve calculations to come up with a good BigM value. Can you think of how you might add such a feature to this model?

A reasonable question at this point would be: "We have the machinery to force *BUILD* to 1 when we build a plane. What forces *BUILD* to zero when we *don't* build a plane?" The fact that *BUILD* appears in the objective with a negative coefficient (we multiply it by *SETUP* and then subtract it from the objective) guarantees this. If a plane was not being built and the corresponding *BUILD* variable was 1, we could get a better solution by simply setting *BUILD* to 0. Since the goal is to maximize the objective, *BUILD* will always be driven to 0 when a plane is not built.

One final feature of our forcing constraints to note is that we have piggybacked the *@BIN* function call onto the *@FOR* statement for the forcing constraints. As you recall from the discussion of set looping functions in *Using Sets*, an *@FOR* function may contain multiple expressions as long as they are separated by a semicolon. We have capitalized on this feature by including the *@BIN* expression as well.

As a final feature, we can make the *QUANTITY* variables general integers with the expression:

```
@FOR( PLANES: @GIN( QUANTITY));
```

The Solution

The formulation in its entirety and a selected portion of the solution appear below. The formulation file may be found in file *PRODMIX*.

```
MODEL:
SETS:
  PLANES:
    PROFIT, SETUP, QUANTITY, BUILD;
  RESOURCES: AVAILABLE;
  RXP( RESOURCES, PLANES): USAGE;
ENDSETS
DATA:
  PLANES    PROFIT  SETUP =
    ROCKET     30     35
    METEOR     45     20
    STREAK     24     60
    COMET      26     70
    JET        24     75
    BIPLANE    30     30;
  RESOURCES AVAILABLE =
    STEEL,800 COPPER,1160 PLASTIC,1780
    RUBBER,1050 GLASS,1360 PAINT,1240;
  USAGE =  1 4 0 4 2 0
           4 5 3 0 1 0
           0 3 8 0 1 0
           2 0 1 2 1 5
           2 4 2 2 2 4
           1 4 1 4 3 4;
ENDDATA

MAX = @SUM( PLANES:
PROFIT * QUANTITY - SETUP * BUILD);
@FOR( RESOURCES( I):
@SUM( PLANES( J):
 USAGE( I, J) * QUANTITY( J)) <=
  AVAILABLE( I);
);
@FOR( PLANES:
 QUANTITY <= 400 * BUILD;
 @BIN( BUILD);
);
@FOR( PLANES: @GIN( QUANTITY));

END
```

Model: PRODMIX

```
Global optimal solution found.
Objective value:                    14764.00
Objective bound:                    14764.00
Infeasibilities:                    0.000000
```

```
Extended solver steps:                    18
Total solver iterations:                 332
Elapsed runtime seconds:                0.11

          Variable           Value        Reduced Cost
 QUANTITY( ROCKET)        96.00000           -30.00000
 QUANTITY( METEOR)        0.000000           -45.00000
 QUANTITY( STREAK)        195.0000           -24.00000
  QUANTITY( COMET)        0.000000           -26.00000
    QUANTITY( JET)        191.0000           -24.00000
QUANTITY( BIPLANE)        94.00000           -30.00000
    BUILD( ROCKET)        1.000000            35.00000
    BUILD( METEOR)        0.000000            20.00000
    BUILD( STREAK)        1.000000            60.00000
     BUILD( COMET)        0.000000            70.00000
       BUILD( JET)        1.000000            75.00000
   BUILD( BIPLANE)        1.000000            30.00000
```

Solution to PRODMIX

Surprisingly, we see from the solution that we build none of the "profitable" Meteors. Can you determine why this is so? On the other hand, the Rocket, Streak, Jet and Biplane are produced, and, as we anticipated, the *BUILD* variable for each of these planes has been correctly set to 1.

Dual Values and IP

In Chapter 1, *Getting Started with LINGO*, we introduced the concept of dual values. The dual values of a solution are the reduced costs of the variables and dual prices on the constraints. We also discussed the useful information that can be obtained from dual values. Unfortunately, in IP models the interpretation of the dual values breaks down. Due to the discreet nature of IP models, the dual values in the solution to an IP model are of no practical use to the average user. Given this, the dual values should be ignored when your model contains integer variables created through the use of *@BIN* or *@GIN*.

Variable Priorities

The time required for branch-and-bound algorithm to converge is highly dependent on the order in which the solver branches on the integer variables. Some integer variables impact the solution more than others. An example would be a plant location model, where we are deciding a) which plants to open, and b) which customers to assign to which plants. Clearly, the decision of opening a plant influences the solution considerably more than the decision of which plant to assign a given customer to. In general, performance will improve if the solver branches on the more critical integer variables first, while branching on the less critical variables later. LINGO provides the *@PRIORITY* function for controlling the branching priority of the variables.
The syntax of the *@PRIORITY* function is:

@PRIORITY(variable_name, relative_priority);

where *variable_name* is the name of the variable and *relative_priority* is a non-negative integer representing the relative priority of the variable. The *@PRIORITY* function may be used in a model anywhere you would normally enter a constraint. The *@PRIORITY* function can be embedded in an

@FOR statement to allow you to easily set the priority of all, or selected, variables of an attribute. If a variable is not assigned a priority, it is assumed to have the lowest priority level of 0.
Some examples of *@PRIORITY* are:

Example 1: `@PRIORITY( X, 100);`
assigns variable *X* a priority of 100,

Example 2: `@PRIORITY( PRODUCE( 5), 10);`
assigns variable *PRODUCE(5)* a priority of 10,

Example 3: `@FOR( DAYS( I): @PRIORITY( START( I), 30));`
assigns all the variables of the *START* a priority of 30.

Summary

You should now be familiar with the use of the variable domain functions *@BIN* and *@GIN*, and how they are used to introduce integer variables into a model. This section has shown how integer variables bring a whole new dimension of power to the mathematical modeler. Given that we have only briefly delved into the topic of modeling with integer variables, the user that would like to become more familiar with the many practical applications of integer programming and their formulations can refer to Schrage (2006), or Winston (1995).

Free Variables

By default, a LINGO variable has a lower bound of zero and an upper bound of infinity. *@FREE* removes the lower bound of zero and lets a variable take negative values, rendering it unconstrained in sign, or *free*. The syntax is:

@FREE(*variable_name*);

where *variable_name* is the name of the variable you wish to make free.

The *@FREE* function may be used in a model anywhere you would normally enter a constraint. The *@FREE* function can be embedded in an *@FOR* statement to allow you to easily make all, or selected, variables of an attribute to be free. Some examples of *@FREE* are:

Example 1: `@FREE( X);`
makes the scalar variable, *X*, free,

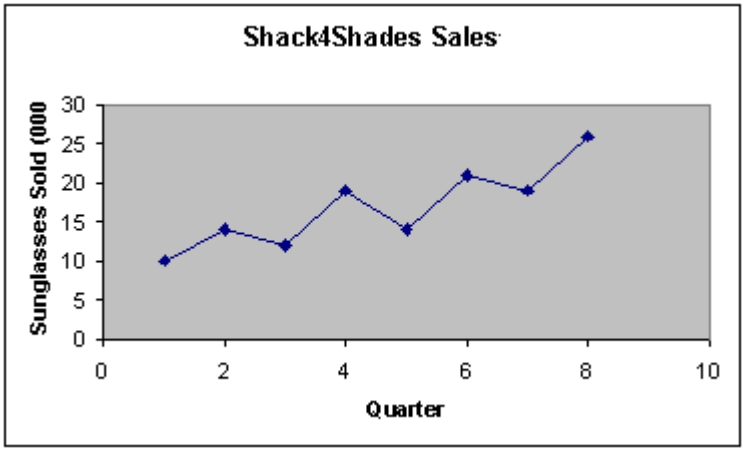
Example 2: `@FREE( QUANTITY( 4));`
makes the variable *QUANTITY(4)* free,

Example 3: `@FOR( ITEMS: @FREE( QUANTITY));`
makes all variables in the *QUANTITY* attribute free.

FREE Variable Example - Forecasting

You are the inventory controller for the successful new Shack4Shades retail chain. Your business specializes exclusively in the retailing of sunglasses to the lover of the outdoors. You need to come up with a model to forecast sales of sunglasses in the coming quarter in order to build up inventory levels.

You have created the following chart of your sales for the last eight quarters:



Looking at this chart, you theorize that sales are growing according to a linear trend line, but with rather sizable seasonal variations. Sales pick up in the summer months when people head to the beaches and again in winter when they head for the ski slopes. Given this, you have come up with the following theoretical function to forecast sales as a function of time:

$$Predicted_Sales(t) = Seasonal_Factor(t) * (Base + Trend * t)$$

where,

Predicted_Sales(t)	represents predicted sales for quarter *t*,
Seasonal_Factor(t)	is one of four multipliers (one for each quarter of the year) to account for seasonal variations,
Base	is the *y*-intercept of the hypothesized linear function, and
Trend	is the slope of the linear function.

You would like to come up with a LINGO model to estimate the six parameters of your function (i.e., the four seasonal factors, the trend line base, and the trend line slope). To do this, you will let LINGO choose values for the parameters that minimize the sum of the squared differences between predicted and observed sales for the historical data.

The Formulation

We will need two primitive sets in our model. The first set will have eight members to represent the quarters that we have historical data for. The second set will have four members corresponding to the four quarters of the year. This second set is used for defining the four seasonal factors. Here is our sets section that incorporates these two sets:

```
SETS:
   PERIODS: OBSERVED, PREDICT, ERROR;
   QUARTERS: SEASFAC;
ENDSETS
```

The three attributes on the *PERIODS* set—*OBSERVED*, *PREDICT*, and *ERROR*—correspond to the observed sales values, predicted sales values, and the prediction error. The prediction error is simply predicted sales minus observed sales. The *SEASFAC* attribute on the *SEASONS* set corresponds to the seasonal sales factors and will be computed by LINGO.

We will also need to add a data section to initialize the set members and the *OBSERVED* attribute with the historical sales data. We can do this with the following:

```
DATA:
   PERIODS = P1..P8;
   QUARTERS = Q1..Q4;
   OBSERVED = 10 14 12 19 14 21 19 26;
ENDDATA
```

Next, we must add a formula to compute the error terms. As mentioned, the error term in a period is the difference between the observed and predicted sales. We can express this in LINGO as:

```
@FOR( PERIODS: ERROR =
 PREDICT - OBSERVED);
```

Our objective is to minimize the sum of the squared error terms, which may be written as:

```
MIN = @SUM( PERIODS: ERROR ^ 2);
```

We choose to use squared error terms as a measure to minimize because we want to weight large errors relatively more heavily. Another option might be to minimize the sum of the absolute values of the errors, which would weight large and small errors proportionally the same.

In order to compute the error terms, we will also need to compute predicted sales. Using our theoretical formula, we compute predicted sales as follows:

```
@FOR( PERIODS( P): PREDICT( P) =
 SEASFAC( @WRAP( P, 4))
  * ( BASE + P * TREND));
```

The *@WRAP* function is used here to allow us to apply the four seasonal factors over a time horizon exceeding four periods. Had we simply used the index *P,* instead of *@WRAP(P, 4),* we would have generated a subscript out of range error. For a more in depth explanation of the use of the *@WRAP* function, please see the staff-scheduling example on page 82.

For esthetic reasons, we would like the seasonal factors to average out to a value of one. We can do this by adding the constraint:

```
@SUM( QUARTERS: SEASFAC) = 4;
```

Finally, it is possible for the error terms to be negative as well as positive. Given that variables in LINGO default to a lower bound of zero, we will need to use the *@FREE* function to allow the error terms to go negative. By embedding the *@FREE* function in an *@FOR* loop, we can apply *@FREE* to all the *ERROR* variables in the statement:

```
@FOR( PERIODS: @FREE( ERROR));
```

The Solution

The entire formulation and excerpts from the solution appear below.

```
MODEL:
SETS:
   PERIODS: OBSERVED, PREDICT, ERROR;
   QUARTERS: SEASFAC;
ENDSETS

DATA:
   PERIODS = P1..P8;
   QUARTERS = Q1..Q4;
   OBSERVED = 10 14 12 19 14 21 19 26;
ENDDATA

MIN = @SUM( PERIODS: ERROR ^ 2);

@FOR( PERIODS: ERROR =
 PREDICT - OBSERVED);

@FOR( PERIODS( P): PREDICT( P) =
 SEASFAC( @WRAP( P, 4))
  * ( BASE + P * TREND));

@SUM( QUARTERS: SEASFAC) = 4;

@FOR( PERIODS: @FREE( ERROR);
 @BND( -1000, ERROR, 1000));

END
```

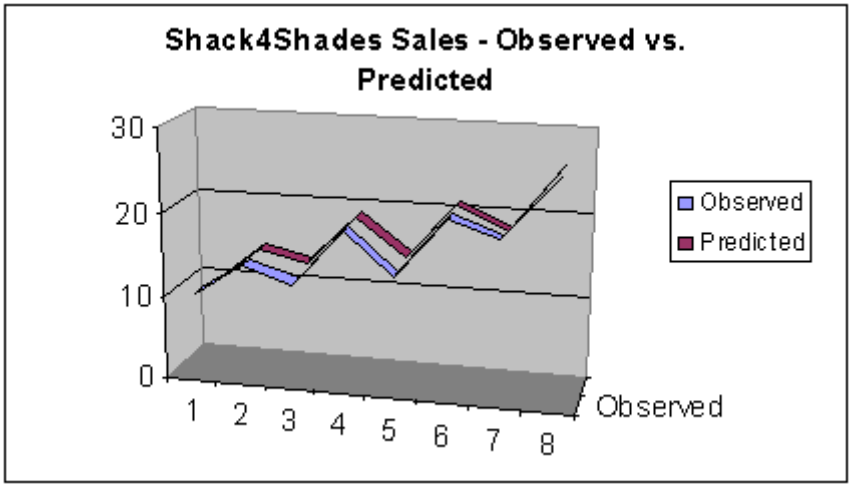
Model: SHADES

```
Local optimal solution found.
Objective value:                    1.822561
Total solver iterations:                  32

        Variable           Value
            BASE        9.718878
           TREND        1.553017
   OBSERVED( P1)        10.00000
   OBSERVED( P2)        14.00000
   OBSERVED( P3)        12.00000
   OBSERVED( P4)        19.00000
   OBSERVED( P5)        14.00000
   OBSERVED( P6)        21.00000
   OBSERVED( P7)        19.00000
   OBSERVED( P8)        26.00000
    PREDICT( P1)        9.311820
    PREDICT( P2)        14.10136
    PREDICT( P3)        12.85213
    PREDICT( P4)        18.80620
    PREDICT( P5)        14.44367
    PREDICT( P6)        20.93171
    PREDICT( P7)        18.40496
    PREDICT( P8)        26.13943
      ERROR( P1)      -0.6881796
      ERROR( P2)       0.1013638
      ERROR( P3)       0.8521268
      ERROR( P4)      -0.1938024
      ERROR( P5)       0.4436688
      ERROR( P6)      -0.6828722E-01
      ERROR( P7)      -0.5950374
      ERROR( P8)       0.1394325
    SEASFAC( Q1)       0.8261096
    SEASFAC( Q2)        1.099529
    SEASFAC( Q3)       0.8938789
    SEASFAC( Q4)        1.180482
```

Solution to SHADES

The solution is: *TREND*, 1.55; *BASE*, 9.72. The four seasonal factors are .826, 1.01, .894, and 1.18. The spring quarter seasonal factor is .826. In other words, spring sales are 82.6% of the average. The trend of 1.55 means, after the effects of season are taken into account, sales are increasing at an average rate of 1,550 sunglasses per quarter. As one would expect, a good portion of the error terms are negative, so it was crucial to use the *@FREE* function to remove the default lower bound of zero on *ERROR*.

Our computed function offers a very good fit to the historical data as the following graph illustrates:



Using this function, we can compute the forecast for sales for the upcoming quarter (quarter 1). Doing so gives:

$$\begin{aligned} \textit{Predicted_Sales}(9) &= \textit{Seasonal_Factor}(1) * (\textit{Base} + \textit{Trend} * 9) \\ &= 0.826 * (9.72 + 1.55 * 9) \\ &= 19.55 \end{aligned}$$

Given this, inventory levels should be brought to a level sufficient to support an anticipated sales level of around 19,550 pairs of sunglasses.

Bounded Variables

Whereas *@FREE* sets the upper and lower bounds of the specified variable to plus and minus infinity (effectively removing any bounds on the variable), the *@BND* function lets you set specific upper and lower bounds on a variable. In other words, *@BND* limits a variable's range within some specified interval. The syntax for *@BND* is:

@BND(*lower_bound*, *variable_name*, *upper_bound*);

where *variable_name* is the variable to be bounded below by the quantity *lower_bound* and bounded above by the quantity *upper_bound*. Both *lower_bound* and *upper_bound* must be either numeric values or variables whose values have been set in a data section or calc section. *@BND* may be used wherever you would normally enter a constraint in a model—including inside an *@FOR* looping function.

In mathematical terms, LINGO interprets this *@BND* function as:

$$\textit{lower_bound} \le \textit{variable_name} \le \textit{upper_bound}$$

It is certainly possible to add constraints in lieu of the *@BND* function, but, from the standpoint of the optimizer, *@BND* is an extremely efficient way of representing simple bounds on variables. Specifying variable bounds using *@BND* rather than explicitly adding constraints can noticeably speed up the solution times for larger models. Furthermore, *@BND* does not count against the limit on the total number of constraints LINGO imposes on some versions. So, in general, it is a good idea to use *@BND* in place of constraints whenever possible.

Some examples of *@BND* are:

Example 1: `@BND( -1, X, 1);`
constrains the variable *X* to lie in the interval [-1,1],

Example 2: `@BND( 100, QUANTITY( 4), 200);`
constrains the variable *QUANTITY(4)* to fall within 100 to 200,

Example 3: `@FOR( ITEMS: @BND( 10, Q, 20));`
sets the bounds on all variables in the *Q* attribute to 10 and 20,

Example 4: `@FOR( ITEMS: @BND( QL, Q, QU));`
sets the bounds on all variables in the *Q* attribute to *QL* and *QU* (*QL* and *QU* must have been previously set to some values in a data section).

SOS Variables

LINGO supports SOS (Special Ordered Sets) variables of Type 1, 2 and 3 via the *@SOS1*, *@SOS2* and *@SOS3* functions, respectively. The properties of the three SOS types are:

SOS Type	Property
SOS1	At most, only one variable belonging to an SOS1 set will be greater than 0.
SOS2	At most, only two variables in an SOS2 set can be different from 0. If two variables are nonzero, then the variables will be adjacent to one another. SOS2 sets are particularly useful for implementing piecewise-linear functions in models.
SOS3	Exactly one variable from a given SOS3 set will be equal to 1. All remaining variables will be equal to 0.

Note: Any variables added to an SOS set will count against the integer variable limit imposed in limited versions of LINGO.

The syntax for the *@SOS* declarations is as follows:

@SOS{1|2|3}('set_name', *variable_reference*);

The *set_name* argument is a unique label, or name, for the particular set of SOS variables. You add additional variables to an SOS set by making subsequent calls to the *@SOS* function using the same set name.

Some examples of SOS sets are:

Example 1:
```
@SOS3( 'SUM_TO_1', X);
@SOS3( 'SUM_TO_1', Y); @SOS3( 'SUM_TO_1', Z);
```

In this example, an SOS Type 3 set forces either *X*, *Y* or *Z* to be equal to 1. The remaining variables will be equal to 0.

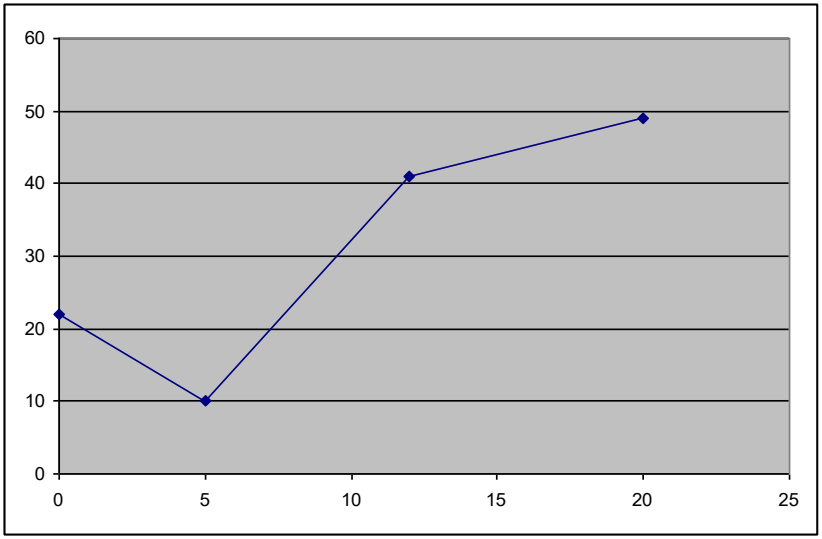
Example 2:
```
@FOR( CUST( J): @FOR( PLANTS( I):
@SOS1( 'SNGSRC_' + CUST( J), SHIP( I, J))));
```

Here, multiple SOS Type 1 sets force each customer to receive shipments from only one plant. There is one SOS1 set created for each customer, each bearing the name SNGSRC_*customer_name.*

An example of using Type 2 sets follows in the next section.

Piecewise Linear Example - Type SOS2 Set

As we mentioned above, SOS2 sets are particularly useful for implementing piecewise-linear functions. Many cost curves exhibit the piecewise-linear property. For example, suppose we want to model the following cost function, where cost is a piecewise-linear function of volume, *X*:



Piecewise-Linear Function Example

The breakpoints of the curve lie at the following points: (0,22), (5,10), (12,41) and (20,49).

The following sample model, *SOSPIECE.LG4,* uses a Type 2 SOS set to model this piecewise-linear function using what is referred to as the *lambda method*:

```
MODEL:

! Demonstrates the lambda method for
  representing arbitrary, piecewise-linear
  curves using an SOS2 set;

! See "Optimization Modeling with Lingo",
  Section 11.2.7;

SETS:
   ! 4 breakpoints in this example;
   B /1..4/: W, U, V;
ENDSETS

DATA:
   ! total cost at the breakpoints;
   V = 22 10 41 49;

   ! the breakpoints;
   U =  0  5 12 20;
ENDDATA

   ! set x to any value in interval--the cost
     variable will automatically be set to the
     correct total cost;
   X = 8.5;

   ! calculate total cost;
   COST = @SUM( B( i): V( i) * W( i));

   ! force the weights (w);
   X  = @SUM( B( I): U( I) * W( i));

   !weights must sum to 1;
   @SUM( B( I): W( I)) = 1;

   ! the weights are SOS2: at most two adjacent
     weights can be nonzero;
   @FOR( B( I): @SOS2( 'SOS2_SET', W( I)));

END
```

Model: SOSPIECE

We defined an attribute, *W,* whose members act as weights, placing us on an particular segment of the curve. For example, if *W(2)=W(3)*=0.5, then we are exactly halfway between the second and third breakpoints : (5,10) and (12,41), i.e., at point (8.5,25.5). In the case where we lie exactly on a breakpoint, then only one of the *W(i)* will be nonzero and equal to 1.

For this strategy to work correctly, only two, at most, of the *W(i)* may be nonzero, and they must be adjacent. As you recall, this is the definition of an SOS2 set, which we create at the end of the model with the expression:

```
! the weights are SOS2: at most two adjacent
  weights can be nonzero;
@FOR( B( I): @SOS2( 'SOS2_SET', W( I)));
```

In particular, each weight *W(i)* is a member of the Type SOS2 set titled *SOS2_SET*.

For this particular example, we have chosen to pick an x-value and then let LINGO compute the corresponding y-value, or cost. Running, the model, as predicted, we see that for an *X* value of 8.5, total cost is 25.5:

Variable	Value
X	8.500000
COST	25.50000
W(1)	0.000000
W(2)	0.5000000
W(3)	0.5000000
W(4)	0.000000

Solution to SOSPIECE

In addition to allowing the solver to work more efficiently, SOS sets also help to reduce the number of variables and constraints in your model. In this particular example, had we not had the SOS2 capability, we would have needed to add an additional 0/1 attribute, *Z*, and the following expressions to the model:

```
! Here's what we eliminated by using @sos2:
! Can be on only one line segment at a time;
w( 1) <= z( 1); w( @size( b)) <= z( @size( b));
@for( b( i) | i #gt# 1 #and# i #lt# @size( b):
   w( i) <= z( i) + z( i + 1)
);
@sum( b( i): z( i)) = 1;
@for( b( i): @bin( z( i)));
```

Note: It may seem that piecewise linearity could be implemented in a more straightforward manner through the use of nested *@IF* functions. Certainly, the *@IF* approach would be more natural than the lambda method presented here. However, *@IF* functions would add discontinuous nonlinearities to this model. This is something to try and avoid, in that such functions are notoriously difficult to solve to global optimality. In the approach used above, we have maintained linearity, which allows LINGO to use its faster, linear solvers, and converge to a globally optimal solution.

Cardinality

Related to the SOS capability discussed above, LINGO also supports *cardinality sets* of variables via the *@CARD* function. The cardinality feature allows you to specify a set of variables with a cardinality of *N,* meaning that, at most, *N* of the variables in the set will be allowed to be nonzero.

As with SOS sets, cardinality sets help the integer solver branch more efficiently, and they reduce the number of variables and constraints in your models. Also, as with SOS sets, each variable added to a cardinality set will count against any integer variable limits imposed on your installation of LINGO.

The syntax for the *@CARD* declarations is as follows:

```
@CARD( 'set_name', variable_reference|set_cardinality);
```

The *set_name* argument is a unique label, or name, for the particular cardinality set. You add additional variables to an SOS set by making subsequent calls to the *@CARD* function using the same set name with a different *variable_reference*. In addition to calling *@CARD* once for each variable in a set, you will need to call *@CARD* once for each set passing an integer value as the second argument. This integer argument is the *set_cardinality,* and may be either an actual integer number or a variable set to an integer value in either a data or calc section.

Some examples of *@CARD* sets are:

Example 1:

```
@CARD( 'PICK2', 2);
@CARD( 'PICK2', X);
@CARD( 'PICK2', Y); @CARD( 'PICK2', Z);
```

In this example, at most, two out of the three variable X, Y, and Z will be nonzero.

Example 2:

```
@FOR( PLANT( I): @CARD( 'OPENLIM', OPEN( I)));
@CARD( 'OPENLIM', NCARD);
```

Here, we limit the maximum number of open plants to *NCARD*, where *NCARD* must be set beforehand to an integer value in either a data or calc section.

Semicontinuous Variables

Many models require certain variables to either be 0 or lie within some nonnegative range, e.g., 10 to 20. Variables with this property are said to be *semicontinuous.* Modeling semicontinuity in LINGO in the past meant having to add an additional 0/1 variable and two additional constraints. LINGO now allows you to establish semicontinuous variables directly with the *@SEMIC* statement.

The syntax for the *@SEMIC* declarations is as follows:

```
@SEMIC( lower_bound, variable_reference, upper_bound);
```

This will restrict the variable, *variable_reference*, to be either 0 or to lie within the range [*lower_bound*, *upper_bound*].

Note: Each semi-continuous variable will be counted against any integer variable limit for your installation.

Some examples of *@SEMIC* usage are:

Example 1: `@SEMIC( 10, X, 20);`
In this example, X will be restricted to being either 0, or to lie within the range [10,20].

Example 2: `@FOR( PLANT( I): @SEMIC( MIN_HOURS, HOURS( I), MAX_HOURS));`
Here, we restrict the operating hours of each plant to be either 0, or to line in the range [*MIN_HOURS,MAX_HOURS*]. Note that *MIN_HOURS* and *MAX_HOURS* must have been set to explicit values beforehand in either a data or calc section.

Below, we have taken our familiar transportation model and modified it, via the use of *@SEMIC*, to restrict shipments from each warehouse to each customer to be either 0, or between 3 and 10.

```
MODEL:
! A 3 Warehouse, 4 Customer Transportation Problem
  that uses the semi-continuous (@SEMIC) to restrict
  nonzero shipments to be between 3 and 10 units.;

SETS:
   WAREHOUSE: CAPACITY;
   CUSTOMER: DEMAND;
   ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

DATA:
   WAREHOUSE,CAPACITY =  WH1,30 WH2,25 WH3,21;
   CUSTOMER,DEMAND =   C1,15 C2,17 C3,22 C4,12;
   COST =  6  2  6  7
           4  9  5  3
           8  8  1  5;
ENDDATA

! The objective;
 [R_OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
 @FOR( CUSTOMER( J): [R_DEM]
  @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
   DEMAND( J));

! The supply constraints;
 @FOR( WAREHOUSE( I): [R_SUP]
  @SUM( CUSTOMER( J): VOLUME( I, J)) <=
   CAPACITY( I));

 @FOR( ROUTES: @SEMIC( 3, VOLUME, 10));

END
```

Model: TRANSEMIC

Solving this model yields the following optimal values for the semicontinuous attribute, *VOLUME:*

```
Global optimal solution found.
Objective value:                              264.0000
Objective bound:                              264.0000
Infeasibilities:                              0.000000
Extended solver steps:                               1
Total solver iterations:                            32

          Variable           Value        Reduced Cost
  VOLUME( WH1, C1)        5.000000            0.000000
  VOLUME( WH1, C2)        10.00000           -6.000000
  VOLUME( WH1, C3)        6.000000            0.000000
  VOLUME( WH1, C4)        0.000000            2.000000
  VOLUME( WH2, C1)        10.00000           -1.000000
  VOLUME( WH2, C2)        0.000000            2.000000
  VOLUME( WH2, C3)        6.000000            0.000000
  VOLUME( WH2, C4)        9.000000           -1.000000
  VOLUME( WH3, C1)        0.000000            2.000000
  VOLUME( WH3, C2)        7.000000            0.000000
  VOLUME( WH3, C3)        10.00000           -5.000000
  VOLUME( WH3, C4)        3.000000            0.000000
```

Positive Semi-Definite Matrices

LINGO's *@POSD* function allows one to specify that a matrix of decision variables must be positive semi-definite. The matrix may be either lower triangular, upper triangular or square. In the case of square matrices, *@POSD* will force the matrix to be symmetric. Alternatively, one can think of this as allowing the user to formulate in terms of decision variables that are matrices rather than just scalars, and where the non-negativity of a scalar variable is replaced by the positive definiteness of the matrix of decision variables. It turns out that the barrier algorithms that are used for second-order cone problems can be generalized to solving models with semi-definite matrix decision variables.

The syntax for *@POSD* is:

@POSD(*matrix*);

where *matrix* refers to a two-dimensional matrix (i.e., attribute on a two-dimensional set) that is either lower triangular, upper triangular or square. An example of the *@POSD* function follows.

Note: The *@POSD* function requires that your LINGO installation have both barrier and conic solver licenses. Attempting to solve a model containing *@POSD* without these license will trigger an error message.

Positive Semi-Definite Matrix Example

To illustrate *@POSD*, suppose we have a covariance matrix for three stocks, AAPL, AMZN and GOOG. This covariance matrix was "computed" by making an educated guess of what the covariances are amongst our three stocks. A requirement for covariance matrices is that they be positive semi-definite (POSD). It turns out that by estimating the covariance matrix, it's possible that the resulting matrix is not truly POSD. To resolve this problem, we will find a new matrix that is close to our original matrix, but with the requirement that the new matrix be positive semi-definite. The model follows:

```
MODEL:

SETS:
 STOCK;
 SXS( STOCK, STOCK):  XG, XP, ERROR;
ENDSETS

DATA:
 STOCK= AAPL   GOOG  AMZN;
! We have some "guesstimates" of individual entries
  of a covariance matrix, however, taken together,
  the resulting matrix happens to not be Positive Definite
  and thus not a valid covariance matrix;
 XG =
       .011   -.013  .012
      -.013    .061  .07
       .012    .07   .06 ;
ENDDATA

! Find a symmetric matrix XP that is close to XG
 but is positive definite;

! Minimize some measure of difference between XG and XP;
 MIN = @SUM( SXS( I, J): ERROR( I, J)^2);
! Measure the errors;
 @FOR( SXS( I, J):
   ERROR( I, J) = XP( I, J) - XG( I, J);
   @FREE( ERROR);
 );

! Off diagonal terms can be negative;
 @FOR( SXS( I, J) | I #NE# J: @FREE( XP( I, J)));

! The new matrix must be positive semi-definite;
 @POSD( XP);

DATA:
 @TEXT() = 'Our initial guess matrix:';
 @TEXT() = @TABLE(XG);
 @TEXT() = '';
 @TEXT() = 'A Positive Semi-Definite matrix close to our guess:';
 @TEXT() = @TABLE(XP);
 @TEXT() = '';
ENDDATA

END
```

Model: POSDCOVMTX

The basic idea behind the model is that the solver should find a new matrix, *XP*, that is close to our guess matrix, *XG*, as measured by the sum of the squared differences of each of the matrix elements, and that XP must be POSD. The following use of *@POSD* forces the POSD requirement on *XP*:

```
! The new matrix must be positive semi-definite;
 @POSD( XP);
```

The data section of the model uses the *@TABLE* function to display both the original non-POSD matrix and the new POSD matrix:

```
Our initial guess matrix:
                 AAPL            GOOG            AMZN
  AAPL   0.1100000E-01  -0.1300000E-01   0.1200000E-01
  GOOG  -0.1300000E-01   0.6100000E-01   0.7000000E-01
  AMZN   0.1200000E-01   0.7000000E-01   0.6000000E-01

A Positive Semi-Definite matrix close to our guess:
                 AAPL            GOOG            AMZN
  AAPL   0.1590567E-01  -0.6982941E-02   0.5975522E-02
  GOOG  -0.6982941E-02   0.6838027E-01   0.6261064E-01
  AMZN   0.5975522E-02   0.6261064E-01   0.6739848E-01
```

4 Data, Init and Calc Sections

Typically, when dealing with a model's data, you need to assign set members to sets and give values to some set attributes before LINGO can solve your model. For this purpose, LINGO gives the user three optional sections, the *data section* for inputting set members and data values, the *init section* for setting the starting values for decision variables, and *the calc section* for performing computations on raw input data.

The DATA Section of a Model

The data section allows you to isolate data from the rest of your model. This is a useful practice in that it facilitates model maintenance and scaling of a model's dimensions.

The data section begins with the keyword *DATA:* (including the colon) and ends with the keyword *ENDDATA*. In the data section, you can have statements to initialize set members and/or the attributes of the sets you declared in a previous sets section. These expressions have the syntax:

object_list = *value_list*;

The *object_list* contains the names of the attributes and/or a set whose members you want to initialize, optionally separated by commas. There can be no more than one set name in *object_list*, while there may be any number of attributes. If there is more than one attribute name in *object_list*, then the attributes must be defined on the same set. If there is a set name in *object_list*, then all attributes in *object_list* must be defined on this set.

The *value_list* contains the values you want to assign to the members of *object_list,* optionally separated by commas. As an example, consider the following model:

```
SETS:
  SET1 /A, B, C/: X, Y;
ENDSETS
DATA:
  X = 1, 2, 3;
  Y = 4, 5, 6;
ENDDATA
```

We have two attributes, *X* and *Y*, defined on the *SET1* set. The three values of *X* are set to 1, 2, and 3, while *Y* is set to 4, 5, and 6. We could have also used the following compound data statement to the same end:

```
SETS:
  SET1 /A, B, C/: X, Y;
ENDSETS
DATA:
  X, Y =
  1, 4,
  2, 5,
  3, 6;
ENDDATA
```

Looking at this example, you might imagine *X* would be assigned the values 1, 4, and 2 because they are first in the values list, rather than the true values of 1, 2, and 3. When LINGO reads a data statement's value list, it assigns the first *n* values to the first position of each of the *n* attributes in the attribute list, the second *n* values to the second position of each of the *n* attributes, and so on. In other words, LINGO is expecting the input data in column form rather than row form.

As mentioned, we can also initialize the set members in the data section. Modifying our sample model to use this approach by moving the set members from the sets section to the data section we get:

```
SETS:
  SET1: X, Y;
ENDSETS
DATA:
  SET1, X, Y =
    A   1  4
    B   2  5
    C   3  6;
ENDDATA
```

This method is, perhaps, the most elegant of the three methods presented above, in that all model data (attribute values and set members) are isolated within the data section. Given that a model's data is subject to frequent change, having all data isolated to one area should make model maintenance simpler. We also refer to this concept of isolating data as *data independence*.

One additional method for inputting two-dimensional tables is through the use of a *data frames.* Consider the following sets section for a transportation model where we are minimizing the cost of shipping product from warehouses to customers:

```
! A 3 Warehouse, 4 Customer
  Transportation Problem;
SETS:
  WAREHOUSE    : CAPACITY;
  CUSTOMER     : DEMAND;
  ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS
```

We could enter the data for a four-customer, three-warehouse model with the data section:

```
DATA:
  CUSTOMER = C1 C2 C3 C4;
  WAREHOUSE = WH1 WH2 WH3;
  COST =
   6  2  6  7
   4  9  5  3
   8  8  1  5
  ;
  CAPACITY =   30, 25, 21;
  DEMAND   = 15 17 22 12;
ENDDATA
```

On the other hand, the data frame style of input allows us to enter the first dimension of a two-dimensional set along with an attribute of the same two-dimensional set, meaning we could use the alternative data section:

```
DATA:
  CUSTOMER = C1 C2 C3 C4;
  WAREHOUSE, COST =
         WH1   6  2  6  7
         WH2   4  9  5  3
         WH3   8  8  1  5
  ;
  CAPACITY =   30, 25, 21;
  DEMAND   = 15 17 22 12;
ENDDATA
```

Note that the *WAREHOUSE* set and *COST* attribute are now both entered in the same data statement. The data frame method of input can be useful when reading in preexisting data sets from external files (e.g., linear regression data sets). The complete version of this model can be found in the LINGO sample models set under the name *TRANFRAME*

Parameters

You are not limited to putting attributes and sets on the left-hand side of data statements. You may also initialize scalar variables in the data section. When a scalar variable's value is fixed in a data section, we refer to it as a *parameter*.

As an example, suppose your model uses an interest rate of 8.5% as a parameter. You could input the interest rate as a parameter in the data section as follows:

```
DATA:
   INTEREST_RATE = .085;
ENDDATA
```

As with set attributes, you can initialize multiple parameters in a single statement. Suppose you also add the inflation rate to your model. You could initialize both the interest rate and inflation rate in the same data statement as follows:

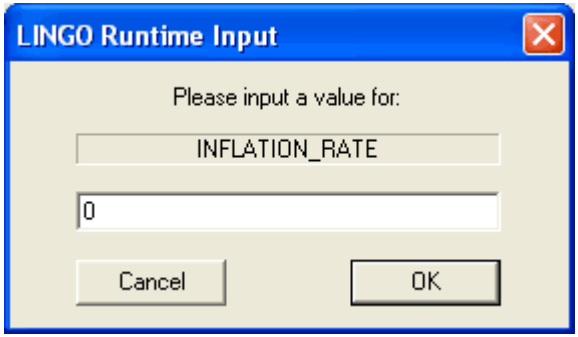
```
DATA:
   INTEREST_RATE, INFLATION_RATE = .085, .03;
ENDDATA
```

What If Analysis

In some cases, you may not be sure what values to input for the data in your model. For example, suppose your model uses the inflation rate as a parameter. You may be uncertain as to the exact rate of inflation in the future, but you know it will most likely fall within a range of 2 to 6 percent. What you would like to do is run your model for various values of the inflation rate within this range to see how sensitive the model's results are to inflation. We refer to this as *what if analysis*, and LINGO has a feature to facilitate this. To set up a parameter for what if analysis, input a question mark (?) as its value in place of a number as in the following example:

```
DATA:
   INFLATION_RATE = ?;
ENDDATA
```

LINGO will prompt you for a value for the *INFLATION_RATE* parameter each time you solve the model. Under Windows, you will receive a dialog box resembling:



Simply input the desired value for the inflation rate and then press the *OK* button. LINGO will then set *INFLATION_RATE* to the value you input and proceed with solving the model.

On platforms other than Windows, LINGO will write a prompt to your screen asking you to input a value for *INFLATION_RATE*. Type in the value and then press the *Enter* key.

In addition to parameters, you can perform what if analysis on individual members of attributes by initializing them to question marks in the data section, as well.

For an example of a model that uses what if analysis to compute the value of a home mortgage, see the *Home Mortgage Calculation* model in Appendix A, *Additional Examples of LINGO Modeling*.

Initializing an Attribute to a Single Value

Suppose you want to fix all the elements of an attribute to a single value. You can do this by entering a single value on the right-hand side of the data statement. LINGO will initialize all the elements of the attribute to this value. To perform what if analysis on the attribute, initialize it to a single question mark and LINGO will prompt you for the values of all the members of the attribute each time the model is solved.

As an example, consider the following excerpt from a model:

```
SETS:
   DAYS / MO, TU, WE, TH, FR, SA, SU/:
    NEEDS;
ENDSETS

DATA:
   NEEDS = 20;
ENDDATA
```

LINGO will initialize all the members of the *NEEDS* attribute to the value 20.

If there are multiple attributes on the left-hand side of the data statement, you will need one value on the right-hand side for each attribute on the left. For instance, let's extend the previous example, so we have an additional attribute called *COST*:

```
SETS:
   DAYS / MO, TU, WE, TH, FR, SA, SU/:
    NEEDS, COST;
ENDSETS

DATA:
   NEEDS, COST = 20, 100;
ENDDATA
```

All seven members of *NEEDS* will be initialized to 20 and all seven members of *COST* to 100.

Omitting Values in a Data Section

You can omit values in a data statement to indicate that you don't want to fix the values of particular members. For instance, suppose you have a manufacturing company and you need to do some capacity planning for the next 5 years. Furthermore, suppose it takes some time to boost capacity. As such, it would be impossible to increase capacity over the next two years. In such a case, you might do something like the following:

```
SETS:
   YEARS /1..5/: CAPACITY;
ENDSETS
DATA:
   CAPACITY = 34, 34, , , ;
ENDDATA
```

We have set *CAPACITY* for the first two years to 34, but have omitted values for the last three years. LINGO will assume, therefore, that it is free to determine the values for *CAPACITY* in the last three years.

Note: You must use commas when omitting values. If you do not use the commas, LINGO will think you did not enter the correct number of values for the attribute, which will trigger an error message.

The INIT Section of a Model

The *INIT* section is another optional section offered by LINGO. In the init section, you enter initialization statements that look much like the data statements found in the data section. The values you input in the init section are used as starting points by LINGO's solver. Unlike the variables that are initialized in the data section, the solver is free to alter the values of variables initialized in the init section.

Note: *Starting points specified in an INIT section are only of use in nonlinear or integer models.* Starting points currently offer no help in purely linear models. If you are not sure whether your model is linear or nonlinear, you can check the count of nonlinear constraints in the solver status window. If there are any nonlinear constraints, then your model is nonlinear. For more information on the nature of nonlinear models and how good starting points can be of assistance, please see Chapter 15, *On Mathematical Modeling*.

Note: As an example, in a set defining a group of stocks, you may have a known price of each stock, but the amount to buy or sell of each stock is unknown. You would typically initialize the price attribute in the data section. If approximate values of the buy and sell attributes are known, you can tell LINGO this information by entering it in the init section. LINGO then uses the values specified as a starting point in its search for the optimal solution. If your starting point is relatively close to an optimal solution, you may save on the solution time required to run your model.

An init section begins with the keyword *INIT:* and ends with the keyword *ENDINIT*. The syntax rules for init statements in the init section are identical to the rules for data section statements. You can have multiple attributes on the left-hand side of a statement, you can initialize an entire attribute to a single value, you can omit values in an attribute, and you can use the question mark to have LINGO prompt you for an initialization value whenever you solve the model.

As an example of how a good starting point may help to reduce solution times, consider the small model:

```
Y <= @LOG(X);
X^2 + Y^2 <=1;
```

The function *@LOG(X)* returns the natural logarithm of *X*. This model has only one feasible point of (X,Y) = (1,0). If we solve this model without an init section, we get the solution:

```
Feasible solution found at step:       12
     Variable           Value
            Y       0.5721349E-03
            X        1.000419
```

Note that it required 12 iterations to solve. Now, let's add an init section to initialize *X* and *Y* to a point close to the solution, so we have:

```
INIT:
   X = .999;
   Y = .002;
ENDINIT

Y <= @LOG(X);
X^2 + Y^2 <=1;
```

Solving this modified model, we get the solution:

```
Feasible solution found at step:        3
     Variable           Value
            X       0.9999995
            Y       0.0000000
```

Note our solution required only 3 iterations compared to the 12 iterations required without the init section.

Note: Variables may also be initialized in a calc section as illustrated in the next section, *The Calc Section of a Model.*

The CALC Section of a Model

In many instances, your model's raw input data will need additional massaging to get it into the proper form. As an example, suppose your raw data consists of daily observations of a number of securities' closing prices. Furthermore, let's suppose that your model ultimately requires the covariance matrix for the securities to be computed from the raw closing price data. You could certainly compute the covariance matrix as part of the constraint section in your model. However, entering simple computations as constraints will make your model artificially large. Another option, although inconvenient, would be to compute the covariance matrix outside of LINGO and pass it to LINGO as external data. Actually, what you would really like is a section in LINGO to perform data manipulation in such a way that it doesn't increase the size of the final optimization model passed through to the solver engine. This is the function of the calc section.

A *CALC* section begins with the keyword *CALC:* and ends with the keyword *ENDCALC*. You may input any expression in a calc section that you would in the constraint section of a model. However, each expression must be in the form of an assignment statement. In an assignment statement, a single variable appears on the left-hand side of an expression, followed by an equality sign, followed by an

arbitrary mathematical expression on the right-hand side. Furthermore, the right-hand side expression may only contain references to variables that are set as part of the model's input data (i.e., set in a previous data section or calc expression.)

As an example, here's a model with a calc section that computes the average of three variables:

```
MODEL:

DATA:
   X, Y, Z = 1, 2, 3;
ENDDATA

CALC:
   AVG = ( X + Y + Z) / 3;
ENDCALC

END
```

Example of a *valid* calc section

Now, suppose we did not know the value of *Y* beforehand. The following model with *Y* dropped from the data section would trigger an error in LINGO. The error occurs because the value of *Y* is an unknown, which violates the requirement that all right-hand side variables in a calc expression must have already had their values established in a previous data or calc section:

```
MODEL:

DATA:
   X, Z = 1, 3;
ENDDATA

CALC:
   AVG = ( X + Y + Z) / 3;
ENDCALC

END
```

Example of an invalid calc section

You may perform *running calculations* in a calc section, which means that you may break complex calc expressions down into a series of smaller expressions. Here we break the computation from above into two steps:

```
MODEL:

DATA:
   X, Y, Z = 1, 2, 3;
ENDDATA

CALC:
   AVG = X + Y + Z;
   AVG = AVG / 3;
ENDCALC

END
```

Example of a running calc expression

There is no limit to the number of times that a variable may appear on the left-hand side of a calc expression. However, the final calc expression for the variable will determine its value in the final solution report.

Calc expressions are computed sequentially in the order in which they appear in the model. So, if one calc expression feeds its value into a subsequent expression, then it must appear before its dependent expression. For example, the following calc section is valid:

```
CALC:
   X = 1;
   Y = X + 1;
ENDCALC
```

while this variation is not valid:

```
CALC:
   Y = X + 1;
   X = 1;
ENDCALC
```

In the second example, *Y* depends on *X*, but *X* is not defined until after *Y*.

Of course, set looping functions may also be used in calc expressions. For example, consider the following portfolio optimization model. In this model, we take the annual returns for three stocks and in a calc section compute the following three pieces of information for the stocks: average return, the covariance matrix, and the correlation matrix. This information is then used in a standard *Markowitz model* to determine an optimal portfolio that meets a desired level of return while minimizing overall risk.

```
MODEL:

SETS:
   STOCKS: AVG_RET, WEIGHT;
   DAYS;
   SXD( DAYS, STOCKS): RETURN;
   SXS( STOCKS, STOCKS): COVR, CORR;
ENDSETS

DATA:
   DAYS   = 1..12;
   TARGET = .15;
   STOCKS =   ATT     GMC      USX;
   RETURN = 0.300   0.225    0.149
            0.103   0.290    0.260
            0.216   0.216    0.419
           -0.046  -0.272   -0.078
           -0.071   0.144    0.169
            0.056   0.107   -0.035
            0.038   0.321    0.133
            0.089   0.305    0.732
            0.090   0.195    0.021
            0.083   0.390    0.131
            0.035  -0.072    0.006
            0.176   0.715    0.908;
```

```
ENDDATA

CALC:
   !Average annual return for each stock;
   @FOR( STOCKS( S):
    AVG_RET( S) =
     ( @SUM( SXD( D, S): RETURN( D, S)) /
      @SIZE( DAYS))
   );
   !Covariance matrix;
   @FOR( SXS( S1, S2):
    COVR( S1, S2) =
     @SUM( DAYS( D):( RETURN( D, S1) - AVG_RET( S1)) *
      ( RETURN( D, S2) - AVG_RET( S2))) / @SIZE( DAYS)
   );
   !Although not required, compute the correlation matrix;
   @FOR( SXS( S1, S2):
    CORR( S1, S2) = COVR( S1, S2) /
    ( COVR( S1, S1) * COVR( S2, S2))^.5;
   );
ENDCALC
!Minimize the risk of the portfolio
 (i.e., its variance);
[R_OBJ] MIN = @SUM( SXS( S1, S2):
 WEIGHT( S1) * WEIGHT( S2) * COVR( S1, S2));
!Must be fully invested;
[R_BUDGET] @SUM( STOCKS: WEIGHT) = 1;
!Must exceed target return;
[R_TARGET] @SUM( STOCKS: AVG_RET * WEIGHT) >= TARGET;

END
```

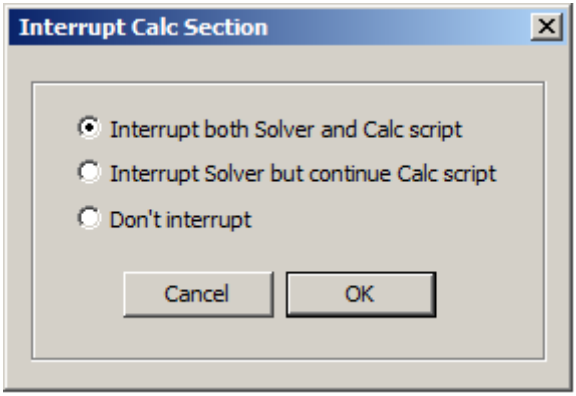
Model: MARKOW

We demonstrated how to initialize variable values in the previous topic on init sections. Variables may also be initialized in a calc section. The basic idea is to set the variable's value using one or more statements in a calc section. Once the variable's value has been set in a calc section, the variable will be marked as fixed in value, and will not be allowed to be changed the next time you issue a solve command. Obviously, we don't want an optimizeable to be a fixed variable. To correct this, we use the *@RELEASE* function to release the fixed variable so that it is once again optimizable. The following example illustrates:

```
MODEL:


SETS:
  S1: X;
ENDSETS

DATA:
  S1 = 1..5;
ENDDATA

CALC:
! Initialize the odd members of X to .5;
  @FOR( S1( I) | @MOD( I, 2) #NE# 0:
     X( I) = .5;
! The previous statement fixed X(I) to .5.
 We must now release it so that it remains
 optimizable;
     @RELEASE( X( I));
  );
ENDCALC

END
```

Interrupting Calc Sections

If you are processing a long-running calc section, say solving several large models in a loop, then at some point you may decide to interrupt processing. When you press the Interrupt Solver button on the Solver Status Window while processing a calc script, LINGO will display the dialog box:



The choices available here are:

- Interrupt both Solver and Calc script — This option interrupts both the solver and the calc script, meaning all processing stops and the solver returns control to the LINGO front-end.

- Interrupt solver but continue Calc script — This interrupts the solver, stopping optimization on the current model. Control, however, is not returned to the front-end, rather it gets returned to the calc script processor allowing the script to continue running and solving additional models.
- Don't interrupt — Here, the calc section picks up again where it left off before the interrupt was requested and processing continues normally.

Summary

You should now be comfortable with adding basic data, init and calc sections to your models. Keep in mind that initialization performed in a data section fixes a variable's value. Initialization done in an init section is used only as a temporary starting point, which may be of benefit in finding solutions to nonlinear models. Initialization in a calc section holds until another calc expression redefining a variable's value is encountered. The benefit of placing computations in a calc section as opposed to placing them in the general constraint section is that calc expressions are treated as side computations and aren't fed into the main solver, thereby improving execution times.

We have only touched on some of the basic features of data and init sections in this chapter. In subsequent sections, you will see how to add hooks in your data and init sections to external files, spreadsheets, and databases.

5 Menu Commands

In this section, we will discuss the pull down menu commands available in the Windows, Mac and Linux versions of LINGO. The *Command-line Commands* section deals with the commands available through LINGO's command-line interface. If you're not using a Windows, Mac or Linux version of LINGO, then you will be primarily interested in the command-line section. If you are using a Windows, Mac or Linux version, then you will be primarily interested in this section. Windows Mac and Linux users may also be interested in the command-line interface if they plan to build command scripts to automate LINGO.

Accessing Menu Commands

Menu commands may be accessed by either selecting them from a pull down menu, pressing the command's button in *the toolbar*, or, if applicable, entering the command's keyboard equivalent (also referred to as its accelerator key).

Menus

LINGO groups commands under the following five menus:

- *File*
- *Edit*
- *Solver*
- *Window*
- *Help*

The *File* menu contains commands that primarily deal with handling input and output. The *Edit* menu contains commands for editing the document in the current window. The *Solver* menu contains commands to solve a model and generate solution reports. The *Window* menu has commands that deal with the mechanics of handling multiple windows. The *Help* menu provides access to LINGO's online help facility.

In addition, the Mac has a *Lingo* menu. This menu is referred to as the application menu, and is a standard feature of Mac applications. The application menu typically contains commands to set preferences/options, the application's about box, and the quit command.

The Toolbar

By default, the toolbar runs along the top of the screen and provides a shortcut for accessing the most commonly used commands in the menus.

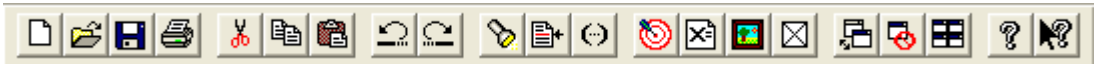
LINGO's toolbar "floats." Thus, you can reposition it by dragging it to any part of the screen. You can also choose to suppress the toolbar by clearing the *Toolbar* button on the *Interface tab* of the *Solver|Options* command.

Each button on the toolbar corresponds to a menu command. Not all menu commands have a toolbar button, but, in general, the most frequently used commands have an equivalent button.

LINGO displays "tool tips" for each button. When you position the mouse over a button, a short description of what the button does appears in a pop up window and in the status bar at the bottom of the screen.

The Windows Toolbar

LINGO's toolbar for its Windows version is illustrated in the following graphic:



Here is a list of the buttons and their equivalent commands:

File Menu:

	File|New		*File|Open*
	File|Save		*File|Print*

Edit Menu:

	Edit|Undo		*Edit|Redo*
	Edit|Cut		*Edit|Copy*
	Edit|Paste		*Edit|Find*
	Edit|Go To Line		*Edit|Match Parenthesis*

Solver Menu:

	Solver|Solve		*Solver|Solution*
	Solver|Options		*Solver|Picture*

Window Menu:

	Window|Send To Back		*Window|Close All*
	Window|Tile		

Help Menu:

	Help|Topics		*Help|Pointer*

The Mac and LinuxToolbar

LINGO's toolbar for its Mac and Linux versions is illustrated in the following graphic:

Here is a list of the buttons and their equivalent commands:

File Menu:

File|New

File|Open

File|Save

File|Print

Edit Menu:

Edit|Undo

Edit|Redo

Edit|Cut

Edit|Copy

Edit|Paste

Edit|Find

Edit|Match Parenthesis

Solver Menu:

Solver|Solve

Solver|Solution

Solver|Options

Solver|Picture

Window Menu:

Window|Close All

Window|Tile

Window|Previous

Help Menu:

Help|Topics

Accelerator Keys

Along with accessing commands via the menus and toolbar, most commands may also be accessed by a single, unique keystroke known as an accelerator. The equivalent accelerator key is listed alongside each command in the menus.

Menu Commands In Brief

In this section, we give a brief listing of the menu commands available in LINGO. The commands are categorized into the five main menus:

- *File*
- *Edit*
- *Solver*
- *Window*
- *Help*

Note:	Not all commands are available under all operating systems. The following command tables will indicate via check marks if a command is available in either Windows, Mac, or Linux.

File Menu Commands

Command				***Description***
New	✓	✓	✓	Opens a new model window.
Open	✓	✓	✓	Opens an existing model previously saved to disk.
Save	✓	✓	✓	Saves the contents of the current window to disk.
Save As	✓	✓	✓	Saves the contents of the current window to a new name.

Close	✓	✓	✓	Closes the current window.
Print	✓	✓	✓	Prints the contents of the current window.
Print Setup	✓	✓	✓	Configures your printer.
Print Preview	✓	✓	✓	Displays the contents of the current window as it would appear if printed.
Log Output	✓			Opens a log file for logging output to the command window.
Take Commands	✓			Runs a command script contained in a file.
Export File	✓	✓	✓	Exports a model in MPS or MPI file format.
License	✓	✓	✓	Prompts you for a new license password to upgrade your system.
Excel	✓			Allows you to load a model from Excel into a LINGO model window, or write a LINGO model to an Excel workbook range. This is useful for users that maintain embedded models in Excel workbooks.
Database User Info	✓			Prompts you for a user id and password for database access via the *@ODBC()* function.
Exit	✓	✓	✓	Exits LINGO.

Edit Menu Commands

Command				***Description***
Undo	✓	✓	✓	Undoes the last change.
Redo	✓	✓	✓	Redoes the last undo command.
Cut	✓	✓	✓	Cuts the current selection from the document.
Copy	✓	✓	✓	Copies the current selection to the clipboard.
Paste	✓	✓	✓	Pastes the contents of the clipboard into the document.
Paste Special	✓			Pastes the contents of the clipboard into the document, allowing choice as to how the object is pasted.
Select All	✓	✓	✓	Selects the entire contents of the current window.
Find	✓	✓	✓	Searches the document for the occurrence of a specified text string.

Find Next	✓	✓	✓	Repeats the find operation for the last string specified.
Replace	✓	✓	✓	Replaces a specified text string with a new string.
Go To Line	✓			Moves the cursor to a specified line number.
Match Parenthesis	✓	✓	✓	Finds the parenthesis that closes a selected parenthesis.
Paste Function	✓	✓	✓	Pastes a template of a selected LINGO function.
Select Font	✓	✓	✓	Specifies a font for a selected block of text.
Insert New Object	✓			Embeds an OLE object into the document.
Links	✓			Controls the links to external objects in your document.
Object Properties	✓			Specifies the properties of a selected, embedded object.

Solver Menu Commands

Command				***Description***
Solve	✓	✓	✓	Solves the model in the current window.
Solution	✓	✓	✓	Generates a solution report window for the current model.
Range	✓	✓	✓	Generates a range analysis report for the current window.
Options	✓	✓	✓	Sets system options.
Generate	✓	✓	✓	Generates the algebraic representation for the current model.
Picture	✓	✓	✓	Displays a graphical picture of the models matrix.
Debug	✓	✓	✓	Tracks down formulation errors in infeasible and unbounded linear programs.
Model Statistics	✓	✓	✓	Displays a brief report regarding the technical details of a model.
Look	✓			Generates a formulation report for the current window.

Window Menu Commands

Command				*Description*
Command Window	✓	✓	✓	Opens a command window for command-line operation of LINGO.
Status Window	✓	✓	✓	Opens the solver's status window.
Close All	✓	✓	✓	Closes all open windows.
Tile	✓	✓	✓	Arranges all open windows into a tiled pattern.
Cascade	✓	✓	✓	Arranges all open windows into a cascading pattern.
Next		✓	✓	Brings the next window to the front.
Previous	✓	✓	✓	Brings the previous window to the front.
Arrange Icons	✓			Aligns all iconized windows at the bottom of the main frame window.

Help Menu Commands

Command				*Description*
Help Topics	✓	✓	✓	Accesses LINGO's online help facility.
Register	✓			Registers your version of LINGO online.
AutoUpdate	✓			Checks to see if an updated copy of LINGO is available for download on the LINDO Systems Web site
About LINGO	✓	✓	✓	Displays the version and size of your copy of LINGO, along Systems.with information on how to contact LINDO

Menu Commands In Depth

In the remainder of this section, we will document all the LINGO commands specific to the Windows, Mac and Linux versions. The commands are categorized by the five main menus:

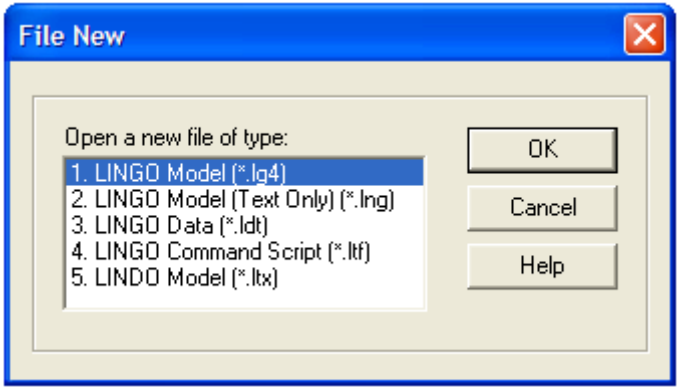
- *File*
- *Edit*
- *Solver*
- *Window*
- *Help*

1. File Menu

LINGO's *File* menu contains commands that generally pertain to the movement of files in and out of LINGO.

File|New

The *New* command opens a new, blank window. When you select the *New* command, you will be presented with the following dialog box:



You may then select the type of file you want to create. The file must be one of the four types:

1. LINGO Model (*.lg4)
The LG4 format was established with release 4.0 of LINGO. LG4 is the primary file format used by LINGO to store models under Windows and is not used on other platforms. This format supports multiple fonts, custom formatting, and Object Linking and Embedding (OLE). LG4 files are saved to disk using a custom binary format. Therefore, these files can't be read directly into other applications or transported to platforms other than Windows. Use the LNG format (discussed next) to port a file to other applications or platforms.

2. LINGO Model (Text Only) (*.lng)
The LNG format is a portable format for storing your models. It is the standard file format used by LINGO on non-Windows platforms. LNG files are saved to disk as ASCII text and may be read into any application or word processor that supports text files. The LNG file format is supported by LINGO on all platforms, and LNG files can be ported from one platform to another. LNG files do not support multiple fonts, custom formatting, or OLE embedding of objects.

3. LINGO Data (*.ldt)
LDT files are data files that are typically imported into LINGO models using the *@FILE* function. *@FILE* can only read text files. Given this, all LDT files are stored as ASCII text. LDT files do not support multiple fonts, custom formatting, or OLE embedding.

4. LINGO Command Script (*.ltf)
LTF files are LINGO command scripts. These are ASCII text files containing a series of LINGO commands that can be executed with the *File|Take Commands* command. For more information on commands that can be used in a LINGO script, refer to *Command-line Commands*. LTF files do not support multiple fonts, custom formatting, or OLE.

5. LINDO Model (*.ltx)
LTX files are model files that use the LINDO syntax. Longtime LINDO users may prefer LINDO syntax over LINGO syntax. LINDO syntax is convenient for quickly entering small to medium-sized linear programs. As long as a file has an extension of .ltx, LINGO will assume that the model is written using LINDO syntax. Readers interested in the details of LINDO syntax may contact LINDO Systems to obtain a LINDO user's manual.

When you simply press either the *New* toolbar button or the F2 key, LINGO assumes you want a model file. Thus, LINGO does not display the file type dialog box and immediately opens a model file of type LG4.

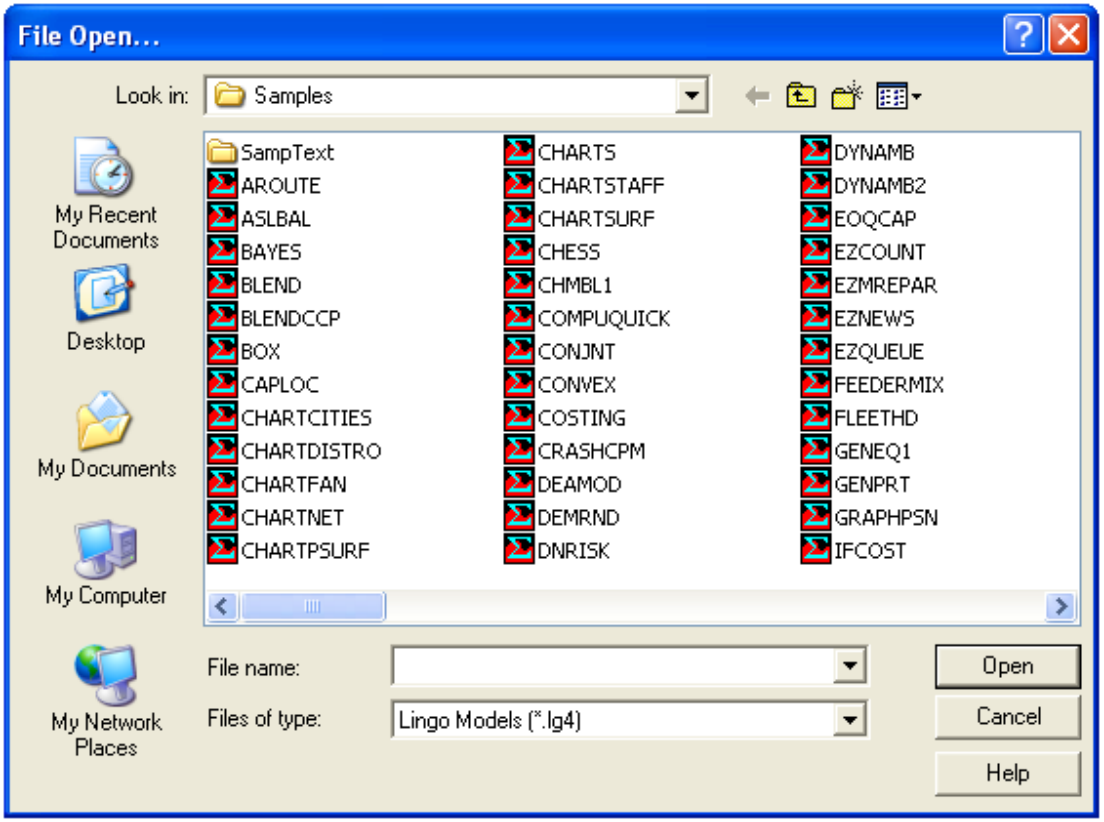
If you have used the *Solver|Options* command to change the default model file format from LG4 to LNG, LINGO will automatically open a model of type LNG when you press either the *New* button or the F2 key.

You may begin entering text directly into a new model window or paste in text from other applications using the Windows clipboard and the *Edit|Paste* command in LINGO.

File|Open

The *Open* command reads a saved file from disk and places it in a LINGO Window. The file can be a LINGO model file (*.LG4), or any other file. If the file is not in LG4 format, it must be in ASCII text format.

After issuing the *Open* command, you will be presented with a dialog box resembling the following:



You can enter a file name in the *File name* edit box, or select a file name from the list of existing files by double-clicking on a file. Press the *Open* button to open the file, the *Cancel* button to exit without opening a file, or the *Help* button for assistance. You may select a different file type from the *Files of type* list box causing LINGO to list only the files of that type. Once you have read in a LINGO model file (a LG4 or LNG file), you may use the *Solver|Solve* command to solve the model.

In addition to its native LG4 and LNG file formats, LINGO supports the following three additional file formats :

- **MPS** - The MPS file format is an industry standard format developed by IBM, which is useful for passing linear and quadratic models from one solver or platform to another.
- **MPI** - The MPI format was developed by LINDO Systems as a portable format for representing arbitrary math programming models.
- **LP** - The LP file format is another industry standard file format, as is MPS, for storing linear and quadratic model. However, with LP format equations are stored using standard algebraic format, making LP files much easier to read and interpret than MPS files. At present, LINGO only supports linear models when reading LP format files.

If the file to be opened has an extension of .MPS, .MPI or .LP, then LINGO will invoke, respectively, its MPS, MPI or LP reader to parse the file. When importing a non-native file, LINGO reads the file from disk, converts it to an equivalent LINGO model, and places the model into a new model window. More details follow immediately below. LINGO can also write MPS and MPI format files (but not LP files); this is discussed in the *File|Export File* section below.

Importing MPS, MPI or LP Files

When LINGO reads an MPS, MPI or LP file, it converts the formulation to an equivalent LINGO model. As an example, consider the following, simple model:

```
ObjRow) Maximize 20X + 30Y
Subject To:
  Row1) X < 50
  Row2) Y < 60
  Row3) X + 2Y < 120
```

The MPS file for this model is:

```
NAME          SAMPLE
OBJSENSE
   MAX
ROWS
 N OBJROW
 L ROW1
 L ROW2
 L ROW3
COLUMNS
    X            ROW3              1.0000000
    X            OBJROW           20.0000000
    X            ROW1              1.0000000
    Y            OBJROW           30.0000000
    Y            ROW2              1.0000000
    Y            ROW3              2.0000000
RHS
    RHS          ROW1             50.0000000
    RHS          ROW2             60.0000000
    RHS          ROW3            120.0000000
ENDATA
```

The MPI version of the model is:

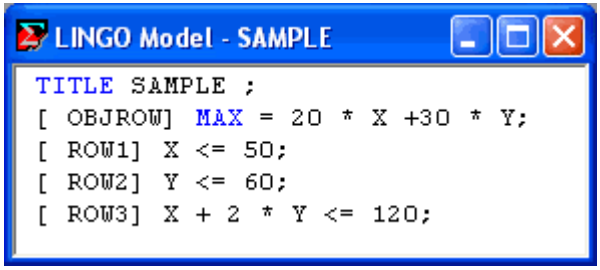
```
BEGINMODEL SAMPLE
! Number of Objective Functions: 1
! Number of Constraints : 3
! Number of Variables : 2
VARIABLES
!   Name Lower Bound  Initial Point Upper Bound   Type
     X           0         1.23457     1e+030       C
     Y           0         1.23457     1e+030       C
OBJECTIVES
OBJROW       MAXIMIZE
   EP_USRCOD              -101
   EP_PUSH_NUM      20
   EP_PUSH_VAR  X
   EP_MULTIPLY
   EP_PUSH_NUM      30
   EP_PUSH_VAR  Y
   EP_MULTIPLY
   EP_PLUS
CONSTRAINTS
 ROW1    L
   EP_USRCOD              -101
   EP_PUSH_VAR  X
   EP_PUSH_NUM      50
   EP_MINUS
 ROW2    L
   EP_USRCOD              -101
   EP_PUSH_VAR  Y
   EP_PUSH_NUM      60
   EP_MINUS
 ROW3    L
   EP_USRCOD              -101
   EP_PUSH_VAR  X
   EP_PUSH_NUM      2
   EP_PUSH_VAR  Y
   EP_MULTIPLY
   EP_PLUS
   EP_PUSH_NUM      120
   EP_MINUS
ENDMODEL
```

And, the LP format version of the model is:

```
\ LP format example
Maximize
 objrow: 20x + 30y
Subject To
 row1: x <= 50
 row2: y <= 60
 row3: x + 2y <= 120
END
```

One thing to notice at this point is that MPS and MPI formats are not very compact methods for storing a model - they are designed for portability, as opposed to efficiency.

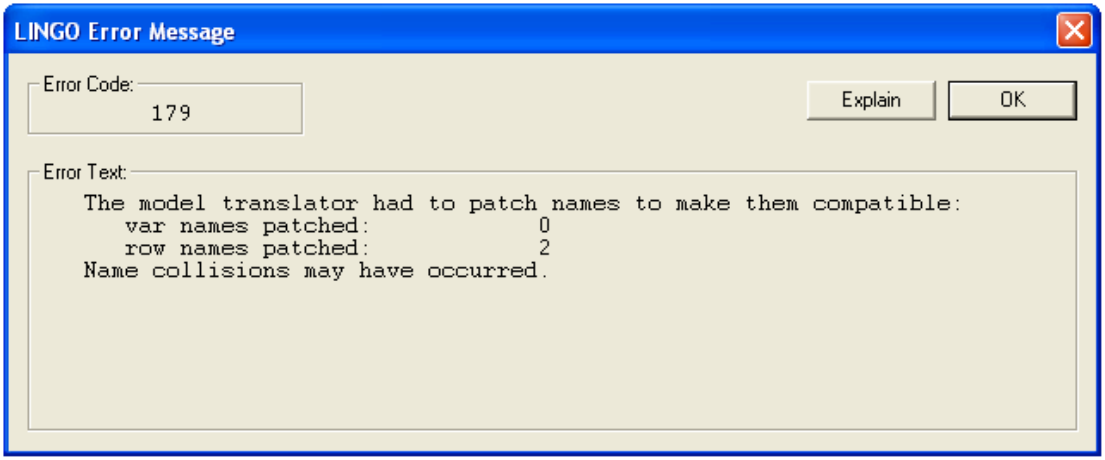
Using the *File|Open* command to read either of these three versions of the model into LINGO, we are presented with the following window containing an equivalent LINGO model:



Note how the model is automatically converted from MPS, MPI or LP format to native LINGO format. Should you wish to save the file again using either MPS or MPI format rather than LINGO format, you must use the *File|Export File|MPS Format…* command.

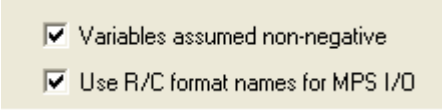
Note:	The MPS, MPI and LP file formats are intended primarily for exporting (importing) models to (from) other applications or platforms. These file formats are purely scalar in nature—all set-based information is lost upon saving a LINGO model in either MPS or MPI format (LINGO does not currently write LP format files). Thus, when saving copies of a model on your own machine, you should always use the File|Save command in order to save models in a native LINGO format (LG4 or LNG) in order to preserve your model in its entirety.

When it comes to acceptable constraint and variable names, the MPS, MPI and LP formats are less restrictive than LINGO. To compensate for this fact, LINGO attempts to patch names when reading a file, so that all the incoming names are compatible with its syntax. LINGO does this by substituting an underscore for any character in a name that is not admissible. In most cases, this will work out OK. However, there is a chance for name collisions where two or more names get mapped into one. For instance, the variable names *X.1* and *X%1* would both get mapped into the single LINGO name *X_1*. Of course, situations such as this entirely alter the structure of the model, rendering it incorrect. However, you will be warned whenever LINGO has to patch a name with the following error message:

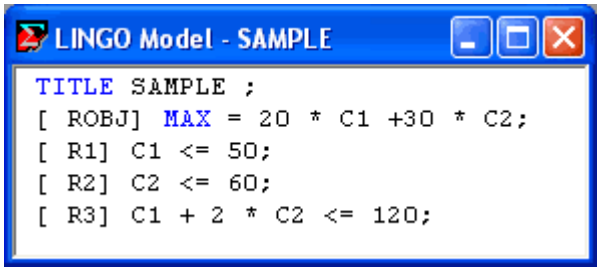


This message displays the number of variable and row names that were patched to get them to conform to LINGO syntax.

If name collisions are a problem, then LINGO has an option that will ensure that all names remain unique. This option involves using *RC format* for names encountered during MPS I/O. *RC format* involves renaming each row (constraint) in a model to be *Rn*, where *n* is the row's index. Similarly, each column (variable) is renamed to *Cn*. In addition, LINGO renames the objective row to be *ROBJ*. To switch to *RC format for MPS names*, run the *Solver|Options* command, select the *General Solver tab*, then click the checkbox titled *Use R/C format names for MPS I/O*, as illustrated here:



As an example, we will once again import the same MPS format model as above. However, this time we will use RC naming conventions. Here is the model as it appears after importing it into LINGO:



Notice how the variable names now use RC format, guaranteeing that name collisions will not occur. Another potential conflict is that MPS and MPI allow variable names to be duplicated as constraint names and vice versa. LINGO does not allow for this. When you go to solve the model, you will receive either error message 28 (Invalid use of a row name), or error message 37 (Name already in use). Once again, you can switch to using RC name format to avoid this conflict.

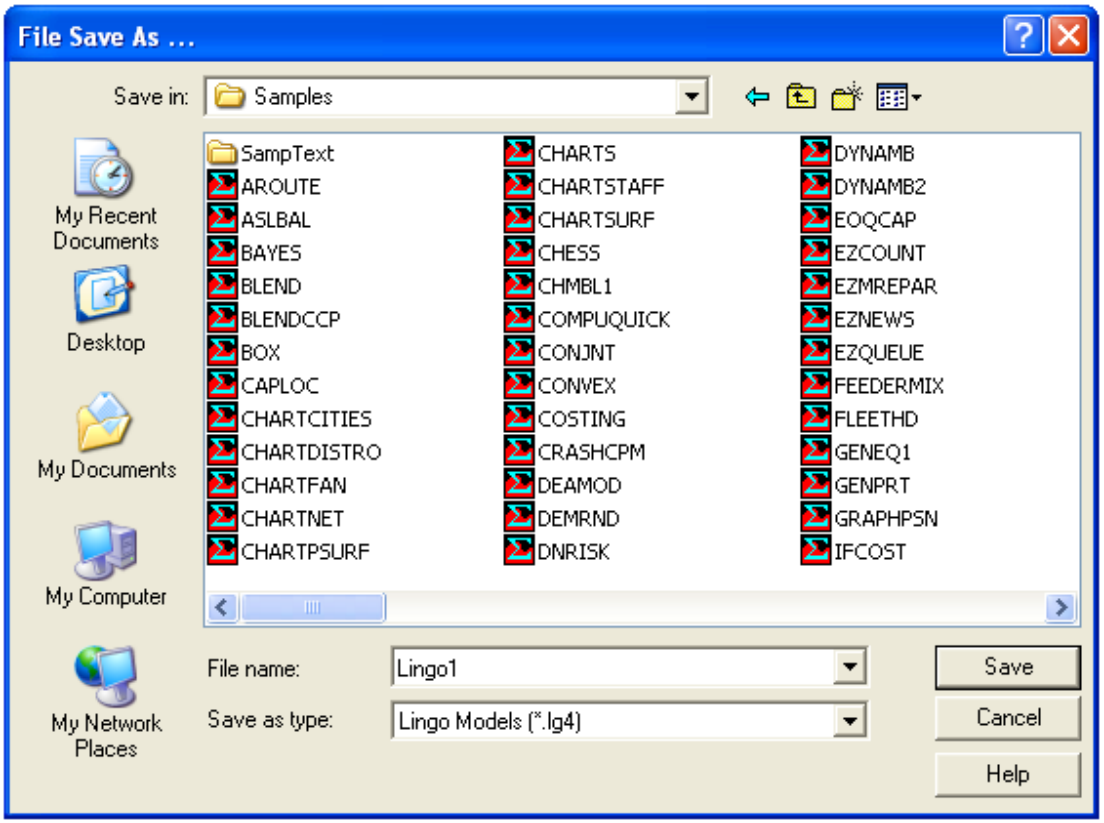
As a final note, LINGO only supports free format MPS files, and does not support fixed format MPS files. Therefore, variable and row names may not contain embedded blanks.

File|Save

The *Save* command saves the contents of the active window to disk using the existing file name for the window. If the window has not been saved before, you will be prompted to provide a name for the file.

File|Save As

The *Save As* command allows you to save the contents of the active window under a new file name. When issuing the *Save As* command, you will be presented with a dialog box that resembles the following:



You can enter a new file name in the *File name* edit box, or select a file name from the list of existing files by double-clicking on it. If you do not specify a file extension, LINGO will append the extension of the default model format to the name. If you want to prevent LINGO from appending an extension to the name, place the file name in double quotes.

Press the *Save* button to save the model, the *Cancel* button to exit without saving, or the *Help* button for assistance.

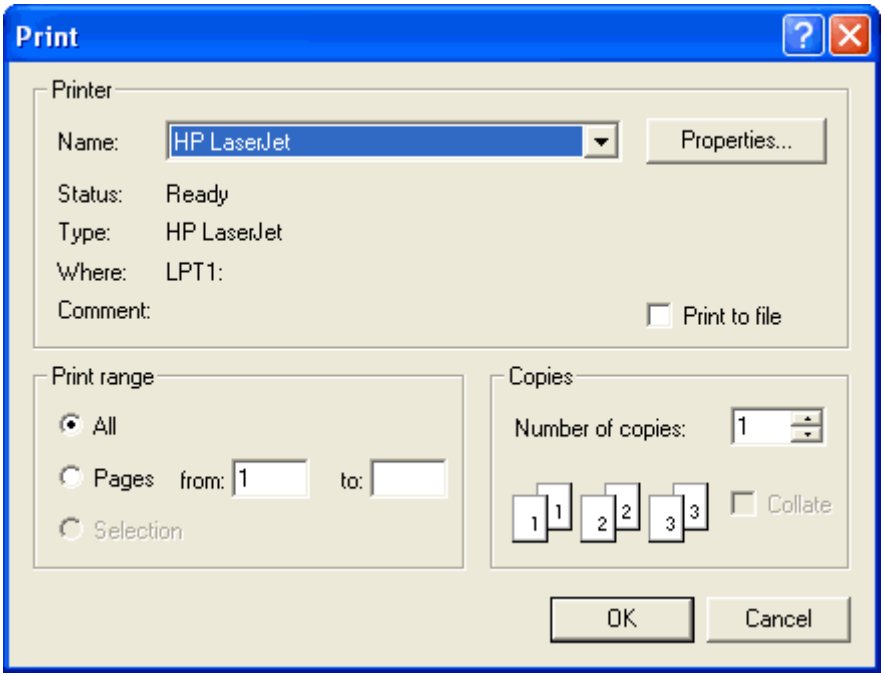
You may select a different file type from the *Save as type* list box. If your model has special fonts or embedded objects, you must save it using the LG4 file format to preserve them. The LG4 format is a special binary format readable only by LINGO. If you wish to create a text copy of your model, then use the LNG file format. For further discussion of the available file formats under LINGO, refer to the *New* command.

File|Close

Use the *Close* command to close the active (frontmost) window. If the window has been modified without being saved, you'll be asked whether you want to save the changes.

File|Print F7

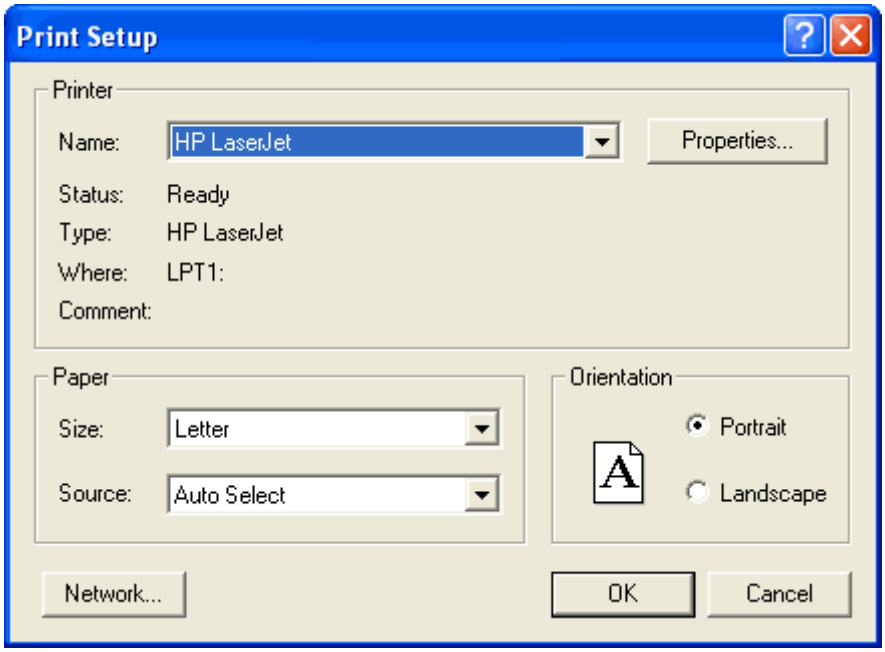
Use the *Print* command to send the contents of the active window to your printer. First, LINGO will display the *Print* dialog box:



Select the printer to route the output to from the *Name* list box. Modify the printer's properties by pressing the *Properties* button. Select a range of pages to print in the *Print Range* group box. If you need multiple copies, input the number desired in the *Number of copies* field and specify if you want the copies collated (assuming your printer is capable of collating). Finally, press the *OK* button to begin printing. Press the *Cancel* button to exit without printing.

File|Print Setup F8

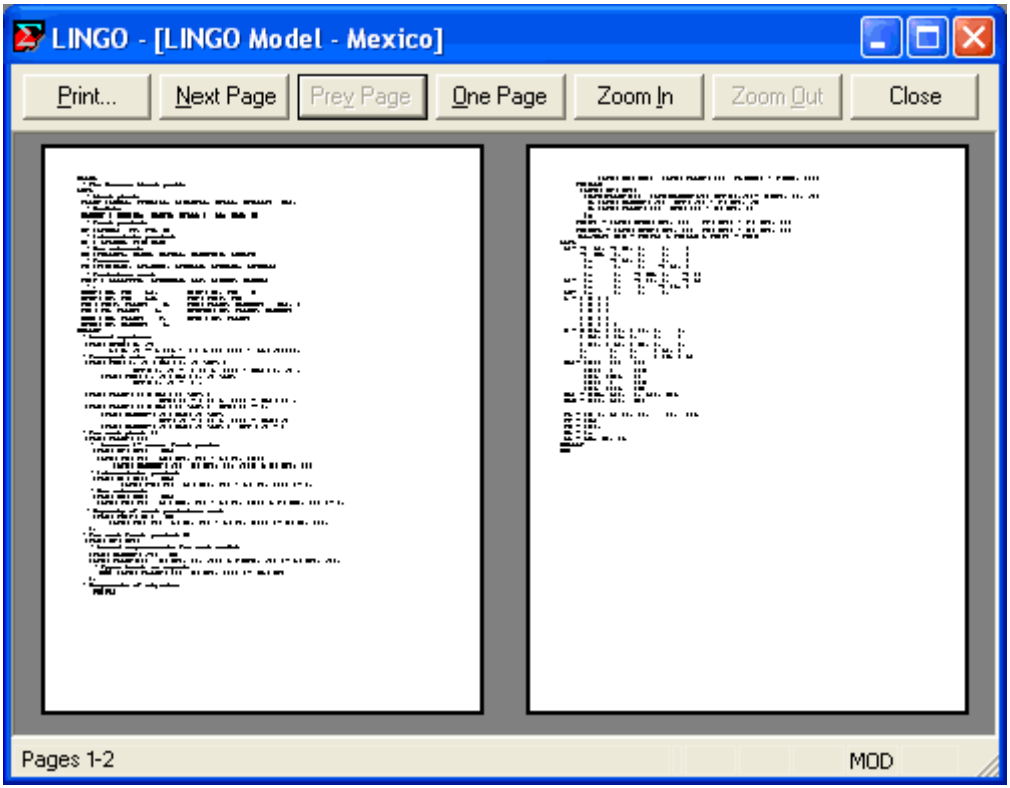
Use the *Print Setup* command to configure your printer. You should see a dialog box that resembles the following:



Select the target printer from the *Name* list box. Press the *Properties* button to set additional printer properties. Select the type of paper and tray from the *Paper* group box. In the *Orientation* group box, select whether you want portrait or landscape output. Press the *Cancel* button to exit without changing the printer configuration. Press the *OK* button to save your changes and exit the *Print Setup* command.

File|Print Preview Shift+F8

Use the *Print Preview* command to display each page of the active window as it will appear when printed. After issuing the *Print Preview* command, the contents of the active window will be placed in a Preview window as follows:



The *Print* button sends the file to the printer. The *Next Page* button brings the next page into the viewer. The *Prev Page* button brings the previous page into the viewer. The *One Page* button puts the viewer into single page mode, while the *Two Page* button puts the viewer into double page mode. The *Zoom In* button is used to have the viewer zoom in on a region of the document. The *Zoom Out* button undoes the effect of a *Zoom In*. Press the *Close* button to close the print viewer and return to the normal command mode of LINGO.

If you would like to change some of the printer specifications, such as landscape output, use the *Print Setup* command before issuing the *Print Preview* command.

File|Log Output

Normally, when you are using LINGO, it is operating in a menu driven mode, where you choose commands from the pull down menus and reports are displayed in individual windows. LINGO can also operate in command mode, where text commands or command script files drive the application

and all output is routed to a window known as the *command window*. All input and output passes through the command window when LINGO is in command mode. You can open a command window at anytime by issuing the *Window|Command Window* command.

In general, you will only be interested in running LINGO in command mode if you are planning to embed LINGO in a larger application. If you do use LINGO in command mode, you will find the command window can only hold a limited amount of output. Should you need to keep a disk based copy of all that transpires in the command window, you will need to use the *Log Output* command.

The *Log Output* command opens a standard Windows file dialog from which you can name the log file. You can echo the output to the command window as well as the file by checking the *Echo to screen* checkbox. If you would like to append output to the end of an existing file, check the *Append output* checkbox.

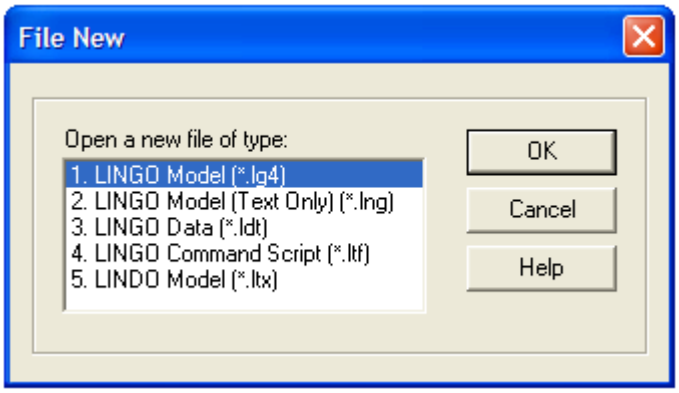
When you have selected a file for logging output, a check mark will appear in the *File* menu before the *Log Output* command. To turn off *Log Output*, select the command again and the check mark will disappear.

File|Take Commands

The *Take Commands* command is used to submit a LINGO command script file for processing. For more information on LINGO's script language, refer to *Command-line Commands*.

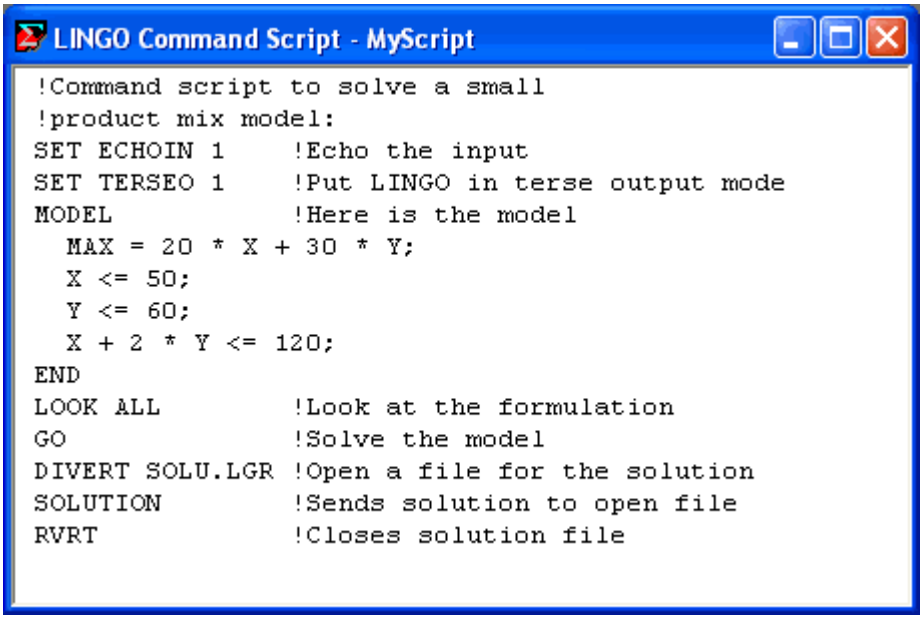
As an example, we will build a small script file that contains a small product mix model and process it using *Take Commands.*

To build a script file, issue the *File|New* command. LINGO will present you with the following dialog box:



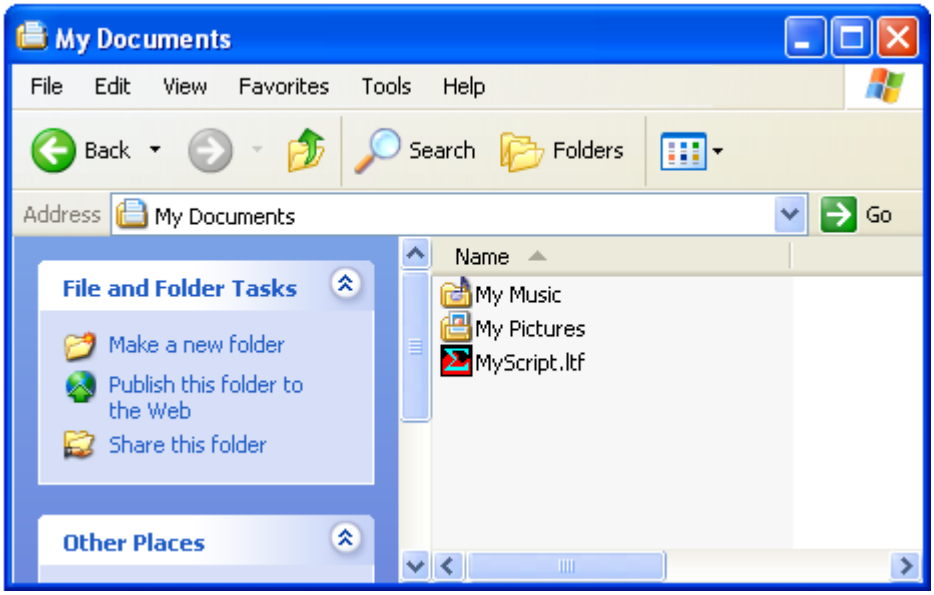
Select item 4, *LINGO Command Script*, and press the *OK* button. LINGO will open a blank script file.

Now, enter the following into the script file:



This is a command script that inputs a small product mix model, solves it, and puts the solution in a text file. Save the command script to a file titled *MySCript.ltf* using the *File|Save As* command.

To run the script, issue the *File|Take Command*s command. You should see the following:



Double-click on the icon for *MyScript.ltf* to begin processing the command script. LINGO's command window will now appear and you should be able to watch LINGO's progress at processing the script by watching commands and output as they are logged in the command window. When LINGO finishes the command script, the command window will resemble the following:

Command Window

```
: : !Command script to solve a small
: !product mix model:
: SET ECHOIN 1     !Echo the input

 Parameter         Old Value      New Value
   ECHOIN              1              1

: SET TERSEO 1     !Put LINGO in terse output mode

 Parameter         Old Value      New Value
   TERSEO              1              1

: MODEL            !Here is the model
?   MAX = 20 * X + 30 * Y;
?   X <= 50;
?   Y <= 60;
?   X + 2 * Y <= 120;
? END
: LOOK ALL         !Look at the formulation

   1]  MAX = 20 * X + 30 * Y;
   2]  X <= 50;
   3]  Y <= 60;
   4]  X + 2 * Y <= 120;

: GO               !Solve the model

 Global optimal solution found at step:             2
 Objective value:                            2050.000

: DIVERT SOLU.LGR !Open a file for the solution
: SOLUTION         !Sends solution to open file
: RVRT             !Closes solution file
:
```

Also of interest is the solution file, *SOLU.LGR*, created as part of our command script. If you open this file, you should find the following solution to the model:

Variable	Value	Reduced Cost
X	50.00000	0.000000
Y	35.00000	0.000000

Row	Slack or Surplus	Dual Price
1	2050.000	1.000000
2	0.000000	5.000000
3	25.00000	0.000000
4	0.000000	15.00000

The output that was routed to the command window can be routed to a file using the *Log Output* command.

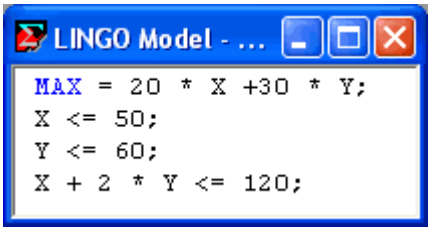
File|Export File

The *File|Export File…* command allows you to export either MPS or MPI files. The MPS file format is an industry standard format developed by IBM, and is useful for passing models from one solver or platform to another. MPI file format was developed by LINDO Systems as a way to store all math programs, from linear models to, in particular, nonlinear models.

Exporting MPS Files

The *File|Export File|MPS format* command generates the underlying algebraic formulation for the current model and then writes it to a selected file in MPS format. MPS format is a common format for representing linear programming models. MPS files can be ported to any solver that reads MPS files—this includes most commercial linear programming packages.

As an example of exporting an MPS file, consider the model:



After issuing the *File|Export File|MPS format* command and opening the file containing the MPS model, we will find:

```
NAME        NO_TITLE
****************************************************
* NOTICE: Generated by the MPS export utility for
* a maximization type problem.
*
* The objective coefficients have flipped signs.
* Interpret the objective value from the solution of
* this model accordingly.
****************************************************
ROWS
N 1
L 2
L 3
L 4
COLUMNS
   X  1            -20
   X  2              1
   X  4              1
   Y  1            -30
   Y  3              1
   Y  4              2
RHS
   RHS1     2              50
   RHS1     3              60
   RHS1     4             120
BOUNDS
ENDATA
```

Note: A model must be linear or quadratic to successfully export it in MPS format.

Note: The MPS file format is intended primarily for exporting models to other applications or platforms. The MPS format is purely scalar in nature—all set-based information is lost upon converting a LINGO model to MPS format. Thus, when saving copies of a model on your own machine, you should always use the File|Save command in order to preserve your model in its entirety.

Note: When exporting a stochastic program to MPS format, LINGO will write a total of three SMPS format files, consisting of the core model, its stochastic declarations and its time structure.

Exporting MPI Files

The MPI (Math Programming Interface) file format was developed by LINDO Systems as a way to store all math programs, from linear models to, in particular, nonlinear models. As with MPS files, the MPI format is scalar based. Thus, you will lose any sets in your model when saving it in this format. Most users will not have a need for MPI formatted files. However, LINDO API users can load these files directly and may find this feature useful.

Note:	When exporting a stochastic program to MPI format, LINGO will write a total of four SMPI format files, consisting of a header file, the core model, its stochastic declarations and its time structure.

File|License

Some versions of LINGO require the user to input a license key. Also, if you upgrade your copy of LINGO, then you will need to enter a new password. The *File|License* command prompts you for a new license key.

When you run the *File|License* command, you will be presented with the dialog box:

Carefully enter the key into the edit field, including hyphens, making sure that each character is correct. Click the *OK* button and, assuming the password was entered correctly, LINGO will display the *About LINGO* dialog box listing the features in the upgraded license. Verify that these features correspond to the license you intended to install.

Note: If you were emailed your license key, then you have the option of cutting-and-pasting it into the dialog box.

File|Excel

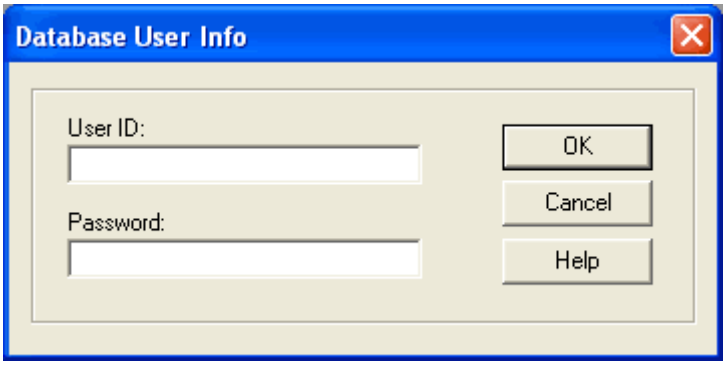
On Windows, LINGO has some useful links to Excel, which are discussed in more detail in the *Interfacing With Spreadsheets* chapter. One of these features involves embedding a LINGO model in an Excel workbook. LINGO may then be accessed directly from Excel via a VBA macro. The end result is that the model appears to be seemlessly linked to Excel, which is helpful for users more familiar with Excel than they are with LINGO.

The *File|Excel* command is useful for LINGO models embedded in Excel, in that it facilitates the movement of models back and forth from Excel for testing within the LINGO GUI. The *File|Excel|Write Model to Range* command sends models from LINGO to a range within a specified Excel workbook. Whereas, the *File|Excel|Read Model from Range* loads a model from an Excel range into a LINGO model window. Examples of these commands may be found in the section *OLE Automation Links from Excel*.

File|Database User Info

On Windows, LINGO allows models to link directly with databases through use of the *@ODBC()* function. Many times, the database you link your model to will require a user id and password. To avoid having to enter your user id and password each time your model is run, you can input them once at the start of your session using this command.

When you run the *File|User Database Info* command, you will be presented with the following dialog box:



Enter any user id and/or password into the appropriate fields. For security reasons, LINGO does not store this information from one session to the next. So, you will need to run this command at the start of each session.

If security is not a concern, and you would like to store your database user information, then you can create an *AUTOLG.DAT* file containing a *DBUID* command and a *DBPWD* command. Commands in the *AUTOLG.DAT* file are executed automatically each time LINGO starts. Thus, *DBUID* and

DBPWD commands contained in an *AUTOLG.DAT* file will restore your database user information at the start of each LINGO run.

File|Exit

Use the *Exit* command to quit LINGO when running on Windows. If any unsaved files are open, you will be prompted to save them before LINGO shuts down.

2. Edit Menu

LINGO's *Edit* menu contains commands that generally pertain to editing and modifying the text within a window.

Edit|Undo

Use the *Undo* Command to undo the last modification made to the contents of a Window. *Undo* can undo all operations except drag-and-drop text moves. LINGO stores a limited amount of undo operations, so you won't be able to depend on LINGO to undo extensive changes.

Edit|Redo

This command will redo the last undo operation. LINGO stores a limited amount of redo operations, so you won't be able to depend on LINGO to redo extensive changes.

Edit|Cut

Use the *Cut* command to clear the selected block of text and place it on the clipboard for pasting. To select a block of text for cutting, place the cursor immediately before the block and press down on the left mouse button. Now, drag the mouse until the cursor appears immediately after the block of text. The text block should now be displayed in reverse video. Now, issue the *Cut* command to remove the selected text from the document, placing it in the Windows clipboard.

Edit |Copy

Use the *Copy* command to copy the selected text to the clipboard for pasting. To select a block of text for copying, place the cursor immediately before the block and press down on the left mouse button. Now, drag the mouse until the cursor appears immediately after the block of text. The text block should now be displayed in reverse video. Now, issue the *Copy* command to place a copy of the selected text in the Windows clipboard.
The *Copy* command is a convenient way to transfer small amounts of data from LINGO to other applications.

Edit|Paste

Use the *Paste* command to replace the current selection in the active window with the contents of the Windows clipboard. The *Paste* command is a convenient way to import small amounts of data from other applications into your LINGO models.

Edit|Paste Special

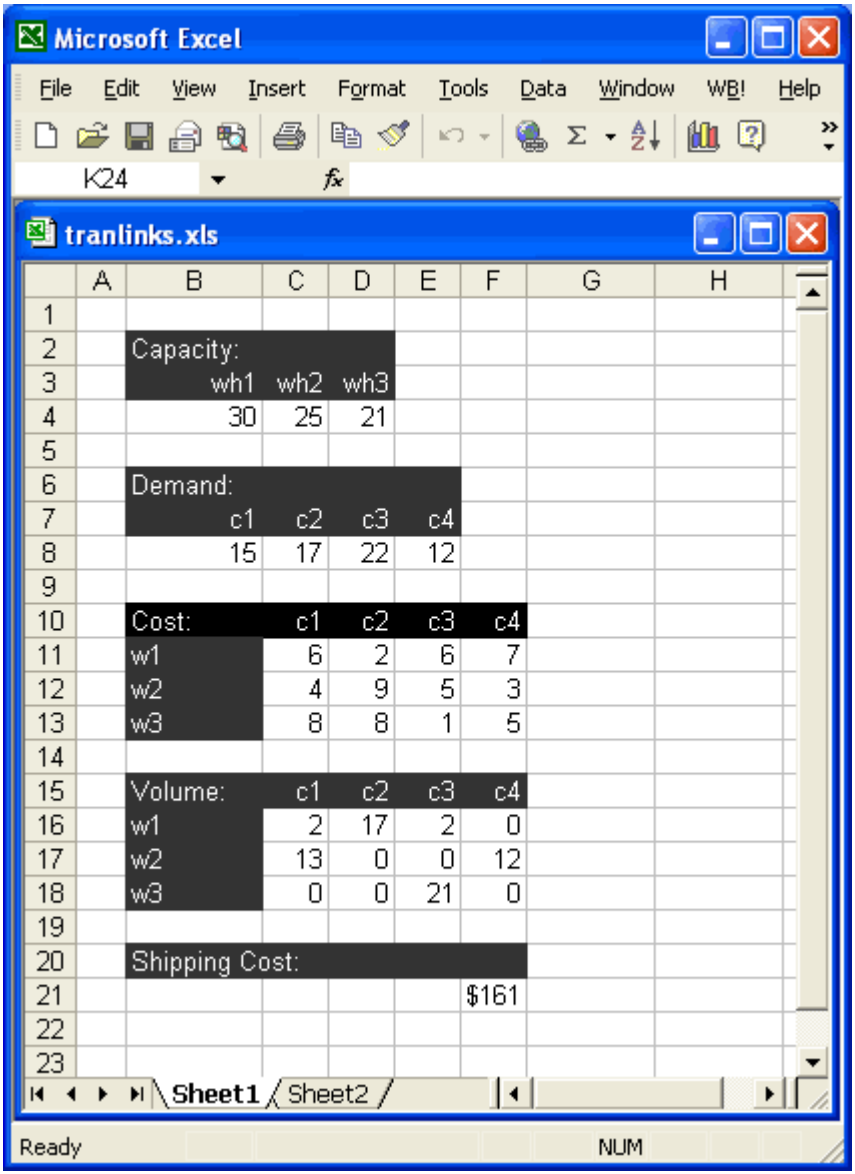
Use the *Paste Special* command to insert the contents from the Windows clipboard into the active window at the cursor insertion point. This command can do much more than insert just plain text as done by the standard *Paste* command. *Paste Special* can be used to insert other objects and links to other objects. This is particularly useful for adding links to supporting data for your model. By inserting a link to your data sources, it is much easier to find and view them.

As an example, suppose we have the following transportation model:

```
! A 3 Warehouse, 4 Customer
Transportation Problem;
SETS:
 WAREHOUSE / WH1, WH2, WH3/: CAPACITY;
 CUSTOMER / C1, C2, C3, C4/: DEMAND;
 ROUTES( WAREHOUSE, CUSTOMER): COST, VOLUME;
ENDSETS
! The objective;
MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
@FOR( CUSTOMER( J):
@SUM( WAREHOUSE( I): VOLUME( I, J)) >=
 DEMAND( J));

! The supply constraints;
@FOR( WAREHOUSE( I): [SUP]
@SUM( CUSTOMER( J): VOLUME( I, J)) <=
 CAPACITY( I));

! Here are the parameters;
DATA:
 CAPACITY = @OLE('D:\LNG\TRANLINKS.XLS');
 DEMAND = @OLE('D:\LNG\TRANLINKS.XLS');
 COST = @OLE('D:\LNG\TRANLINKS.XLS');
 @OLE('D:\LNG\TRANLINKS.XLS') = VOLUME;
ENDDATA
```

As we can see from the data section, we are importing data from the Excel file *TRANLINKS.XLS* and writing the solution back out to the same file.

A nice feature would be to insert a link to the spreadsheet into our model file. This way, we could view the data and the solution without having to start Excel and load the spreadsheet. To do this, open Excel and load the spreadsheet as we have done here:

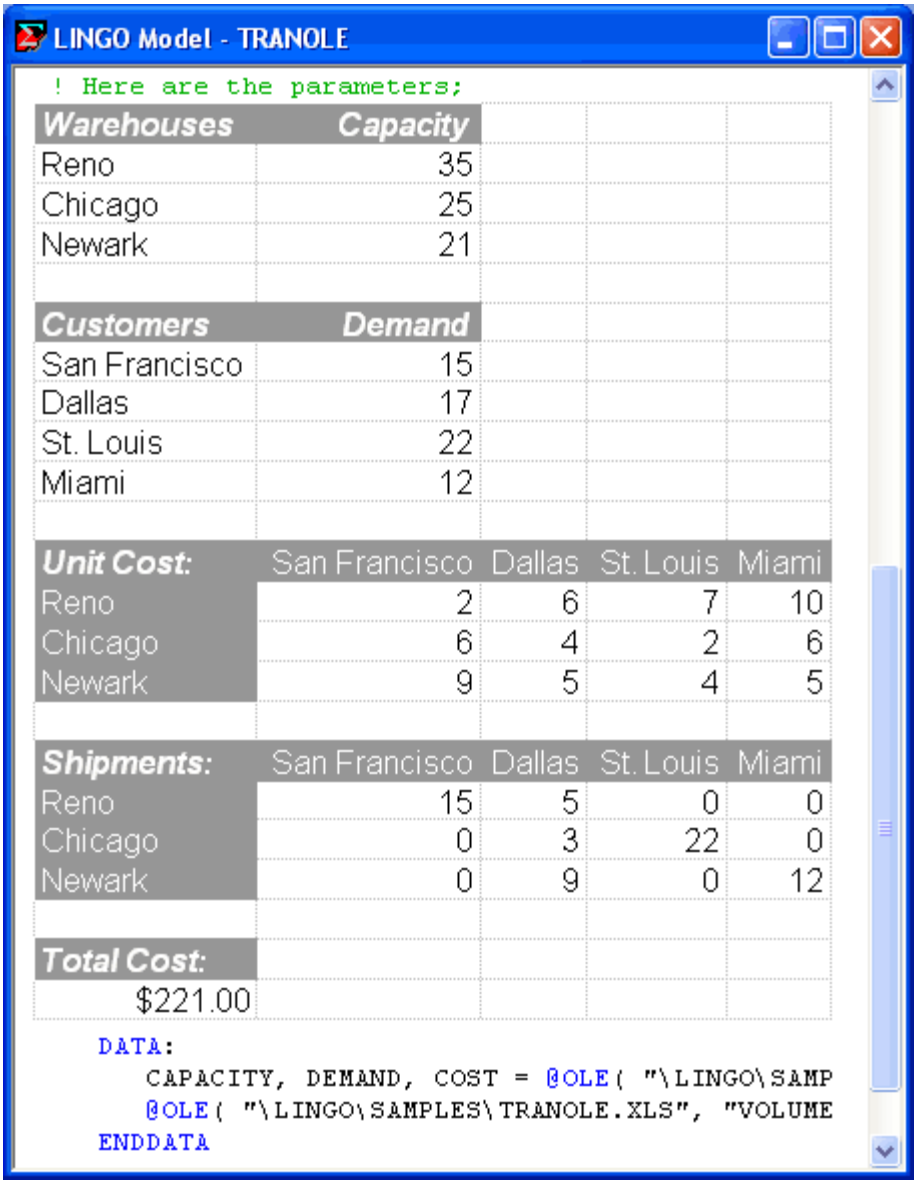


For complete information on importing data from Excel, see *Interfacing with Spreadsheets*.

Now, select the range B2:F21 in the spreadsheet. Next, from Excel's *Edit* menu, choose the *Copy* command. Now, click on LINGO, place the cursor right before the data section, and give the

Edit|Paste Special command. Click on the *Paste Link* button in the dialog box, so you see the following:

Finally, click the *OK* button and you should be able to see the spreadsheet contents in the LINGO model:



This link will be saved as part of your LINGO file. Therefore, whenever you open the model, the spreadsheet will be visible. Note that whenever you reopen the LINGO model, you may want to open the link, so the contents are updated automatically.

You can do this by selecting the spreadsheet in the LINGO model, giving the *Edit|Links* command, and pressing the *Open Links* button in the dialog box.

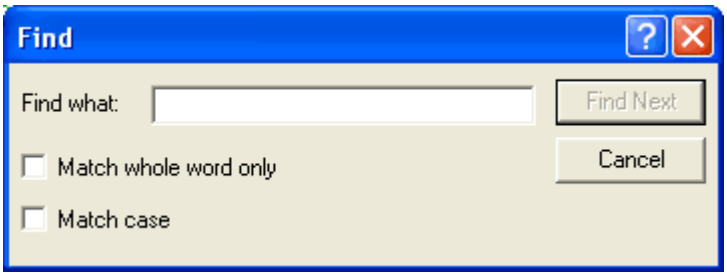
As a final note, all embedded links and objects are ignored by LINGO's compiler. Thus, you are free to insert links and objects wherever you choose in a model.

Edit|Select All

Use the *Select All* command to select the entire contents of the active window. This is useful when you want to copy the entire contents of the window elsewhere, or if you want to delete the contents of the window.

Edit|Find

Use the *Find* command to search for a desired string of text in the active window. When you issue the *Find* command, you should see the following dialog box:



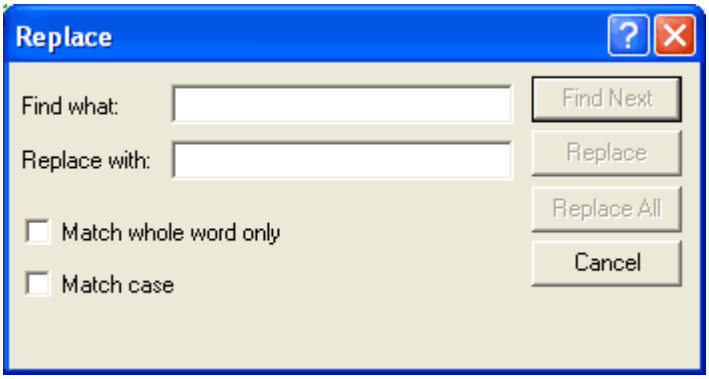
Enter the text you wish to search for in the *Find what* box. Check the *Match whole word only* box to have LINGO find only whole words of text (i.e., don't search for occurrences of the text embedded in other words). Check the *Match case* box to have LINGO search only for instances of the text with the same capitalization. Click the *Find Next* button to find the next instance of the text.

Edit|Find Next

Use the *Find Next* command to find the next instance of the text most recently searched for using the *Find* command in the active window.

Edit|Replace

Use the *Replace* command to replace one string of text with another in the active window. When you issue the *Replace* command, you will see the following dialog box:

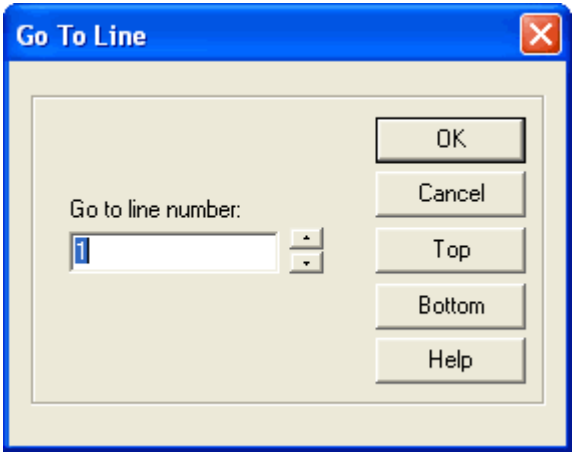


Enter the name of the text you want to replace in the *Find what* box. Enter the text you want to replace the old text with in the *Replace with* box. Clicking the *Find Next* button will cause LINGO to find the next occurrence of the old text. Clicking the *Replace* button will cause the next occurrence of the old text to be replaced by the new text. The *Replace All* button will replace all occurrences of the old text with the new text throughout the entire document.

Check the *Match whole word only* box to have LINGO replace only whole words of the text (i.e., don't replace occurrences of the text embedded in other words). Check the *Match case* box to have LINGO replace only instances of the text with the same capitalization.

Edit|Go To Line

Use the *Go To Line* command to jump to a selected line in the active window. When you issue the *Go To Line* command, you will see the following dialog box:



Enter a line number in the *Go to line number* box. Then, press the *OK* button and LINGO will jump to the desired line number. Press the *Top* button to go to the top of the document, or the *Bottom* button to go to the bottom.

Edit|Match Parenthesis

Select a parenthesis in a document. Then, use the *Match Parenthesis* command to find the closing parenthesis for selected parenthesis.

This command is useful when using nested statements such as:

```
@FOR( FXA( I, J):
   JP( I, J) = MPF( I) * CAGF( I, J);
   JP( I, J) = MPA( J) * CFGA( I, J));
```

where it may be difficult to find the close of a given parenthesis.

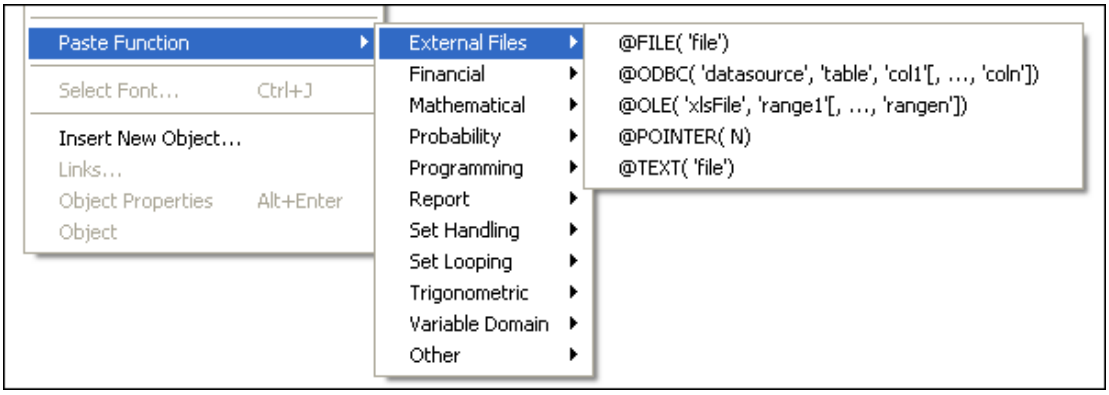
If no parenthesis is selected prior to issuing the *Match Parenthesis* command, LINGO will select the parenthesis nearest to the current cursor position.

In addition to this command, there is one other way to find matching parentheses. LINGO will highlight matching parentheses in red when the *Match Paren* option is enabled under the *Solver|Options* command. By placing the cursor immediately after one of the parentheses of interest, you will notice that the color of the parenthesis changes from black to red. LINGO will simultaneously display the matching parenthesis in red. These parentheses will remain displayed in red until you move the cursor to another position, at which point they will be returned to a black color.

Edit|Paste Function

Use the *Paste Function* command to paste any of LINGO's built-in functions at the current insertion point. Choose the category of the LINGO function you want to paste from the secondary menu, and then select the function from the cascading menu.

In the following illustration, we have chosen the *External Files* category from the secondary menu.



On the right are all the functions that deal with external files. By selecting one of these functions, LINGO will paste a template for the selected function into your document, with a suggestive placeholder for each of the function's arguments. You should then replace the argument placeholders with actual arguments that are relevant to your model.

Edit|Select Font

Use the *Select Font* command to select a new font, size, style, color, or effect in which to display the selected text. You may find it easier to read models and solution reports if you select a monospaced font such as Courier. Custom fonts are preserved only when saving in the LG4 file format. (Refer to the *File|New* command for a description of LINGO's various file types.)

Note:	Under Windows you may also change the font color, however, you cannot change the display color if syntax coloring is enabled. If you need to use specific display colors in your document, you will need to disable syntax coloring.

Edit|Insert New Object

Use the *Insert New Object* command to insert an object or a link to an object into your model. As with the *Edit|Paste Special* command, this command is helpful in that it allows you to insert links to your model's data sources. Unlike the *Paste Special* command, which links to portions of an external object, the *Insert New Object* command can add a link to an entire object.

As an example, suppose you have the following staff scheduling model:

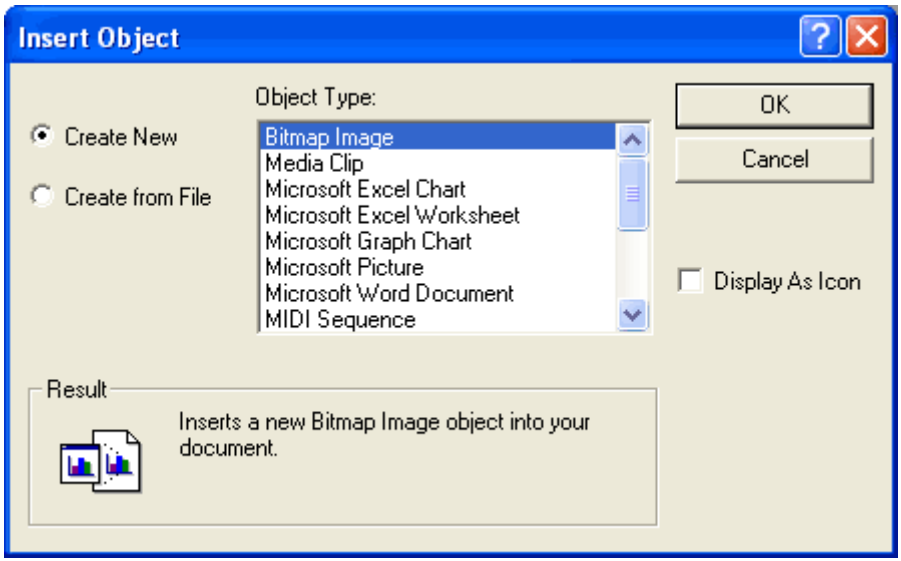
LINGO Model - stafODBC

```
SETS:
   DAY / @ODBC( 'STAFFING', 'STAFFING',
    'DAY')/: NEED, START;
ENDSETS

! The objective;
  MIN = @SUM( DAY: START);

! The constraints;
  @FOR( DAY( TODAY):
   @SUM( DAY( COUNT)| COUNT #LE# 5:
    START( @WRAP(TODAY - COUNT+1,
     @SIZE( DAY)))) >= NEED( TODAY);
   @GIN( START);
  );

DATA:
  NEED = @ODBC( 'STAFFING',
   'STAFFING', 'REQUIREMENTS');
  @ODBC( 'STAFFING',
   'STAFFING', 'START') = START;
ENDDATA
```

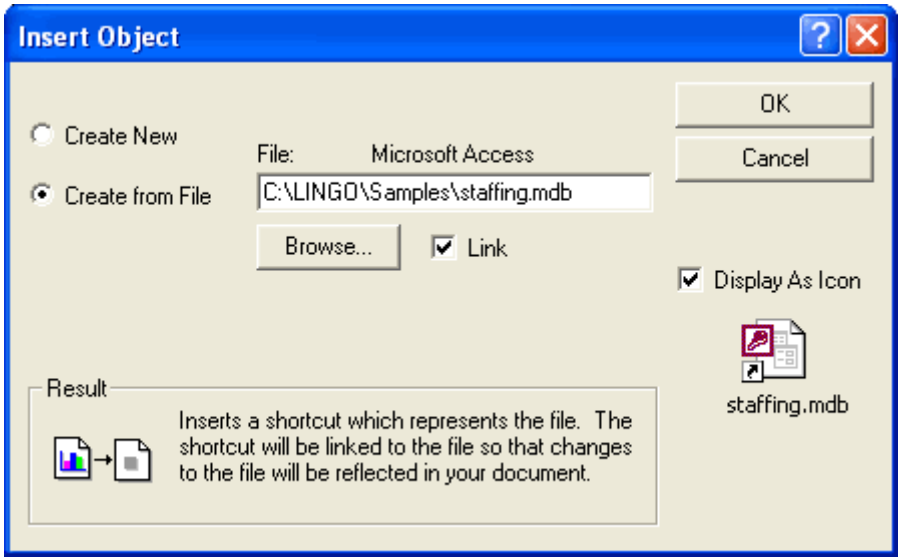
From the model's data section, we see that we are using the *@ODBC* function to retrieve the values for the *NEED* attribute from an ODBC data source titled *STAFFING.* We are also using the *@ODBC* function to send the optimal values for the *START* attribute back to the same data source. Because this data source is an integral part of our model, it would be nice to place a link to it in our model, so we can retrieve it easily each time we want to refer to it. We can do this with the *Edit|Insert New Object* command as follows:

1. Position the cursor in the model where you would like the icon for the link to appear (Note, the LINGO parser ignores links to external objects, so you can insert the link anywhere you like.)

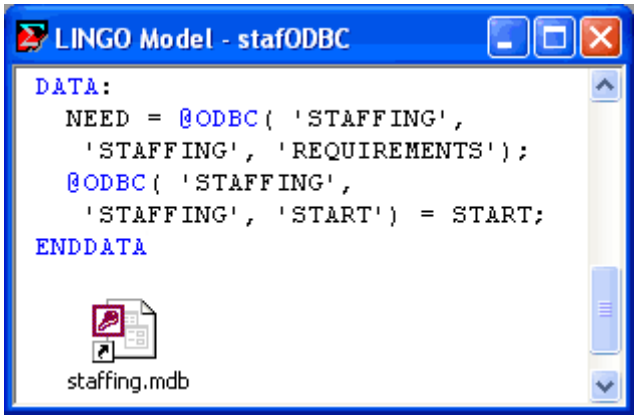
2. Issue the *Edit|Insert New Object* command. You should see the following dialog box:



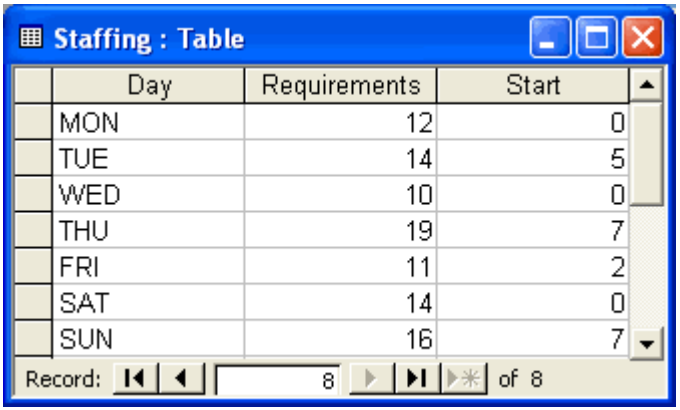
3. Select the *Create from File* radio button.
4. Type in the name of the database file containing your data.
5. Click the *Display As Icon* button, so the box now resembles:



6. Finally, click on the *OK* button, and an icon representing the linked database will appear in your LINGO model as pictured below:



Now, whenever you want to edit or view the supporting database, all you need do is double-click on the icon. In this case, Microsoft Access will start and load the staffing database, so you will see the following on the screen:



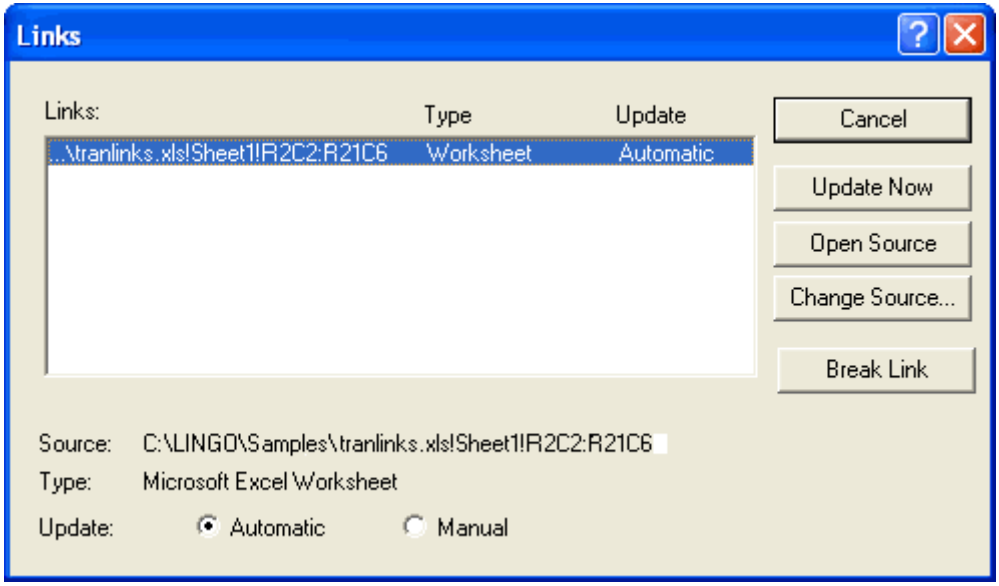
Day	Requirements	Start
MON	12	0
TUE	14	5
WED	10	0
THU	19	7
FRI	11	2
SAT	14	0
SUN	16	7

As a final note, keep in mind that linked objects are preserved only when a model is saved in LG4 format (see the *File|New* command for details on the LG4 file format.)

For complete information on exchanging data and solution values with data sources, see *Interfacing with Databases*.

Edit|Links

Use the *Links* command to modify the properties of the links to external objects in a LINGO document. The dialog box appears as follows:



Select the *Automatic* radio button to have LINGO automatically update the object when the source file is changed. The *Manual* radio button allows you to update the object only when you select the *Update Now* button.

The *Open Source* button is used to open the connection to an automatic link. Once the link has been opened, any changes to the source document will be reflected in the view of the object in your LINGO model.

The *Change Source* button is used to attach the link to a different source file.

Finally, the *Break Link* button is used to break the connection to the external object.

Edit|Object Properties

Select a linked or embedded object in your model by single-clicking it, and then you can use the *Object Properties* command to modify the properties of the object. Properties you will be able to modify include:

- display of the object,
- the object's source,
- type of update (automatic or manual),
- opening a link to the object,
- updating the object, and
- breaking the link to the object.

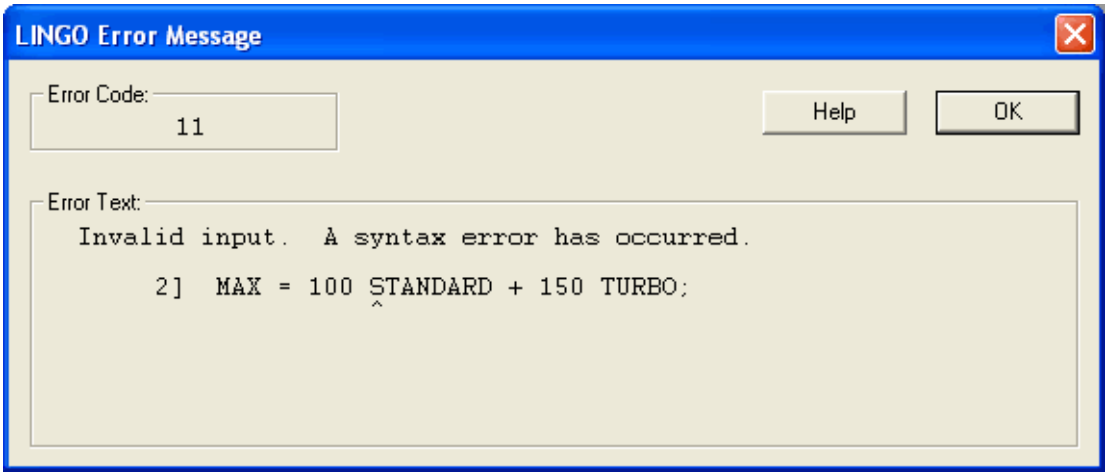
3. Solver Menu

The *Solver* menu contains commands that generally pertain to solving a model and generating reports. This menu also contains the *Options* command for customizing LINGO's configuration.

Solver|Solve

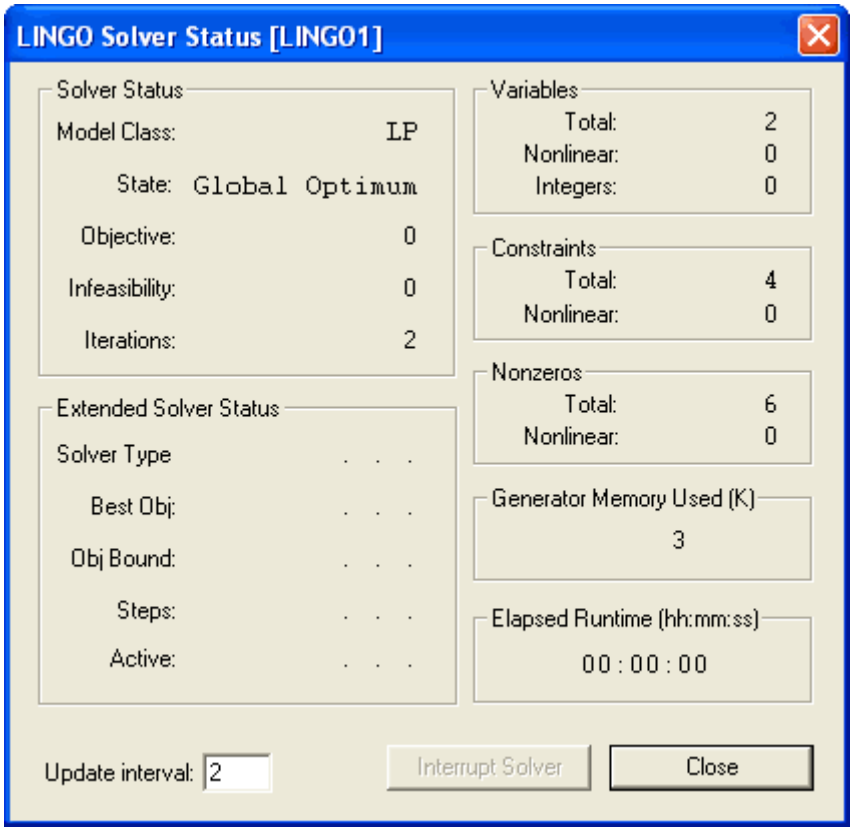
Use the *Solve* command to have LINGO solve the model in the active window. The *Solve* command is available only for model windows—report, script, and data windows cannot be solved.

When you solve a model, LINGO first examines the model's syntax to determine if it is valid. If LINGO finds a mistake in the syntax, you will be presented with a dialog box similar to the following:



In the *Error Text* box, LINGO prints the line number where the syntax error occurred, the text of the line, and points to where LINGO determines that the error occurred. In most cases, LINGO is good at pointing to where the error occurred. Sometimes, however, the error may not be located exactly where LINGO is pointing. Be sure to examine neighboring lines for possible flaws as well. In this particular example, the syntax error occurred in line 2, where we forgot to insert the multiplication signs (*) between the two coefficients and variable names.

When you issue the *Solve* command (assuming your model has no further syntax errors), LINGO will post the *solver status window.* This window contains information about the composition of your model and keeps you posted as to the progress of the solver. The *solver status window* resembles the following:



For more information on the various fields in the solver status window, refer to the section *Solver Status Window*.

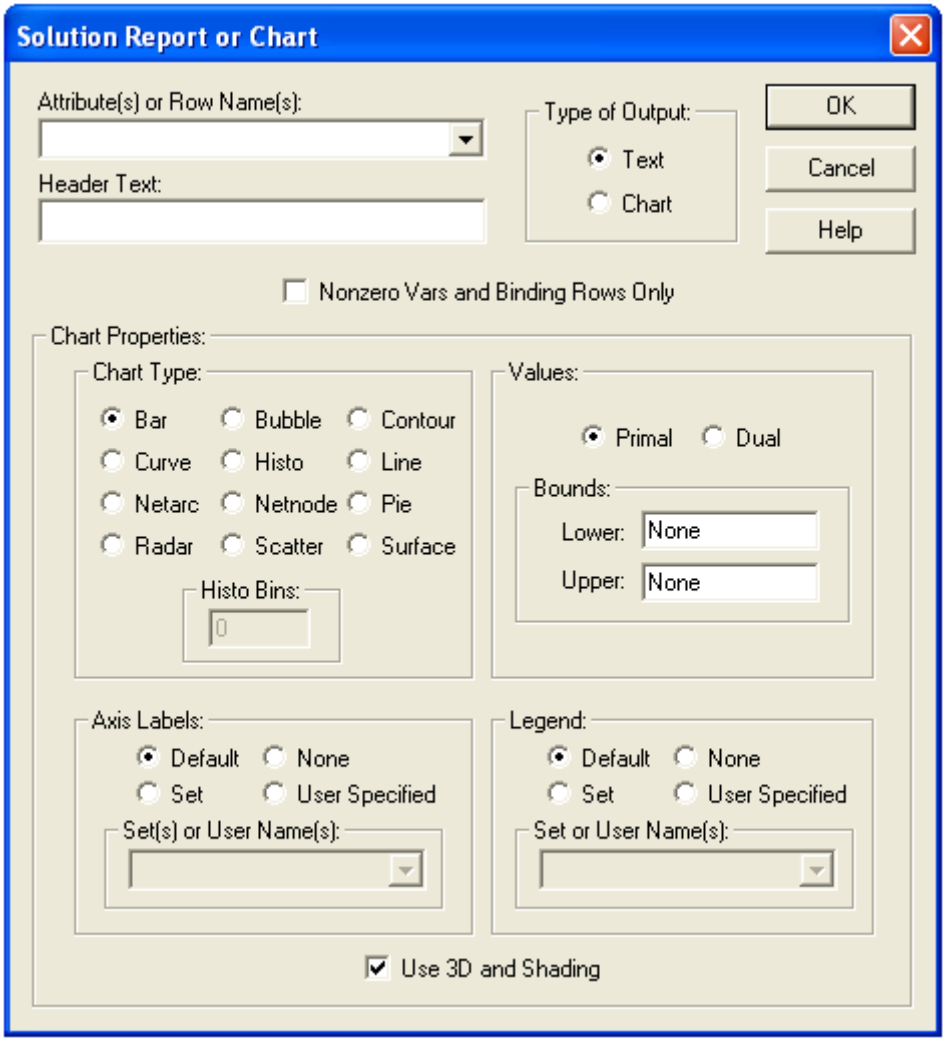
Once the solver has completed processing your model, it will create a new window containing the solution report for your model. You can scroll through this window to examine its contents, save it to a text file, or queue it to your printer. The following is a sample solution report window:

Solution Report - BOX

```
Local optimal solution found.
Objective value:                              50.96508
Infeasibilities:                              0.000000
Extended solver steps:                               5
Total solver iterations:                            51
Elapsed runtime seconds:                          0.17

Model Class:                                       NLP

Total variables:                     3
Nonlinear variables:                 3
Integer variables:                   0

Total constraints:                   6
Nonlinear constraints:               6

Total nonzeros:                     15
Nonlinear nonzeros:                 15

          Variable           Value        Reduced Cost
                 D        23.03096            0.000000
                 W        9.562198            0.000000
                 H        6.865657            0.000000

               Row    Slack or Surplus      Dual Price
              COST        50.96508           -1.000000
           SURFACE       0.3349107E-04      -0.2342588E-01
            VOLUME       0.1439074E-03      -0.1329934E-01
          NOTNARRO       0.1348107E-06        2.298548
           NOTHIGH       0.1999999            0.000000
          FOOTPRNT        31.77341            0.000000
```

Solver|Solution

Use the *Solution* command to generate a solution report for the active window. After selecting the model window that you want to generate a solution for, issue the *Solver|Solution* command and you will be presented with this dialog box:



By clicking on the appropriate radio button on the *Type of Output* box, the solution report may be viewed in either text or chart format. If you select *Text*, LINGO will create a new window containing the solution in text format. If you select *Chart*, LINGO will create a new window containing the solution in one of several different graphical formats. Current supported chart formats are: bar, bubble, contour, curve, histogram, line, pie, radar, scatter and surface.

Note: LINGO maintains only one solution in memory. This is the solution to the last window you issued the *Solver|Solve* command for. If you try to issue the *Solution* command for a window that LINGO does not currently have a solution, you will receive an error message. Thus, if you plan to work with two or more models that take a long time to solve, be sure to save copies of your solutions. This will allow you to refer to them later without having to re-solve your models.

Note: At present, the charting features of the *Solution* command are only supported in Windows versions of LINGO. However, Mac and Linux versions of LINGO can generate charts using the *@CHART* function set.

Text Solution Reports

In the A*ttribute(s) or Row Name(s)* list box, select an attribute or row name for which you would like a report. If you do not select a name in this box, LINGO will generate a full solution report that includes *all* attributes and rows. You may also specify multiple objects in this box, in which case, each object will be included in the report.

Both the multi-character (*) and single character (%) wildcards are supported in the A*ttribute(s) or Row Name(s)* field. For example, inputting "X*" would cause all variables and rows beginning with the letter X to be displayed, while "X%1" will display all variables and rows with three-character names that begin with letter X and end with the digit 1.

In the *Header Text* box, enter whatever text (e.g., "Values for X") you would like to appear at the head of the report.

Check the *Nonzeros Vars and Binding Rows Only* box to see a report that contains only the variables with a nonzero value and constraints that are binding.

When you click OK, LINGO creates a new solution window containing the solution report. You can use Cut and Paste commands to move the contents of the report to other applications. You may also use the File|Save command to save the report to a text file.

Charting Solutions

If you choose to have the solution displayed as a chart, the box titled *Chart Properties* will be undimmed, which allows you to select options influencing the display of the chart.

In the *Chart Type* box, you have the option of selecting a bar, bubble, contour, curve, histogram, line, netarc, netnode, pie, radar, scatter or surface chart.

One easy way to familiarize yourself with the various chart types is to open and run the sample model *CHARTS.LG4*, which displays a sample of each of the chart types.

If you've selected a histogram chart, then you will also have the option of specifying the number of bins in the histogram in the *Histo Bins* box. If the number of bins is set to 0, then LINGO will select a reasonable number of bins for the given data set.

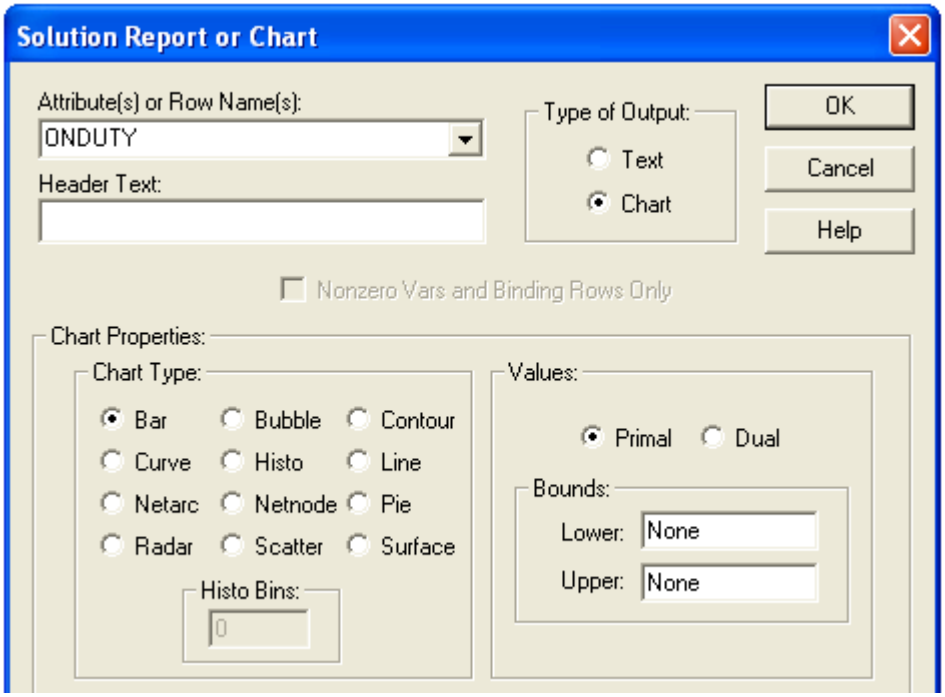
In the *Values* box, you can select to graph either primal or dual values.

The *Attribute(s) or Row Name(s)* field works in the same way as mentioned in the previous section, with the additional feature that a colon may be inserted in between object names to indicate that you wish to display multiple series on the same graph. Each series of data will be displayed using a different color. The chart types that support multiple series are: bar, bubble, curve, line, radar, scatter. Contour, histogram, pie and surface charts do not support multiple data series. As an example, consider the following staff scheduling model, similar to the one discussed above in section *A Staff Scheduling Problem*:

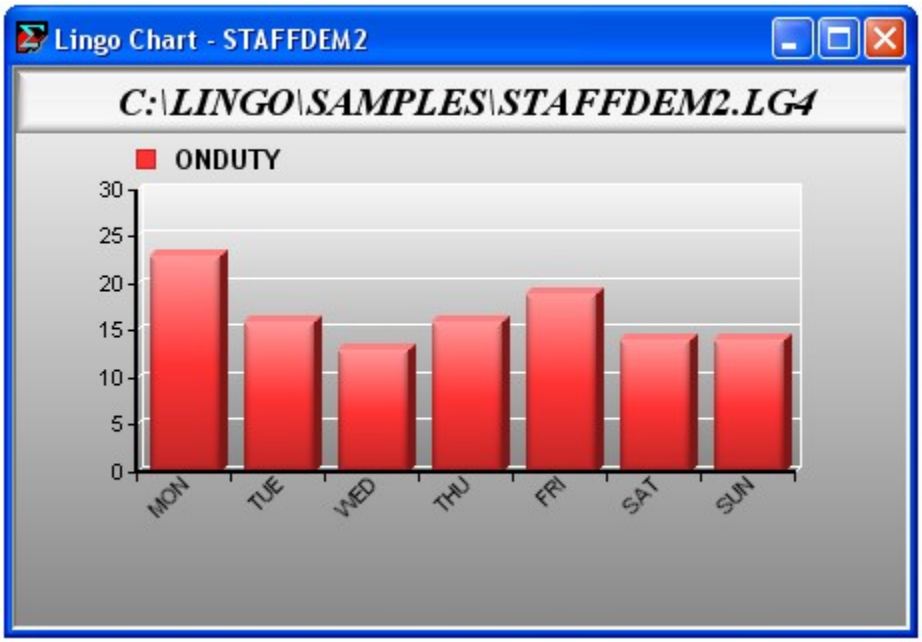
```
MODEL:

SETS:
   DAYS: REQUIRED, START, ONDUTY;
ENDSETS

DATA:
   AYS = MON TUE WED THU FRI SAT SUN;
   REQUIRED = 20 16 13 16 19 14 12;
ENDDATA

MIN = @SUM( DAYS( I): START( I));

@FOR( DAYS( J):

   ONDUTY( J) =
     @SUM( DAYS( I) | I #LE# 5:
       START( @WRAP( J - I + 1, 7)));

   ONDUTY( J) >= REQUIRED( J)
);

END
```

Model: STAFFDEM2

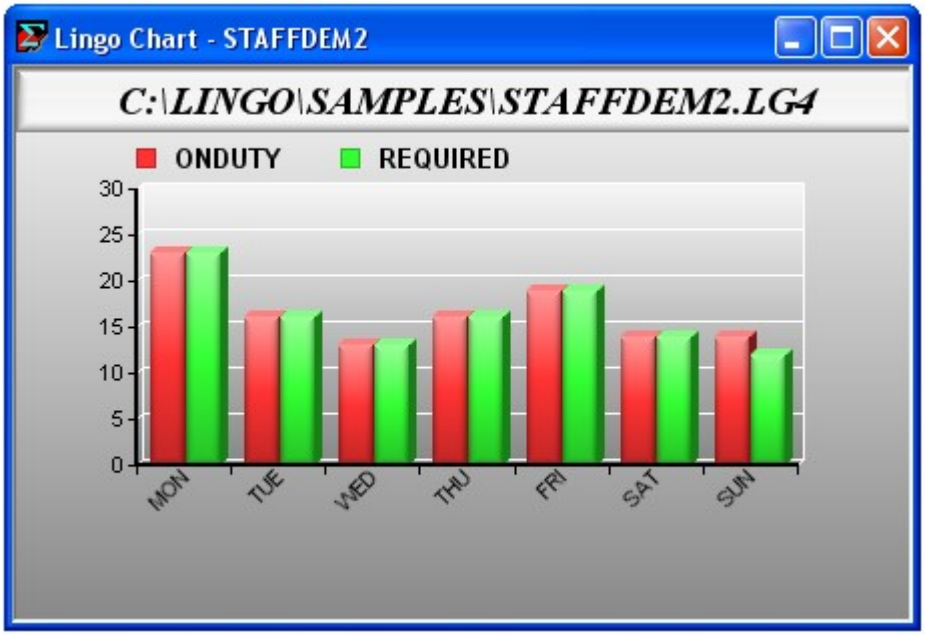
The *ONDUTY* attribute tells us how many employees are working on each day of the week. To display a bar chart of this information, you would fill out the *Solver|Solution* dialog box as below:



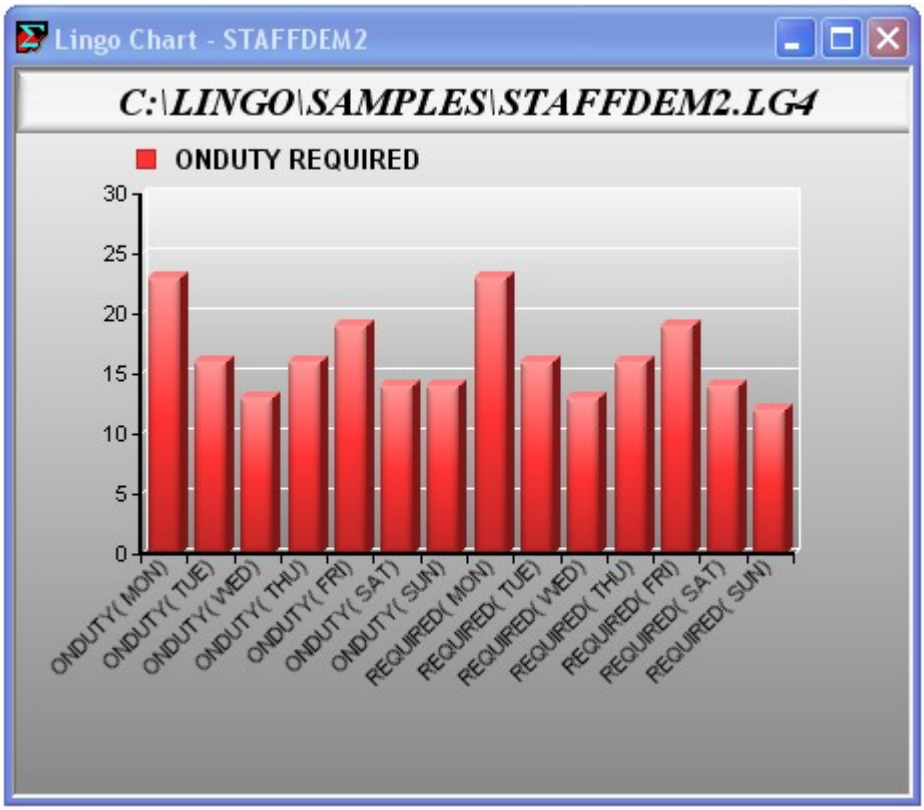
After pressing *OK*, a new window will open with the desired chart:



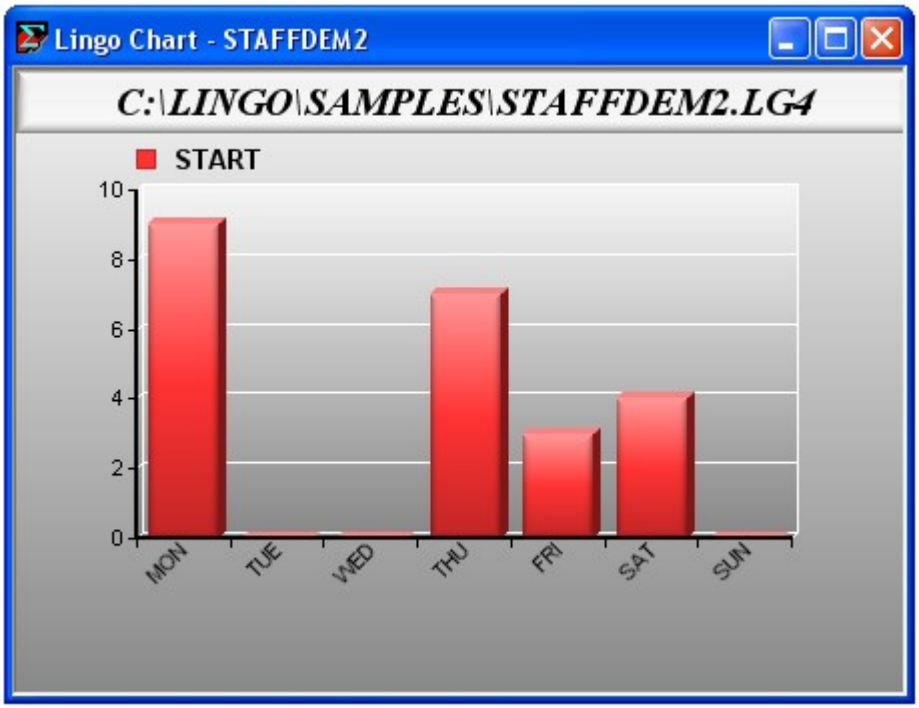
Along with charting the number on duty, it would also be useful to include the number of staffers required on each day. This would allow us in one glance to quickly see the days (if any) where we are over staffed. To do this, change the contents of *Attribute(s) or Row Name(s)* field from "ONDUTY" to "ONDUTY : REQUIRED". This tells LINGO we want to view both on duty and required staffing levels, with the colon inserted to indicate they should be displayed as separate series. Doing this yields the multi-bar chart:


Lingo Chart - STAFFDEM2
C:\LINGO\SAMPLES\STAFFDEM2.LG4
ONDUTY
REQUIRED
30
25
20
15
10
5
0
MON
TUE
WED
THU
FRI
SAT
SUN

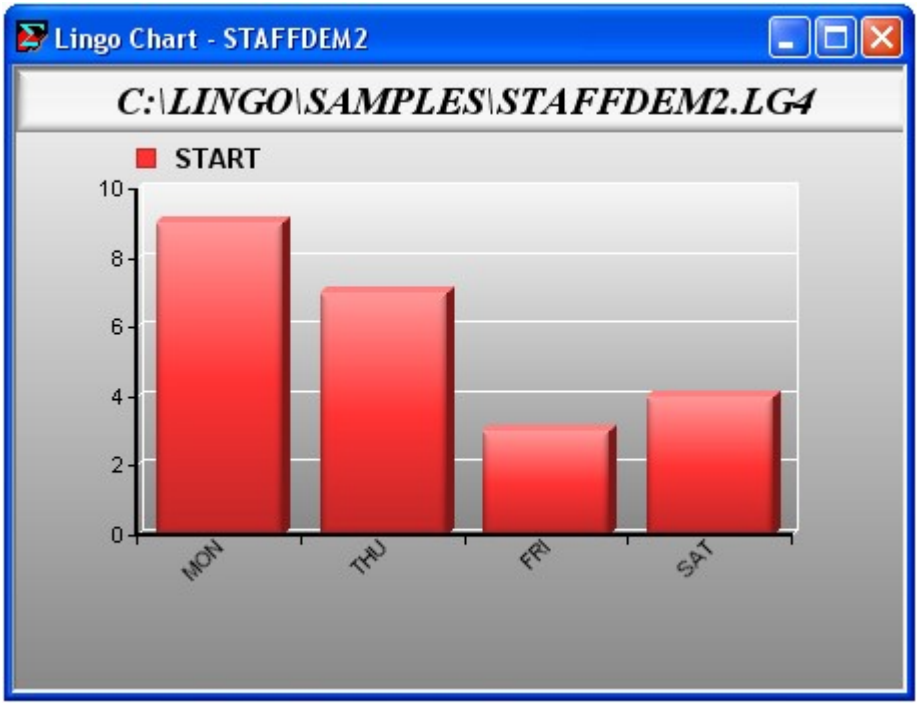
Here we see that we just meet our staffing needs Monday through Saturday and are over staffed slightly on Sunday. Had we not inserted the colon separator between the attribute fields, then LINGO would have displayed the two attributes as a single series:



The *Bounds* box gives you the option of placing bounds on the values to be included in the graph. If a number is entered in the *Lower* bound field, LINGO will only display points in the graph that are greater-than-or-equal-to the value. Conversely, if a value is placed in the *Upper* bound field, LINGO will only graph points less-than-or-equal-to the bound. For example, if we display a bar chart showing the number of employees starting each day of the week we'd see the following:



Notice there are several "holes" in the chart due to no one starting on Tuesday, Wednesday and Sunday. Entering a lower bound of .1 in the *Bounds* box removes the holes in our chart:



For charts with one-dimensional data (bar, histogram, line, pie and radar), the bounds will be applied to all data points in the first data series. If a point is eliminated in the first series, then the corresponding points in any additional series will also be eliminated, regardless of whether or not they lie within the specified bounds.

Charts with two-dimensional data points (curve and scatter) and three-dimensional data (bubble, contour and surface) will have the bound applied only to the *x*-axis data. If the *x*-axis value lies outside the bounds, then the entire point will be eliminated from the chart. Of these higher dimension charts, only curve scatter and bubble allow for multiple series. In this case, the bounds will be applied to the *x*-axis value of each series, as opposed to just the first.

Note: Bounds are not currently applied to the two network chart types - netarc and netnode.

Next in the Chart Properties box are options for controlling how the axes and legends are labeled. In both cases, the choices are:

- *Default* - LINGO tries to make intelligent choices in choosing an appropriate labeling scheme.
- *None* - No labels are displayed.
- *Set* - Use one of the model's sets for labeling. Multiple sets may be specified, if needed, to supply sufficient labels.
- *User Specified* - Labels are entered explicitly into the Set(s) or User Name(s) field, with individual labels being separated with colons.

The final option in the *Chart Properties* box is the *Use 3D and Shading* checkbox. This option is on by default and will result in more modern looking charts that utilize shading and 3-dimensional effects. Disable this option to display simpler, 2-dimensional charts that may display better on certain printers.

Higher Dimension Charts

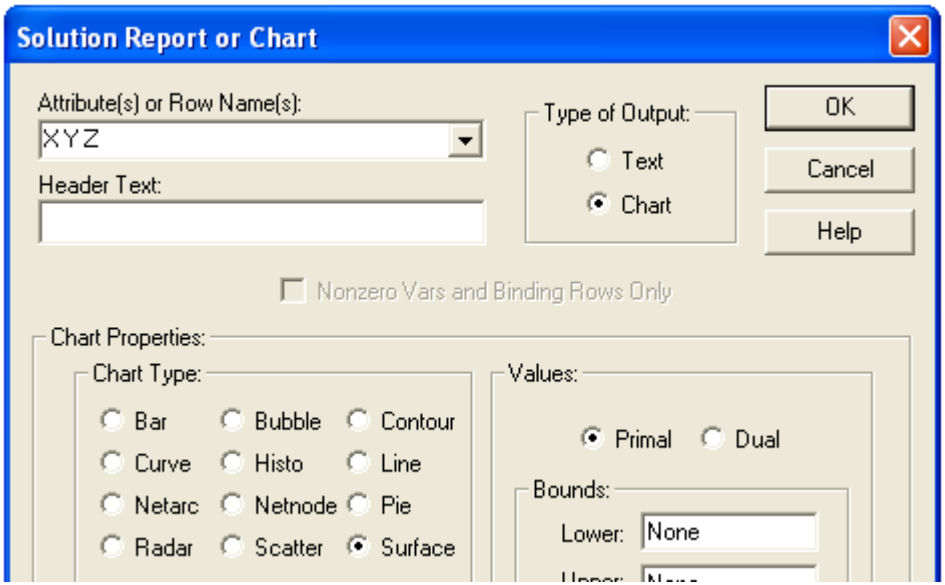
In addition to the standard charts with one-dimensional data, LINGO offers 2-dimensional curve and scatter charts, as well as 3-dimensional bubble, contour and surface charts. When a chart requires more than one dimension of data, you can supply the data in either of two ways. The first method is to supply one attribute for each dimension. So, for example, a 3-dimensional surface chart might be specified using the three attributes *XVALS*, *YVALS* and *ZVALS*. The first attribute always stores the *x*-axis data, the second stores the *y*-axis and, if needed, the third attribute stores the *z*-axis values. The second option is to provide a data series in an *m* x *n* table/attribute, where *m* is the number of data points and *n* is the dimension of the chart type.

As an example, suppose we have the following model that generates points of the surface *X * SIN(Y) + Y * SIN(X)*:

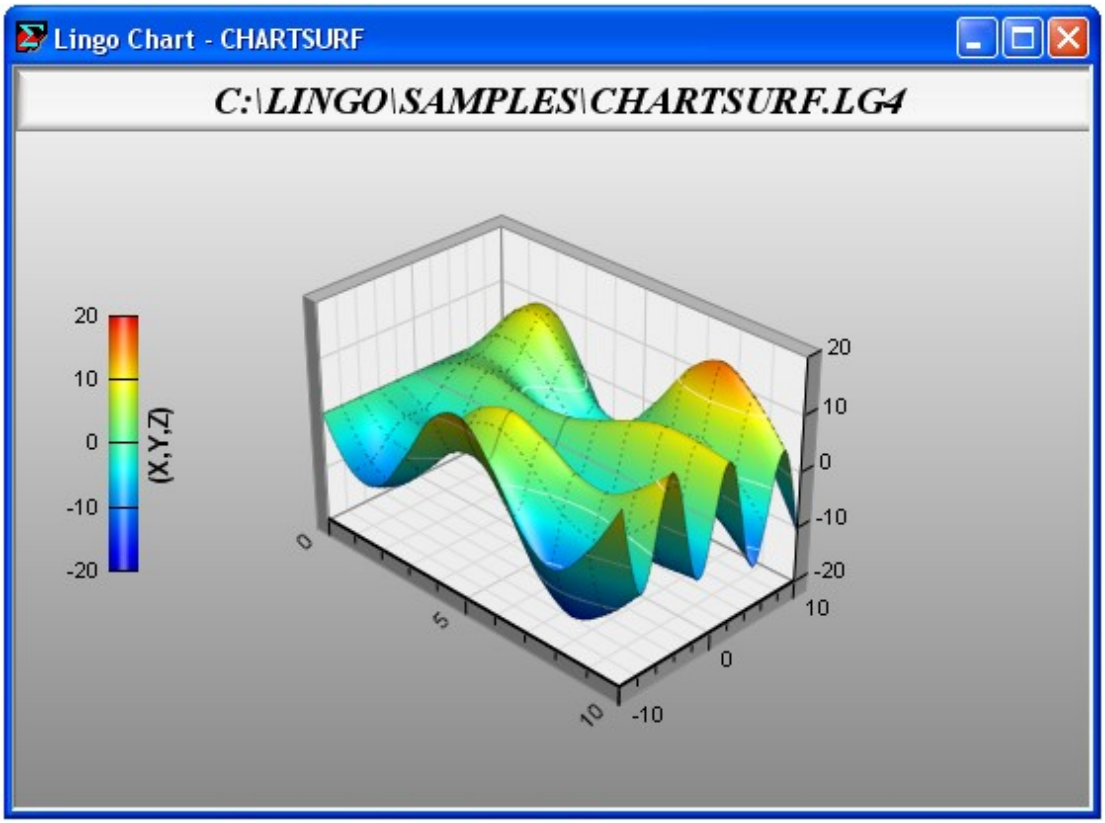
```
MODEL:

SETS:
 POINTS /1..21/;
 POINTS2( POINTS, POINTS): X, Y, Z;
ENDSETS

CALC:
 XS = @FLOOR( -(@SIZE( POINTS) / 2) + .5);
 YS = XS;
 @FOR( POINTS2( I, J):
   X( I, J) = XS + I - 1;
   Y( I, J) = YS + J - 1;
   Z( I, J) = X( I, J) * @SIN( Y( I, J)) +
   Y( I, J ) * @SIN( X( I, J));
 );
ENDCALC

END
```

Model: CHARTSURF

A surface chart requires 3-dimensional data, which, in this case, is contained in the *X*, *Y* and *Z* attributes. To request a surface chart after solving the model, we fill out the *Solver|Solution* dialog box as follows:



Note that we've listed each of the three attributes in the *Attribute(s) or Row Name(s)* field. Given that this is a 3-dimensional graph, the three attributes will be used to create a single chart. Clicking on *OK* then gives us the chart:



Below we list the dimensions of each of the various chart types and whether or not they allow for multiple data series:

Chart Type	Dimension	Supports Multiple Series
Bar	1	Yes
Histogram	1	No
Line	1	Yes
Pie	1	No
Radar	1	Yes
Curve	2	Yes
Netarc	2	Yes
Netnode	2	Yes
Scatter	2	Yes
Bubble	3	Yes
Contour	3	No
Surface	3	No

Network Charts

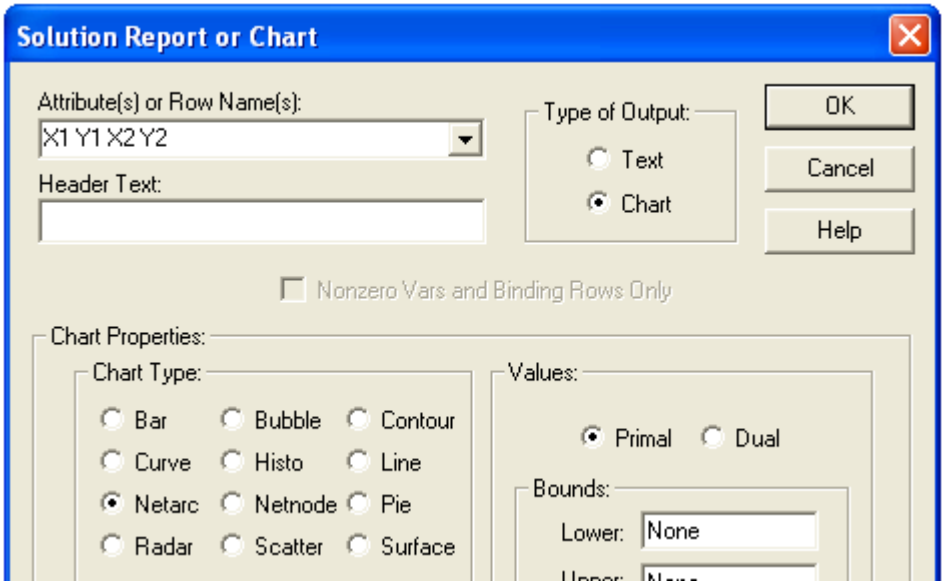
Network charts are specified slightly differently from other charts. Although network charts are technically 2-dimensional charts, they require 4 attributes of data.

LINGO supports two different formats for specifying network charts -- *netarc* and *netnode.* In netarc format, each arc is specified by two (*X,Y*) pairs, indicating the two end points of the arc. Whereas in netnode format, each node in the network is specified with one (*X,Y*) pair, and each arc between the nodes is specified by pair of arc indices from the arcs set. An example of both formats follows:

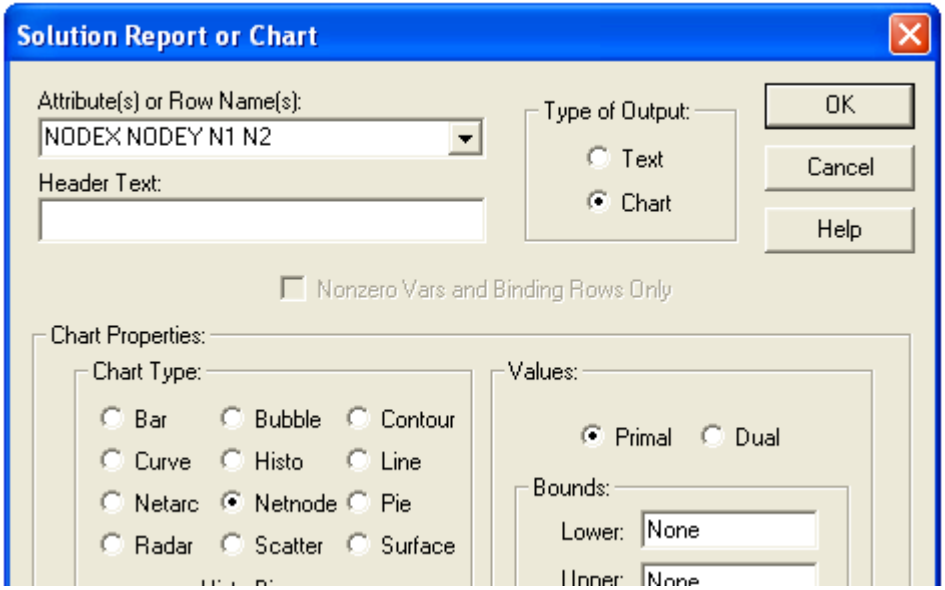
```
MODEL:
! Illustrates the two formats for network charts:
  NETARC and NETNODE;
SETS:
 NODES /N1..N4/: NODEX, NODEY;
 NODESINNET /1..6/: N1, N2;
 ARCS( NODES, NODES) /
 N1,N2, N2,N3, N3,N4, N4,N1, N4,N2, N1, N3
 /: X1, Y1, X2, Y2;
ENDSETS

DATA:
  !List locations of all the nodes;
   NODEX, NODEY =
      10,    10,
      20,    10,
      20,    40,
      10,    40;
  !And list the indices of the nodes
   that have arcs between them;
   N1, N2 =
    1,  2,
    2,  3,
    3,  4,
    4,  1,
    4,  2,
    1,  3;
ENDDATA

CALC:
  !Load the arc coordinates into X1, Y1, X2, Y2;
@FOR( ARCS( I, J):
  X1( I, J) = NODEX( I);
  Y1( I, J) = NODEY( I);
  X2( I, J) = NODEX( J);
  Y2( I, J) = NODEY( J);
);
ENDCALC

END
```

Model: CHARTNET

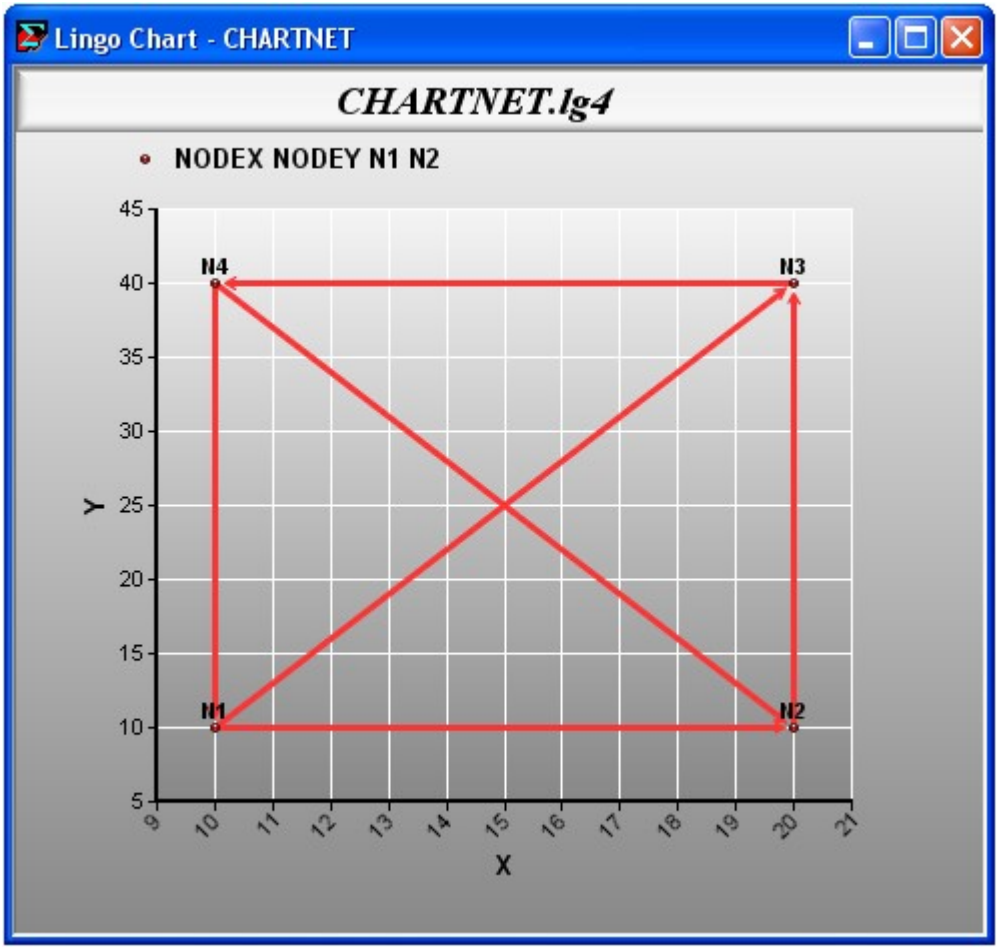
The *CHARTNET* model above uses both network chart formats for specifying a rectangular network of four nodes with each node pair being joined by an arc. After solving the model, you can display the network using netarc format by running the *Solver|Solution* command and filling out the dialog box using the netarc attributes (*X1*, *Y1*, *X2* and *Y2*) and clicking on the "Netarc" radio button as follows:



To display the network using netnode format, you would fill out the dialog box as below, using the netnode attributes (*NODEX*, *NODEY*, *N1* and *N2*) along with checking off the "Netnode" radio button:



In both cases, you should then generate a network chart resembling:



Solver|Range

Use the *Range* command to generate a range report for the model in the active window. A range report shows over what ranges you can: 1) change a coefficient in the objective without causing any of the optimal values of the decision variables to change, or 2) change a row's constant term (also referred to as the right-hand side coefficient) without causing any of the optimal values of the dual prices or reduced costs to change.

Note: The solver computes range values when you solve a model. Range computations must be enabled in order for the solver to compute range values. Range computations are not enabled by default. To enable range computations, run the *Solver|Options* command, select the *General Solver Tab*, and, in the *Dual Computations* list box, choose the Prices and Ranges option. Range computations can take a fair amount of computation time, so, if speed is a concern, you don't want to enable range computations unnecessarily.

Note: The example model below, when solved, yields the range report that follows.

```
[OBJECTIVE] MAX = 20 * A + 30 * C;
[ALIM]      A <= 60;
[CLIM]      C <= 50;
[JOINT]     A + 2 * C  <= 120;
```

Here is the range report:

```
Ranges in which the basis is unchanged:

              Objective Coefficient Ranges
             Current    Allowable    Allowable
Variable  Coefficient     Increase     Decrease
      A     20.00000     INFINITY     5.000000
      C     30.00000     10.00000     30.00000

                  Righthand Side Ranges
    Row      Current    Allowable    Allowable
                 RHS     Increase     Decrease
   ALIM     60.00000     60.00000     40.00000
   CLIM     50.00000     INFINITY     20.00000
  JOINT     120.0000     40.00000     60.00000
```

The first section of the report is titled *Objective Coefficient Ranges.* In the first column, titled *Variable*, all the optimizable variables are listed by name. The next column, titled *Current Coefficient*, lists the current coefficient of the variable in the objective row. The next column, *Allowable Increase*, tells us the amount that we could increase the objective coefficient without changing the optimal values for the variables. The final column, *Allowable Decrease*, lists the amount that the objective coefficient of the variable could decrease before the optimal values of the variables would change. Information on the allowable increases and decreases on objective coefficients can be useful when you need answers to questions like, "How much more (less) profitable must this activity be before we should be willing to do more (less) of it?".

Referring to the *Objective Coefficient Ranges* report for our example, we can say, as long as the objective coefficient of *A* is greater-than-or-equal-to 15, the optimal values of the variables will not change. The same may be said for the objective coefficient of variable *C*, as long as it falls within the range of [0,40].

Note: Ranges are valid only if you are planning to alter a single objective or right-hand side coefficient. The range information provided by LINGO cannot be applied in situations where one is simultaneously varying two or more coefficients. Furthermore, ranges are only lower bounds on the amount of change required in a coefficient to actually force a change in the optimal solution. You can change a coefficient by any amount up to the amount that is indicated in the range report without causing a change in the optimal solution. Whether the optimal solution will actually change if you exceed the allowable limit is not certain.

The second section of the range report is titled *Right-hand Side Ranges*. The first column, *Row*, lists the names of all the optimizable rows, or constraints, in the model. The second column, *Current RHS*, gives the constant term, or right-hand side value, for the row. The next two columns, *Allowable Increase* and *Allowable Decrease*, tell us how far we can either increase or decrease the right-hand side coefficient of the row without causing a change in the optimal values of the dual prices or reduced costs. If you recall, the dual prices on rows are, effectively, shadow prices that tell us at what price we should be willing to buy (or sell) our resources for. The dual prices do not, however, tell us what quantity we should be willing to buy (or sell) at the dual price. This information is obtained from the allowable increases and decreases on the right-hand side coefficients for the row. So, for our example, the dual prices and reduced costs will remain constant as long as the right-hand side of row *ALIM* falls within the range [20,120], the right-hand side of *CLIM* is greater-than-or-equal-to 30, and the right-hand side of *JOINT* is in [60,160].

Note: We preceded all the constraints in our model with a name enclosed in square brackets. This is an important practice if you wish to generate range reports. If you do not name your constraints, LINGO assigns them a name that corresponds to the internal index of the constraint. This internal index will not always correspond to the order of the constraint in the text of the original model. So, to make the Right-hand Side Ranges section of range reports meaningful, be sure to name all your constraints. (See the section *Constraint Names* for details on assigning constraint names.)

Note: If a variable is nonlinear in the objective, its value in the Current Coefficient column will be displayed as *NONLINEAR*. Similarly, if a row is nonlinear, the value in the *Current RHS* column will be displayed as *NONLINEAR*.

Coefficients that can be increased or decreased indefinitely will display a range of *INFINITY*.

Fixed variables are substituted out of a model and will not appear in a range report. Rows that contain only fixed variables are also substituted out of models and will also not appear in range reports. As an example, suppose we changed the following inequality in our sample model from:

```
[ALIM]      A <= 60;
```

to the equality:

```
[ALIM]      A = 60;
```

LINGO can now solve directly for the value of A. The variable A is considered fixed, as is the row ALIM (since it contains no optimizable variables.) Given this, the variable A will no longer appear in the Objective Coefficient Ranges section of the range report and the row ALIM will not appear in the Right-hand Side Ranges section. We can verify this by examining the updated range report:

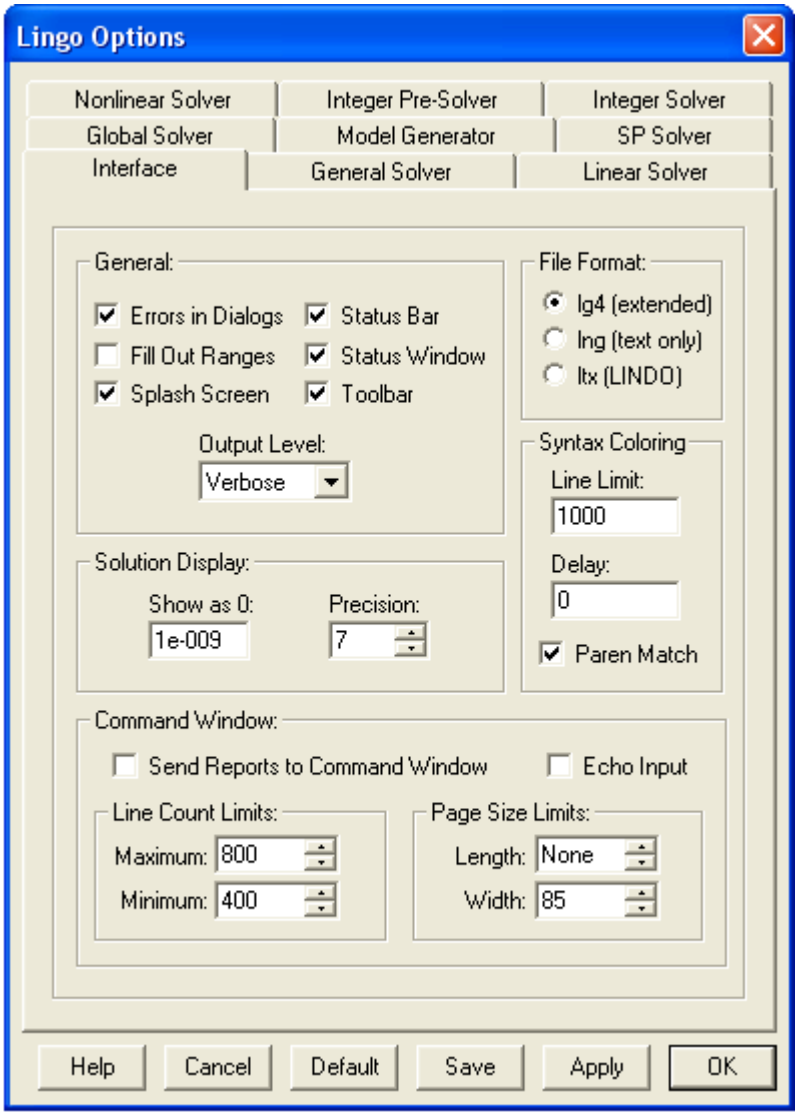
```
Ranges in which the basis is unchanged:

              Objective Coefficient Ranges
              Current    Allowable    Allowable
Variable  Coefficient     Increase     Decrease
       C     30.00000     INFINITY     30.00000

                  Righthand Side Ranges
     Row      Current    Allowable    Allowable
                  RHS     Increase     Decrease
    CLIM     50.00000     INFINITY     20.00000
  J OINT     60.00000     40.00000     60.00000
```

Note: LINGO maintains the range report for only one model in memory. This is the report for the window that you last issued the *Solver|Solve* command for. If you try to issue the *Range* command for a window that LINGO does not currently have range information for, you will receive an error message. If you plan to work with two or more models that take a long time to solve, be sure to save copies of your range reports to disk, so you can refer to them later without having to re-solve your models.

Note: The barrier crossover option must be enabled if you plan to do range analysis. Range computations cannot be performed if the final solution is not a basic solution.

Solver|Options

Use the *Solver|Options* command to change a number of parameters that affect LINGO's user interface, as well as the way LINGO solves your model. When issuing the *Options* command, you will be presented with the following dialog box:



Set these parameters to your personal preference and press the *Apply* button to set them for the extent of the current LINGO session. The currently selected settings are also applied when you click the *OK* button, with the one difference being that the *OK* button closes the dialog box. Click the *Save* button if you would like the current parameter settings to be maintained for use in subsequent LINGO sessions. The original default settings can be restored at any time by clicking the *Default* button.

There are nine tabs on the *Options* dialog box:

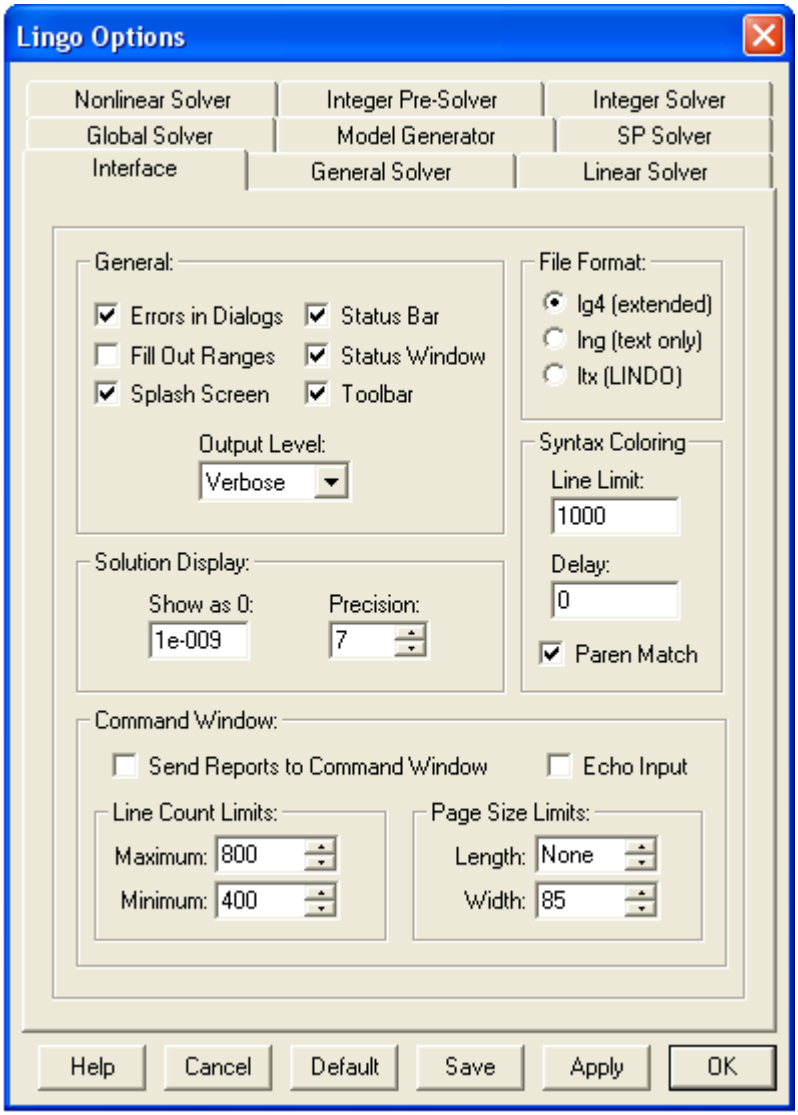
- *Interface*
- *General Solver*
- *Linear Solver*
- *Nonlinear Solver*
- *Integer Pre-Solver*
- *Integer Solver*
- *Global Solver*
- *Model Generator*
- *SP Solver.*

The first time you run the *Options* command during a session, the *Interface* tab will be selected. The *Interface* and *General Solver* tabs contain options of interest to most users. The remaining tabs (*Linear Solver, Nonlinear Solver, Integer Pre-Solver, Integer Solver, Global Solver* and *SP Solver*) contain advanced options that tend to be of interest primarily to the expert user. Follow the links above for more details on the options available under each tab.

Note: LINGO uses the LINDO API as its solver engine. The LINDO API has a wealth of advanced parameter settings to control its various solvers. Most of the more relevant parameters may be set through the *Solver|Options* command. However, some of the more advanced parameters must be set using the *APISET* command.

Interface Tab

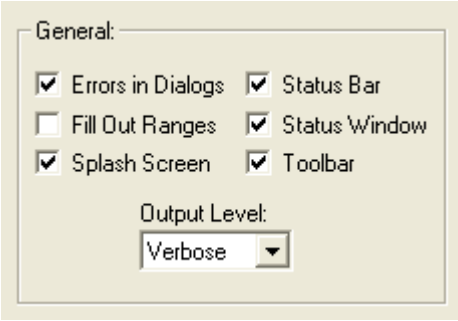
The *Interface* tab on the *Options* dialog box:



can be used to control the appearance of LINGO, LINGO's output, and the default file format.

General Box

The *General* box on the *Interface* tab:



allows you to set the following general options:

- *Errors In Dialogs,*
- *Status Bar,*
- *Fill Out Ranges and Tables*,
- *Status Window,*
- *Splash Screen,*
- *Toolbar, and*
- *Output Level.*

Errors In Dialogs

If the *Errors In Dialogs* box is checked, LINGO will display error messages issued by the solver in a modal dialog box. This dialog box must be cleared before LINGO proceeds with any other operation. In some instances, you may have LINGO embedded in other applications, where it may not be desirable, or possible, to have users clearing error dialogs. By unchecking this option, LINGO will route the solver's error messages to the command window, where they will be displayed, and no user intervention will be required to clear the messages.

The default is for solver errors to be displayed in dialog boxes.

Note:	This option allows you to route only those error messages generated by LINGO's solver to the report window. Error messages displayed by LINGO's interactive front-end will always be posted in dialog boxes.

Status Bar

If the *Status Bar* box is checked, LINGO displays a status bar along the bottom of the main frame window. Among other things, the status bar displays the time of day, location of the cursor, menu tips, and the current status of the program. To remove the status bar from the screen, clear the *Status Bar* checkbox.

The default is for LINGO to display the status bar.

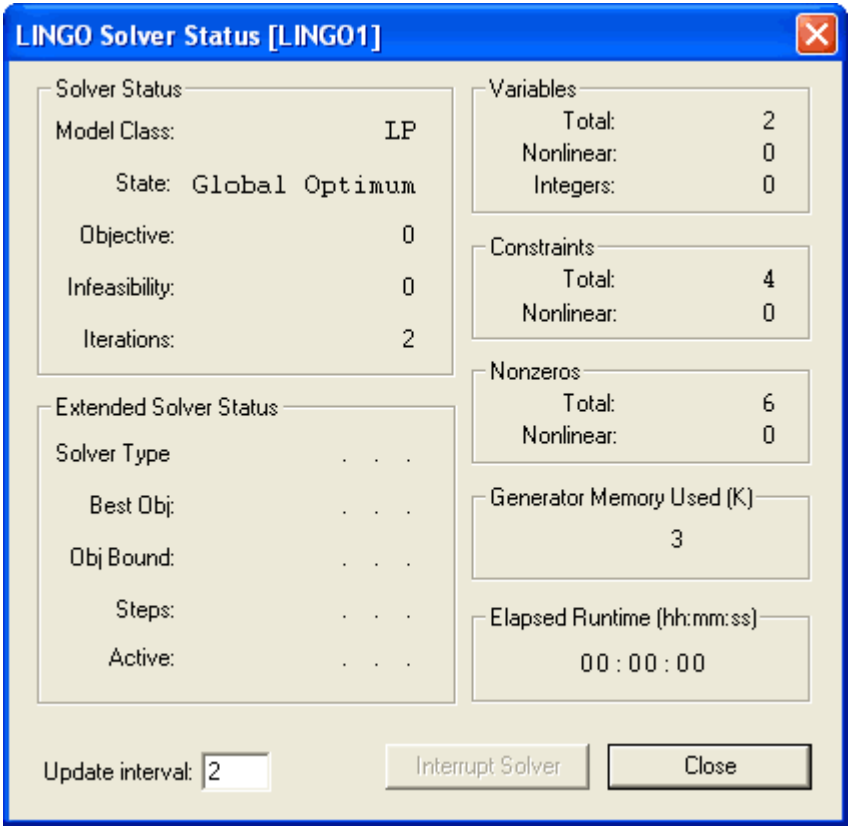
Fill Out Ranges

Windows versions LINGO can export a model's solution to Excel and/or databases. When exporting to Excel, LINGO sends solutions to user defined ranges in a workbook. Solutions exported to a database are sent to tables within the database. In either case, the target range or table may contain more space for values than you are actually exporting. In other words, there may be cells at the end of ranges or records at the end of tables that will not be receiving exported values from LINGO. The *Fill Out Ranges* option determines how these extra cells and records are treated. When the *Fill Out Ranges* option is enabled, LINGO overwrites the extra values with null entries. Conversely, when the option is not enabled, LINGO leaves any extra values untouched.

The *Fill Out Ranges* option is disabled by default.

Status Window

If the Status Window box is checked, LINGO displays a solver status window whenever you issue the *Solver|Solve* command. This window resembles the following:



The solver status window is useful for monitoring the progress of the solver and the dimensions of your model. It is updated every *n* seconds, where *n* is the value in the *Update interval* field in the lower right corner of the window. For a detailed description of the various fields in the solver status window, see the section *Solver Status Window*.

The default is for the solver status window to be displayed.

Splash Screen

If the *Splash Screen* box is checked, LINGO will display its splash screen each time it starts up. The splash screen lists the release number of LINGO and the software's copyright notice. If you disable this option, LINGO will not display the splash screen.

The default is for the splash screen to be displayed.

Output Level

You can use the *Output Level* setting to control the amount of output LINGO generates. There are four settings available:

- ***Verbose***—Causes LINGO to display the maximum amount of output, including full solution reports.
- ***Terse***—Less output than *Verbose,* with full solution reports suppressed. This is a good output level if you tend to solve large models. LINGO also suppresses Export Summary Reports generated when exporting data to spreadsheets or databases.
- ***Errors Only***—All output is suppressed, with the exception of error messages.
- ***Nothing***—LINGO suppresses all output. This level may be useful when taking advantage of the programming capabilities in LINGO, in which case, you will add statements to your model to generate all required output. Keep in mind all output is suppressed in this mode, meaning that error messages are also not displayed. Therefore, the *Nothing* output level should only be used once a model has been thoroughly debugged.

The default is for LINGO to be in verbose mode.

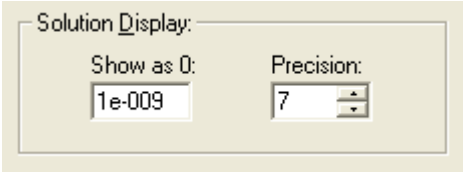
Toolbar

If the *Toolbar* box is checked, LINGO displays a toolbar containing buttons, which act as shortcuts to various commands contained in the LINGO menu. For definitions of the buttons on the toolbar, please see the *The Toolbar* section. If the *Toolbar* checkbox is unchecked, LINGO does not display its toolbar.

The default is for LINGO to display its toolbar.

Solution Display Box

The *Solution Display* box on the *Interface* tab:



allows you to set the following options:

- *Show as 0*, and
- *Precision*.

Show as 0

On occasion, due to round-off error, some of the values returned by LINGO's solver will be very small (less than 1e-9.) In reality, the true values of these variables are either zero or so small as to be of no consequence. These tiny values can be distracting when interpreting a solution report. The *Show as 0*

parameter can be used to suppress small solution values. Any solution value less-than-or-equal-to *Show as 0* will be reported as being zero.

The default value for *Show as 0* is 1e-9.

Precision

LINGO defaults to displaying seven significant digits for values in standard solution reports. In some cases, you may desire more or less than seven digits of precision, in which case, you will need to adjust the *Precision* parameter.

For example, suppose we have a very simple model consisting of the single expression:
X = 1/3;

The default solution report for this model using seven digits of precision would contain:

```
Variable           Value
      X       0.3333333

     Row           Slack or Surplus
      1        0.000000
```

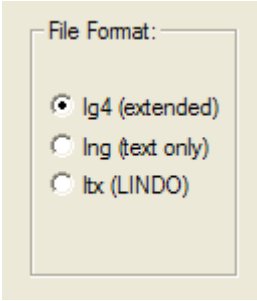
Increasing the *Precision* parameter to 16 and resolving yields the following solution report with 16 significant digits:

```
Variable           Value
      X       0.3333333333333333

     Row           Slack or Surplus
      1        0.000000000000000
```

The default value for *Precision* is seven significant digits.

File Format Box

The *File Format* box on the *Interface* tab:



is used to select the default file format that LINGO uses to save models to disk. There are three different formats to choose from: *LG4*, *LNG*, or *LTX*.

The *LG4* format is a binary format readable only by LINGO and is available only on Windows versions of LINGO. This format enables you to have custom formatting, multiple fonts in your models, and to use LINGO as an OLE server and container. Files saved in the *LG4* format are readable only by Windows versions of LINGO.

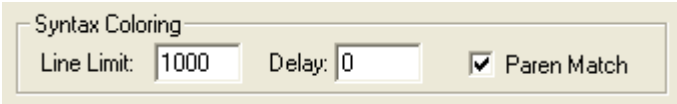
The *LNG* format is a text-based format. Thus, models saved in the *LNG* format can be read into other applications. *LNG* format models are transferable to other platforms running LINGO. Models saved in *LNG* format cannot contain custom formatting or embedded objects.

LTX files are model files that use the LINDO syntax. Longtime LINDO users may prefer LINDO syntax over LINGO syntax. LINDO syntax is convenient for quickly entering small to medium sized linear programs. As long as a file has an extension of .ltx, LINGO will assume that the model is written using LINDO syntax. Readers interested in the details of LINDO syntax may contact LINDO Systems to obtain a LINDO user's manual.

The default file format is *LG4* for Windows versions of LINGO. All other platforms default to *LNG* format.

Syntax Coloring Box

The *Syntax Coloring* box on the *Interface* tab:



is used to control the syntax coloring capability in LINGO's editor. LINGO's editor is "syntax aware". In other words, when it encounters LINGO keywords, it displays them in blue. Comments are displayed in green, and all remaining text is displayed in black. Matching parentheses are also highlighted in red when you place the cursor immediately following a parenthesis.

The controls available in this box are: *Line Limit, Delay,* and *Paren Match*.
Syntax coloring can take a long time if you have very large files. The *Line Limit* field sets the maximum acceptable file size for syntax coloring. Files with line counts exceeding this parameter will not be syntax colored. Setting this parameter to 0 will disable the syntax coloring feature. The default is 1000 lines.

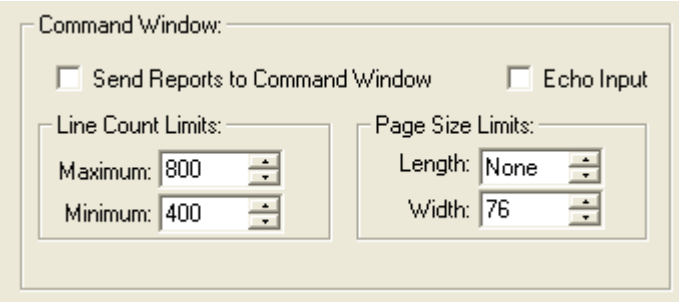
The *Delay* field sets the number of seconds LINGO waits after the last keystroke was typed before re-coloring modified text. Users on slower machines may want to set this higher to avoid having syntax coloring interfere with typing. Users on faster machines may want to decrease this value, so text is re-colored quickly. The default is 0 seconds.

If the *Paren Match* box is checked, LINGO will highlight matching parentheses in red when you place the cursor immediately following a parenthesis. In other words, by placing the cursor immediately after one of the parentheses of interest, you will notice that the color of the parenthesis changes from black to red. LINGO will simultaneously display the matching parenthesis in red. These parentheses

will remain displayed in red until you move the cursor to another position, at which point they will be returned to a black color. The default is for parenthesis matching to be enabled.

Command Window Box

The *Command Window* box on the *Interface* tab:



is used to customize the configuration of LINGO's command window.

LINGO's command window can be opened using the *Window|Command Window* command. This gives the user a command-line interface to LINGO. This interface is identical to ones used by LINGO on platforms other than Windows. The command window is also useful for testing LINGO command scripts. For more information on the commands available under LINGO's command-line interface, refer to *Command-line Commands*.

Send Reports to Command Window

If the *Send Reports to Command Window* box is checked, LINGO will send any reports it generates to the command window rather than to individual report windows. This is useful if you'd like to have two or more LINGO generated reports contained in a single window.

The default is to not send reports to the command window.

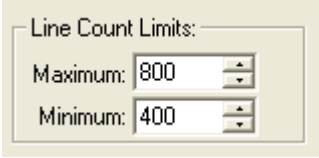
Echo Input

When you run a LINGO command script with *File|Take Commands*, the commands LINGO processes are normally not displayed. If the Echo Input box is checked, processed commands will be displayed in the command window. This can be a useful feature when you are trying to develop and debug a LINGO command script.

The default is to not echo input.

Line Count Limits

The *Line Count Limits* box on the *Interface* tab:



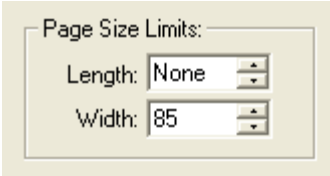
is used to control the total number of output lines that can be stored in the command window. When LINGO sends output to the command window, it places it at the bottom of the window. All previous output is scrolled up to make way for the new output. The *Maximum* field sets the maximum number of output lines allowed in the command window. When LINGO hits this limit, it starts removing lines from the top of the command window until there are *n* lines left, where *n* is the value of the *Minimum* field.

In general, output to the command window will become slower as the maximum and minimum line counts are increased, or the difference between the maximum and minimum is decreased. If you have a long session you need to save, you can use the *File|Log Output* command to log all command window output to disk.

The default value for *Line Count Limits* is 800 lines maximum and 400 lines minimum.

Page Size Limits

The *Page Size Limit* box on the *Interface* tab:



is used to control the page length and width of the command window.

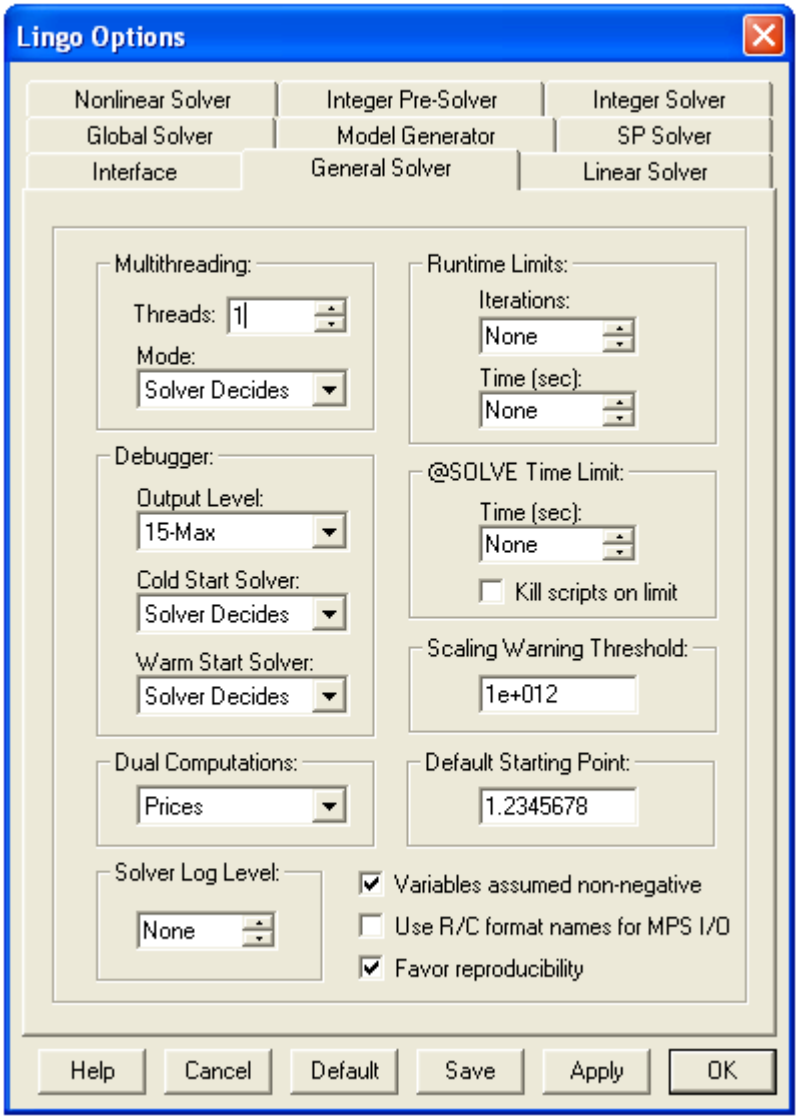
If you would like LINGO to pause after a certain number of lines have been written to the command window, you can do so by setting the *Length* field in the *Page Size Limits* box. When LINGO hits this limit, it will display the following button on your screen:

LINGO will wait until you press the *More* button to display any subsequent output in the command window. The default is *None*, meaning no page length limit is imposed.

When LINGO generates reports, it limits output lines to a certain width. In some reports, lines will be wrapped, so they fall within the line limit. In other reports, lines may be truncated. Because LINGO concatenates variable names in performing set operations, a variable name such as *SHIPMENTS(WAREHOUSE1, CUSTOMER2)* may result. This could be truncated in a solution report if too narrow an output width is used. You can control this line width limit through the *Width* field of the *Page Size Limits* box. You may set it anywhere between 64 and 200, with the default being 85.

General Solver Tab

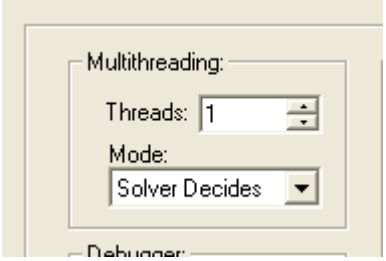
The *General Solver* tab on the *Options* dialog box, shown here:



can be used to control several general parameters related to the functioning of LINGO's solver.

Multithreading Box

The *Multithreading* box on the *General Solver* tab:



gives you control over LINGO's use of multithreading. With multithreading, LINGO's model generator and its solvers can utilize multiple processors to improve performance.

The *Threads* option controls the maximum number of threads, or processors, to utilize at any given time. Interesting settings for the thread count are:

- *Max Available* — LINGO determines the number of processors on the system and sets the thread limit to this number.
- *1* — Only one processor will be used, causing LINGO to run in single-threaded mode.
- *N* — Here, *N* is some positive integer greater than 1 and LINGO will set the maximum number of threads to *N*.

LINGO defaults to running in single-threaded mode with a thread limit of 1.

The second option in the *Multithreading* box is *Mode*, which controls the multithreading mode that LINGO operates in. LINGO offers multicore extensions to its model generator and solvers. The multicore extensions are of two types: *concurrent* and *parallel*. Concurrent algorithms run two or more different serial algorithms on multiple copies of the same model, using a separate thread for each algorithm, terminating as soon as the winner thread finishes. These “different algorithms” may in fact be the same algorithm type, but using different strategies and/or parameters. Parallel algorithms, on the other hand, parallelize computationally intensive portions of the serial algorithm to distribute the workload across multiple threads.

The following multicore extensions are currently available in LINGO:

LINGO Component	*Model Class*	*Parallel Option*	*Concurrent Option*
Barrier Solver	Linear Programs	Yes	No
BNP Solver	Mixed Integer Programs	Yes	No
Global Solver	Nonlinear Programs	Yes	No
Integer Solver	Mixed Integer Programs	Yes	Yes
Linear Solver	Linear Programs	No	Yes
Model Generator	All	Yes	No
Multistart Solver	Nonlinear Programs	Yes	No
Stochastic Solver	Stochastic Programs	Yes	No

The available settings for the *Mode*, or multithreading mode, parameter are as follows:

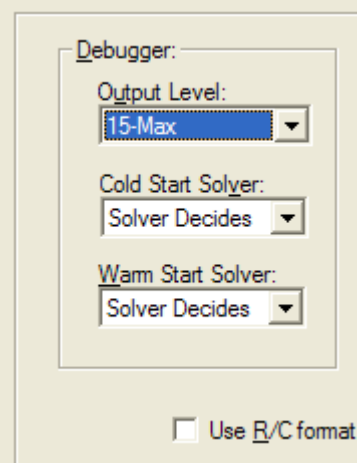
- *Solver Decides* — The best available multithreading strategy, either parallel or concurrent, will be selected for each step of the solution process.
- *Off in Solver* — Multithreading will be disabled in the solvers, but allowed in the model generator if the number of threads is greater than 1.
- *Prefer Parallel* — If a parallel option is available for a particular solution step, then it will be selected, otherwise, a concurrent option will be selected when available.
- *Parallel Only* - If a parallel option is available for a particular solution step, then it will be selected, otherwise, the step will be executed in single-thread mode.
- *Prefer Concurrent* - If a concurrent option is available for a particular solution step, then it will be selected, otherwise, a parallel option will be selected when available.
- *Concurrent Only* - If a concurrent option is available for a particular solution step, then it will be selected, otherwise, the step will be executed in single-thread mode.

Note: If the maximum thread count parameter, *Threads*, is set to 1, then the multithreading mode setting will be ignored, and LINGO will execute in single-threaded mode.

Note: Setting the maximum thread count parameter, *Threads*, to *N* does not guarantee than *N* cores will be fully utilized throughout the entire solve. There may be serial sections inside parallel code that get executed from time to time, causing only one core to be utilized. Concurrent algorithms may also have fewer than *N* different serial algorithms to launch, meaning fewer than *N* cores will be occupied.

Debugger Box

The *Debugger* box on the *General Solver* tab:



gives you control over the output level and the solver used as part of the model debugging command, *Solver|Debug*. The debugger is very useful in tracking down problems in models that are either infeasible or unbounded.

The *Output Level* option controls how much output the model debugger generates. Possible output levels range from 1 (minimum output) to 15 (maximum output). In general, you will want to generate as much output as possible. The only reason to restrict the amount of output would be to speed debugging times on large models.

The default setting for the debugger output level is 15.

The *Cold Start Solver* and *Warm Start Solver* options control the solver used on linear models for cold starts (starting without an existing basis in memory) and warm starts (restarting from an existing basis) during the debugging process. In either case, the available options are:

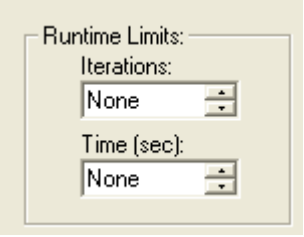
- *Solver Decides* — LINGO selects the solver it believes is the most appropriate,
- *Primal* — the primal simplex solver will be used,
- *Dual* — the dual simplex solver will be used, and
- *Barrier* — the barrier solver will be used (requires a barrier solver license).

With some models, you may find that choosing a particular solver improves overall performance of the debugger.

LINGO defaults to *Solver Decides* for both the cold and warm debug solver.

Runtime Limits Box

The *Runtime Limits* box on the *General Solver* tab:



is used to control the length of time the solver spends on your model.

The first field, *Iterations*, allows you to place an upper limit on the number of iterations the solver will perform. An *iteration* is the fundamental operation performed by the solver. At the risk of oversimplification, it is a process that involves forcing a variable, currently at 0, to become nonzero until some other variable is driven to zero, improving the objective as we go. In general, larger models will take longer to perform an iteration and nonlinear models will take longer than linear models. The default iteration limit is *None*, meaning no limit is imposed on the iteration count.

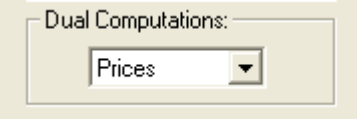
The second field in the *Runtime Limits* box, *Time (sec)*, is a limit on the amount of elapsed time the solver is allowed when optimizing a model. The default time limit is *None*, meaning no limit is imposed on the length of time the solver can run.

If the solver hits either of these limits, it returns to normal command mode. If the model contains integer variables, LINGO will restore the best integer solution found so far. You may need to be patient, however, because the solver may have to perform a fair amount of work to reinstall the current best solution after it hits a runtime limit.

Note: When the solver is interrupted, the only time it will return a valid solution is when an incumbent solution exists, as indicated by a *Best Objective* value in the *Extended Solver Status* box of LINGO's solver status window. Interrupting a model without and incumbent solution will result in an undefined solution. Models that can be interrupted include any model with integer variables, or nonlinear models solved with either the global or multistart solvers.

Dual Computations

The *Dual Computations* box on the *General Solver* tab:



is used to control the level of dual computations performed by the solver.
The choices for this option are:

- *None*,
- *Prices*,
- *Prices & Ranges*, and
- *Prices, Opt Only*.

When the *None* option is selected, LINGO does not compute any dual and range information. This option yields the fastest solution times, but is suitable only if you don't require any dual information. In fact, the *Solver|Range* command will not execute when dual computations are turned off.

When the *Prices* option is selected, LINGO computes dual values, but not the ranges on the duals. When *Prices & Ranges* is selected, LINGO computes both dual prices and ranges.

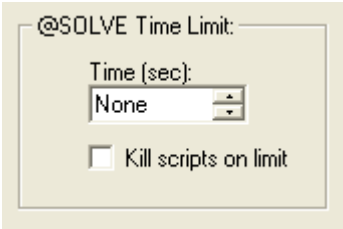
When the *Prices, Opt Only* option is selected, LINGO computes dual values on the optimizable rows only—fixed rows simply receive a dual value of 0. Ranges are also not computed under this option. This can be a useful option if LINGO is spending a lot of time in the "Computing Duals…" phase of the solution process. This phase is devoted to the computation of dual values on the fixed rows.

LINGO defaults to the *Prices* option (computing all the dual prices but not ranges.)

Note: If solution times are a concern, you should avoid unnecessarily enabling range computations.

@SOLVE Time Limit Box

The *@SOLVE Time Limits* box on the *General Solver* tab:



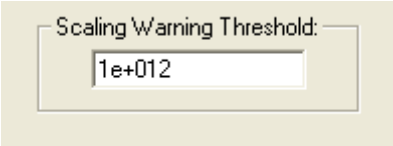
is used to set time limits on the runtime of *@SOLVE* commands, which is a command available in calc sections for solving sub-models. The time limit, if specified, will be applied to each individual *@SOLVE* command encountered in calc sections.

When the *None* option is selected for the *Time* field, LINGO does not impose a time limit. Any nonnegative value will be treated as a runtime limit, in seconds, for each *@SOLVE* command. If the time limit is hit, the *@SOLVE* command will be interrupted, and the best solution found, up to that point, will be returned.

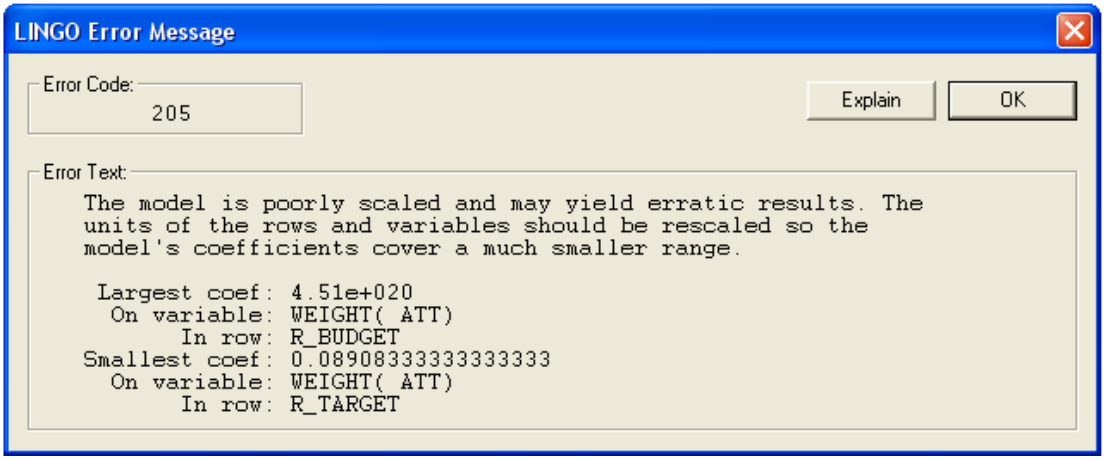
LINGO defaults to the no time limit on *@SOLVE* commands and will not kill scripting when interrupts occur.

Scaling Warning Threshold

The *Scaling Warning Threshold* parameter on the *General Solver* tab:



controls when LINGO displays its scaling warning message:



When LINGO generates a model, it keeps track of the largest and smallest coefficient values in the model. LINGO then computes the ratio of the largest to smallest coefficient, and if the absolute value of this ratio exceeds the setting for *Scaling Warning Threshold,* LINGO will display the above error message.

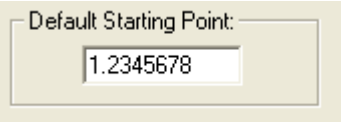
The scaling warning is to reminder that, due to finite precision, round-off error in arithmetic calculations can result when the largest-to-smallest coefficient ratio becomes too large. Not all models will necessarily experience round-off problems when the scaling warning is issued, however, it is something you should be on the lookout for.

The recommended technique for eliminating the scaling warning is to rescale the model so as to reduce the extreme coefficient ratio. Many times, just scaling units of measure (e.g., dollars to millions of dollars) is enough the bring a model's scaling into line. If this is not possible, or your model is performing well in light of the poor scaling, you may choose to increase the *Scaling Warning Threshold* in order to suppress the warning message.

The default value for the *Scaling Warning Threshold* is 1.e12.

Default Starting Point

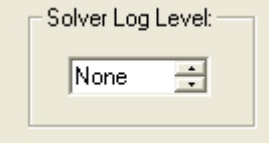
The *Default Starting Point* parameter on the *General Solver* tab:



may be used to set the default starting value for variables. This can be useful in some nonlinear programs, where you want the solver to start in a particular neighborhood close to good solutions to improve runtimes.

Solver Log Level

The *Solver Log Level* parameter on the *General Solver* tab:



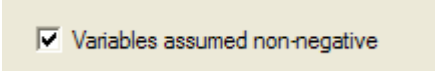
can be used to control the amount of output the solver engines display while running. By default, LINGO does not display much solver log output. However, some technical users may be curious to see this information. Below is a selection of some log output from a small integer program:

```
Number of constraints:       2    le:       1, ge:       0, eq:
   1, rn:       0 (ne:0)
Number of variables  :       9    lb:       1, ub:       0, fr:
   0, bx:       8 (fx:0)
Number of nonzeroes  :      12    density:  0.0067(%)     , sb:
   0

Abs. Ranges     :         Min.          Max.    Condition.
Matrix Coef. (A):      1.00000       7.00000       7.00000
Obj. Vector  (c):      1.00000      10.00000      10.00000
RHS Vector   (b):     15.00000      15.00000       1.00000
Lower Bounds (l):  1.0000e-100   1.0000e-100       1.00000
Upper Bounds (u):      1.00000       1.00000       1.00000
BadScale Measure: 0

Binary variables     :       8 (in 2 constraints)
Integer variables    :       0 (in 0 constraints)

Startpoint info (not feasible):
 Objvalue                          : 3.500000e+01  (startpoint)
 Infeasibility of solution         : 1.8e+01
 Integer infeasibility of solution: 0.0e+00

Feasible obj value found in MIP prerelax heu=0.000000
```

Setting the log level to higher integer values may result in a higher level of output. In general, a level of 1 is perhaps the most useful and meaningful. Setting the log level to 'None' disables solver logs, which is the default setting.

Variables Assumed Non-Negative

When enabled, the *Variables Assumed Non-Negative* checkbox on the *General Solver* tab:

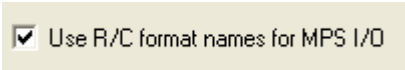


tells LINGO to place a default lower bound of 0 on all variables. In other words, unless otherwise specified, variables will not be allowed to go negative. Should you want a variable to take on a negative value, you may always override the default lower bound of 0 using the *@BND()* function. If this option is disabled, then LINGO's default assumption is that variables are unconstrained and may take on any value, positive or negative. Unconstrained variables are also referred to as be being *free*.

By default, LINGO enables the non-negative option, thereby setting a default lower bound of 0 on all variables.

Use R/C Format Names for MPS I/O

The *Use R/C format names for MPS I/O* checkbox on the *General Solver* tab:

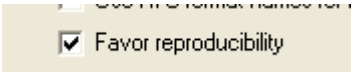


tells LINGO to convert all variable and row names to RC notation when performing MPS file format I/O. Refer to section titled *Importing MPS Files* for a discussion of why this option may be useful.

By default, LINGO disables the use of RC format names.

Favor Reproducibility

The *Favor reproducibility* checkbox on the *General Solver* tab:



allows you to indicate if your preference is for reproducible solutions across multiple runs. In general, being able to reproduce results is a desirable outcome. However, some models may run faster if we opt to not favor reproducible runs.

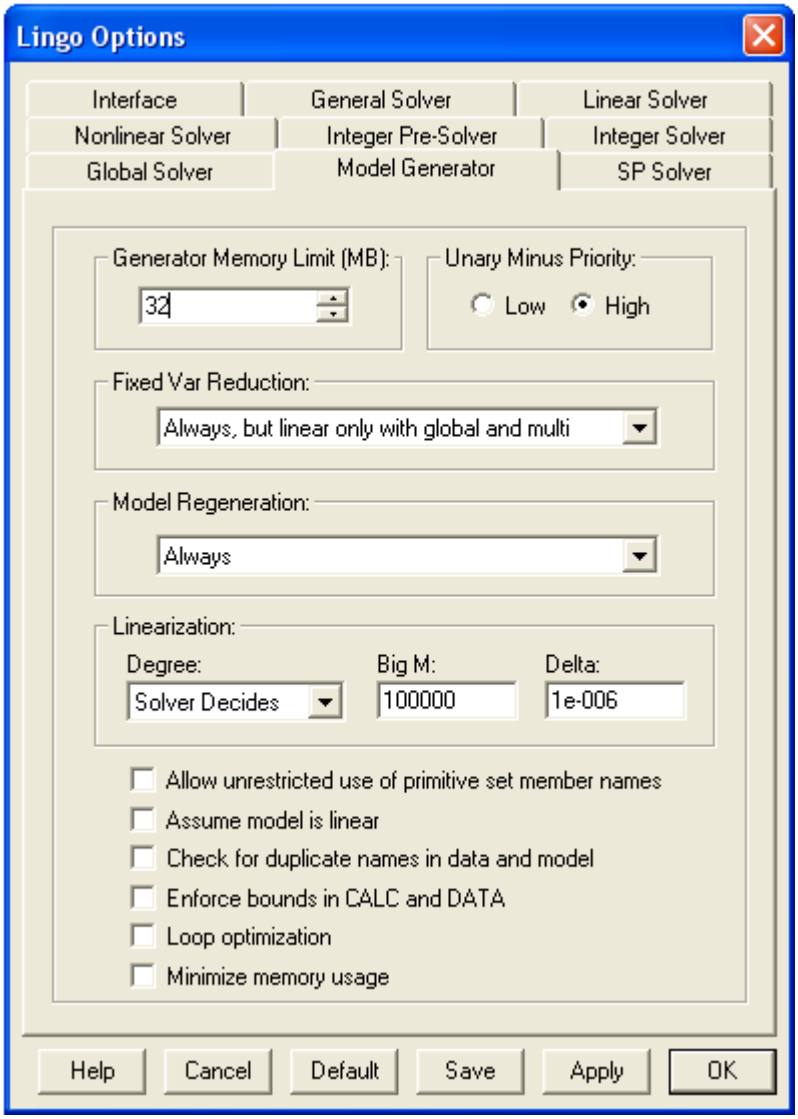
A number of solver steps have been found to perform better when using elapsed time as a measure of work performed. The problem with this approach is that the actual amount of work that gets performed over a fixed time interval will vary slightly across runs depending on a machine's load factor. Many models have alternate optimal solutions, with equal objectives but different variable values. LINGO is indifferent as to which solution is selected; it just wants to find a feasible solution with the best objective value, regardless of the variables values. The end result of this variability in work performed is that a different solution path may be selected from one run to the next, in which case, you may end up with a different alternate optimum from a previous run.

On the other hand, enabling the *Favor reproducibility* option causes LINGO to use fixed measures of work (such as iteration counts), allowing solutions to be reproducible across runs when using the same machine.

By default, LINGO enables the *Favor reproducibility* option.

Model Generator Tab

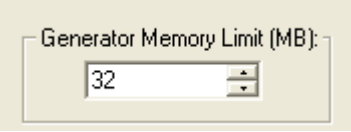
The *Model Generator* tab on the *Options* dialog box, shown here:



is used to control several parameters related to the generation of the model. The model generator takes the expressions in your LINGO model and converts them to a format understood by the solver engines that find the actual solutions to the model.

Generator Memory Limit

The *Generator Memory Limit* box on the *Model Generator* tab:



is used to control the amount of memory set aside to use as workspace for generating a model.

Large models may run out of generator memory when you attempt to solve them. In this case, you will receive the error message "The model generator ran out of memory." To avoid this error, increase the amount of memory in the *Generator Memory Limit* field. You will then need to click the *Save* button and restart LINGO. Since LINGO sets aside this memory when it starts, changes in LINGO's generator memory limit *are not established* until you restart the program.

To determine exactly how much generator memory LINGO was able to successfully allocate, run the *Help|About LINGO* command. The *About LINGO* dialog box displays the amount of generator memory allocated at startup.

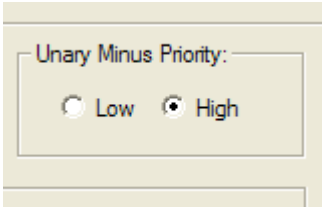
The memory allocated to LINGO's generator will not be available to the various solver engines contained in LINGO. Thus, you should not allocate overly excessive amounts of memory to the generator.

If you set LINGO's generator memory limit to *None*, LINGO will allocate all available memory when it starts up. This is not a recommended practice. The default size for the workspace is 32Mb.

Note:	By setting LINGO's generator memory limit abnormally high, both LINGO and Windows will resort to swapping virtual memory to and from the hard drive, which can slow down your machine dramatically and result in poor performance. In general, set the memory allocation to a level high enough to comfortably handle your largest models, but not too much higher than that. You can view the amount of memory used in the allotted workspace at any time by opening the solver status window and examining the *Generator Memory Used* field.

Unary Minus Priority

The *Unary Minus Priority* box on the *Model Generator* tab:



is used to set the priority of the unary minus operator. The two available options are *High* and *Low*.

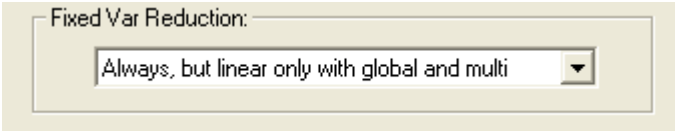
There are two theories as to the priority that should be assigned to the unary minus (i.e., negation) operator in mathematical expressions. On the one hand, there is the Excel practice that the unary minus operator should have the highest priority, in which case, the expression -3^2 would evaluate to +9. On the other hand, there is the mathematicians' preference for assigning a lower priority to unary minus than is assigned to exponentiation, in which case, -3^2 evaluates to -9.

Note that regardless which relative priority is used, one can force the desired result through the use of parenthesis.

LINGO defaults to the Excel approach of setting a higher priority (*High*) on negation than on exponentiation.

Fixed Var Reduction

The *Fixed Var Reduction* box on the *Model Generator* tab:



is used to control the degree to which fixed variables are substituted out of the ultimate math program passed to the solver engines.

For example, consider the model:

```
MAX= 20*X + 30*Y + 12*Z;
X = 2*Y;
X + Y + Z <= 110;
Y = 30;
```

If we run the *Solver|Generate* command, we see that LINGO is able to reduce this model down to the equivalent, but smaller model:

```
MAX= 12 * Z + 2100;
Z <= 20;
```

From the third constraint of the original model, it is obvious that *Y* is fixed at the value 30. Plugging this value for *Y* into the first constraint, we can conclude that *X* has a value of 60. Substituting these two fixed variables out of the original formulation yields the reduced formulation above.

In most cases, substituting out fixed variables yields a smaller, more manageable model. In some cases, however, you may wish to avoid this substitution. An instance in which you might want to avoid substitution would be when equations have more than one root. When multiple roots are present, reduction may select a suboptimal root for a particular equation. On the other hand, the global and multistart solvers are adept at handling equations containing multiple roots. Thus, when using these solvers one may wish to forgo fixed variable reduction.

The available options are:

- *None*,
- *Always*,
- *Always, but linear only with global and multi*, and
- *Linear rows only*.

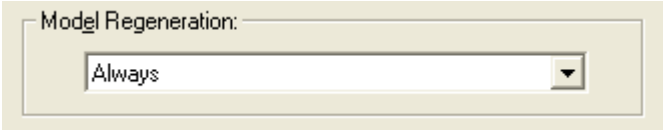
Selecting *None* disables all fixed variable reduction. Selecting *Always* enables reduction. When *Always, but linear only with global and multi* is selected, LINGO always enables reduction except when either the global or multistart solvers are selected, in which case it will only perform reduction on rows where the key variable appears linearly. The *Linear rows only* option always limits reduction to rows in which the key variable is linear.

Note: You should be careful when turning off fixed variable reduction. If the model generator is unable to substitute out fixed variables, you may end up turning a linear model into a more difficult nonlinear model.

LINGO defaults to selecting *Always, but linear only with global and multi* for fixed variable reduction.

Model Regeneration

The *Model Regeneration* box on the *Model Generator* tab:



is used to control the frequency with which LINGO regenerates a model. Commands that will trigger the model generator are:

- *Solver|Solve,*
- *Solver|Generate,*
- *Solver|Model Statistics,*
- *Solver|Picture*,
- *Solver|Debug,* and
- *File|Export_File.*

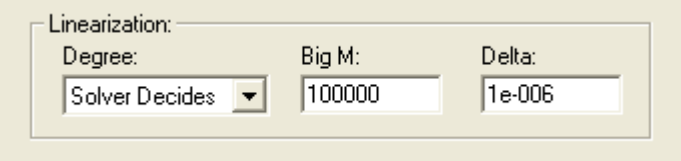
The choices available under this option are:

- *Only when text changes* — LINGO regenerates a model only when a change has been made to the model's text since the last generation took place.
- *When text changes or with external references* — LINGO regenerates whenever a change is made to the model text or when the model contains references to external data sources (e.g., text files, databases, or spreadsheets.)
- *Always* — (default) LINGO *always* regenerates the model each time information regarding the generated model is needed.

LINGO defaults to always regenerating the model.

Linearization

The *Linearization* box on the *Model Generator* tab:



controls the linearization option in LINGO. Many nonlinear operations can be replaced by linear operations that are mathematically equivalent. The ultimate goal is to replace all the nonlinear operations in a model with equivalent linear ones, thereby allowing use of the faster and more robust linear solvers. We refer to this process as linearization. For more information on linearization, please refer to the section *On Mathematical Modeling*.

Degree determines the extent to which LINGO will attempt to linearize models.

The available options are:

- *Solver Decides,*
- *None,*
- *Math Only,*
- *Math and Logic.*

Under the *None* option, no linearization occurs. With the *Math Only* option, LINGO linearizes the mathematical functions: *@ABS(), @FLOOR(), @IF(), @MAX(), @MIN(), @SIGN(). @SMAX()*, and *@SMIN()* along with any products of binary and continuous variables. The *Math and Logic* option is equivalent to the *Math* option plus LINGO will linearize all logical operators (*#LT#, #LE#, #EQ#, #GT#, #GE#,* and *#NE#*). Under the *Solver Decides* option, LINGO will do maximum linearization if the number of variables is less-than-or-equal-to 12, otherwise, LINGO will not perform any linearization. LINGO defaults to the *Solver Decides* setting.

The *Delta Coefficient* is a tolerance indicating how closely you want the additional constraints added as part of linearization to be satisfied. Most models won't require any changes to this parameter. However, some numerically challenging formulations may benefit from increasing *Delta* slightly. LINGO defaults to a *Delta* of 1.e-6.

When LINGO linearizes a model, it will add *forcing constraints* to the mathematical program generated to optimize your model. These forcing constraints are of the form:

$$f(x) \leq M \bullet y$$

where M is the *BigM Coefficient* and y is a 0/1 variable. The idea is that if some activity in the variables is occurring, then the forcing constraint will drive y to take on the value of 1. Given this, if we set the *BigM* value to be too small, we may end up with an infeasible model. Therefore, the astute

reader might conclude that it would be smart to make *BigM* quite large, thereby minimizing the chance of an infeasible model. Unfortunately, setting *BigM* to a large number can lead to numerical stability problems in the solver resulting in infeasible or sub-optimal solutions. So, getting a good value for the *BigM Coefficient* may take some experimentation.

As an example of linearization, consider the following model:

```
model:
sets:
  projects: baths, sqft, beds, cost, est;
endsets
data:
projects, beds, baths,     sqft,  cost =
p1           5      4      6200   559608
p2           2      1       820   151826
p3           1      1       710   125943
p4           4      3      4300   420801
p5           4      2      3800   374751
p6           3      1      2200   251674
p7           3      2      3400   332426
;
enddata
min = @max( projects: @abs( cost - est));
@for( projects:
est = a0 + a1 * beds + a2 * baths + a3 * sqft
);
end
```

Model: COSTING

This model estimates the cost of home construction jobs based on historical data on the number of bedrooms, bathrooms, and square footage. The objective minimizes the maximum error over the sample project set. Both the *@MAX()* and *@ABS()* functions in the objective are non-smooth nonlinear functions, and, as a result, can present problems for LINGO's default, local search NLP solver.

Running the model under the default settings with linearization disabled, we get the following result:

```
Local optimal solution found at step:      91
Objective value:                     3997.347
   Variable         Value     Reduced Cost
         A0      37441.55         0.000000
         A1      27234.51         0.000000
         A2      23416.53         0.000000
         A3      47.77956         0.000000
```

Enabling linearization and re-optimizing yields the substantially better solution:

```
Global optimal solution found at step:  186
Objective value:                    1426.660
   Variable         Value     Reduced Cost
         A0      46814.64         0.000000
         A1      22824.18         0.000000
         A2      16717.33         0.000000
         A3      53.74674         0.000000
```

Note that the maximum error has been reduced from 3,997 to 1,426!

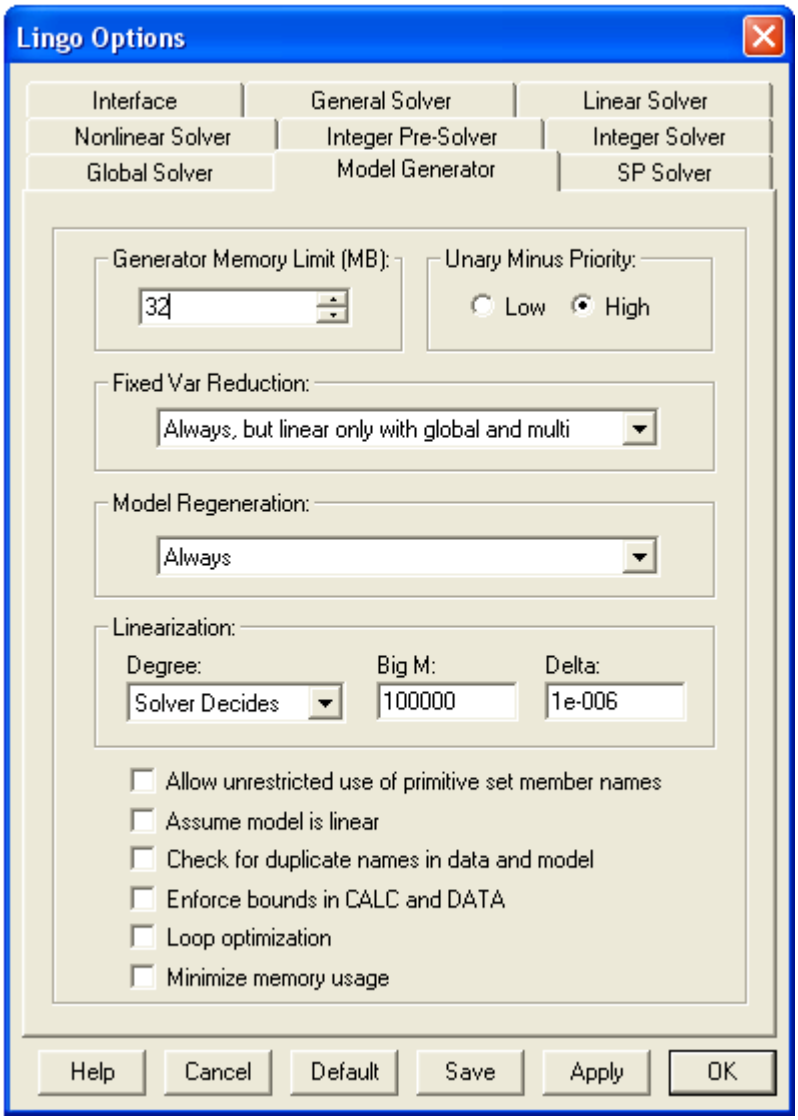
Linearization will substantially increase the size of your model. The sample model above, in un-linearized form, has a mere 8 rows and 11 continuous variables. On the other hand, the linearized version has *51 rows*, *33 continuous variables*, and *14 binary variables*! Although linearization will cause your model to grow in size, you will tend to get much better solution results if the model can be converted entirely to an equivalent linear form.

Note: Linearization will be of most use when a nonlinear model can be 100% linearized. If LINGO can only linearize a portion of your model, then you may actually end up with a more difficult nonlinear model.

The linearization option is set to *Solver Decides* by default.

Allow Unrestricted Use of Primitive Set Member Names Box

The *Allow unrestricted use of primitive set member names* checkbox on the *Model Generator* tab:



allows for backward compatibility with models created in earlier releases of LINGO.

In many instances, you will need to get the index of a primitive set member within its set. Prior to release 4 of LINGO, you could do this by using the primitive set member's name directly in the model's equations. This can create problems when you are importing set members from an external source. In this case, you will not necessarily know the names of the set members beforehand. When one of the imported primitive set members happens to have the same name as a variable in your model,

unintended results can occur. More specifically, LINGO will not treat such a variable as optimizable. In fact, it would treat it as if it were a constant equal to the value of the index of the primitive set member!

In short, different primitive set names can potentially lead to different results. Therefore, starting with release 4.0 of LINGO, models such as the following were no longer permitted:

```
MODEL:
SETS:
DAYS /MO TU WE TH FR SA SU/;
ENDSETS
INDEX_OF_FRIDAY = FR;
END
```

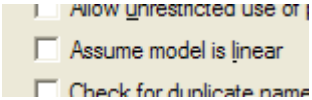
If you want the index of FR in the DAYS set, you should now use the *@INDEX* function:

```
INDEX_OF_FRIDAY = @INDEX( DAYS, FR);
```

By default, LINGO disables the use of primitive set member names.

Assume Model is Linear

The *Assume model is linear* checkbox on the *Model Generator* tab:

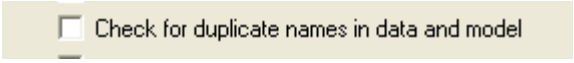


can be used for minimizing memory usage on models that are entirely linear. When this option is in effect, the model generator can take steps to dramatically reduce overall memory consumption without sacrificing performance. In fact, if all your models are linear, we recommend that you enable this option permanently as the default for your installation. The one restriction is that the model must prove to be *entirely linear*. If a single nonlinearity is detected, you will receive an error message stating that the model is nonlinear and model generation will cease. At which point, you should clear this option and attempt to solve the model again.

By default, the *Assume model is linear* option is disabled.

Check for Duplicate Names

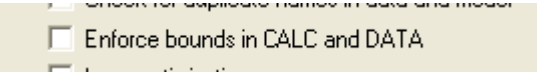
The *Check for duplicate names in data and model* checkbox on the General Solver tab:



allows you to test your LINGO models from older releases for instances where primitive set members appear in the model's equations. The next time you run a model, LINGO will issue an error message if duplicate names appear as set members and as variables in the model.
By default, this option is disabled.

Enforce Bounds In CALC and DATA

The *Enforce Bounds In CALC and DATA* checkbox on the *General Solver* tab:



causes LINGO to check the values of any variables input in calc and data sections against their bounds. Normally, variable bounds apply only to optimizable variables that appear in the model's constraints and not to variables fixed to constant values in calc and data sections. If you would also like LINGO to check variables fixed in calc and data sections against their bounds, then you will need to enable this option.

By default, this option is disabled.

Loop Optimization

The *Loop optimization* checkbox on the *Model Generator* tab:

is used to either enable or disable LINGO's *loop optimization* step. Loop optimization reformulates expressions containing set looping functions in order to make them more efficient, while maintaining mathematical equivalency. The end goal of loop optimization is to minimize the number of passes through the inner loop of any nested loops in an expression.

As an example, consider the following transportation model fragment, that just contains constraints for satisfying customer demand:

```
MODEL:
! A transportation problem fragment;
SETS:
  WAREHOUSE /1..50/ : CAPACITY;
  CUSTOMER /1..5000/ : DEMAND;
  ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The demand constraints;
@FOR( CUSTOMER( J):
   @SUM( ROUTES( I, J): VOLUME( I, J)) >= DEMAND( J)
);

END
```

Transportation Model Fragment with Inefficient Constraints

In the demand constraints expression there are two loops — an outer *@FOR()* over the *CUSTOMER* set and an inner *@SUM()* over the *ROUTES* set. As written, the inner loop must be executed 5000*50*5000=*1.25 billion times*.

Note that a valid reformulation of these demand constraints would be:

```
! The demand constraints;
@FOR( CUSTOMER( J):
   @SUM( WAREHOUSE( I): VOLUME( I, J)) >= DEMAND( J)
);
```

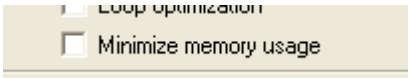
With the expression rewritten in this manner, the inner loop will now only be executed 50*5000 times, reducing the total number of passes by a factor of 5000. LINGO's loop optimizer seeks out such inefficient loops and, invisible to the user, rewrites them before the model gets passed to the generator. Of course, the end result of such reformulations are faster generation times. In fact, the speedup from reformulation can be quite dramatic in many cases. In this particular example, the reformulated version ran over 400 times faster in the model generator.

Note that in some cases, particularly with models that primarily contain sparse sets, loop optimization may actually increase runtimes. So, you should compare runtimes with, and without, the feature being enabled.

By default, the *Loop optimization* feature is enabled.

Minimize Memory Usage

The Minimize memory usage checkbox on the General Solver tab:

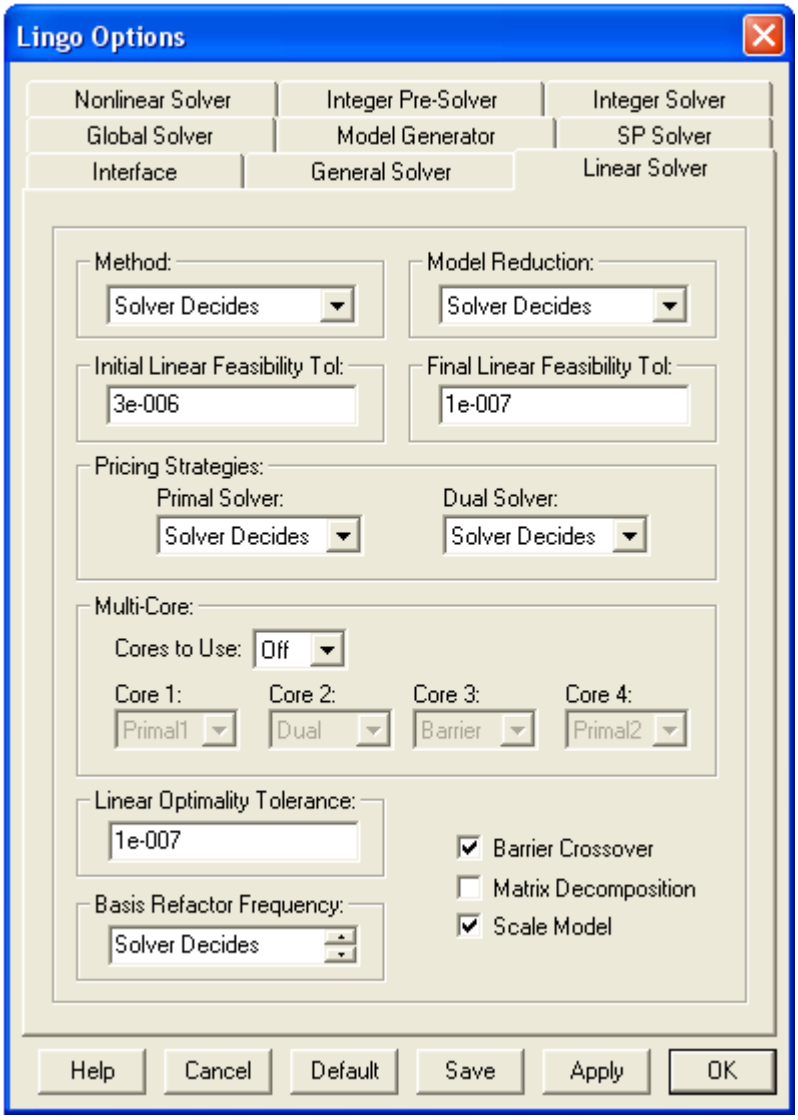


may be used to guide LINGO’s memory usage. Enabling *Minimize memory usage* causes LINGO to opt for less memory usage when solving a model. The downside is that opting for less memory may result in longer runtimes.

LINGO defaults to disabling *Minimize memory usage.*

Linear Solver Tab

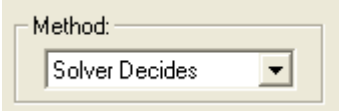
The *Linear Solver* tab on the *Options* dialog box, pictured here:



can be used to control several options for tailoring the operation of LINGO's linear solver. The linear solver is used on linear models and mixed integer linear models as part of the branch-and-bound process.

Method Box

The *Method* box on the *Linear Solver* tab:



is used to control the algorithm LINGO's linear solver employs.

The current choices are:

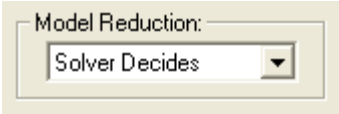
- *Solver Decides* — LINGO selects the algorithm it determines is most appropriate.
- *Primal Simplex* — LINGO uses a primal simplex algorithm.
- *Dual Simplex* — LINGO uses a dual simplex algorithm.
- *Barrier* — LINGO uses a barrier algorithm (i.e., interior point.)

The simplex algorithm moves around the exterior of the feasible region to the optimal solution, while the interior point algorithm, or barrier solver, moves through the interior of the feasible region. In general, it is difficult to say which algorithm will be fastest for a particular model. A rough guideline is primal simplex tends to do better on sparse models with fewer rows than columns. Dual simplex does well on sparse models with fewer columns than rows. And the barrier solver works best on densely structured models or very large models.

The barrier solver is available only as an additional option to the LINGO package. Furthermore, if the model has any integer variables, the barrier solver will be used for solving the LP at the initial root node of the branch-and-bound tree, but may or may not be used on subsequent nodes. From a performance point-of-view, the barrier solver's impact will be reduced on integer models.
LINGO defaults to the *Solver Decides* option.

Model Reduction Box

The *Model Reduction* box on the *Linear Solver* tab:



is used to control the amount of model reduction performed by LINGO's linear solver.

Your options are:

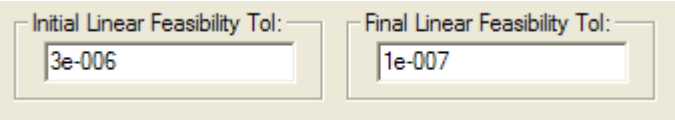
- *Off* — Disables reduction,
- *On* — Reduction is used on all models, and
- *Solver Decides* — LINGO decides whether or not to use reduction.

When this option is enabled, LINGO attempts to identify and remove extraneous variables and constraints from the formulation before solving. In certain cases, this can greatly reduce the size of the final model to be solved. Sometimes, however, reduction merely adds to solution times without trimming back much on the size of the model.

LINGO defaults to the *Solver Decides* option.

Feasibility Tolerance Boxes

The *Initial Linear Feasibility Tol* and the *Final Linear Feasibility Tol* boxes on the *Linear Solver* tab:



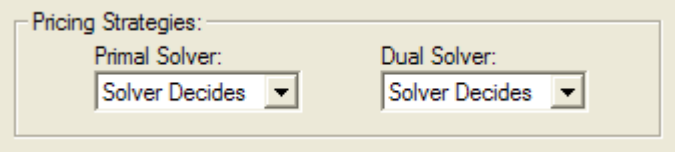
are used to control the feasibility tolerances for the linear solver. These tolerances are related to how closely constraints must be satisfied in linear models. In general, if your models are well formulated, you should not have to modify these tolerances. However, access to these tolerances is provided for the expert user.

Due to the finite precision available for floating point operations on digital computers, LINGO can't always satisfy each constraint exactly. Given this, LINGO uses these two tolerances as limits on the amount of violation allowed on a constraint while still considering it "satisfied". These two tolerances are referred to as the *Initial Linear Feasibility Tolerance* (ILFT) and the *Final Linear Feasibility Tolerance* (FLFT.) The default values for these tolerances are, respectively, .000003 and .0000001.

The ILFT is used when the solver begins iterating. In the early stages of the solution process, having the solver less concerned with accuracy issues can boost performance. When LINGO thinks it has an optimal solution, it switches to the more restrictive FLFT. At this stage in the solution process, you want a relatively high degree of accuracy. Thus, the FLFT should be smaller than the ILFT.
One instance where these tolerances can be of use is when LINGO returns a solution that is almost feasible. You can verify this by checking the values in the *Slack or Surplus* column in the model's solution report. If there are only a few rows with small negative values in this column, then you have a solution that is close to being feasible. Loosening (i.e., increasing) the ILFT and FLFT may help you get a feasible solution. This is particularly true in a model where scaling is poor (i.e., very large and very small coefficients are used in the same model), and the units of measurement on some constraints are such that minor violations are insignificant. For instance, suppose you have a budget constraint measured in millions of dollars. In this case, a violation of a few pennies would be of no consequence. Short of the preferred method of re-scaling your model, loosening the feasibility tolerances may be the most expedient way around a problem of this nature.

Pricing Strategies Box

The *Pricing Strategies* box on the *Linear Solver* tab:



is used to control the pricing strategy used by LINGO's simplex solvers. Pricing determines the relative attractiveness of the variables during the simplex algorithm.

For the *Primal Solver*, you have the following choices:

- *Solver Decides* — LINGO selects the pricing method it believes is the most appropriate.
- *Partial* — LINGO prices out a small subset of variables at each iteration and intermittently prices out all the variables to determine a new subset of interesting variables.
- *Devex* — Devex prices out all columns at each iteration using a steepest-edge approximation (see below.)

Partial pricing tends to yield faster iterations. Devex, while slower, results in fewer overall iterations and can be helpful when models are degenerate. Thus, it is difficult to determine which method is superior beforehand.

For the *Dual Solver*, you have these options:

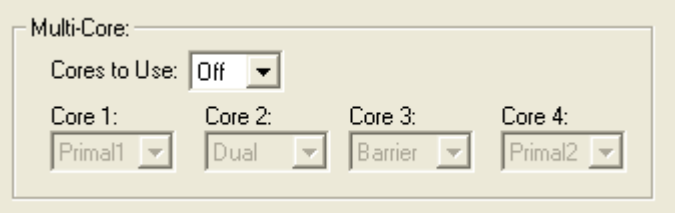
- *Solver Decides* — LINGO selects the pricing method it believes is the most appropriate.
- *Dantzig* — The dual simplex solver will tend to select variables that offer the highest absolute rate of improvement to the objective regardless of how far other variables may have to move per unit of movement in the newly introduced variable.
- *Steepest-Edge* — The dual solver spends a little more time selecting variables by looking at the total improvement in the objective when adjusting a particular variable.
- *Devex* — Devex prices out all columns at each iteration using a steepest-edge approximation.
- *Approximate Devex* — An simplified implementation of true Devex pricing.

Dantzig pricing generally yields faster iterations, however, the other variables in the model may quickly hit a bound resulting in little gain to the objective. With the steepest-edge option, each iteration will tend to lead to larger gains in the objective resulting in fewer overall iterations, however, each iteration will tend to take more compute time due to increased time spent in pricing. The Devex options approximate true steepest-edge pricing.

LINGO defaults to *Solver Decides* for both the primal and dual solvers.

Multi-Core Box

The *Multi-Core* box on the *Linear Solver* tab:



may be used to perform parallel solves of linear programs on multiple cores. One of four different linear solvers is chosen for each core. LINGO will take the solution from the solver that finishes first and then interrupt the remaining solver threads.

The idea behind this approach is that different linear solvers will have relatively better or worse performance on different classes of models. However, it may be difficult to predict beforehand the solver that is most likely to outperform. So, by enabling multi-core solves, we guarantee that we will always get top performance, even without knowledge beforehand of which solver is likely to run the fastest.

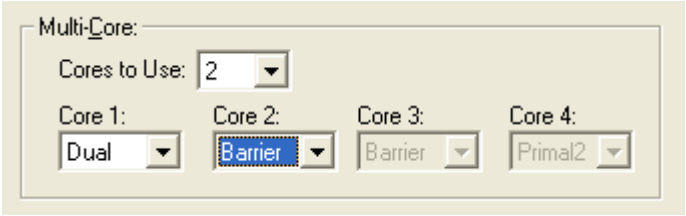
Note: The multi-core feature requires that your machine have at least one core free for each solver you wish to run. Using this feature with an inadequate number of cores will tend to decrease overall performance.

For the *Cores to Use* parameter, you have the following choices: *Off, 2, 3,* or 4. When the default *Off* option is selected, the multi-core feature is disabled, and LINGO will run only one solver on linear programs, namely the one specified as part of the *Solver Method* option detailed above. When either option *2, 3*, or *4* is selected, LINGO will run linear solvers in the requested number of cores.

When selecting 2 or more cores, you will have the option to specify which of the linear solvers to use in each of the running cores as part of the *Core1* - *Core4* list boxes. The available linear solvers are:

- *Primal1* - Primal simplex algorithm 1
- *Dual* - Dual simplex algorithm
- *Barrier* - Barrier/Interior point solver (available as a option)
- *Primal2* - Primal simplex algorithm 2, installed as part of the Barrier option

As an example, the settings of the *Multi-Core* box below are requesting to run LP solvers in two cores, with core 1 running the dual simplex solver and core 2 running the barrier solver:

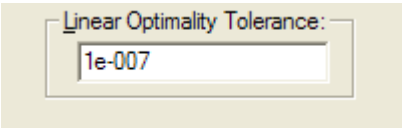


While LINGO is solving linear programs it normally displays solver statistics in the Solver Status Window. This will also be true with muti-core solves. However, LINGO reports the statistics from only one of the solvers, specifically, the solver selected to run in Core 1. Once optimization is complete, LINGO will populate the Solver Status Window with statistics from the solver that finished first. Finally, as part of the solution report, LINGO will display a line indicating the solver that finished first. In the solution report excerpt below, we see that the barrier solver was the first to completion:

```
First returning solver: BARRIER
Global optimal solution found.
Objective value:                        -247203.9
Infeasibilities:                         0.000000
Total solver iterations:                       68
Elapsed runtime seconds:                    28.35
```

Linear Optimality Tolerance Box

The *Linear Optimality Tolerance* box on the *Linear Solver* tab:

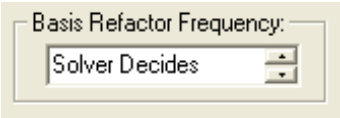


allows you to control the setting for the linear optimality tolerance. This tolerance is used to determine whether a reduced cost on a variable is significantly different from zero. You may wish to loosen this tolerance (make it larger) on poorly scaled and/or large models to improve performance.

The default setting for the *Linear Optimality Tolerance* is 1.e-7.

Basis Refactor Frequency Box

The *Basis Refactor Frequency* box on the *Linear Solver* tab:

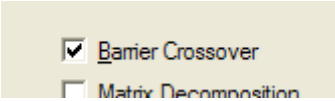


allows you to control how frequently the linear solver refactors the basis matrix. The options are either *Solver Decides* or some positive integer quantity. If an integer value, *N*, is selected, then the linear solver will refactor every *N* iterations. Numerically tough and/or poorly scaled models may benefit from more frequent refactoring. However, refactoring too frequently will cause the solver to slow down.

The default setting for the *Basis Refactor Frequency* is *Solver Decides*, which will typically result in refactoring about once every 100 iterations.

Barrier Crossover

The *Barrier Crossover* box on the *Linear Solver* tab:



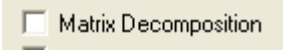
is used to control whether or not the barrier solver performs a *crossover* operation. Unlike simplex algorithms, the barrier solver does not automatically find basic (cornerpoint) solutions. Very roughly speaking, basic solutions have the nice mathematical property that exactly *m* variables will have nonzero values, where *m* is the number of constraints. The crossover procedure takes the barrier's non-basic solution, and, through the use of a simplex solver, converts the non-basic solution to a basic one. If the basic solution property is not important for your models, then you may wish to disable crossovers to improve performance when using the barrier solver.

Note:	Barrier crossover *must be enabled* if you plan to do range analysis. Range computations cannot be performed if the final solution is not a basic solution.

The default is to perform crossovers.

Matrix Decomposition

The *Matrix Decomposition* box on the *Linear Solver* tab:



allows you to enable the matrix decomposition feature.

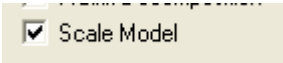
Many large-scale linear and mixed integer problems have constraint matrices that are totally decomposable into a series of block structures. If total decomposition is possible, LINGO will solve

the independent problems sequentially and report a solution for the original model, resulting in dramatic speed improvements.

LINGO defaults to not using matrix decomposition.

Scale Model

The *Scale Model* box on the *Linear Solver* tab:



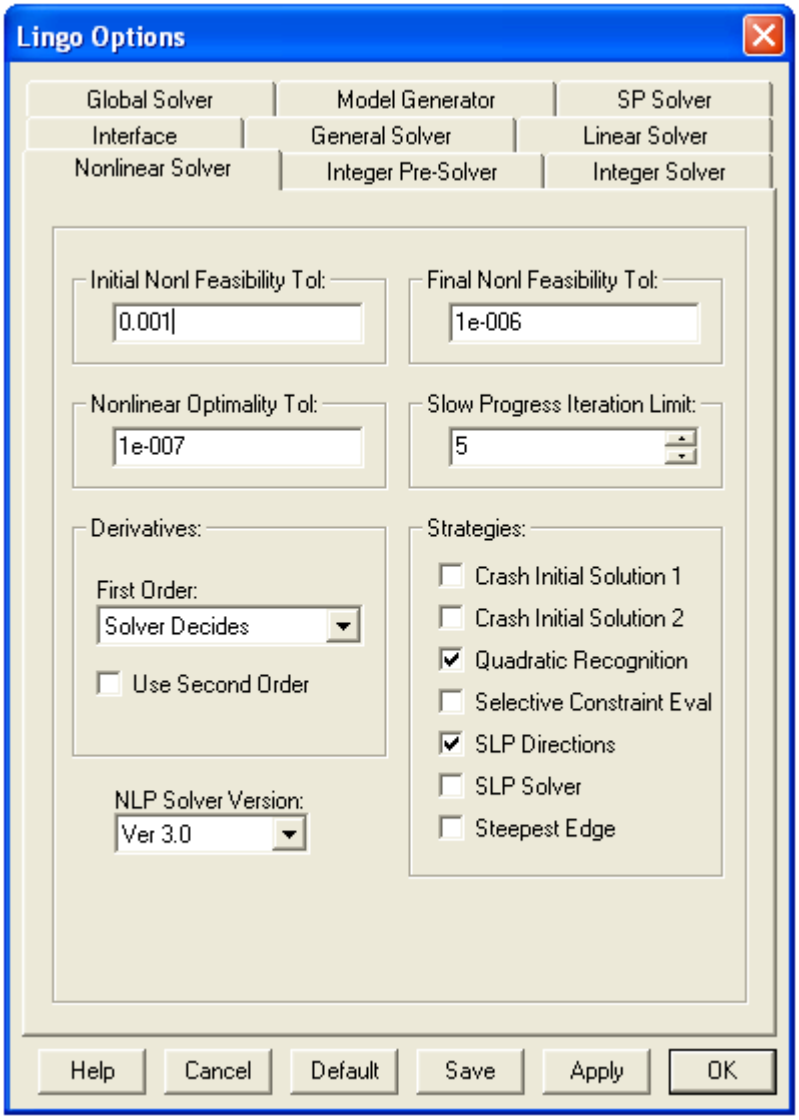
allows you to enable the matrix scaling option.

This option rescales the coefficients in the model's matrix, so the ratio of the largest to smallest coefficients is reduced. This reduces the chances of round-off error, which leads to greater numerical stability and accuracy in the linear solver.

LINGO defaults to using scaling.

Nonlinear Solver Tab

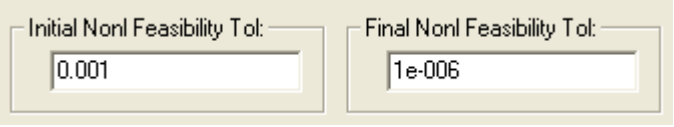
The *Nonlinear Solver* tab on the *Options* dialog box, pictured here:



controls several options that affect the operation of LINGO's solver on nonlinear models.

Initial and Final Nonlinear Feasibility Tols

The *Initial Nonlinear Feasibility* tolerance and the *Final Nonlinear Feasibility* tolerance boxes on the *Nonlinear Solver* tab:

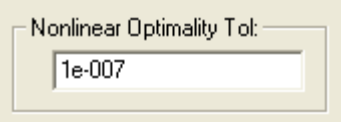


are used to control the feasibility tolerances for the nonlinear solver in the same manner that the *Initial Linear* and *Final Linear Feasibility* tolerances are used by the linear solver. For information on how and why these tolerances are useful, refer to the *Feasibility Tolerances* section on the *Linear Solver Tab*.

Default values for these tolerances are, respectively, .001 and .000001.

Nonlinear Optimality Tolerance

The *Nonlinear Optimality Tol* box on the *Nonlinear Solver* tab:



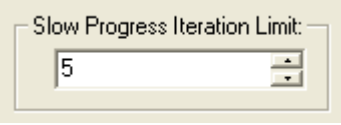
is used to control the adjustments to variables as described below.

While solving a model, the nonlinear solver is constantly computing a *gradient*. The gradient gives the rate of improvement of the objective function for small changes in the variables. If the gradient's rate of improvement computation for a given variable is less-than-or-equal-to the *Nonlinear Optimality Tolerance*, further adjustments to the variable's value are not considered to be beneficial. Decreasing this tolerance towards a limit of 0 will tend to make the solver run longer and may lead to better solutions for poorly formulated or poorly scaled models.

The default value for *Nonlinear Optimality Tolerance* is .0000001.

Slow Progress Iteration Limit

The *Slow Progress Iteration* Limit (SPIL) box on the *Nonlinear Solver* tab:



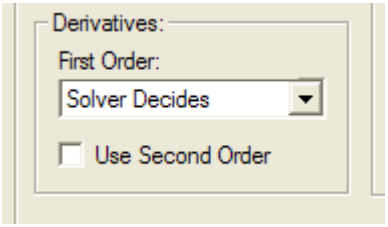
is used to terminate the solution process if little or no progress is being made in the objective value.

Specifically, if the objective function's value has not improved significantly in *n* iterations, where *n* is the value of SPIL, the nonlinear solver will terminate the solution process. Increasing this tolerance's value will tend to force the solver to run longer and may be useful in models with relatively "flat" objective functions around the optimal solution.

The default value for SPIL is 5 iterations.

Derivative Computation

The *Derivatives* box on the *Nonlinear Solver* tab:



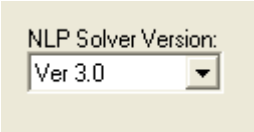
controls how derivatives are used in the nonlinear solver. There are two settings available: *First Order* and *Use Second Order*.

The *First Order* option determines how the nonlinear solver computes first order derivatives. There are two general methods available: *numerical* or *analytical derivatives*. Analytical derivatives are computed directly by symbolically analyzing the arithmetic operations in a constraint. Numerical derivatives are computed using finite differences. There are two types of numerical derivatives available using either central differences or *forward differences*. There are also two types of analytical derivatives available: backward analytical and forward analytical. Finally, a *Solver Decides* option is also available. LINGO defaults to the *Solver Decides* setting for the *First Order* option, which presently involves LINGO using backward analytical derivatives. However, one of the other choices may be more appropriate for certain classes on nonlinear models. We suggest you try the various derivative options to see which works best for your particular models.

The *Use Second Order* option determines if the nonlinear solver will use second order derivatives. If used, second order derivatives will always be computed analytically. Computing second order derivatives will take more time, but the additional information they provide may lead to faster runtimes and/or more accurate solutions. LINGO defaults to not using second order derivatives.

Nonlinear Solver Version

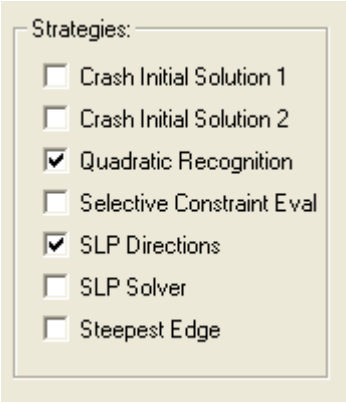
The *Nonlinear Solver Version* option on the *Nonlinear Solver* tab:



may be used to select the version of the nonlinear solver. Presently, there are two options: *Version 3.0* and *Version 4.0.* LINGO defaults to using Version 3.0, but you may find that on some models Version 4.0 can perform better.

Strategies Box

The *Strategies* box on the *Nonlinear Solver* tab:



allows you to set the following options:

- *Crash Initial Solution 1,*
- *Crash Initial Solution 2,*
- *Quadratic Recognition,*
- *Selective Constraint Eval,*
- *SLP Directions,*
- *SLP Solver,* and
- *Steepest Edge.*

Crash Initial Solution 1 and 2

LINGO has two solution "crashing" methods that use different heuristics for generating starting points for nonlinear models. If at least one of the *Crash Initial Solution boxes* is checked, LINGO's nonlinear solver will invoke a heuristic for generating a "good" starting point when you solve a model. If this initial point is relatively good, subsequent solver iterations should be reduced along with overall runtimes. It's possible, however, that runtimes may slow if the selected point turns out to be poor. It is

difficult to know beforehand if a particular crashing technique, if any, will help performance. You will need to experiment with your model to determine the best settings for the two crash procedures.

LINGO defaults to not crashing an initial solution.

Quadratic Recognition

If the *Quadratic Recognition* box is checked, LINGO will use algebraic preprocessing to determine if an arbitrary nonlinear model is actually a quadratic programming (QP) model. If a model is found to be a QP model and convex, then it can be passed to the faster quadratic solver. Note that the QP solver is not included with the standard, basic version of LINGO, but comes as part of the Barrier option.

LINGO defaults to using quadratic recognition.

Selective Constraint Evaluation

If the *Selective Constraint Eval* box is checked, LINGO's nonlinear solver will only evaluate constraints on an as needed basis. Thus, not every constraint will be evaluated during each iteration. This generally leads to faster solution times, but can also lead to problems in models that have functions that are undefined in certain regions.

LINGO may not evaluate a constraint for many iterations only to find that it has moved into a region where the constraint is no longer defined. In this case, there may not be a valid point for the solver to retreat to, and the solution process terminates with an error. Turning off selective constraint evaluation eliminates these errors.

LINGO defaults to not using *Selective Constraint Eval*.

SLP Directions

If the *SLP Directions* box is checked, LINGO's nonlinear solver will use successive linear programming (SLP) to compute new search directions. This technique uses a linear approximation in search computations in order to speed iteration times. In general, however, the number of total iterations will tend to rise when *SLP Directions* are used.

LINGO defaults to using *SLP Directions*.

SLP Solver

If the *SLP Solver* box is checked, LINGO uses a successive linear programming (SLP) algorithm for its nonlinear solver. This technique uses a linear approximation of the true nonlinear model within successive, small regions. This then allows the use of the fast linear solver for optimizing each linear subregion. In general, the SLP solver will not be well suited for highly nonlinear models. However, it may offer better performance than the standard nonlinear solver when a model has few nonlinear variables, such that the model is "mostly linear".

LINGO defaults to not enabling the *SLP Solver* option.

Steepest Edge

If the *Steepest Edge* box is checked, LINGO's nonlinear solver will use the steepest-edge strategy when selecting variables to iterate on.

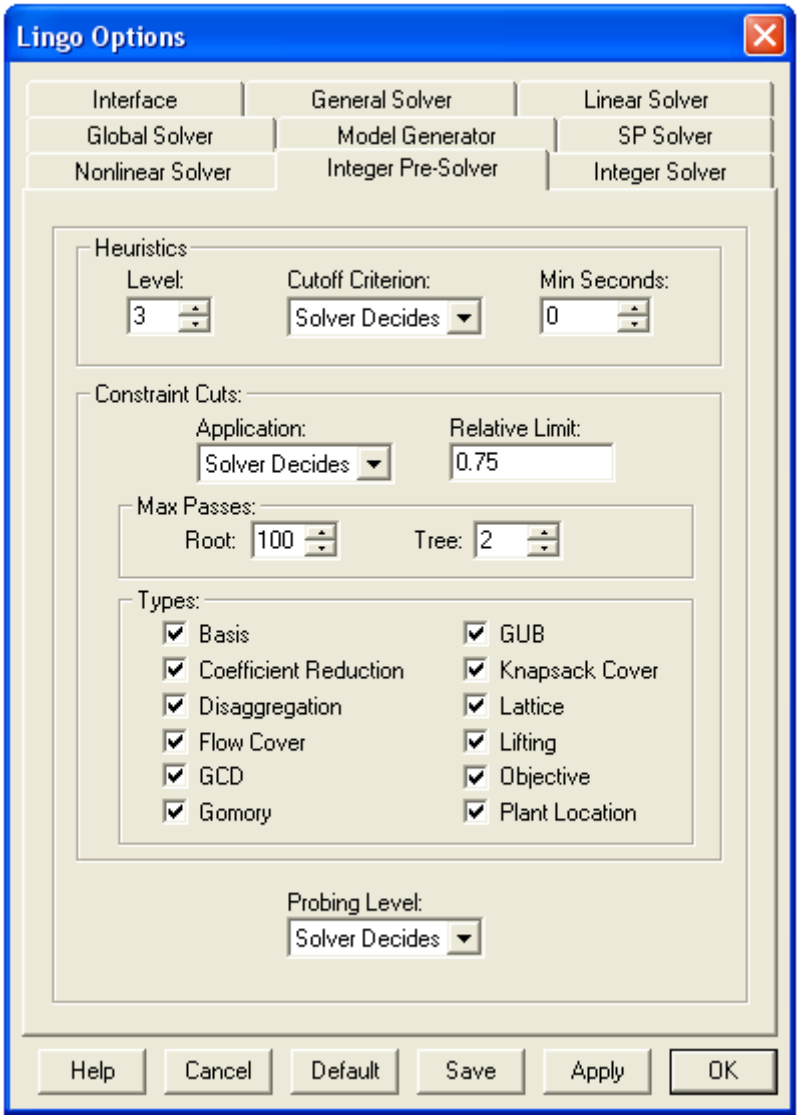
When LINGO is not in steepest-edge mode, the nonlinear solver will tend to select variables that offer the highest *absolute* rate of improvement to the objective, regardless of how far other variables may have to move per unit of movement in the newly introduced variable. The problem with this strategy is that other variables may quickly hit a bound, resulting in little gain to the objective.

With the steepest-edge option, the nonlinear solver spends a little more time in selecting variables by looking at the rate that the objective will improve *relative* to movements in the other nonzero variables. Thus, on average, each iteration will lead to larger gains in the objective. In general, the steepest-edge option will result in fewer iterations. However, each iteration will take longer.

LINGO defaults to not using the *Steepest Edge* option.

Integer Pre-Solver Tab

The *Integer Pre-Solver* tab on the *Options* dialog box, pictured here:



is used to control several options for tailoring the operation of LINGO's integer programming pre-solver. The integer pre-solver does a great deal of model reformulation, so that the final formulation passed to the branch-and-bound solver may be solved as fast as possible.

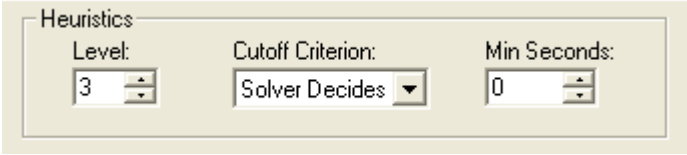
The reformulated model is always mathematically equivalent to the original formulation, but it is structured in such a way that it is best suited for solution by the branch-and-bound integer

programming algorithm. The pre-solver also performs a number of heuristic techniques to find valid integer solutions before entering the branch-and-bound solver.

Note: The integer pre-solver pertains only with integer models (i.e., models that make use of the *@BIN* and *@GIN* functions to restrict one or more variables to integer values.)

Heuristics Box

The *Heuristics* box on the *Integer Pre-Solver* tab:



controls the level of integer programming heuristics used by the integer solver. These heuristics use the continuous solution at each node in the branch-and-bound tree to attempt to quickly find a good integer solution.

The *Level* field controls the level and number of heuristics that are applied, and may run from 0 (none) to 100 (highest level.)

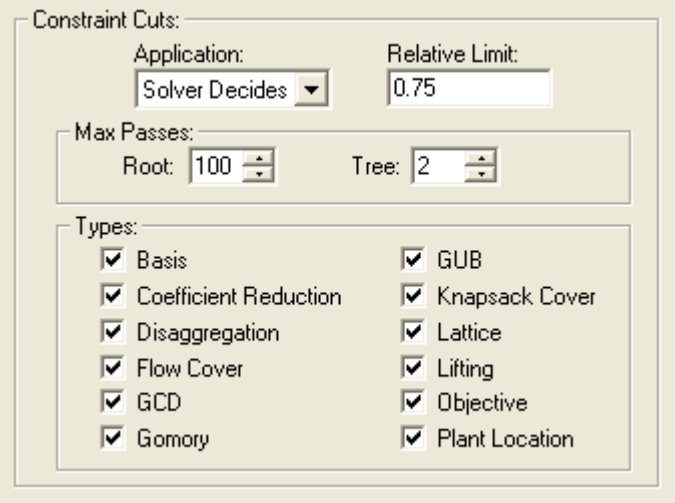
The *Cutoff Criterion* is used to control the criterion for terminating heuristics. Choices here are *Solver Decides, Time,* and *Iterations.* Under the *Time* setting, LINGO terminates heuristics after a certain amount of elapsed time. The *Iterations* option terminates heuristics after a certain number of iterations.

In general, the *Time* setting results in the fastest performance. However, due to shifting computational loads on a machine, solution paths may change under the *Time* setting from one solve to the next, potentially resulting in non-reproducible solutions. If reproducibility of a runs is a concern, then the *Iterations* option should be selected. Under the *Solver Decides* setting, LINGO chooses the most appropriate strategy.

The *Min Seconds* field specifies the minimum amount of time to spend on heuristics at each node. The default settings are 3 for *Level, Solver Decides* for *Cutoff Criterion*, and 0 for *Min Seconds.*

Constraint Cuts Box

The tolerances contained in the *Constraint Cuts* box on the *Integer Pre-Solver* tab:



can be used to control the solver's cut generation phase on linear models.

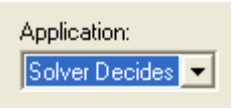
LINGO's integer programming solver performs extensive evaluation of your model in order to add *constraint cuts*. Constraint cuts are used to "cut" away sections of the feasible region of the continuous model (i.e., the model with integer restrictions dropped) that are not contained in the feasible region to the integer model.

On most integer models, this will accomplish two things. First, solutions to the continuous problem will tend to be more naturally integer. Thus, the branch-and-bound solver will have to branch on fewer variables. Secondly, the bounds derived from intermediate solutions will tend to be tighter, allowing the solver to "fathom" (i.e., drop from consideration) branches higher in the branch-and-bound tree. These improvements should dramatically speed solution times on most integer models. However, on some poorly formulated models, disabling one or more of the cut types may help in finding feasible solutions.

All constraint cut types are enabled by default.

Application

In the *Application* drop-down box of the *Constraint Cuts* box:



you can control the nodes in the solution tree where the branch-and-bound solver adds cuts.

If you pull down the selection list, you will find the three options:

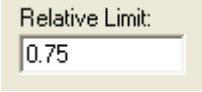
- *Root Only*,
- *All Nodes*, and
- *Solver Decides.*

Under the *Root Only* option, the solver appends cuts only at the first node, or root node, in the solution tree. With the *All Nodes* option, cuts are appended at each node of the tree. The *Solver Decides* option causes the solver to dynamically decide when it is best to append cuts at a node.

The default is to let the solver decide when to append cuts. In general, this will offer superior performance. There may be instances, however, where one of the other two options prevails.

Relative Limit

In the *Relative Limit* field of the *Constraints Cuts* box:

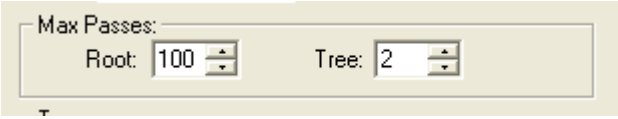


you can control the number of constraint cuts that are generated by the integer pre-solver. Most integer programming models benefit from the addition of some constraint cuts. However, at some point additional cuts take more time to generate than they save in solution time. For this reason, LINGO imposes a relative limit on the number of constraint cuts that are generated.

The default limit is set to .75 times the number of true constraints in the original formulation.

Max Passes

In the *Max Passes* box of the *Constraint Cuts* box:

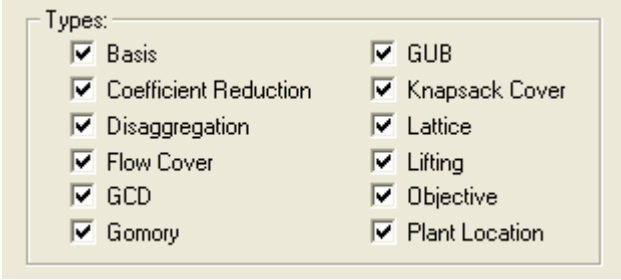


you can control the number of iterative passes the integer pre-solver makes through a model to determine appropriate constraint cuts to append to the formulation. In general, the benefits of each successive pass decline. At some point, additional passes will only add to the total solution time. Thus, LINGO imposes a limit on the maximum number of passes.

The default limit is 100 passes at the root node of the branch-and-bound tree, and 2 passes at all subsequent nodes.

Types

The *Types* box of the *Constraint Cuts* box:



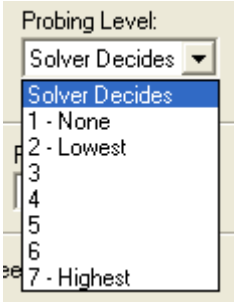
is used to enable or disable the different strategies LINGO uses for generating constraint cuts. LINGO uses twelve different strategies for generating constraint cuts. The default is for all cut generation strategies to be enabled.

It is beyond the scope of this manual to go into the details of the various strategies. Interested readers may refer to any good text on integer programming techniques. In particular, see Nemhauser and Wolsey (1988.)

Probing Level

The *Probing Level* option on the *Integer Pre-Solver* tab can be used on mixed integer linear programs to perform an operation known as *probing*. Probing involves taking a close look at the integer variables in a model and deducing tighter variable bounds and righthand side values. In many cases, probing can tighten an integer model sufficiently to speed overall solution times. In other cases, however, probing may not be able to do much tightening, and the overall solution time will increase due to the extra time spent probing.

Pulling down the selection list for the *Probing Level* field:

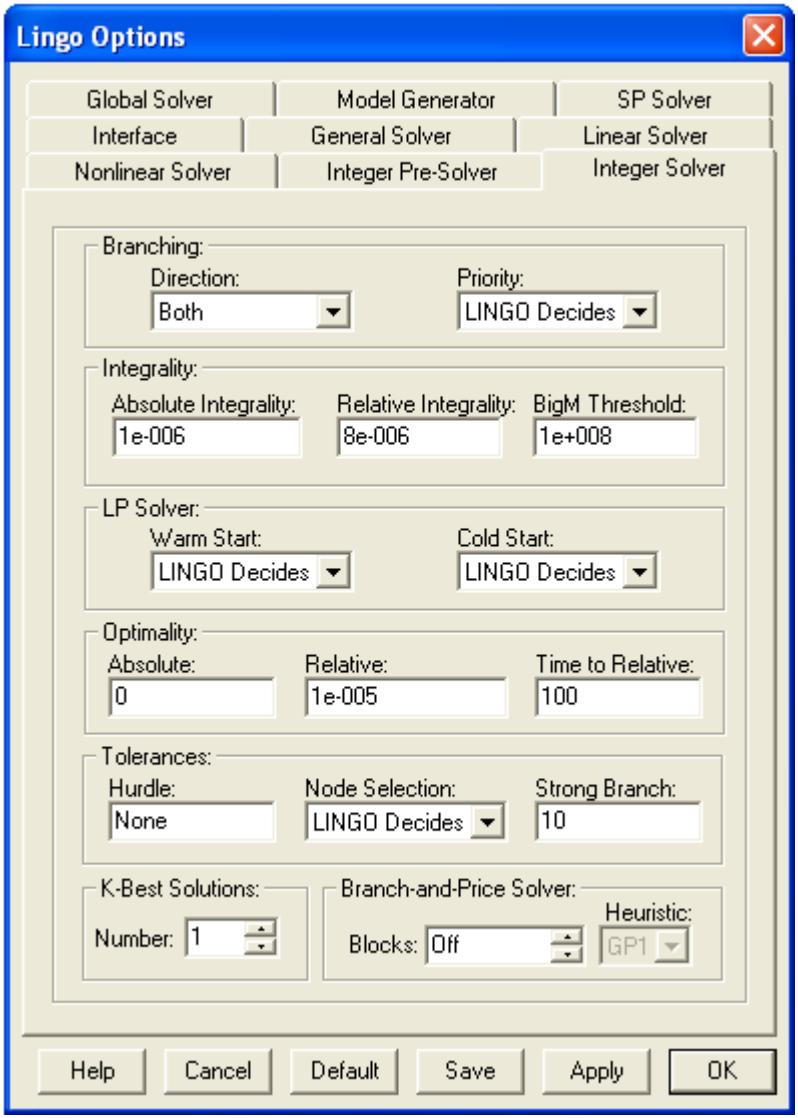


you will see that you can choose one of eight different probing levels. A probing level of 1 means probing is disabled, while levels 2 through 7 indicate successively higher degrees of probing. The

default setting for this option, *Solver Decides,* leaves the decision up to LINGO to select the probing level.

Integer Solver Tab

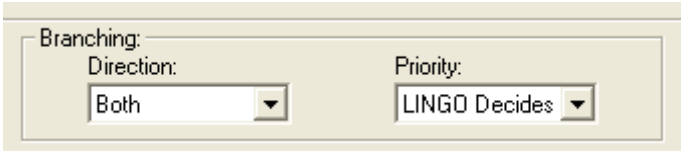
The *Integer Solver* tab on the *Options* dialog box:



can be used to control several tolerances for tailoring the operation of LINGO's branch-and-bound solver used on integer models (i.e., models that make use of the *@BIN* and *@GIN* functions to restrict one or more variables to integer values.)

Branching Box

The *Branching* box on the *Integer Solver* tab:



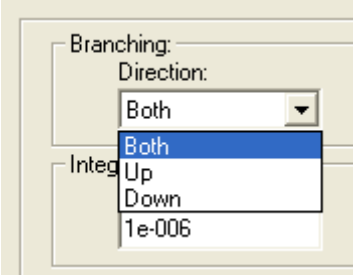
contains the following two options for controlling the branching strategy used by LINGO's branch-and-bound solver:

- *Direction*, and
- *Priority*.

Direction

LINGO uses a branch-and-bound solution procedure when solving integer programming models. One of the fundamental operations involved in the branch-and-bound algorithm is *branching* on variables. Branching involves forcing an integer variable that is currently fractional to either the next greatest or the next lowest integer value. As an example, suppose there is a general integer variable that currently has a value of 5.6. If LINGO were to branch on this variable, it would have to choose whether to set the variable first to 6 or 5. The *Direction* option controls how LINGO makes this branching decision.

If you pull down the drop-down box for the *Direction* option, you'll find the following:



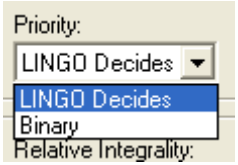
The default setting, *Both*, involves LINGO making an intelligent guess as to whether it should branch up or down first on each individual variable. If *Up* is selected, LINGO will always branch up first. If *Down* is selected, LINGO will always branch down first. In most cases, the *Both* option will result in the best performance.

Priority

When branching on variables, the branch-and-bound procedure can give priority to branching on the binary variables first, or it can make an intelligent guess as to the next best variable to branch on,

regardless of whether it is binary or general. The *Priority* field controls how LINGO makes this branching decision.

If you pull down the drop-down box for *Priority*, you willl find the following:

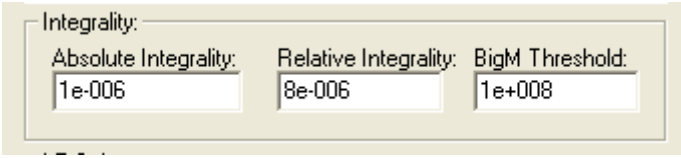


Select *Binary* to have LINGO give branching priority to the binary variables. Select *LINGO Decides* to have LINGO select the next integer variable for branching based on an intelligent guess, regardless of whether it is binary or general.

The default is *LINGO Decides,* which should generally give the best results.

Integrality Box

Due to the potential for round-off error on digital computers, it is not always possible for LINGO to find exact integer values for the integer variables. The *Integrality* box on the *Integer Solver* tab:



contains the following three options for controlling the amount of deviation from integrality that will be tolerated:

- *Absolute Integrality*,
- *Relative integrality*, and
- *BigM Threshold*.

Absolute Integrality

The *Absolute Integrality* tolerance is used by LINGO as a test for integrality in integer programming models. Due to round-off errors, the "integer" variables in a solution may not have values that are *precisely* integer. The absolute integrality tolerance specifies the absolute amount of violation from integrality that is acceptable. Specifically, if X is an "integer" variable and I is the closest integer to X, then X would be accepted as being integer valued if:

$$|X - I| \leq \textit{Absolute Integrality Tolerance.}$$

The default value for the absolute integrality tolerance is .000001. Although one might be tempted to set this tolerance to 0, this may result in feasible models being reported as infeasible.

Relative Integrality

The *Relative Integrality* tolerance is used by LINGO as a test for integrality in integer programming models. Due to round-off errors, the "integer" variables in a solution may not have values that are *precisely* integer. The relative integrality tolerance specifies the relative amount of violation from integrality that is acceptable. Specifically, if I is the closest integer value to X, X will be considered an integer if:

$$\frac{|X - I|}{|X|} \leq \textit{Relative Integrality Tolerance.}$$

The default value for the relative integrality tolerance is .000008. Although one might be tempted to set this tolerance to 0, this may result in feasible models being reported as infeasible.

BigM Threshold

Many integer programming models have constraints of the form:

$$f(x) \leq M * z$$

where $f(x)$ is some function of the decision variables, M is a large constant term, and z is a binary variable. These types of constraints are called *forcing constraints* and are used to force the binary variable, z, to 1 when $f(x)$ is nonzero. In many instances, the binary variable is multiplied by a fixed cost term in the objective; a fixed cost that is incurred when a particular activity, represented by $f(x)$, occurs. The large constant term, M, Is frequently referred to as being a *BigM* coefficient.

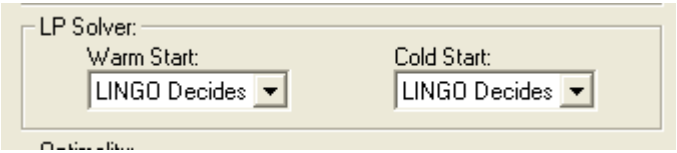
Setting BigM too small can lead to infeasible or suboptimal models. Therefore, the BigM value will typically have to be rather large in order to exceed the largest activity level of $f(x)$. When BigM is large, the solver may discover that by setting z slightly positive (within normal integrality tolerances), it can increase $f(x)$ to a significant level and thereby improve the objective. Although such solutions are technically feasible to tolerances, they are invalid in that the activity is occurring without incurring its associated fixed cost.

The BigM threshold is designed to avoid this problem by allowing LINGO to identify the binary variables that are being set by forcing constraints. Any binary variable with a coefficient larger than the BigM threshold will be subject to a much tighter integrality tolerance.

The default value for the *BigM Threshold* is 1.e8.

LP Solver Box

In a mixed linear integer programming model, LINGO's branch-and-bound solver solves a linear programming model at each node of the solution tree. LINGO has a choice of using the primal simplex, dual simplex, or barrier solver (assuming the barrier option was purchased with your license) for handling these linear programs. The *LP Solver* box on the *Integer Solver* tab:



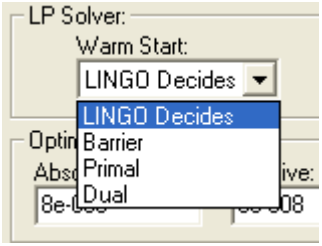
contains the following two options for controlling this choice of linear program solver:

- *Warm Start*, and
- *Cold Start*.

Warm Start

The *Warm Start* option controls the linear solver that is used by the branch-and-bound solver at each node of the solution tree when a previous solution is present to use as a "warm start." The *Cold Start* option, discussed below, determines the solver to use when a previous solution *does not exist.*

If you pull down the drop-down box for *Warm Start*, you'll find the following:



The available options are:

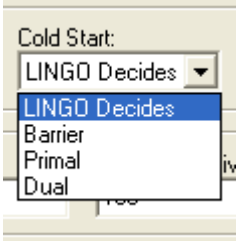
- *LINGO Decides* – LINGO chooses the most appropriate solver.
- *Barrier* – LINGO uses the barrier method, assuming you have purchased a license for the barrier solver. Otherwise, the dual solver will be used.
- *Primal* – LINGO uses the primal solver exclusively.
- *Dual* – LINGO uses the dual solver exclusively.

In general, *LINGO Decides* will yield the best results. The barrier solver can't make use of a pre-existing solution, so *Barrier* usually won't give good results. In general, *Dual* will be faster than *Primal* for reoptimization in branch-and-bound.

Cold Start

The *Cold Start* option controls the linear solver that is used by the branch-and-bound solver at each node of the solution tree when a previous solution is *not* present to use as a "warm start". The *Warm Start* option, duscussed above, determines the solver to use when a previous solution *does exist.*

If you pull down the drop-down box for *Cold Start*, you'll find the following:



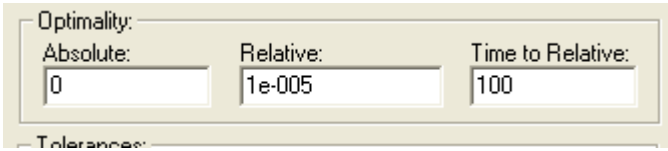
The available options are:

- *LINGO Decides* – LINGO chooses the most appropriate solver at each node.
- *Barrier* – LINGO uses the barrier method, assuming you have purchased a license for the barrier solver. Otherwise the dual solver will be used.
- *Primal* – LINGO uses the primal solver exclusively.
- *Dual* – LINGO uses the dual solver exclusively.

In general, *LINGO Decides* will yield the best results. However, experimentation with the other options may be fruitful.

Optimality Box

The *Optimality* box on the *Integer Solver* tab:



is used to control three tolerances: *Absolute*, *Relative*, and *Time to Relative*. These tolerances control how close you want the solver to come to the optimal solution. Ideally, we'd always want the solver to find the best solution to a model. Unfortunately, integer programming problems are very complex, and the extra computation required to seek out the absolute best solution can be prohibitive. On larger integer models, the alternative of getting a solution within a few percentage points of the true optimum after several minutes of runtime, as opposed to the true optimum after several days, makes the use of these tolerances quite attractive.

Absolute Optimality

The *Absolute Optimality Tolerance* is a positive value *r*, indicating to the branch-and-bound solver that it should only search for integer solutions with objective values at least *r* units better than the best integer solution found so far. In many integer programming models, there are huge numbers of branches with roughly equivalent potential. This tolerance can help to keep the branch-and-bound solver from being distracted by branches that can't offer a solution significantly better than the incumbent solution.

In general, you shouldn't have to set this tolerance. Occasionally, particularly on poorly formulated models, you might need to increase this tolerance slightly to improve performance. In most cases, you should experiment with the relative optimality tolerance rather than the absolute optimality tolerance in order to improve performance.

The default value for the absolute optimality tolerance is 0.

Relative Optimality

The *Relative Optimality Tolerance* is a value *r*, ranging from 0 to 1, indicating to the branch-and-bound solver that it should only search for integer solutions with objective values at least 100**r*% better than the best integer solution found so far.

The end results of modifying the search procedure in this way are twofold. First, on the positive side, solution times can be improved tremendously. Second, on the negative side, the final solution obtained by LINGO may not be the true optimal solution. You will, however, be guaranteed the solution is within 100**r*% of the true optimum.

Typical values for the relative optimality tolerance would be in the range .01 to .05. In other words, you would be happy to get a solution within 1% to 5% of the true optimal value. On large integer models, the alternative of getting a solution within a few percentage points of the true optimum after several *minutes* of runtime, as opposed to the true optimum after several *days*, makes the use of an optimality tolerance quite attractive.

Note: Generally speaking, the relative optimality tolerance is the tolerance that will most likely improve runtimes on integer models. You should be sure to set this tolerance whenever near optimal solutions are acceptable. Do keep in mind that when you set this option, LINGO may not return the true global optimum. However, you will be guaranteed the solution is within 100*r% of the true optimum.

The default for the relative optimality tolerance is 5e-8.

Time To Relative

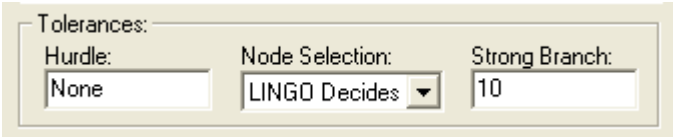
If an integer programming model is relatively easy to solve, then we would like to have the solver press on to the true optimal solution without immediately resorting to a relative optimality tolerance. On the other hand, if, after running for a while, it becomes apparent that the optimal solution won't be immediately forthcoming, then you might want the solver to switch to using a relative optimality tolerance.

The *Time to Relative* tolerance can be used in this manner. This tolerance is the number of seconds before the branch-and-bound solver begins using the relative optimality tolerance. For the first *n* seconds, where *n* is the value of the time to relative tolerance, the branch-and-bound solver will not use the relative optimality tolerance and will attempt to find the true optimal solution to the model. Thereafter, the solver will use the relative optimality tolerance in its search.

The default value for the time to relative tolerance is 100 seconds.

Tolerances Box

The *Tolerances* box on the *Integer Solver* tab:



contains three miscellaneous tolerances for controlling the branching strategy used by the branch-and-bound solver on integer programming models. The three tolerances are *Hurdle*, *Node Selection*, and *Strong Branch*.

Hurdle

If you know the objective value of a solution to a model, you can enter it as the *Hurdle* tolerance. This value is used in the branch-and-bound solver to narrow the search for the optimum. More specifically, LINGO will only search for integer solutions in which the objective is better than the hurdle value.

This comes into play when LINGO is searching for an initial integer solution. LINGO can ignore branches in the search tree with objective values worse than the hurdle value, because a better solution exists (i.e., the solution whose objective value equals the hurdle tolerance) on some alternate branch.

Depending on the problem, a good hurdle value can greatly reduce solution time. Once LINGO finds an initial integer solution, however, the hurdle tolerance no longer has an effect. At this point, the *Relative Optimality* tolerance comes into play.

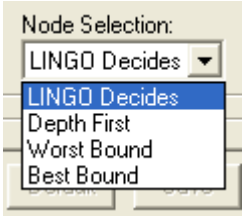
Note:	Be sure when entering a hurdle value that a solution exists that is at least as good or better than your hurdle. If such a solution does not exist, LINGO will not be able to find a feasible solution to the model.

The default hurdle value is *None*. In other words, a hurdle value is not used by the solver.

Node Selection

The branch-and-bound solver has a great deal of freedom in deciding how to span the branch-and-bound solution tree. The *Node Selection* option allows you to control the order in which the solver selects branch nodes in the tree.

If you examine the pull down list for *Node Selection*, you will see the following:



The four choices function as follows:

- *LINGO Decides* – This is the default option. LINGO makes an educated guess as to the best node to branch on next.
- *Depth First* – LINGO spans the branch-and-bound tree using a depth first strategy.
- *Worst Bound* – LINGO picks the node with the worst bound.
- *Best Bound* – LINGO picks the node with the best bound.

In general, *LINGO Decides* will offer the best results. Experimentation with the other options may be beneficial with some classes of models.

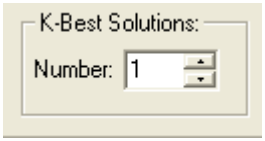
Strong Branch

The *Strong Branch* option uses a more intensive branching strategy during the first *n* levels of the branch-and-bound tree, where *n* is the value in *Strong Branch*. During these initial levels, LINGO picks a subset of the fractional variables as branching candidates. LINGO then performs a tentative branch on each of the variables in the subset, selecting as the final candidate the variable that offers the greatest improvement in the bound on the objective. Although strong branching is useful in tightening the bound quickly, it does take additional computation time. Therefore, you may want to try different settings to determine what works best for your model.

The default strong branch setting is 10 levels.

K-Best Solutions Box

The K-Best Solutions box on the *Integer Solver* tab:



is used to set the number of solutions desired as part of the *K-Best solutions feature* of LINGO's mixed integer solver. Whenever this value is greater than 1, say *K,* LINGO will will return up to *K* unique solutions to the model. These solutions will have the property that they are the next best solutions available in terms of their objective values. Less than *K* solutions may be returned if a sufficient number of feasible solutions do not exist. An example of the *K*-Best feature follows.

K-Best Solutions Example

In order to illustrate the K-Best feature, we will be using a variant of the knapsack model discussed above in the *Binary Integer Variables* section. You may want to refer back to the earlier discussion if you are not familiar with the knapsack model.

Here's our model:

```
MODEL:


SETS:
  ITEMS: INCLUDE, WEIGHT, RATING;
  MYFAVORITES( ITEMS);
ENDSETS

DATA:
  KNAPSACK_CAPACITY = 15;

ITEMS     WEIGHT      RATING =
BRATS       3            1
BROWNIES    3            1
BEER        3            1
ANT_REPEL   7            1
BLANKET     4            6
FRISBEE     1            6
SALAD       5           10
WATERMELON  7            9;

MYFAVORITES = BRATS BROWNIES BEER;
ENDDATA

MAX = @SUM( ITEMS: RATING * INCLUDE);

@SUM( ITEMS: WEIGHT * INCLUDE) <=
KNAPSACK_CAPACITY;

@FOR( ITEMS: @BIN( INCLUDE));

NUMFAVE = @SUM( MYFAVORITES: INCLUDE);

END
```

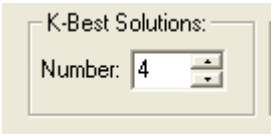
Model: KBEST

In this example, we are packing a picnic basket for a picnic we will be taking with a friend. Our friend's ratings of the candidate picnic items is given in the data section above. It turns out that our friend is health conscious and does not care much for bratwurst, brownies nor beer. This is unfortunate, because these happen to be our favorite items, which we indicate with a new subset of *ITEMS* called *MYFAVORITES.*

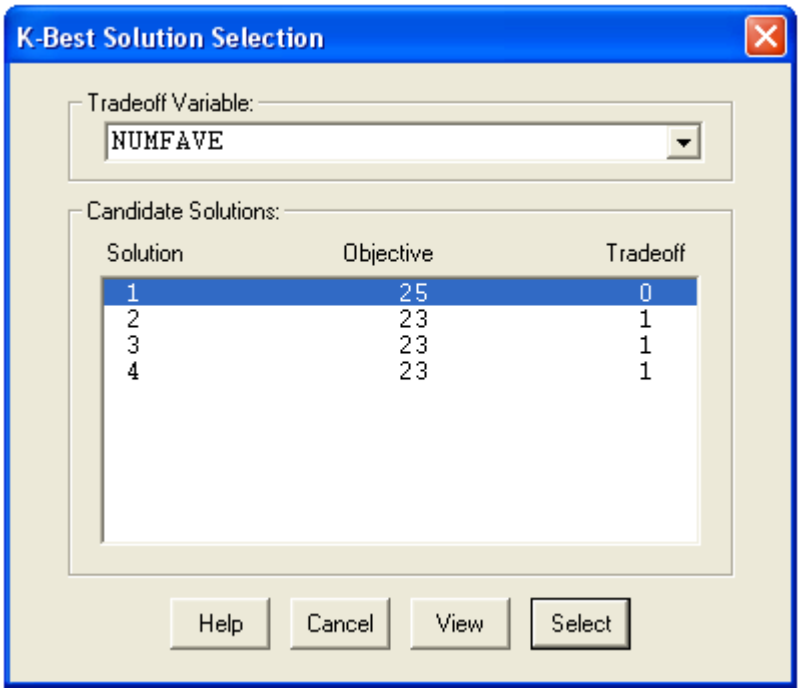
If we solve the model as is, thus solely maximizing our friend's preferences, we get the following solution:

```
Global optimal solution found.
Objective value:         25.00000

            Variable           Value
             NUMFAVE          0.000000
     INCLUDE( BRATS)          0.000000
  INCLUDE( BROWNIES)          0.000000
      INCLUDE( BEER)          0.000000
 INCLUDE( ANT_REPEL)          0.000000
   INCLUDE( BLANKET)          0.000000
   INCLUDE( FRISBEE)          1.000000
     INCLUDE( SALAD)          1.000000
INCLUDE( WATERMELON)          1.000000
```

As indicated by the *NUMFAVE* variable, none of our favorite items are included in the optimal basket. Now, we like our friend a lot, and we want him to be happy. However, we are wondering if there isn't another combination of items that our friend might like almost as much that includes at least one of our favorite items. To investigate this question, we set the *Desired Number* parameter of the *K-Best Solutions* box on the *Solver|Options Integer Solver* tab to 4:



This means that we would like LINGO to generate the 4 best solutions to the model. We then click *OK* and then run the *Solver|Solve* command. At which point, the integer solver sees that the K-Best feature is being requested, and it automatically generates the 4 best solutions to the model. At which point, we are presented with the following dialog box:



In the *Candidate Solutions* window we see that the solver was able to find 4 feasible next-best solutions to the model. The solutions are ranked in order by their objective values.

There is also a column labeled *Tradeoff*, which lists the value in each solution of a designated tradeoff variable. Any scalar variable in a model can be selected as the tradeoff variable. In this example, there is only one scalar variable, *NUMFAVE,* so it is automatically selected as the tradeoff variable. The idea behind the tradeoff variable is that it allows you to weigh the tradeoffs in a model's objective value with a secondary goal. In this case, our secondary goal is the number of our favorite items in the picnic basket. In particular, we see that there are three solutions with slightly worse objective values (23 vs. 25) that include one of our favorite items. For example, if we selected solution 2 and pressed the *View* button, we'd see the following solution containing one of our favorite items, bratwurst:

```
Objective value:                23.00000

             Variable            Value
   KNAPSACK_CAPACITY           15.00000
              NUMFAVE          0.000000
      INCLUDE( BRATS)          0.000000
   INCLUDE( BROWNIES)          0.000000
       INCLUDE( BEER)          0.000000
  INCLUDE( ANT_REPEL)          0.000000
    INCLUDE( BLANKET)          0.000000
    INCLUDE( FRISBEE)          1.000000
      INCLUDE( SALAD)          1.000000
 INCLUDE( WATERMELON)          1.000000
```

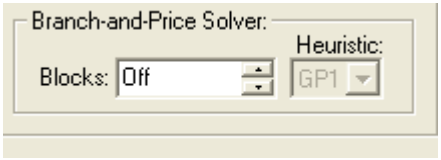
The following buttons are available along the bottom edge of the K-Best dialog box:

- *Help* – Displays online help regarding the K-Best feature.
- *Cancel* – Cancels out of K-Best mode, closing the dialog box.
- *View* – Displays any solutions selected in the *Candidate Solutions* box.
- *Select* – Allow you to select one of the candidate solutions as the final solution to the model.

These buttons allow you to examine selected solutions returned by the K-Best solver. Once you find a solution you believe to be the best, you can select it as the final solution. Once a final solution is selected, all subsequent solution reports will be based on that particular solution.

Branch-and-Price Solver

The *Branch-and-Price Solver* box on the *Integer Solver* tab:



contains two parameters - *Blocks* and *Heuristic* - for controlling the *branch-and-price* (BNP) solver. The BNP solver is a mixed integer programming solver for solving models with block structures like the following:

Minimize: $\sum c(k) * x(k)$
Subject To:
$\sum A(k) * x(k) = d$ (linking constraints)
$x(k)$ in $X(k)$, for all k (decomposition structure)

where d, $c(k)$ and $x(k)$ are vectors and $A(k)$ is a matrix with appropriate dimensions. $x(k)$ contains decision variables and X(k) denotes a linear feasible domain for x(k).

The BNP solver is a hybrid of branch-and-bound, column generation, and Lagrangean relaxation methods. It can help to find either the optimal solution or a better lower bound (the Lagrangean bound) for a minimization problem. Based on the decomposition structure, the solver divides the original problem into several subproblems, or blocks, and solves them (almost) independently, exploiting parallel processing if multiple cores are available.

BNP may perform better than the default MIP solver if: a) the number of linking constraints is small, b) the number of blocks is large and they are of approximately the same size, and c) the number of available processors (or cores) is large, e.g., 4 or more. Also, there may be some models for which BNP finds a good solution and good bound more quickly than the default MIP algorithm, although it may take longer to prove optimality.

The *Blocks* option for the BNP solver controls the number of subproblems, or blocks, that the model will be partitioned into. Possible setting for the *Blocks* parameter are:

- *Row Names* – Row names are constructed in such a way as to specify each row's block (an example is given below).
- *Off* – This will disable the BNP solver, in which case, the standard MIP solver will be used to solve all mixed integer linear programs.
- *Specified* – The user explicitly specifies each row's block using the @BLOCKROW function.
- *N* – A positive integer, greater-than-or-equal-to 2, indicating the number of independent blocks to try and partition the model into via one of the graph partitioning algorithms provided by LINGO. The actual heuristic used is chosen with the Heuristic parameter.

The default setting for *Blocks* is *Off*, i.e., the BNP solver will not be used on integer programming models.

The *Block Heuristic* parameter controls the heuristic used to partition the model into blocks. You may currently select from two graph partitioning algorithms named simply *GP1* and *GP2*, with the default setting being *GP1*.

As an example, consider the following model:

```
MIN =  x1 + x2 + x3 + x4 + x5 + x6;
[c1]   x1 + x2 + x3 + x4 + x5 + x6      >=3; !linking constraint;
[c2]   x1 + x2                          <=1; !block 1;
[c3]         x2 + x3                    <=1; !block 1;
[c4]                x4 + x5 + x6        <=2; !block 2;
[c5]                x4 + x6             <=1; !block 2;
@bin( x1); @bin( x2); @bin( x3);
@bin( x4); @bin( x5); @bin( x6);
```

The above model has six variables and five constraints. Constraint 1 will be the only linking constraint, with linking constraints referred to as being in block 0. Constraints 2 and 3 constitute the first independent subproblem, or block 1. Constraints 4 and 5 form block 2. Thus, for this particular model, you would want to set the *Blocks* parameter to be 2, corresponding to the two independent subproblem blocks. For the partitioning heuristic, you may choose either *GP1* or *GP2.* When we solve this model, note that the beginning of the solution report returned by LINGO contains the following information:

```
Global optimal solution found.
Objective value:                 3.000000
Objective bound:                 3.000000
Infeasibilities:                 0.000000
Extended solver steps:                  0
Total solver iterations:               14
Number of branch-and-price blocks:      2
```

The interesting feature to note is the line:

```
Number of branch-and-price blocks: 2
```

indicating that LINGO partitioned the model in to two independent blocks and invoked the BNP solver.

Note: Note that in some cases the number of blocks listed in the solution report may be less than the number of blocks requested. This occurs when the partitioning heuristic is unable to find a partition with the full number of desired blocks.

Note: The BNP solver can run the independent subproblems on separate threads to improve performance. So, if your machine has multiple cores, be sure to set the thread limit to at least the number of blocks. Refer to the *Threads* parameter on the *General Solver Tab*, discussed above. For this particular small example with its two independent blocks, you'd want to set the thread limit to at least 2.

The graph partitioning algorithms provided by LINGO can generally determine good partitioning schemes, however, for larger models, they may not be able to determine an optimal partitioning. In this case, you may prefer to explicitly specify a model's block structure. LINGO provides two ways to do

this by either a) specifying the block structure as part of a model's row names, or b) specifying the block structure using the *@BLOCKROW* function. Examples of both follow.

To specify a row's block using its row name, you should begin the row name with the string "BNP_N", where N is the row's block number. The block number should be some non-negative integer, with a value of 0 indicating that the row belongs to the set of linking constraints. For example, using the model above, we could have specified the block structure in the row names as follows:

```
MIN =  x1 + x2 + x3 + x4 + x5 + x6;
[bnp_0_c1]    x1 + x2 + x3 + x4 + x5 + x6 >=3; !linking constraint;
[bnp_1_c2]    x1 + x2                      <=1; !block 1;
[bnp_1_c3]          x2 + x3                <=1; !block 1;
[bnp_2_c4]                 x4 + x5 + x6   <=2; !block 2;
[bnp_2_c5]                 x4 + x6        <=1; !block 2;
@bin( x1); @bin( x2); @bin( x3);
@bin( x4); @bin( x5); @bin( x6);
```

You will also need to select the *Row Names* option for the *Blocks* parameter. When specifying block structure in row names, the *Heuristic* parameter is not relevant and will be grayed out.

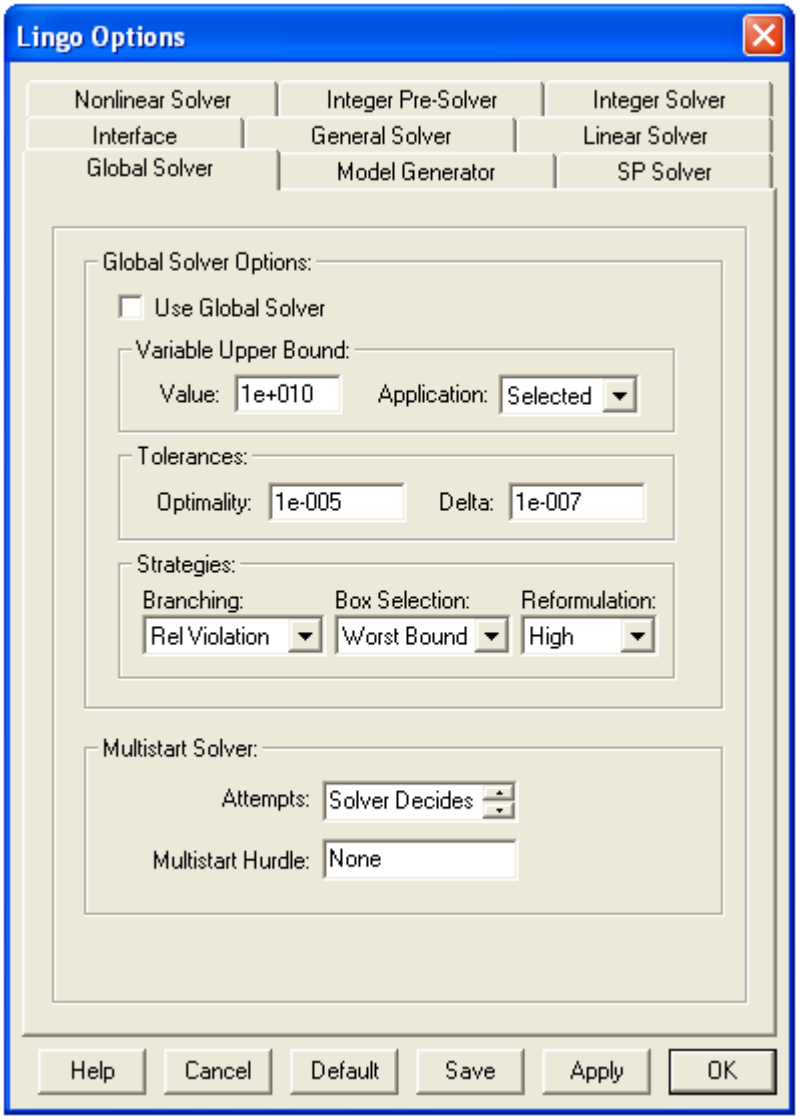
Alternatively, you could use the *@BLOCKROW(BLOCK_NUMBER, ROW_NAME)* function to specify block structure. Again, using our same sample model, we would enter:

```
MIN =  x1 + x2 + x3 + x4 + x5 + x6;
[c1]   x1 + x2 + x3 + x4 + x5 + x6      >=3; !linking constraint;
[c2]   x1 + x2                          <=1; !block 1;
[c3]         x2 + x3                    <=1; !block 1;
[c4]               x4 + x5 + x6         <=2; !block 2;
[c5]               x4 + x6              <=1; !block 2;
@bin( x1); @bin( x2); @bin( x3);
@bin( x4); @bin( x5); @bin( x6);
@blockrow( 0, c1);
@blockrow( 1, c2);
@blockrow( 1, c3);
@blockrow( 2, c4);
@blockrow( 2, c5);
```

In this case, you will need to select the *Specified* option for the *Blocks* parameter. When specifying block structure via *@BLOCKROW*, the *Heuristic* parameter is not relevant and will be grayed out.

Global Solver Tab

The *Global Solver* tab on the *Options* dialog box:



is used to control the operation of LINGO's global solver capabilities. Please keep in mind that the global solver toolkit is an add-on option to LINGO. You must specifically purchase the global solver option as part of your LINGO license in order to make use of its capabilities.

LINGO exploits the convex nature of linear models to find globally optimal solutions. However, we aren't as fortunate with nonlinear models. LINGO's default NLP solver uses a local search procedure.

This can lead to LINGO stopping at locally optimal points when a model is non-convex, perhaps missing a global point lying elsewhere. You may refer to *On Mathematical Modeling* for more information on how and why this can happen. The global solver toolkit contains features designed to sift through the local points in search of the globally optimal point.

The two primary features in LINGO's global toolkit are a *global solver* and a *multistart solver*. The global solver uses range bounding and reduction techniques within a branch-and-bound framework to convert a non-convex model into a series of smaller, convex models. This divide-and-conquer strategy ultimately results in convergence to the guaranteed globally optimal point. The multistart solver, on the other hand, uses a heuristic approach of restarting the NLP solver several times from different initial points. It is not uncommon for a different starting point to lead to a different local solution point. Thus, if we restart from enough unique points, saving the best local solution as we go, then we stand a much better chance of finding the true global solution.

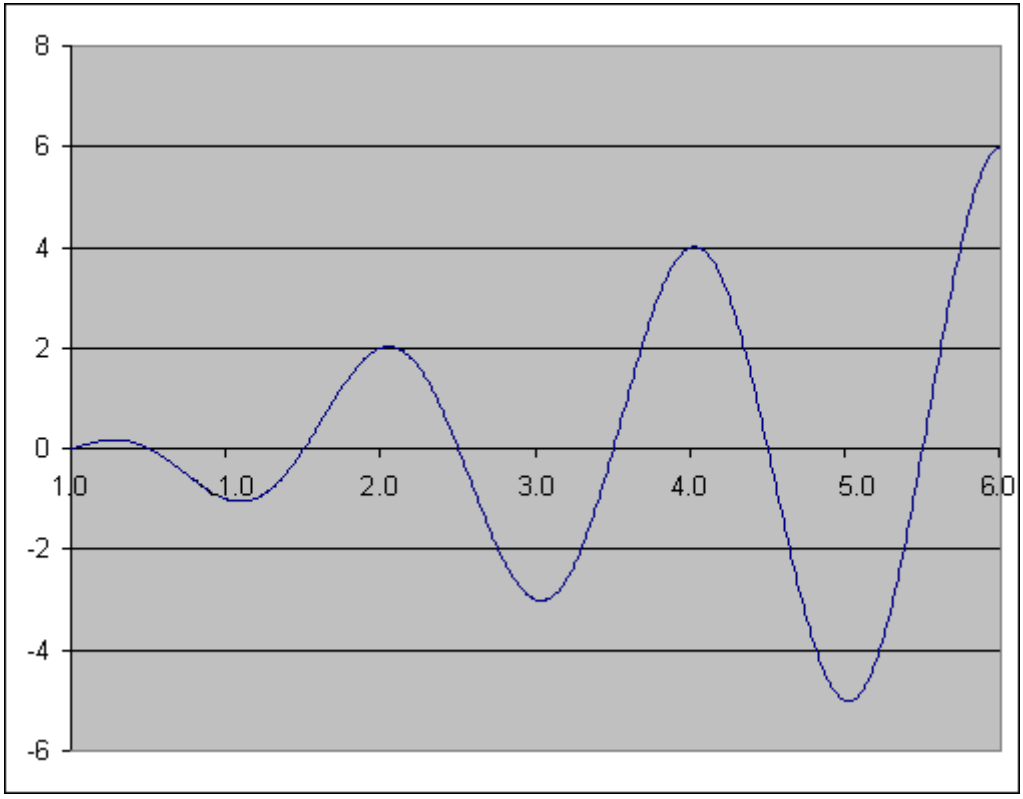
Use Global Solver

If the *Use Global Solver* box is checked, LINGO will invoke the global solver when you solve a nonlinear model. Many nonlinear models are non-convex and/or non-smooth (for more information see Chapter 15, *On Mathematical Modeling*.) Nonlinear solvers that rely on local search procedures (as does LINGO's default nonlinear solver) will tend to do poorly on these types of models. Typically, they will converge to a local, sub-optimal point that may be quite distant from the true, global optimal point. Global solvers overcome this weakness through methods of range bounding (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branch-and-bound framework to find global solutions to non-convex models.

The following example illustrates the usefulness of the global solver. Consider the simple, yet highly nonlinear, model:

```
MODEL:
  MIN = X * @COS( 3.1416 * X);
  @BND( 0, X, 6);
END
```

The graph of the objective function is as follows:



The objective function has three local minimal points over the feasible range. These points are summarized in the following table:

Point	*X*	*Objective*
1	1.09	-1.05
2	3.03	-3.02
3	5.02	-5.01

Clearly, the third local point is also the globally best point, and we would like the NLP solver to converge to this point. Below is the solution LINGO produces if the default nonlinear solver is invoked:

```
Local optimal solution found at step:        11
Objective value:                        -1.046719

Variable           Value          Reduced Cost
       X        1.090405        0.1181082E-07

     Row    Slack or Surplus     Dual Price
       1           -1.046719      -1.000000
```

Unfortunately, as you can see, we converged to the least preferable of the local minimums. However, after enabling the global solver by checking the *Use Global Solver* box, we do obtain the global solution:

```
Local optimal solution found at step:        35
Objective value:                        -5.010083

Variable           Value          Reduced Cost
       X        5.020143        -0.7076917E-08

     Row    Slack or Surplus     Dual Price
       1           -5.010083      -1.000000
```

Note: There is one drawback to using the global solver; it runs considerably slower than the default nonlinear solver. Therefore, the preferred option is to always try and write smooth, convex nonlinear models. By doing this, the faster, default local solver can be successfully invoked.

Keep in mind that the global solver supports most, but not all, of the functions available in the LINGO language. The following is a list of the nonlinear functions not currently supported by the global solver:

- *All probability distributions* — cumulative, inverse and pdf, with the exception of the normal distribution, which is fully supported
- *@PFS()* — Poisson finite source
- *@PPL()* — Poisson linear loss
- *@USER()* — User supplied function

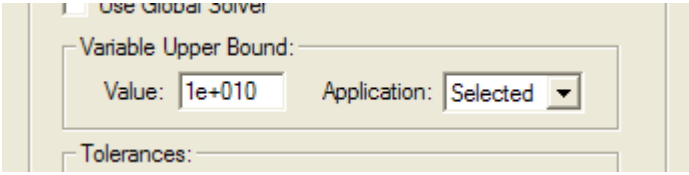
Note: The global solver will not operate on models containing one or more unsupported nonlinear operations that reference optimizable quantities; the default NLP solver will be called in this case.

Note: The global solver can run on multiple cores to improve performance. So, if your machine has multiple cores, be sure to set the thread limit to something higher than 1, with the ideal setting being *N*, where *N* is the number of available cores on your machine. Refer to the *Threads* parameter on the *General Solver Tab*, discussed above, for more information on setting the thread limit.

The global solver is disabled by default.

Variable Upper Bound Box

The *Variable Upper Bound* box:

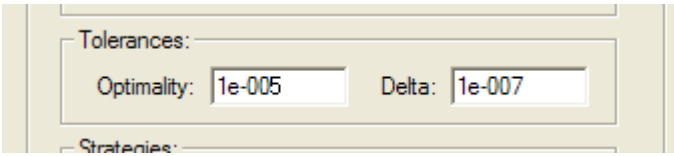


sets the default variable bounds while the global solver is running. If this parameter is set to *d*, then variables will not be permitted to assume values outside the range of [-*d, d*]. Setting this parameter as tightly as possible in the *Value Field* restricts the global solver from straying into uninteresting regions and will reduce run times. The default value for the *Value Field* is 1.e10.

The *Application* list box has three options available: *None, All* and *Selected.* Selecting *None* removes the variable bounds entirely, and is not recommended. The *All* setting applies the bound to all variables. Finally, the *Selected* setting causes the global solver to apply the bound after an initial solver pass to find the first local solution. The bound will only be applied to a variable if it does not cutoff the initial local solution. LINGO defaults to the *Selected* setting.

Tolerances Box

The *Tolerances* box:



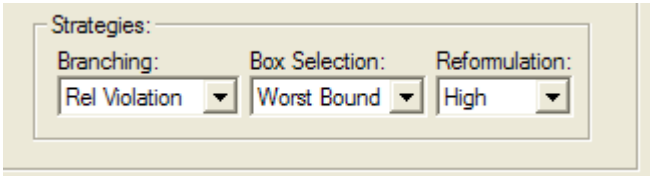
contains two tolerances used by the global solver: *Optimality* and *Delta*.

The *Optimality* tolerance specifies by how much a new solution must beat the objective value of the incumbent solution in order to become the new incumbent. The default value for *Optimality* 1.e-5.

The *Delta* tolerance specifies how closely the additional constraints, added as part of the global solver's convexification process, must be satisfied. The default value for *Delta* is 1.e-7.

Strategies Box

The *Strategies* box:



allows you to control three strategies used by the global solver: *Branching*, *Box Selection* and *Reformulation*.

The *Branching* strategy consists of six options to use when branching on a variable for the first time:

- *Absolute Width,*
- *Local Width,*
- *Global Width,*
- *Global Distance,*
- *Absolute Violation,* and
- *Relative Violation.*

The default setting for *Branching* is *Relative Violation.*

The *Box Selection* option specifies the strategy to use for choosing between all active nodes in the global solver's branch-and-bound tree. The choices are: *Depth First* and *Worst Bound,* with the default being *Worst Bound.*

The *Reformulation* option sets the degree of algebraic reformulation performed by the global solver. Algebraic reformulation is critical for construction of tight, convex sub-regions to enclose the nonlinear and nonconvex functions. The available settings are *None, Low, Medium* and *High*, with *High* being the default.

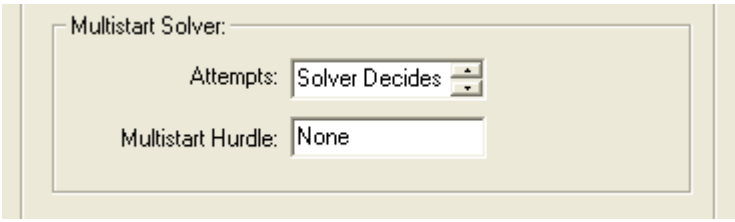
Multistart Solver

LINGO exploits the convex nature of linear models to find globally optimal solutions. However, we aren't as fortunate with nonlinear models. With NLP models, LINGO's default NLP solver uses a local search procedure. This can lead to LINGO stopping at locally optimal points, perhaps missing a global point lying elsewhere. You may refer to *On Mathematical Modeling* for more information on how and why this can happen.

A strategy that has proven successful in overcoming this problem is to restart the NLP solver several times from different initial points. It is not uncommon for a different starting point to lead to a different local solution point. Thus, if we restart from enough unique points, saving the best local

solution as we go, then we stand a much better chance of finding the true global solution. We refer to this solution strategy as *multistart.*

The Multistart Solver box on the *Global Solver* tab:



is contains two options, *Attempts* and *Multistart Hurdle*.

The *Attempts* option is used to set the number of times the multistart solver restarts the standard NLP solver in its attempt to find successively better local solutions. Each new starting point is intelligently generated to maximize the chances of finding a new local point. The default option, *Solver Decides*, entails restarting 5 times on small NLPs and disabling multistart on larger models. Setting multistart to 1 causes the NLP solver to be invoked only once, effectively disabling multistart. Setting multistart to any value greater than 1 will cause the NLP solver to restart that number of times on all NLPs. In general, we have found that setting the number of multistarts to around 5 tends to be adequate for most models. Highly nonlinear models may require a larger setting.

The *Multistart Hurdle* option sets a goal for the multistart solver. If the multistart solver finds a solution as good, or better, than the hurdle value, the solver will interrupt and return that solution. This is useful on long-running models, where you may be happy with any solution better than, say, 100.

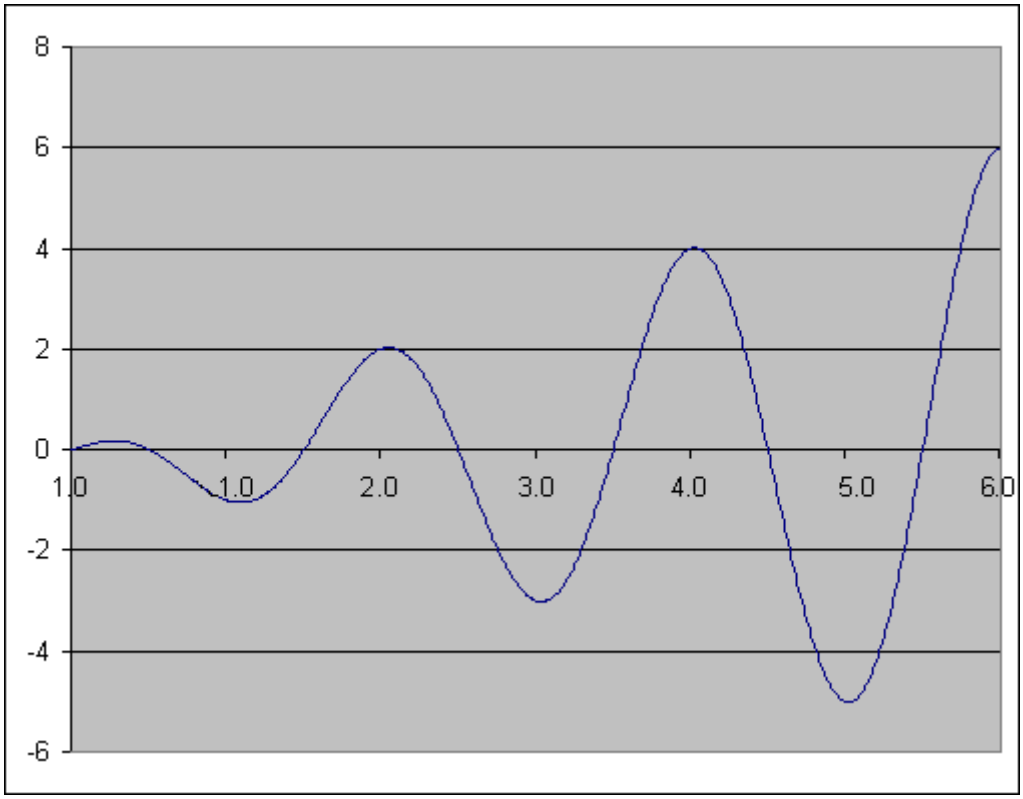
Note: If the Attempts limit for the multistart solver is hit before the HRDLMS value, then the solver will also interrupt.

Note: Keep in mind that multistart will dramatically increase runtimes, particularly if a large number of restarts is selected. Thus, one should avoid using multistart unnecessarily on convex models that will converge to a global point in a single pass without any additional prodding.

The following example illustrates the usefulness of multistart. Consider the simple, yet highly nonlinear, model:

```
MODEL:
   MIN = X * @COS( 3.1416 * X);
   @BND( 0, X, 6);
END
```

The graph of the objective function is as follows:



The objective function has three local minimal points over the feasible range. These points are summarized in the following table:

Point	*X*	*Objective*
1	1.09	-1.05
2	3.03	-3.02
3	5.02	-5.01

Clearly, the third local point is also the globally best point, and we would like the NLP solver to converge to this point. Below is the solution you will get from LINGO if the multistart option is disabled:

```
Local optimal solution found at step:        11
Objective value:                        -1.046719

Variable           Value        Reduced Cost
       X        1.090405       0.1181082E-07

     Row     Slack or Surplus    Dual Price
       1           -1.046719     -1.000000
```

Unfortunately, as you can see, we converged to the least preferable of the local minimums. However, after setting the number of multistarts to five and re-solving, we do obtain the global solution:

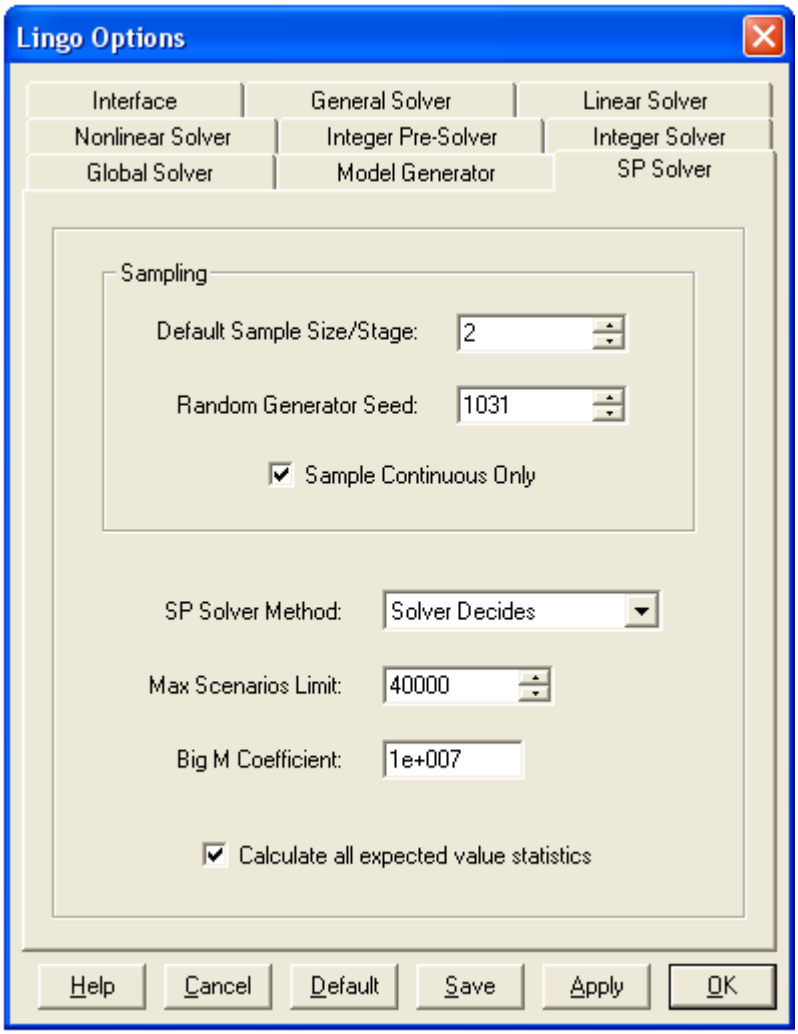
```
Local optimal solution found at step:        39
Objective value:                        -5.010083

Variable           Value        Reduced Cost
       X        5.020143      -0.7076917E-08

     Row     Slack or Surplus    Dual Price
       1           -5.010083     -1.000000
```

Note: Unlike the global solver, the multistart solver can only claim its solution to be locally optimal. This is because there may always be a better solution out there that the multistart solver may, or may not, be able to find with additional runs. The global solver, on the other hand, can claim global optimality by having partitioned the original model into a series of smaller, convex models.

Note: The multistart solver can run on multiple cores to improve performance. So, if your machine has multiple cores, be sure to set the thread limit to something higher than 1, with the ideal setting being N, where N is the number of available cores on your machine. Refer to the *Threads* parameter on the *General Solver* tab, discussed above, for more information on setting the thread limit.

SP Solver Tab

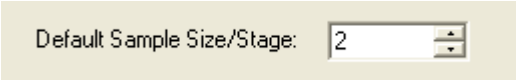
The *SP Solver tab* on the *Options* dialog box:



is used to control the operation of LINGO's SP (Stochastic Programming) solver. For more information on SP, refer to Chapter 14, *Stochastic Programming.* Please keep in mind that the SP solver is an add-on option to LINGO. You must specifically purchase the SP solver option as part of your LINGO license in order to make use of its capabilities.

Default Sample Size/Stage

The *Default Sample Size/Stage* parameter on the *SP Solver* tab:



is used to control the default sample size for random variables in stochastic programming (SP) whose outcomes are determined via sampling.

In many SP models, LINGO will generate a set of sample values for the some or all of the random variables. This is particularly true when you have one or more continuous random variables, which have an infinite number of possible outcomes. In such a case, sampling is the only viable alternative.

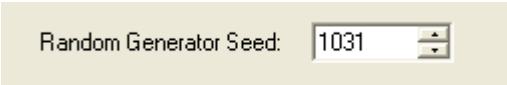
One way to specify the desired sample size for each stage is via the *@SPSAMPSIZE* function used directly in the model's text. If, on the other hand, all or most stages should have the same sample size, then you can use the *Default Sample Size/Stage* parameter to control the default sample size. Any stage which has not had its sample size specified with *@SPSAMPSIZE* will default to a sample size equal to the *Default Sample Size/Stage* parameter.

Note: In general, we prefer larger sample sizes to smaller ones in order to reduce sampling error. However, SP models can become quite large if sample sizes aren't kept reasonably small. This is particularly true for multiperiod models. For example, suppose we have a model with just one random variable and ten periods/stages. If the default sample size is set to 3, then there will be 3^10=59,049 possible scenarios. With this many scenarios, it would only take a handful of decision variables to end up with an underlying deterministic equivalent model approaching one million variables.

The *Default Sample Size/Stage* parameter defaults to a value of 2.

Random Generator Seed

The *Random Generator Seed* parameter on the *SP Solver* tab:



is used to establish the seed for the random number generator used in generating samples for random variables in stochastic programming (SP) models.

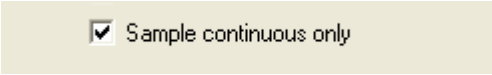
In many SP models, LINGO will generate a set of sample values for the random variables. This is particularly true when you have one or more continuous random variables, which leads to an infinite number of possible outcomes. In such a case, sampling is the only viable alternative.

The *Random Generator Seed* parameter defaults to a value of 1031.

Note:	The seed parameter has no effect when running demo versions of LINGO. Demo versions always use the default seed value, regardless of this parameter's setting.

Sample Continuous Only

The *Sample Continuous Only* option on the *SP Solver* tab:



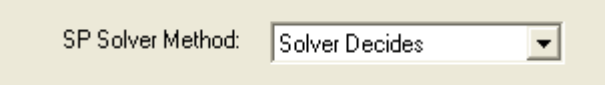
is used to control whether LINGO samples continuous distributions only for the random variables in stochastic programs, or if it samples both continuous and discrete random variables. Obviously, continuous random variables must be subject to sampling, given the infinite number of possible outcomes. On the other hand, we have a choice when it comes to discretely distributed random variables. If this option is enabled, then LINGO will generate one outcome for each density point in the discrete distribution of the random variable. These outcomes will be weighted by their corresponding probabilities.

Note:	If there are many density points in the distributions for the discrete random variables, the deterministic equivalent generated to solve the SP may become exceedingly large, in which case, disabling this option may be required. Once this option is disabled, discrete random variables will be sampled in the same way as the continuous random variables.

The Sample Continuous Only option defaults to being on.

SP Solver Method

The *SP Solver* Method option:



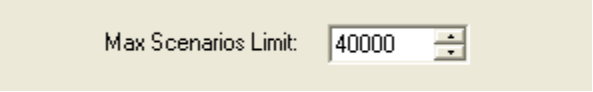
on the *SP Solver Tab* is used to select the solution method for a stochastic programming (SP) model. Presently, the available choices are:

- *Solver Decides* - LINGO decides the most appropriate method for solving the SP model.
- *Deterministic Equivalent* - LINGO generates and directly solves the deterministic equivalent (DE) of the SP model.
- *Nested Benders* - The DE for realistic SP models can get to be quite large, in that the core model is replicated once for every scenario. Fortunately, the DE displays strong block-angular structure, which makes it adaptable to decomposition techniques, such as nested Bender decomposition (NBD). Note that the model must be entirely linear to use NBD.

The default setting for the *SP Solver Method* option is *Solver Decides*.

Max Scenarios Limit

The *Max Scenarios Limit* parameter on the *SP Solver* tab:

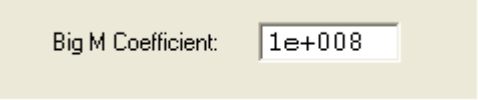


is used to establish a limit on the maximum number of scenarios in a stochastic programming (SP) model before forcing automatic sampling of the random variables.

The *Max Scenarios Limit* defaults to a value of 40,000.

SP Big M Coefficient

The *Big M Coefficient* parameter on the *SP Solver* tab:



is used by the SP solver in constructing forcing constraints that may be required in the deterministic models generated by the solver. Forcing constraints are generally added to force binary variables to 1 when some activity occurs. For example, suppose x is a continuous variable and z is a binary variable. A forcing constraint that would drive z to 1 whenever x is positive would be:

$$x \leq M * z$$

where M is some large number. M in this case is the *Big M coefficient*.

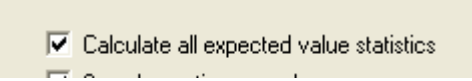
There are two things to note about M. First, if M is too small, a forcing constraint can become infeasible, making the entire model infeasible. In general, M should be at least as large as the largest possible value for x so as not to introduce an infeasibility. However, from an algorithmic point of view, M should not be too large. If M is unrealistically large, it will be tougher for the integer solver to converge, plus it also introduces the potential round off error.

The default setting for *M* should be sufficient for most models. However, if your SP model is infeasible for no known reason, you may want to try increasing *M*. On the other hand, if your SP is running slow, you may want to try reducing *M*.

The *Big M Coefficient* defaults to a value of 100,000,000.

Calculate All Expected Value Statistics

The *Calculate All Expected Value Statistics* option on the *SP Solver* tab:



controls whether LINGO displays information regarding the expected values for a number of statistics when solving stochastic programming (SP) models. To illustrate, when solving the SPGAS.LG4 model when this option is enabled, you will see the following expected values at the top of the solution report:

```
Expected value of:
  Objective (EV):                                          1400.000
  Wait-and-see model's objective (WS):                     1326.667
  Perfect information (EVPI = |EV - WS|):                  73.33333
  Policy based on mean outcome (EM):                       1479.444
  Modeling uncertainty (EVMU = |EM - EV|):                 79.44444
```

These values are a guide as to how the stochastic nature of the model is impacting the objective value. The following is a brief description of these expected values:

Expected Value of Objective (EV) - is the expected value for the model's objective over all the scenarios, and is the same as the reported objective value for the model.

Expected Value of Wait-and-See Model's Objective (WS) - reports the expected value of the objective if we could wait and see the outcomes of all the random variables before making our decisions. Such a policy would allow us to always make the best decision regardless of the outcomes for the random variables, and, of course, is not possible in practice. For a minimization, it's true that *WS* <= *EV*, with the converse holding for a maximization. Technically speaking, WS is a relaxation of the true SP model, obtained by dropping the nonanticipativity constraints.

Expected Value of Perfect Information (EVPI) - is the absolute value of the difference between EV and WS. This corresponds to the expected improvement to the objective were we to obtain perfect information about the random outcomes. As such, this is a expected measure of how much we should be willing to pay to obtain perfect information regarding the outcomes of the random variables.

Expected Value of Policy Based On Mean Outcome (EM) - is the expected true objective value if we (mistakenly) assume that all random variables will always take on exactly their mean values. EM is computed using a two-step process. First, the values of all random variables are fixed at their means, and the resulting deterministic model is solved to yield the optimal values for the stage 0 decision variables. Next, a) the stage 0 variables are fixed at their optimal values from the previous step, b) the random variables are freed up, c) the nonanticipativity constraints are dropped, and d) this wait-and-see is solved. EM is the objective value from this WS model.

Expected Value of Modeling Uncertainty (EVMU) - is the absolute value of the difference EV - EM. It is a measure of what we can expect to gain by taking into account uncertainty in our modeling analysis, as opposed to mistakenly assuming that random variables always take on their mean outcomes.

Note: The above approach for computing EM and EVMU makes unambiguous sense only for models with a stage 0 and a stage 1. If there are later random variables in stages 2, 3, etc., then there are complications. For example, for decisions in later stages, we have seen the outcomes from the random variables in earlier stages, so considering these random variables to take on their mean value does not make sense. For models with a stage 0 and stage 1, EVMU will be an accurate measure of the expected value of modeling uncertainty. For models with additional stages beyond 0 and 1, EVMU will merely be an approximation of the true expected value of modeling uncertainty.

Note: Computing these expected value statistics can be very time consuming for large models. If speed is an issue, you may wish to disable this feature on the *Solver|Options|SP S*olver tab.

The next component of the solution lists various statistics regarding the class and size of the model.

Solver|Generate

Once you remove all the syntax errors from your LINGO model, there is still one very important step required: *model verification*. LINGO's set-based modeling capabilities are very powerful, and they allow you to generate large, complex models quickly and easily. However, when you first develop a model you will need to verify that the model being generated matches up to the model you actually intended to generate. Many set-based models can be quite complex, and it is highly likely that logic errors may creep into one or more expressions, thereby causing your generated model to be flawed.

The *Solver|Generate* command is very useful for debugging such errors. It expands all of the model's compact set-based expressions and then writes out the full scalarbased equivalent of the LINGO model. The expanded model report explicitly lists all the generated constraints and variables in your model. You will find that the *Generate* report can be an invaluable tool in tracking down errors.

When selecting the *Generate* command, you will be presented with a pop-up menu prompting you for one of the following options:

- *Display model*,
- *Don't display model*,
- *Display nonlinear rows*,
- *Dual model,*
- *Explicit Deteq,* and
- *Scenario*.

Display Model

If you choose the *Display model* option of the *Solver|Generate* command, LINGO will place a copy of the generated model in a new window, which you may scroll through to examine, print, or save to disk.

As an example of the output from the *Generate* command, consider the transportation model developed in Chapter 1:

```
MODEL:
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
  WAREHOUSES: CAPACITY;
  VENDORS: DEMAND;
  LINKS( WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS
DATA:
  !set members;
  WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
  VENDORS = V1 V2 V3 V4 V5 V6 V7 V8;
  !attribute values;
  CAPACITY = 60 55 51 43 41 52;
  DEMAND = 35 37 22 32 41 32 43 38;
  COST = 6 2 6 7 4 2 5 9
         4 9 5 3 8 5 8 2
         5 2 1 9 7 4 3 3
         7 6 7 3 9 2 7 1
         2 3 9 5 7 2 6 5
         5 5 2 2 8 1 4 3;
ENDDATA
! The objective;
[OBJECTIVE] MIN = @SUM( LINKS( I, J):
  COST( I, J) * VOLUME( I, J));
! The demand constraints;
  @FOR( VENDORS( J): [DEMAND_ROW]
   @SUM( WAREHOUSES( I): VOLUME( I, J)) =
    DEMAND( J));
! The capacity constraints;
  @FOR( WAREHOUSES( I): [CAPACITY_ROW]
   @SUM( VENDORS( J): VOLUME( I, J)) <=
    CAPACITY( I));
END
```

Model: WIDGETS

The objective will generate one expression, there should be one demand constraint generated for each of the eight vendors and one supply constraint generated for each of the six warehouses, for a grand total of 15 rows in the expanded model. Running the generate command to verify this reveals the following report:

```
MODEL:
[OBJECTIVE] MIN= 6 * VOLUME_WH1_V1 + 2 * VOLUME_WH1_V2 + 6
*
VOLUME_WH1_V3 + 7 * VOLUME_WH1_V4 + 4 * VOLUME_WH1_V5 + 2 *
VOLUME_WH1_V6 + 5 * VOLUME_WH1_V7 + 9 * VOLUME_WH1_V8 + 4 *
VOLUME_WH2_V1 + 9 * VOLUME_WH2_V2 + 5 * VOLUME_WH2_V3 + 3 *
VOLUME_WH2_V4 + 8 * VOLUME_WH2_V5 + 5 * VOLUME_WH2_V6 + 8 *
VOLUME_WH2_V7 + 2 * VOLUME_WH2_V8 + 5 * VOLUME_WH3_V1 + 2 *
VOLUME_WH3_V2 + VOLUME_WH3_V3 + 9 * VOLUME_WH3_V4 + 7 *
VOLUME_WH3_V5 + 4 * VOLUME_WH3_V6 + 3 * VOLUME_WH3_V7 + 3 *
VOLUME_WH3_V8 + 7 * VOLUME_WH4_V1 + 6 * VOLUME_WH4_V2 + 7 *
VOLUME_WH4_V3 + 3 * VOLUME_WH4_V4 + 9 * VOLUME_WH4_V5 + 2 *
VOLUME_WH4_V6 + 7 * VOLUME_WH4_V7 + VOLUME_WH4_V8 + 2 *
VOLUME_WH5_V1 + 3 * VOLUME_WH5_V2 + 9 * VOLUME_WH5_V3 + 5 *
VOLUME_WH5_V4 + 7 * VOLUME_WH5_V5 + 2 * VOLUME_WH5_V6 + 6 *
VOLUME_WH5_V7 + 5 * VOLUME_WH5_V8 + 5 * VOLUME_WH6_V1 + 5 *
VOLUME_WH6_V2 + 2 * VOLUME_WH6_V3 + 2 * VOLUME_WH6_V4 + 8 *
VOLUME_WH6_V5 + VOLUME_WH6_V6 + 4 * VOLUME_WH6_V7 + 3 *
VOLUME_WH6_V8 ;
[DEMAND_ROW_V1] VOLUME_WH1_V1 + VOLUME_WH2_V1 +
VOLUME_WH3_V1 + VOLUME_WH4_V1 + VOLUME_WH5_V1 +
VOLUME_WH6_V1 = 35 ;
[DEMAND_ROW_V2] VOLUME_WH1_V2 + VOLUME_WH2_V2 +
VOLUME_WH3_V2 + VOLUME_WH4_V2 + VOLUME_WH5_V2 +
VOLUME_WH6_V2 = 37 ;
[DEMAND_ROW_V3] VOLUME_WH1_V3 + VOLUME_WH2_V3 +
VOLUME_WH3_V3 + VOLUME_WH4_V3 + VOLUME_WH5_V3 +
VOLUME_WH6_V3 = 22 ;
[DEMAND_ROW_V4] VOLUME_WH1_V4 + VOLUME_WH2_V4 +
VOLUME_WH3_V4 + VOLUME_WH4_V4 + VOLUME_WH5_V4 +
VOLUME_WH6_V4 = 32 ;
[DEMAND_ROW_V5] VOLUME_WH1_V5 + VOLUME_WH2_V5 +
VOLUME_WH3_V5 + VOLUME_WH4_V5 + VOLUME_WH5_V5 +
VOLUME_WH6_V5 = 41 ;
[DEMAND_ROW_V6] VOLUME_WH1_V6 + VOLUME_WH2_V6 +
VOLUME_WH3_V6 + VOLUME_WH4_V6 + VOLUME_WH5_V6 +
VOLUME_WH6_V6 = 32 ;
[DEMAND_ROW_V7] VOLUME_WH1_V7 + VOLUME_WH2_V7 +
VOLUME_WH3_V7 + VOLUME_WH4_V7 + VOLUME_WH5_V7 +
VOLUME_WH6_V7 = 43 ;
[DEMAND_ROW_V8] VOLUME_WH1_V8 + VOLUME_WH2_V8 +
VOLUME_WH3_V8 + VOLUME_WH4_V8 + VOLUME_WH5_V8 +
VOLUME_WH6_V8 = 38 ;
[CAPACITY_ROW_WH1] VOLUME_WH1_V1 + VOLUME_WH1_V2 +
VOLUME_WH1_V3 + VOLUME_WH1_V4 + VOLUME_WH1_V5 +
VOLUME_WH1_V6 + VOLUME_WH1_V7 + VOLUME_WH1_V8 <= 60 ;
[CAPACITY_ROW_WH2] VOLUME_WH2_V1 + VOLUME_WH2_V2 +
```

```
VOLUME_WH2_V3 + VOLUME_WH2_V4 + VOLUME_WH2_V5 +
VOLUME_WH2_V6 + VOLUME_WH2_V7 + VOLUME_WH2_V8 <= 55 ;
[CAPACITY_ROW_WH3] VOLUME_WH3_V1 + VOLUME_WH3_V2 +
VOLUME_WH3_V3 + VOLUME_WH3_V4 + VOLUME_WH3_V5 +
VOLUME_WH3_V6 + VOLUME_WH3_V7 + VOLUME_WH3_V8 <= 51 ;
[CAPACITY_ROW_WH4] VOLUME_WH4_V1 + VOLUME_WH4_V2 +
VOLUME_WH4_V3 + VOLUME_WH4_V4 + VOLUME_WH4_V5 +
VOLUME_WH4_V6 + VOLUME_WH4_V7 + VOLUME_WH4_V8 <= 43 ;
[CAPACITY_ROW_WH5] VOLUME_WH5_V1 + VOLUME_WH5_V2 +
VOLUME_WH5_V3 + VOLUME_WH5_V4 + VOLUME_WH5_V5 +
VOLUME_WH5_V6 + VOLUME_WH5_V7 + VOLUME_WH5_V8 <= 41 ;
[CAPACITY_ROW_WH6] VOLUME_WH6_V1 + VOLUME_WH6_V2 +
VOLUME_WH6_V3 + VOLUME_WH6_V4 + VOLUME_WH6_V5 +
VOLUME_WH6_V6 + VOLUME_WH6_V7 + VOLUME_WH6_V8 <= 52 ;
END
```

Model: Generated Model Report for WIDGETS

As expected, there are 15 rows in the generated model: [*OBJECTIVE*], [*DEMAND_ROW_V1*] through [*DEMAND_ROW_V8*], and [*CAPACITY_ROW_WH1*] through [*CAPACITY_ROW_WH6*].

As a side note, it's interesting to compare the generated model to the original, set-based model. The generated model is the expanded version of the model, and has all sets and vector variables removed, resulting in a fully scalar model. The scalar-based, generated model is mathematically equivalent to the original vector-based model. However, we think most would agree that the set-based model is much easier to comprehend, thereby illustrating one of the primary benefits of modern algebraic languages over more traditional, scalar-based languages.

In addition to verifying that the correct number of rows is being generated, you should also examine each of the rows to determine that the correct variables are appearing in each row along with their correct coefficients.

Note: The reports generated by the *Solver|Generate* command are valid LINGO models. You may load Generate reports into a model window and solve them as you would any other LINGO model.

One thing to keep in mind when examining generated model reports is that the LINGO model generator performs *fixed variable reduction*. This means that any variables that are fixed in value are substituted out of the generated model. For example, consider the simple model:

```
MODEL:
  MAX = 200 * WS + 300 * NC;
  WS = 60;
  NC <= 40;
  WS + 2 * NC <= 120;
END
```

If we generate this model we get the following, reduced model:

```
MODEL:
  MAX= 300 * NC + 12000 ;
  NC <= 40 ;
  2 * NC <= 60 ;
END
```

At first glance, it seems as if both the first constraint and the variable *WS* are missing from the generated model. Note that by the first constraint in the original model (`WS = 60`), *WS* is fixed at a value of 60. The LINGO model generator exploits this fact to reduce the size of the generated model by substituting *WS* out of the formulation. The final solution report will still contain the values for all the fixed variables, however, the fixed variables will not appear in the generated model report. If you would like to suppress fixed variable reduction so that all variables appear in your generated model, you may do so via the *Fixed Var Reduction option*.

Don't Display Model

If you choose the *Don't display model* option, LINGO will generate the model without displaying it, but will store the generated model for later use by the appropriate solver. This is a useful command for verifying that your model contains no errors, while not actually having to pass it to the solver to be optimized.

Display Nonlinear Rows

If you choose the *Display Nonlinear Rows* option of the *Generate* command, then LINGO will generate the model and then display only those rows that contain nonlinearities. Please refer to the *Types of Constraints* section for a discussion of linear vs. nonlinear expressions.

In general, one would prefer to always have purely linear models. Linear models solve faster and will always converge to a global solution. Therefore, when developing a model, you will be interested in carefully evaluating nonlinearities to see if they can either be eliminated or rewritten in a linear fashion. The *Display Nonlinear Rows* is helpful in tracking down a model's nonlinearities.

Dual Model

The third option of the *Solver|Generate* command, *Dual Model,* displays the dual formulation of the current model. Every linear programming model has a corresponding, mirror-image formulation called the *dual*. If the original model has *M* constraints and *N* variables, then its dual will have *N* constraints and *M* variables.

Some interesting properties of the dual are that any feasible solution to the dual model provides a bound on the objective to the original, primal model, while the optimal solution to the dual has the same objective value as the optimal solution to the primal problem. It's also true that the dual of the dual model is, once again, the original primal model. You may wish to refer to any good linear programming text for a further discussion of duality theory.

If you run the *Solver|Generate|Dual Model* command on the Widgets model shown above, you will receive the following formulation:

```
MODEL:
MAX = 35 * DEMAND_ROW_V1 + 37 * DEMAND_ROW_V2 + 22 *
DEMAND_ROW_V3 + 32 * DEMAND_ROW_V4 + 41 * DEMAND_ROW_V5 +
32 *
DEMAND_ROW_V6 + 43 * DEMAND_ROW_V7 + 38 * DEMAND_ROW_V8 +
60 *
CAPACITY_ROW_WH1 + 55 * CAPACITY_ROW_WH2 + 51 *
CAPACITY_ROW_WH3 + 43 * CAPACITY_ROW_WH4 + 41 *
CAPACITY_ROW_WH5 + 52 * CAPACITY_ROW_WH6;
[ VOLUME_WH1_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH1 <= 6;
[ VOLUME_WH1_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH1 <= 2;
[ VOLUME_WH1_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH1 <= 6;
[ VOLUME_WH1_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH1 <= 7;
[ VOLUME_WH1_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH1 <= 4;
[ VOLUME_WH1_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH1 <= 2;
[ VOLUME_WH1_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH1 <= 5;
[ VOLUME_WH1_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH1 <= 9;
[ VOLUME_WH2_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH2 <= 4;
[ VOLUME_WH2_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH2 <= 9;
[ VOLUME_WH2_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH2 <= 5;
[ VOLUME_WH2_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH2 <= 3;
[ VOLUME_WH2_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH2 <= 8;
[ VOLUME_WH2_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH2 <= 5;
[ VOLUME_WH2_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH2 <= 8;
[ VOLUME_WH2_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH2 <= 2;
[ VOLUME_WH3_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH3 <= 5;
[ VOLUME_WH3_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH3 <= 2;
[ VOLUME_WH3_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH3 <= 1;
[ VOLUME_WH3_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH3 <= 9;
[ VOLUME_WH3_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH3 <= 7;
[ VOLUME_WH3_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH3 <= 4;
[ VOLUME_WH3_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH3 <= 3;
[ VOLUME_WH3_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH3 <= 3;
[ VOLUME_WH4_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH4 <= 7;
[ VOLUME_WH4_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH4 <= 6;
[ VOLUME_WH4_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH4 <= 7;
[ VOLUME_WH4_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH4 <= 3;
[ VOLUME_WH4_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH4 <= 9;
[ VOLUME_WH4_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH4 <= 2;
[ VOLUME_WH4_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH4 <= 7;
[ VOLUME_WH4_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH4 <= 1;
[ VOLUME_WH5_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH5 <= 2;
[ VOLUME_WH5_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH5 <= 3;
[ VOLUME_WH5_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH5 <= 9;
[ VOLUME_WH5_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH5 <= 5;
[ VOLUME_WH5_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH5 <= 7;
[ VOLUME_WH5_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH5 <= 2;
[ VOLUME_WH5_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH5 <= 6;
[ VOLUME_WH5_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH5 <= 5;
```

```
[ VOLUME_WH6_V1] DEMAND_ROW_V1 + CAPACITY_ROW_WH6 <= 5;
[ VOLUME_WH6_V2] DEMAND_ROW_V2 + CAPACITY_ROW_WH6 <= 5;
[ VOLUME_WH6_V3] DEMAND_ROW_V3 + CAPACITY_ROW_WH6 <= 2;
[ VOLUME_WH6_V4] DEMAND_ROW_V4 + CAPACITY_ROW_WH6 <= 2;
[ VOLUME_WH6_V5] DEMAND_ROW_V5 + CAPACITY_ROW_WH6 <= 8;
[ VOLUME_WH6_V6] DEMAND_ROW_V6 + CAPACITY_ROW_WH6 <= 1;
[ VOLUME_WH6_V7] DEMAND_ROW_V7 + CAPACITY_ROW_WH6 <= 4;
[ VOLUME_WH6_V8] DEMAND_ROW_V8 + CAPACITY_ROW_WH6 <= 3;
@FREE( DEMAND_ROW_V1); @FREE( DEMAND_ROW_V2);
@FREE( DEMAND_ROW_V3); @FREE( DEMAND_ROW_V4);
@FREE( DEMAND_ROW_V5); @FREE( DEMAND_ROW_V6);
@FREE( DEMAND_ROW_V7); @FREE( DEMAND_ROW_V8);
@BND( -0.1E+31, CAPACITY_ROW_WH1, 0);
@BND( -0.1E+31, CAPACITY_ROW_WH2, 0);
@BND( -0.1E+31, CAPACITY_ROW_WH3, 0);
@BND( -0.1E+31, CAPACITY_ROW_WH4, 0);
@BND( -0.1E+31, CAPACITY_ROW_WH5, 0);
@BND( -0.1E+31, CAPACITY_ROW_WH6, 0);
END
```

Dual Formulation: WIDGETS

You will notice that in the dual formulation the variables from the primal model become the rows of the dual. Similarly, the rows in the primal become the variables in the dual.

Note: The row names from the primal problem will become the variable names in the dual formulation. For this reason, it is strongly recommended that you name all the rows in the primal model. If a row is unnamed, then a default name will be generated for the corresponding dual variable. The default name will consist of an underscore followed by the row's internal index. These default names will not be very meaningful, and will make the dual formulation difficult to interpret.

Explicit Deteq

The fourth option of the *Solver|Generate* command, *Explicit Deteq,* displays the explicit deterministic equivalent (DE) for the current model, assuming the current model is a stochastic program (SP). Viewing the DE can be very helpful in determining if you have properly set up the SP components of your model. You can refer to Chapter 14, *Stochastic Programming*, for more information on SP. When LINGO solves an SP model, it is really solving the deterministic equivalent of the original SP.

The DE contains one instance of the original core model for each scenario, where the random variables in each instance are replaced by their sampled values for the particular scenario. These core instances are tied together by a probabilistically weighted objective of all the objectives form the individual scenarios, plus a set of *nonanticipativity constraints*. The nonanticipativity constraints enforce the condition that all scenarios with the same history must implement the same decisions, otherwise, the DE would have the ability to anticipate future events.

As an example, if you were to load the SP gas buying sample model, *SPGAS1.LG4*, into LINGO and run the *Solver|Generate|Explicit Deteq* command, you will receive the following report, containing the model's DE:

```
MODEL:
TITLE _DETEQ;
[ _1] MIN = 0.3333333333333333 * PURCHASE_COST_SC1
+ 0.3333333333333333 * HOLD_COST_SC1 + 0.3333333333333333 *
PURCHASE_COST_SC2 + 0.3333333333333333 * HOLD_COST_SC2
+ 0.3333333333333333 * PURCHASE_COST_SC3 + 0.3333333333333333
* HOLD_COST_SC3;
[ _2_SC01] PURCHASE_COST_SC1 - 5 * PURCHASE_1_SC1 - 5 *
PURCHASE_2_SC1 = 0;
[ _3_SC01] HOLD_COST_SC1 - INVENTORY_1_SC1 -
INVENTORY_2_SC1 = 0;
[ _4_SC01] INVENTORY_1_SC1 - PURCHASE_1_SC1 = - 100;
[ _5_SC01] - INVENTORY_1_SC1 + INVENTORY_2_SC1 -
PURCHASE_2_SC1 = - 100;
[ _2_SC02] PURCHASE_COST_SC2 - 5 * PURCHASE_1_SC2 - 6 *
PURCHASE_2_SC2 = 0;
[ _3_SC02] HOLD_COST_SC2 - INVENTORY_1_SC2 -
INVENTORY_2_SC2 = 0;
[ _4_SC02] INVENTORY_1_SC2 - PURCHASE_1_SC2 = - 100;
[ _5_SC02] - INVENTORY_1_SC2 + INVENTORY_2_SC2 -
PURCHASE_2_SC2 = - 150;
[ _2_SC03] PURCHASE_COST_SC3 - 5 * PURCHASE_1_SC3 - 7.5 *
PURCHASE_2_SC3 = 0;
[ _3_SC03] HOLD_COST_SC3 - INVENTORY_1_SC3 -
INVENTORY_2_SC3 = 0;
[ _4_SC03] INVENTORY_1_SC3 - PURCHASE_1_SC3 = - 100;
[ _5_SC03] - INVENTORY_1_SC3 + INVENTORY_2_SC3 -
PURCHASE_2_SC3 = - 180;
[ _NAC01] INVENTORY_1_SC1 - INVENTORY_1_SC2 = 0;
[ _NAC02] PURCHASE_1_SC1 - PURCHASE_1_SC2 = 0;
[ _NAC03] INVENTORY_1_SC1 - INVENTORY_1_SC3 = 0;
[ _NAC04] PURCHASE_1_SC1 - PURCHASE_1_SC3 = 0;
END
```

Deterministic Equivalent: SPGAS1

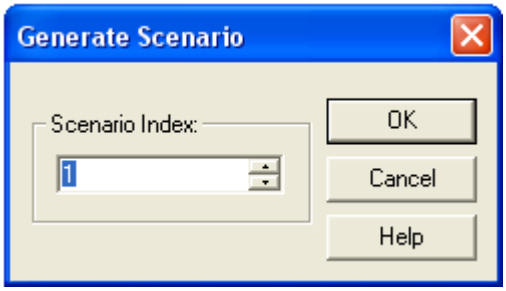
The first row of the DE is the probabilistically weighted objective over the three scenarios (Normal, Cold and Very Cold). Note that in the original core model, we had a variable called *PURCHASE_COST*, which was used to compute the cost of the gas purchases. In the DE we now have three *PURCHASE_COST* variables, or one for each of the three scenarios. LINGO appends the strings *_SC1*, *_SC2* or *_SC3* to *PUCHASE_COST* so as to indicate the appropriate scenario. Each of the constraints from the core model are also replicated across each of the scenarios with the appropriate scenario tag appended to each row's name. Finally, at the bottom of the model, there are four constraints beginning with the string: *_NAC*, which are the nonanticipativity constraints.

It's worthwhile pointing out that this report on the DE for *SPGAS1* is a valid LINGO model. In fact, you can copy this report into a new model window and solve it directly, in which case, you will get the optimal solution to the original SP model.

Scenario

As mentioned in the previous section, the ability to generate the deterministic equivalent (DE) of a stochastic programming (SP) model can be very useful when initially formulating and debugging an SP model. The only problem is that the DE can become quite unwieldy for all but the simplest SPs. For example, if the core model has 500 variables and there are 1,000 scenarios, then the DE will have 500,000 variables. Tracking down problems in such a large model can be difficult. Fortunately, in most cases when an SP is either infeasible or unbounded, LINGO will report the index of the first scenario that is causing the problem. With such information, we can focus our attention on the isolated scenario that's causing the problem. This is where the *Scenario* command comes in - it allows us to generate the formulation for an individual scenario, as opposed to the entire DE, which should substantially cut down on the burden of our debugging task.

As an example, we will work again with the *SPGAS.LG4* gas buying model. After loading the model and issuing the *Solver|Generate|Scenario* command, we are prompted for the index of the scenario we wish to view:



For our purposes, suppose we are interested in viewing scenario 2, in which case, we enter a 2 in the edit box and then press the *OK* button. LINGO the displays the following report:

```
MODEL:
TITLE Scenario 2;
[ _1] MIN = PURCHASE_COST + HOLD_COST;
[ _2] PURCHASE_COST - 5 * PURCHASE_1 - 6 * PURCHASE_2 = 0;
[ _3] HOLD_COST - INVENTORY_1 - INVENTORY_2 = 0;
[ _4] INVENTORY_1 - PURCHASE_1 = - 100;
[ _5] - INVENTORY_1 + INVENTORY_2 - PURCHASE_2 = - 150;
END
```

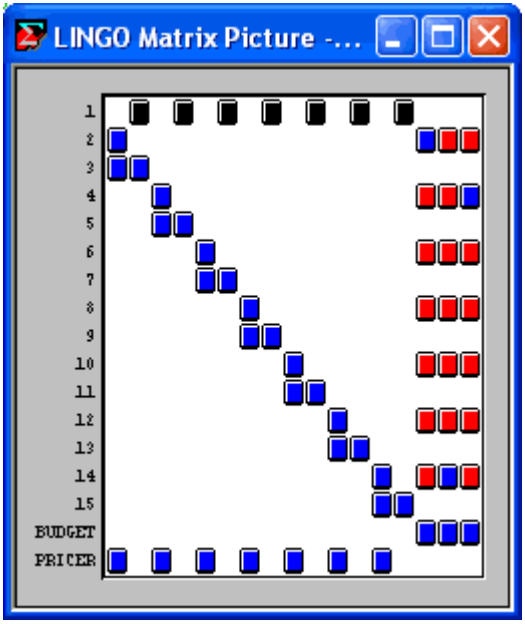
You will note that this is the core model, with the period 2 random variable *DEMAND* replaced by its sampled value of 150 in row 5. Nonanticipativity constraints are not generated as part of individual scenario models.

Solver|Picture

The *Solver|Picture* command displays a model in matrix form. Viewing the model in matrix form can be helpful in a couple of instances. First and perhaps most importantly, is the use of nonzero pictures in debugging formulations. Most models have strong repetitive structure. Incorrectly entered sections of the model will stand out in a model's matrix picture. Secondly, a nonzero picture can be helpful

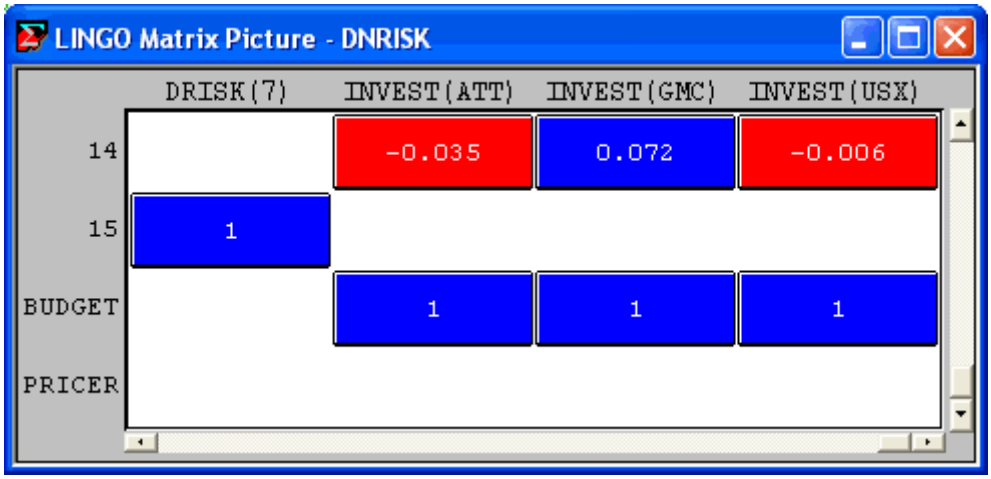
when you are attempting to identify special structure in your model. As an example, if your model displays strong block angular structure, then algorithms that decompose the model into smaller fragments might prove fruitful.

As an example, we loaded the *DNRISK.LG4* model from LINGO's sample model set. Issuing the *Picture* command, we see the following:



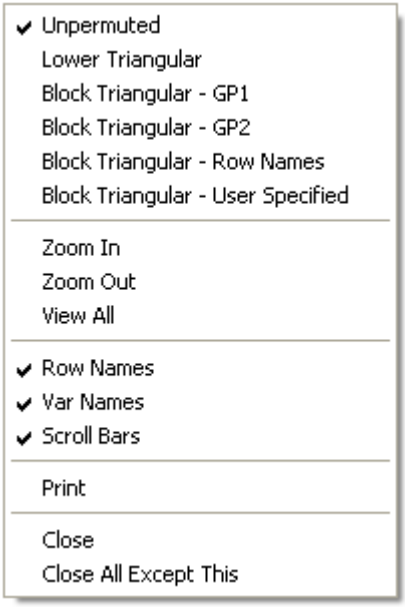
Positive coefficients are represented with blue tiles, negatives with red, and variables that appear in a row nonlinearly show up as black tiles.

You can zoom in on a selected range in the matrix for closer viewing. To do this, place the cursor on the upper left corner of the range you wish to view, press and hold down the left mouse button. Next, drag the mouse to the lower right-hand corner of the desired range. Now, release the left mouse button and LINGO will zoom in on the selected range. As an example, here is a view of the matrix after zooming in on a 4x4 range:



Note, we have zoomed in far enough to be able see the actual coefficient values, row names, and variable names. Scroll bars have also appeared to allow scrolling through the matrix.

The matrix picture window supports a number of additional interactive features. To access these features, place the cursor over the matrix picture, and press and hold the right mouse button. This will bring up the following menu:



A brief description of these features follows:

- *Unpermuted* - Displays the matrix in its original form
- *Lower Triangular* - Displays the matrix picture to lower triangular, or almost lower triangular form
- *Block Triangular - GP1* - The matrix is displayed in block triangular form using LINGO'S GP1 graph partitioning heuristic
- *Block Triangular - GP2* - The matrix is displayed in block triangular form using LINGO'S GP2 graph partitioning heuristic
- *Block Triangular - Row Names* - The matrix is displayed using a block structure specified by the user as part of the row names
- *Block Triangular - User Specified* - The matrix is displayed using a block structure specified by the user via the @BLKROW function
- *Zoom In* - Zooms the view in centered around the current cursor position
- *Zoom Out* - Zooms the view out centered around the current cursor position
- *View All* - Zooms all the way out to give a full view of the matrix
- *Row Names* - Toggles the display of row names on and off
- *Var Names* - Toggles the display of variable names on and off
- *Scroll Bars* - Toggles scroll bars on and off
- *Print* - Prints the matrix picture
- *Close* - Closes the matrix picture window

The *Lower Triangular* option is an interesting feature. The rows and columns are automatically permuted to place the matrix into mostly lower-triangular form. If a matrix is mostly lower triangular, then, in general, the model should prove relatively easier to solve. This is opposed to a model that contains a high degree of simultaneity that can't be permuted into lower triangular form.

Refer to the *BNP Solver* section above for more information on the various Block Triangular options (*GP1*, *GP2*, *Row Names* and *User Specified*).

Solver|Debug

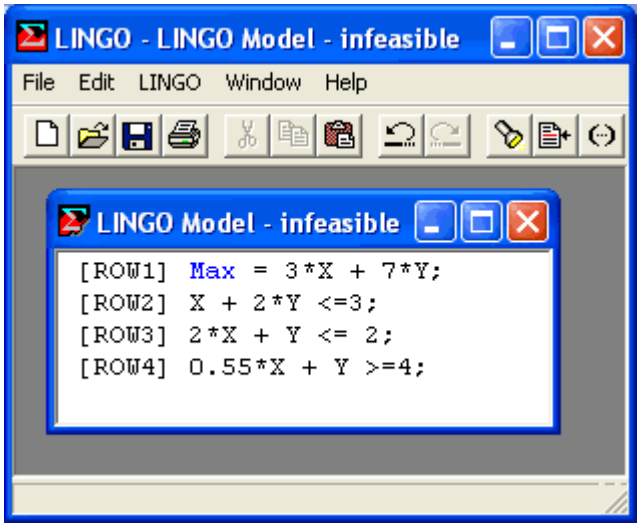
In the ideal world, all models would return an optimal solution. Unfortunately, this is not the case. Sooner or later, you are bound to run across either an infeasible or unbounded model. This is particularly true in the development phase of a project when the model will tend to suffer from typographical errors.

Tracking down an error in a large model can prove to be a daunting task. The *Debug* command is useful in narrowing the search for problems in both infeasible and unbounded linear programs. A small portion of the original model is isolated as the source of the problem. This allows you to focus your attention on a subsection of the model in search of formulation or data entry errors.

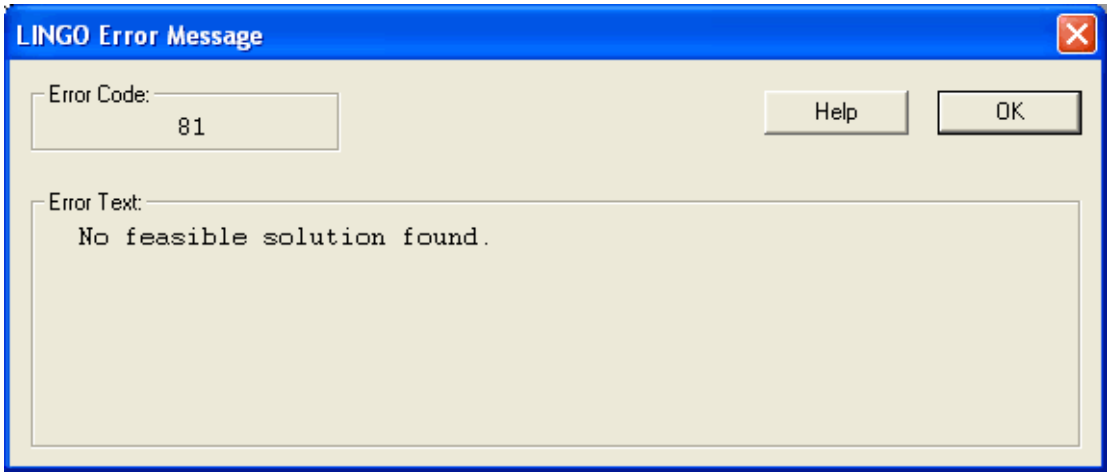
The *Debug* command identifies two types of sets: *sufficient* and *necessary*. Removing any sufficient set object from the model is sufficient to fix the *entire model*. Not all models will have a sufficient set. In which case, they will have a necessary set with the property that removing any object from this set fixes the remaining objects *within that set*.

As an example, suppose you have an infeasible model. If the complete model would be feasible except for a bug in a single row, that row will be listed as part of the sufficient set. If the model has a necessary set, then, as long as all of them are present, the model will remain infeasible.

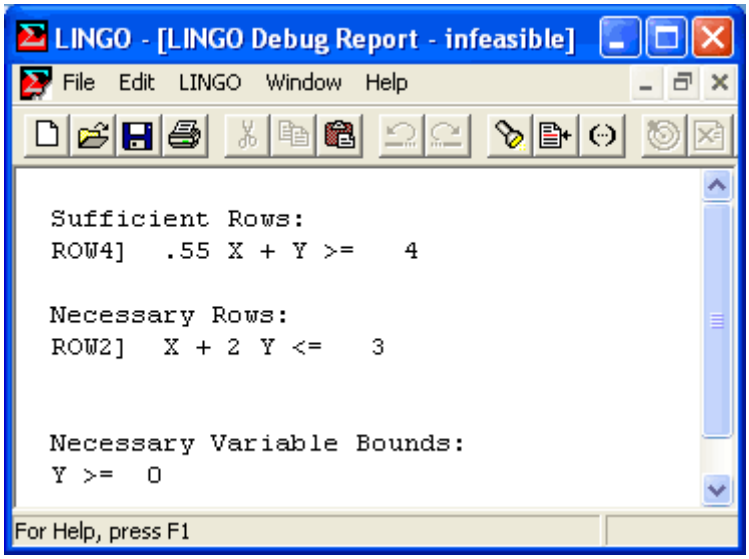
The following example illustrates. The coefficient .55 in row 4 should have been 5.5:



When we attempt to solve this formulation, we get the following error:

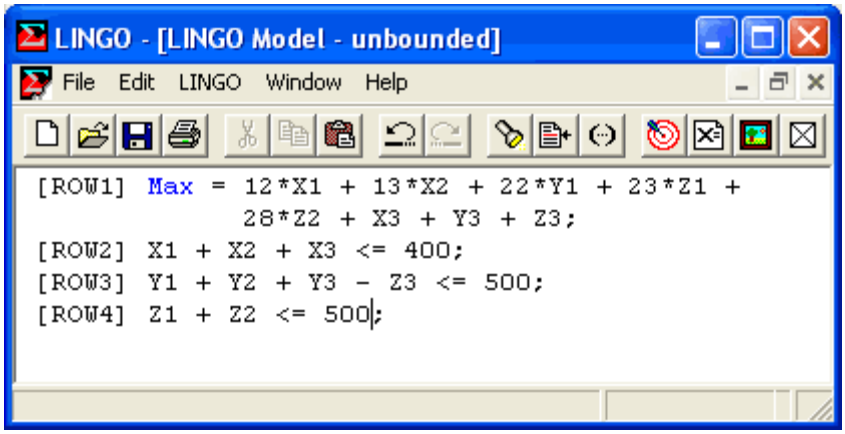


Next, if we run the *Solver|Debug* command, we are presented with the following report:

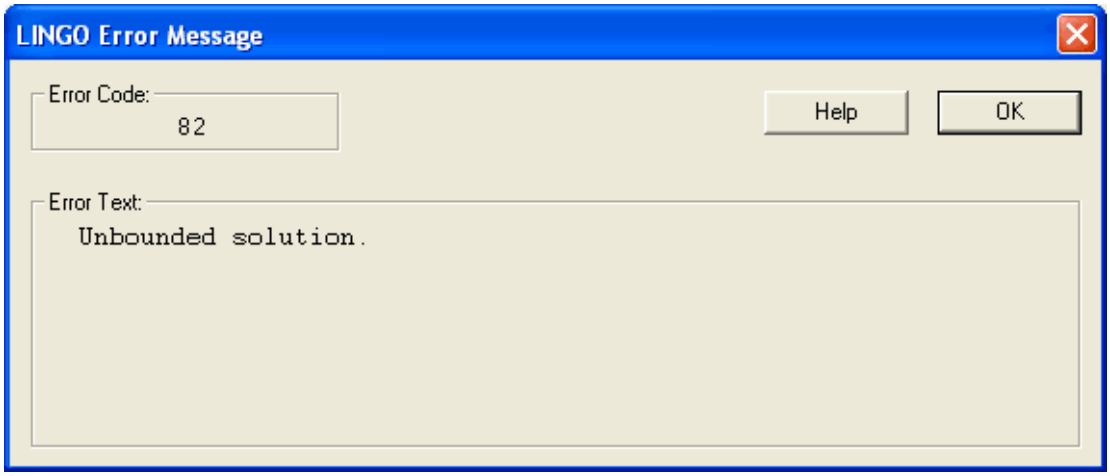


The *Debug* command has correctly identified that the erroneous *ROW4*, when eliminated, is sufficient to make the entire model feasible.

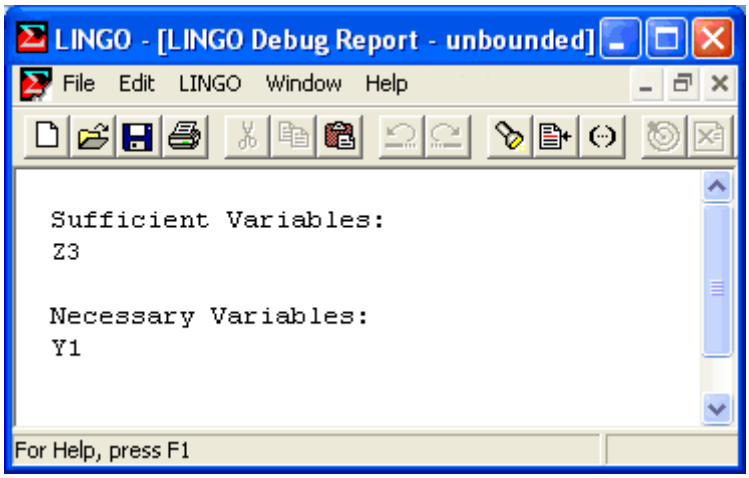
Debug operates in a similar manner for unbounded models. In the following example, we introduced an error by placing a minus sign instead of a plus sign in front of variable *Z3* in *ROW3*. A look at *ROW3* reveals that *Z3* can be increased indefinitely, leading to an unbounded objective.



The resulting model is unbounded and, when issuing the *Solver|Solve* command, we receive the unbounded error message:



Issuing the *Debug* command, we receive the following breakdown:



The *Debug* command has successfully determined that bounding Z3 is sufficient to bound the entire model.

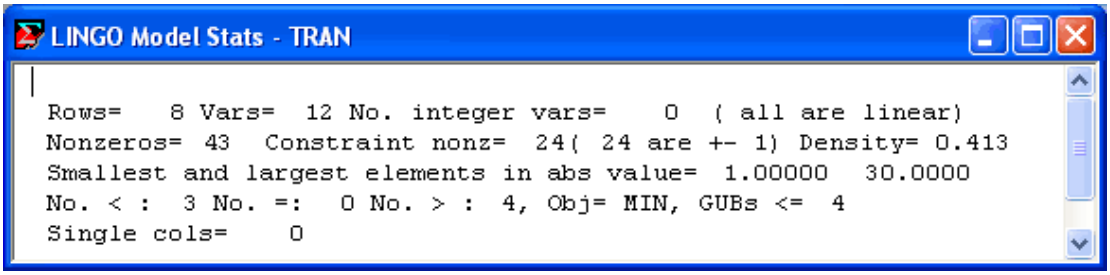
Typically, the *Debug* command helps to substantially reduce the search effort. The first version of this feature was implemented in response to a user who had an infeasible model. The user had spent a day searching for a bug in a model with 400 constraints. The debug feature quickly found a necessary set with 55 constraints, as well as one sufficient set constraint. The user immediately noticed that the right-hand side of the sufficient set constraint was incorrect.

Note: Prior to release 10.0 of LINGO, the debugger was only capable of processing linear models. Starting with release 10.0, all classes of models (LP, QP, IP and NLP) may now be debugged.

Solver|Model Statistics

The *Model Statistics* command lists summary statistics for your model. The statistics vary slightly depending on whether the model you're working with is linear or nonlinear.

In the following example, we open the linear transportation model, *TRAN.LG4,* issue the *Model Statistics* command, and then discuss some of the details of the report. Here is the output generated by *Model Statistics* for *TRAN.LG4:*



The statistics report consists of five lines.

In line one, the number of rows (constraints), variables (columns), and integer variables are shown. The report also specifies when the model is linear by stating that all variables are linear.

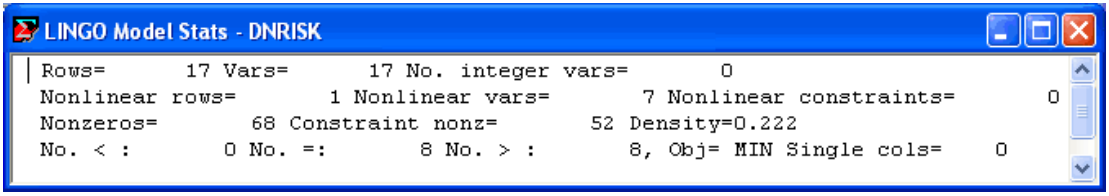
Line two of the report gives a count of the number of nonzero coefficients appearing in the model. The first count is the number of nonzero coefficients in the entire model. The *Constraint nonz* count is the number of coefficients on the left-hand sides of all the constraints, excluding the objective and nonzero right-hand side coefficients. Next, is a count of the number of constraint coefficients that are plus or minus one. In general, a linear programming model is easier to solve when the percentage of +/-1 coefficients increases. Finally, LINGO reports a density figure, which is defined as: *(total nonzeros) / [(number of rows) * (number of columns + 1)]*. For large models, densities under .01 are common. High densities can mean that a problem will take longer to solve.

Line three lists the smallest and largest coefficients in the model in absolute value. For stability reasons, the ratio of the largest coefficient to the smallest should, ideally, be close to 1. Also, in absolute terms, it is best to keep coefficient values in the range of 0.0001 to 100,000. Values outside this range can cause numerical difficulties for the solver.

Line four lists the number of constraints by type (<, =, and >), the sense of the objective, and an upper bound on the number of *Generalized Upper Bound* (GUB) constraints. A GUB constraint is a constraint that does not intersect with the remainder of the model. Given this, the GUB statistic is a measure of model simplicity. If all the constraints were nonintersecting, the problem could be solved by inspection by considering each constraint as a separate problem.

Line five lists the number of variables that appear in only one row. Such a variable is effectively a slack. If you did not explicitly add slack variables to your model and the single column count is greater than zero, then it suggests a misspelled variable name.

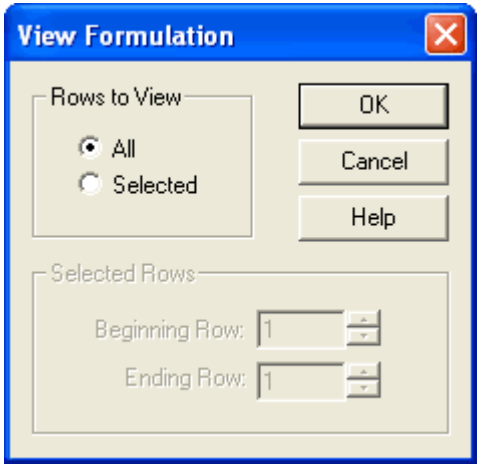
The following report was generated by the *Model Statistics* command for the nonlinear model, *DNRISK.LG4*:



The statistics report for nonlinear models drops information about the range of coefficient values, the number of +/-1 coefficients, and the GUB upper bound. A count of the number of nonlinear variables and rows is added in line two. The nonlinear rows count includes the objective, while the nonlinear constraint count does not.

Solver|Look

Use the *Look* command to generate a report containing your model's formulation. The *Look* command's dialog box, pictured below, lets you choose *All* or *Selected Rows* for viewing.



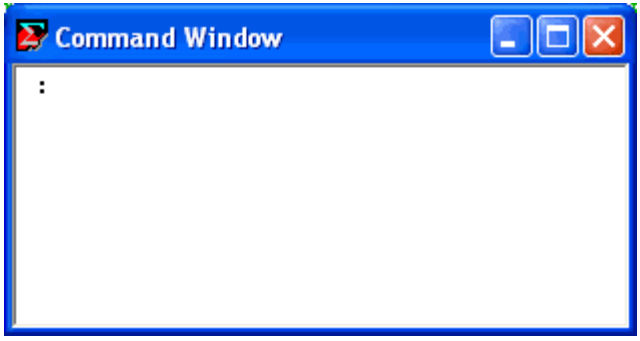
When you choose *Selected Rows*, the *Beginning* and *Ending* row text boxes are available for entry. You must enter the indices of the range of rows you wish displayed. LINGO will display the requested lines with line numbers in a new window.

4. Window Menu

The *Window* menu contains commands that generally pertain to managing open windows.

Window|Command Window

In addition to the pull down menu commands, LINGO's features can also be accessed through a command language. For more details on the command language refer to *Command-line Commands*. A script file that contains LINGO commands may be run using the *File|Take Commands* command or by entering the *TAKE* command in LINGO's command window. Alternatively, you can interactively enter script commands into LINGO's command window. The *Window|Command Window* command opens LINGO's command window. The following window should appear on your screen:



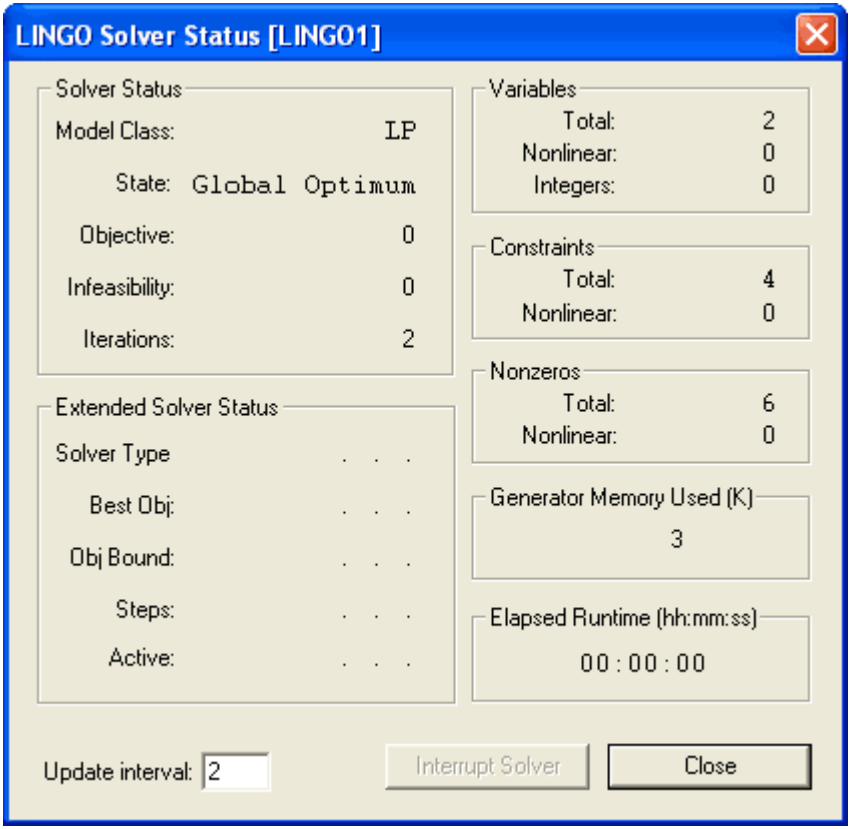
You may enter any valid LINGO script commands to the colon prompt in the upper left corner of the window. In the following example, we enter a small model with the *MODEL* command, display the formulation with the *LOOK ALL* command, then solve it using the *GO* command (user input is shown in bold type):

```
Command
: model
? max = 20*x + 30*y;
? x <= 50;
? y <= 60;
? x + 2*y <= 120;
? end
: look all

  MODEL:
   1]max = 20* x + 30*y;
   2]x <= 50;
   3]y <= 60;
   4]x + 2*y <= 120;
  END

: go
  Global optimal solution found.
  Objective value:                              2050.000
  Total solver iterations:                             0


                      Variable           Value        Reduced Cost
                             X        50.00000            0.000000
                             Y        35.00000            0.000000

                           Row    Slack or Surplus      Dual Price
                             1        2050.000            1.000000
                             2        0.000000            5.000000
                             3        25.00000            0.000000
                             4        0.000000            15.00000

:
```

In general, you will probably prefer to use the pull down menus and toolbar when using LINGO interactively. The command window interface is primarily provided for users wishing to interactively test command scripts.

Window|Status Window

When you invoke LINGO's *Solve* command, a status window is displayed on your screen that resembles the following:



This window allows you to monitor the progress of the solver. You can close the status window at any time. If you close the status window, it may be reopened with the *Window|Status Window* command.

If you would like to prevent LINGO from opening a status window, see the *Solver|Options* command. For more information on the interpretation and use of the status window, see the section *Solver Status Window*.

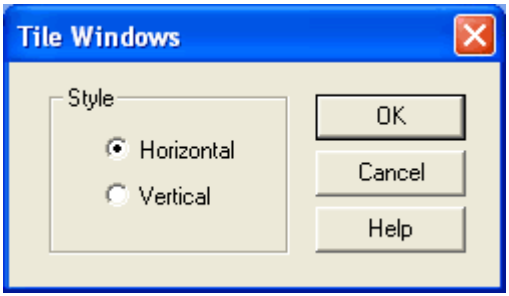
Window|Close All

The *Window|Close All* command closes all open windows. If you made a change to a model window without saving it, you will be prompted to save the model before it is closed.

Window|Tile

The *Window|Tile* command arranges all the open windows in a tiled pattern. Each window is resized, so all windows appear on the screen and are of roughly the same size.

When you issue the *Window|Tile* command, you will see the dialog box:



You have the choice of tiling the windows horizontally or vertically. If you tile horizontally (vertically), LINGO will maximize the horizontal (vertical) dimension of each window.

If there are more than three open windows, LINGO will tile the windows, but the choice of horizontal or vertical will no longer make a difference.

Window|Cascade

The *Window|Cascade* command arranges all open windows in a cascade pattern starting in the upper left corner of the mainframe window. The currently active window remains on top.

Window|Previous

The *Window|Previous* command brings the previous window in the window ordering to the top. This command is useful when switching between a model and a solution window.

Window|Next

The *Window|Next* command brings the next window in the window ordering to the top. This command is useful when switching between a model and a solution window.

Window|Arrange Icons

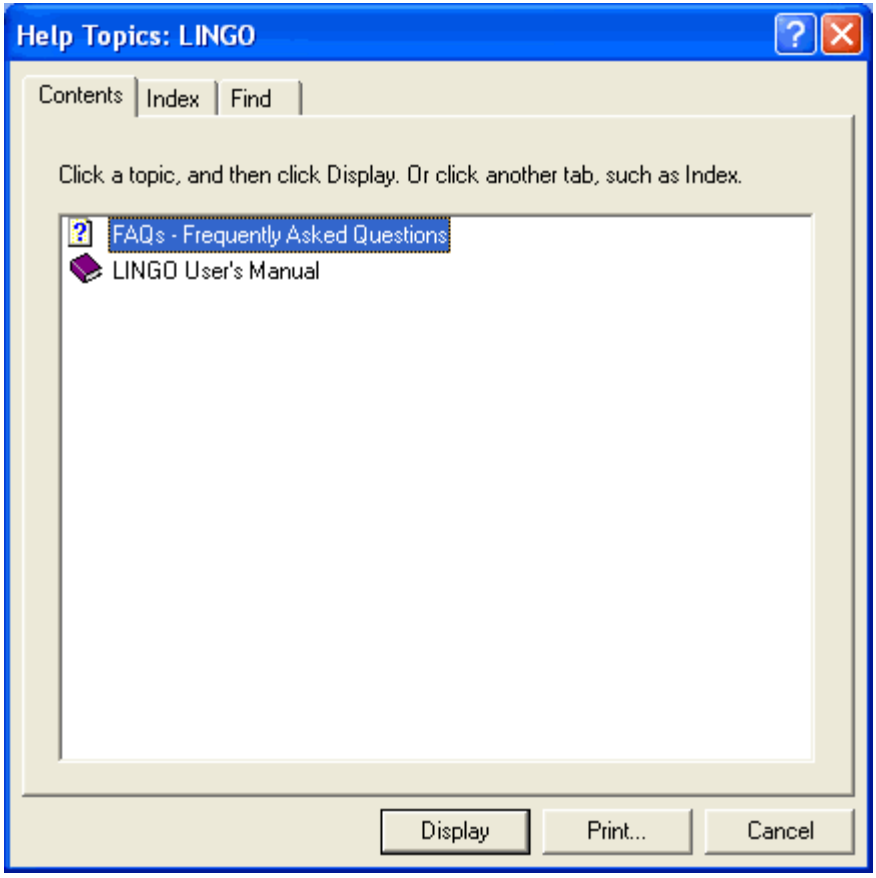
If you have minimized any open windows, so they appear as icons on the screen, you can issue the *Window|Arrange Icons* command to line all the icons up in the lower left-hand corner of the frame window.

5. Help Menu

The *Help* menu contains commands that generally pertain to LINGO's online help system, copyright notice, and version specific information.

Help|Help Topics

A portion of the dialog box displayed by the *Help Topics* command is displayed below:



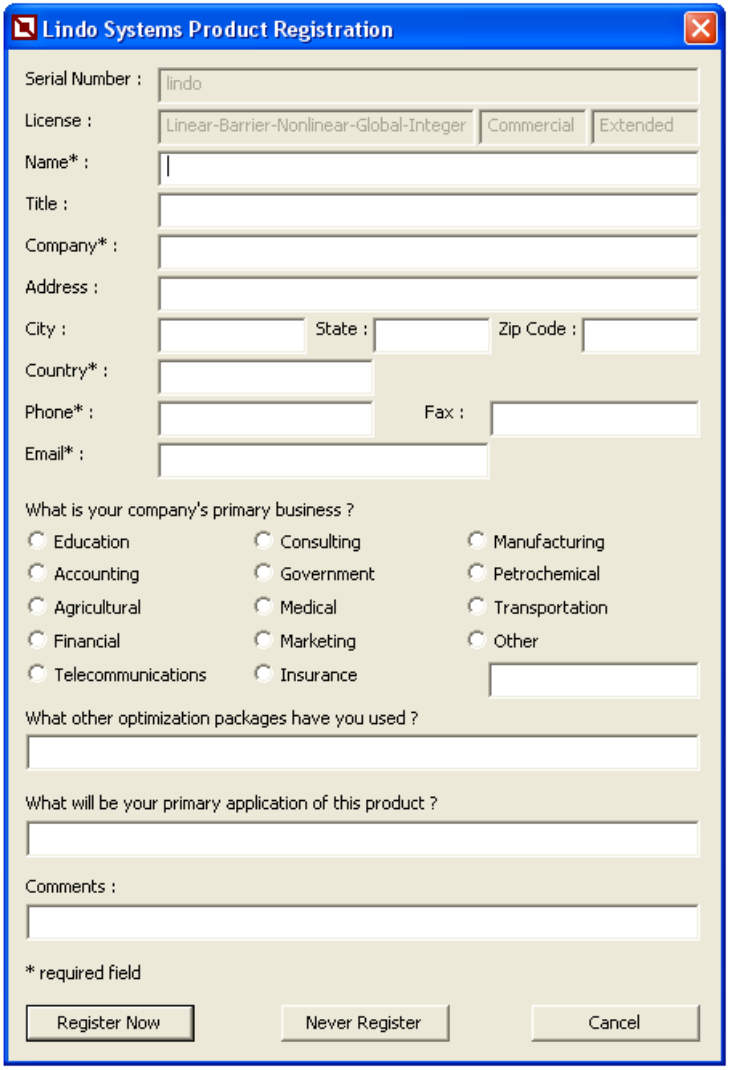
Select the *Contents* tab to display a table of contents for the Help system. You can select any of the topics of interest by double-clicking on them.

Select the *Index* tab to display an index of topics for the Help system. Select an item for viewing by double-clicking on it.

Go to the *Find* tab to search the Help system for a particular item.

Help|Register

Use the *Help|Register* command to register your version of LINGO online. ***You will need a connection to the Internet open for this command to work.*** When you issue the *Register* command, you will be presented with the following dialog box:



Enter your personal information and select the *Register Now* button. Your information will be sent directly to LINDO Systems via the Internet.

Once your registration is complete, the following dialog box will appear on your screen:

Select the *OK* button to be returned to the main LINGO environment.

LINDO Systems is constantly working to make our products faster and easier to use. Registering your software with LINDO ensures that you will be kept up-to-date on the latest enhancements and other product news. You can also register through the mail or by fax using the registration card included with your software package.

Help|AutoUpdate

Turn the *Help|AutoUpdate* command on to have LINGO automatically check every time you start the LINGO software whether there is a more recent version of LINGO available for download on the LINDO Systems website. *You will need a connection to the Internet open for this command to work.*

When you issue the *AutoUpdate* command or start a version of LINGO with *AutoUpdate* enabled, LINGO will search the Internet to see if an updated version of the LINGO software is available for download. If you currently have the most recent version, then you will be returned to the main LINGO environment. If you have an outdated version of the software, you will be presented with the following dialog box:

at which point, you may wish to go to the LINDO Systems Web site, www.lindo.com, to download the latest build of the software.

If you want to disable the *AutoUpdate* feature, then select the *Disable AutoUpdate* button from the *AutoUpdate* dialog box.

The AutoUpdate feature is disabled by default.

Help|About LINGO

When you issue the *About LINGO* command, you will be presented with a dialog box resembling the following:

The first box lists release information of your copy of LINGO.

The second box tells you where you can get in touch with LINDO Systems.

The third box, titled *Limits for this Installation*, lists various capacity limits of your version and the current number of bytes allocated to LINGO's model generator. The maximum sized problem your LINGO software can handle depends on the version you have. The current limits for the various versions are:

Version	*Total Variables*	*Integer Variables*	*Nonlinear Variables*	*Global Variables*	*Constraints*
Demo/Web	300	30	30	5	150
Solver Suite	500	50	50	5	250
Super	2,000	200	200	10	1,000
Hyper	8,000	800	800	20	4,000
Industrial	32,000	3,200	3,200	50	16,000
Extended	*Unlimited*	*Unlimited*	*Unlimited*	*Unlimited*	*Unlimited*

For more information on the definitions of these limits see section *Maximum Problem Dimensions*. In addition to the maximum problem limits, this box also lists the amount of memory allocated to LINGO's model generator. You can adjust the size of generator memory allocation on the General Solver tab of the *Solver|Options* dialog box.

The fourth box titled License Expiration lists the date at which your license expires. If your license does not have an expiration date, this field will display Perpetual.

The box labeled License Usage lists whether your license is for commercial or educational use. Educational licenses are restricted to use by faculty, staff, and students of educational institutions for instructional or research purposes. Commercial licenses are not restricted to any particular use.

The box titled Licenses lists the number of users licensed to use your copy of LINGO.
The *API Version* box lists the version number of the LINDO API in use by your copy of LINGO. The LINDO API is the library of solver tools used by LINGO to optimize your models.

The *License Location* box displays the location of the license file in effect for the current LINGO session, while the *Config Location* box gives the location of LINGO's configuration file. The configuration file is where LINGO stores non-default option settings, which are controlled by the *Solver|Options* command.

The final box, *Additional License Information*, contains information relevant to your particular license. In most cases, your LINGO serial number can be found in this field. Scrolling through this field, you will also find information as to the optional features included with your license (e.g., the barrier, conic, nonlinear, global and stochastic solvers.)

Help|Pointer

Press this button to switch the cursor into *Help* mode. Once the cursor is in *Help* mode, you can select a menu command or a toolbar button and LINGO will display help information on the selected item.

Mac LINGO Applications Menu

Mac versions of Lingo have an additional menu referred to as the *Application Menu.* This menu appears before the *File* menu and is named *Lingo.* The *Application* menu is a common feature of Mac applications and tends to contain similar commands across different applications. LINGO's application menu contains the following commands:

Command	***Description***
About LINGO	Displays the version and size of your copy of LINGO, along with information on how to contact LINDO Systems. This is equivalent to the *Help|About* Lingo command discussed above.
Preferences	Controls LINGO's option settings. This is equivalent to the *Solver|Options* command discussed above.
Services	Gives you access to various OS X settings to control the operation of your Mac.
Hide Lingo	Hides LINGO's main window, allowing you to view the windows of other running apps.
Hide Others	Hides all application windows except LINGO's window.
Show All	Shows the windows of all running apps.
Quit Lingo	Exits LINGO.

6 Command-Line Commands

This chapter discusses all of the command-line commands available to the LINGO user. On platforms other than Windows based PC's, the user interfaces with LINGO entirely through text commands issued to LINGO's command-line colon prompt.

If you are using a Windows version of LINGO, you will primarily be interested in the previous chapter, *Windows Commands,* which details the commands available in the pull down menus of LINGO's Windows version. However, in Windows versions, LINGO command-line commands may be entered using the command window (see the *Window|Command Window* section in Chapter 5, *Windows Commands*) and may also be used to build command scripts. Command scripts may be run automatically at startup or whenever the user desires. Command scripts are useful to both the Windows user and users on other platforms. Thus, the Windows user may find this chapter of interest, too.

We will begin by briefly listing all the command-line commands according to their general function. This will be followed up by an in-depth explanation of the commands.

The Commands In Brief

1. Information

CAT	lists categories of available commands
COM	lists available commands by category
HELP	provides brief help on commands
MEM	provides statistics about model generator memory usage
VERSION	displays LINGO's version information

2. Input

FRMPS	retrieves a model in free MPS format
MODEL	begins input of a new model
RLPF	retrieves a model in LP format
RMPI	retrieves a model in MPI format
RMPS	retrieves a model in fixed MPS format
TAKE	runs a command script from an external file

3. Display

DUAL	generates and displays the dual formulation for the model
GEN	generates the algebraic formulation for the model
HIDE	password protects the current model
LISTFIX	displays a report on variables that the model generator determined as being fixed in value
LOOK	displays the current model
PICTURE	displays a picture of the model's nonzero structure
SCENE	generates the algebraic formulation for a specified scenario of a larger stochastic program
SHOWNL	generates the entire model, but only displays rows containing nonlinearities
STATS	gives summary statistics about the properties of a generated model
XDETEQ	generates the algebraic formulation for a explicit deterministic equivalent of a stochastic program

4. File Output

DIVERT	opens a file for receiving output
RVRT	closes a file previously opened with *DIVERT*
SAVE	saves the current model to disk
SMPI	exports a model in MPI format
SMPS	sends a copy of the current model to a file in MPS format

5. Solution

DEBUG	tracks down formulation errors in infeasible and unbounded models
GO	solves the current model
NONZ	generates a nonzeros only solution report
RANGE	generates a range analysis report
SOLU	generates a solution report

6. Problem Editing

ALTER	edits the contents of the model
DELETE	deletes a selected row from the model
EXTEND	adds rows to the end of the current model

7. Conversational Parameters

PAGE	sets the page/screen length
PAUSE	pauses for keyboard input
TERSE	output level
VERBOSE	switches to verbose output mode
WIDTH	sets terminal display and input width

8. Tolerances

APISET	allows access to advanced parameters in the LINDO API, which is the solver library used by LINGO
DBPWD	sets the password for database access via *@ODBC*
DBUID	sets your user id for database access via *@ODBC*
FREEZE	saves current tolerance settings to disk
SET	overrides a number of LINGO defaults and tolerances

9. Miscellaneous

!	inserts a comment
QUIT	exits LINGO
TIME	displays current elapsed time since start of session

The Commands In Depth

Each LINGO command-line command is discussed in detail in this section. Commands are grouped by category based upon their general function.

Note: User input in the examples below is indicated through the use of **bold** typeface.

1. Information

The *Information* category contains commands related to on-line information.

CAT

The *CAT* command displays the nine categories of commands available in LINGO. You will be prompted to input a number corresponding to one of the categories. If you input a number, LINGO will display the commands available under the corresponding category. To exit out of the command, input a blank line.

COM

The *COM* command lists all the command-line commands available in LINGO by category.

HELP

The *HELP* command combined with another LINGO command gives you information on the command specified. The information is usually quite brief, but is often all that is needed.

The *HELP* command without an argument will give you general information about your version of LINGO, along with the maximum number of constraints and variables that your version of LINGO can handle.

MEM

The *MEM* command displays statistics about the model generator's memory usage. The following is some sample output from the *MEM* command:

```
: MEM
 Total generator memory           5242880
 Peak generator memory usage        12048
 Current generator memory usage      1312

 Total handles                         96
 Peak handle usage                      9
 Current handle usage                   5

 Total bytes moved                   1552
 Total blocks moved                     6
 Total heap compacts                    0

 Fragmentation ratio                0.002

:
```

The *Total generator memory* figure is the amount of memory LINGO has allocated for a working memory heap for model generation. You can control the size of the heap using the *SET* command. *Peak generator memory usage* refers to the maximum amount of memory the model generator used during the current session. *Current memory usage* lists the amount of working memory currently in use by the model generator.

Total handles is the maximum number of memory blocks LINGO can allocate. *Peak handle usage* lists the maximum number of memory blocks LINGO allocated at any one time during this session. *Current handle usage* represents the number of memory blocks currently in use by the model generator.

Total bytes moved lists the number of memory bytes the generator has had to move so far in order to reallocate memory. *Total blocks moved* lists the number of memory blocks moved due to reallocation. *Total heap compacts* lists the number of times the generator has had to compact the heap to make room for growing memory needs. If the number of heap compacts is abnormally large, you should allocate more working memory using the *SET* command.

The *Fragmentation ratio* is a statistic measuring how fragmented the memory heap is. A value of 1 would indicate high fragmentation, whereas a value of 0 indicates no fragmentation.

VERSION

The *VERSION* command displays the installed LINGO's version information. The following shows sample output from the *VERSION* command:

```
: VERSION

 LINGO/WIN64 19.0.1 (17 Jun 2020), LINDO API 13.0.4099.173
 Licensee info: licensee@anywhere.com

:
```

Here we see the main version number and release date for LINGO, the version number of the LINDO API solver engine being used, as well as licensee specific information (typically an email address).

2. Input

The *Input* category contains commands that initiate input into LINGO

FRMPS / RMPS

The *FRMPS* and *RMPS* commands are used to read MPS formatted models. The MPS file format is an industry standard format developed by IBM, and is useful for passing models from one solver or platform to another. *FRMPS* reads an MPS file in free format, while *RMPS* reads fixed format MPS files.

When LINGO reads an MPS file, it converts the formulation to an equivalent LINGO model. As an example, consider the following, simple model:

```
ObjRow) Maximize 20X  + 30Y
Subject To:
  Row1) X < 50
  Row2) Y < 60
  Row3) X + 2Y < 120
```

An equivalent MPS file for this model is:

```
NAME           SAMPLE
OBJSENSE
    MAX
ROWS
  N OBJROW
  L ROW1
  L ROW2
  L ROW3
COLUMNS
    X         ROW3         1.0000000
    X         OBJROW      20.0000000
    X         ROW1         1.0000000
    Y         OBJROW      30.0000000
    Y         ROW2         1.0000000
```

```
    Y         ROW3         2.0000000
RHS
    RHS       ROW1        50.0000000
    RHS       ROW2        60.0000000
    RHS       ROW3       120.0000000
ENDATA
```

As an aside, one thing to notice about the MPS representation is that it is not a very compact method for storing a model.

In the following session, we read this MPS file into LINGO and then display the model with the *LOOK* command. Note how the model is automatically converted from MPS format to LINGO format:

```
: rmps c:\sample.mps
: look all

   1] TITLE  SAMPLE;
   2] [ OBJROW] MAX = 20 * X + 30 * Y;
   3] [ ROW1]  X <= 50;
   4] [ ROW2]  Y <= 60;
   5] [ ROW3]  X + 2 * Y <= 120;

:
```

Should you wish to save the file again using MPS format rather than LINGO format, you may use the *SMPS* command.

When it comes to acceptable constraint and variable names, MPS format is less restrictive than LINGO. MPS allows for embedded blanks and other additional characters in names. To compensate for this fact, LINGO attempts to patch names when reading an MPS file so that all the incoming names are compatible with its syntax. LINGO does this by substituting an underscore for any character in a name that is not admissible. In most cases, this will work out OK. However, there is a chance for name collisions where two or more names get mapped into one. For instance, the variable names *X.1* and *X%1* would both get mapped into the single LINGO name *X_1*. Of course, situations such as this entirely alter the structure of the model rendering it incorrect.

You will be warned whenever LINGO has to patch a name with the following error message:

```
 [Error Code:  179]

 The model translator had to patch names to make them compatible:
    var names patched:            1
    row names patched:            0
 Name collisions may have occurred.
```

This message displays the number of variable and row names that were patched to get them to conform to LINGO syntax.

If name collisions are a problem, then LINGO has an option that will ensure that all names remain unique. This option involves using *RC format* for names encountered during MPS I/O. RC format

involves renaming each row (constraint) in a model to be *Rn*, where *n* is the row's index. Similarly, each column (variable) is renamed to *Cn*. In addition, LINGO renames the objective row to be *ROBJ*. To switch to RC format for MPS names, you will need to use the *SET* command as follows:

```
: SET RCMPSN 1
```

This will cause LINGO to use RC naming conventions for all MPS reads and saves. To cancel the use of RC names, type:

```
: SET RCMPSN 0
```

As an example, we will once again read the same MPS format model we read above, but this time we will switch to RC naming conventions.

```
: set rcmpsn 1

 Parameter         Old Value     New Value
   RCMPSN              0             1

: rmps c:\sample.mps
: look all

   1] TITLE  SAMPLE;
   2] [ ROBJ] MAX = 20 * C1 + 30 * C2;
   3] [ R1]  C1 <= 50;
   4] [ R2]  C2 <= 60;
   5] [ R3]  C1 + 2 * C2 <= 120;
```

Notice how the variable names now use RC format, guaranteeing that name collisions will not occur. Another potential conflict is that MPS allows variable names to be duplicated as constraint names, and vice versa. LINGO does not allow for this. When you go to solve the model, you will either receive error message 28 (Invalid use of a row name), or error message 37 (Name already in use). However, once again, you can switch to using RC format for names to avoid this conflict.

MODEL

Use the *MODEL* command to begin inputting a new model into LINGO. LINGO prompts for each new line of the model with a question mark. When you are through entering the model, enter *END* on a single line by itself. LINGO will then return to normal command mode (indicated by the colon prompt).

In the following example, we enter a small model with the *MODEL* command, display it with the *LOOK* command, and then solve it with the *GO* command:

```
: MODEL
? !How many years does it take
? to double an investment growing
? 10% per year?;
? 1.1 ^ YEARS = 2;
? END
: LOOK ALL

   1]!How many years does it take
   2]to double an investment growing
   3]10% per year?;
   4]1.1 ^ YEARS = 2;

: GO

 Feasible solution found at step:    0

              Variable           Value
                 YEARS        7.272541

                   Row    Slack or Surplus
                     1            0.000000

:
```

RMPI

The *RMPI* command is used to read MPI (Math Programmming Interface) formatted models. The MPI file format was developed by LINDO Systems as a portable format for storing arbitrary math programming models.

When LINGO reads an MPI file, it converts the formulation to an equivalent LINGO model. As an example, consider the following, simple model:

```
ObjRow) Maximize 20X  + 30Y
Subject To:
  Row1) X < 50
  Row2) Y < 60
  Row3) X + 2Y < 120
```

The equivalent MPI file for this model is:

```
BEGINMODEL   SAMPLE
! Number of Objective Functions:         1
! Number of Constraints        :         3
! Number of Variables          :         2
VARIABLES
!  Name                  Lower Bound        Initial Point
Upper Bound     Type
   X                              0              1.23457
1e+030        C
   Y                              0              1.23457
1e+030        C
OBJECTIVES
  OBJROW     MAXIMIZE
    EP_USRCOD            -101
    EP_PUSH_NUM       20
    EP_PUSH_VAR  X
    EP_MULTIPLY
    EP_PUSH_NUM       30
    EP_PUSH_VAR  Y
    EP_MULTIPLY
    EP_PLUS
CONSTRAINTS
  ROW1    L
    EP_USRCOD            -101
    EP_PUSH_VAR  X
    EP_PUSH_NUM       50
    EP_MINUS
  ROW2    L
    EP_USRCOD            -101
    EP_PUSH_VAR  Y
    EP_PUSH_NUM       60
    EP_MINUS
  ROW3    L
    EP_USRCOD            -101
    EP_PUSH_VAR  X
    EP_PUSH_NUM       2
    EP_PUSH_VAR  Y
    EP_MULTIPLY
    EP_PLUS
    EP_PUSH_NUM       120
    EP_MINUS
ENDMODEL
```

As an aside, one thing to notice about the MPI representation is that it is not a very compact method for storing a model—MPI is designed for portability, as opposed to efficiency.

In the following session, we read this MPI file into LINGO and then display the model with the *LOOK* command. Note how the model is automatically converted from MPI format to native LINGO format:

```
: rmpi c:\sample.mpi
: look all

   1] TITLE  SAMPLE;
   2] [OBJROW] MAX = 20 * X + 30 * Y;
   3] [ROW1] X <= 50;
   4] [ROW2] Y <= 60;
   5] [ROW3] X + 2 * Y <= 120;

:
```

Should you wish to save the file again using MPI format rather than LINGO format, you may use the *SMPI* command.

Note: The MPI file format is intended primarily for exporting models to other applications or platforms. The MPI format is purely scalar in nature—all set-based information is lost upon converting a LINGO model to MPI format. Thus, when saving copies of a model on your own machine, you should always use the *SAVE* command in order to save models in native LINGO format in order to preserve your model in its entirety.

When it comes to acceptable constraint and variable names, MPI format is less restrictive than LINGO. MPI allows for embedded blanks and other additional characters in names. To compensate for this fact, LINGO attempts to patch names when reading an MPI file so that all the incoming names are compatible with its syntax. LINGO does this by substituting an underscore for any character in a name that is not admissible. In most cases, this will work out OK. However, there is a chance for name collisions where two or more names get mapped into one. For instance, the variable names *X.1* and *X%1* would both get mapped into the single LINGO name *X_1*. Of course, situations such as this entirely alter the structure of the model rendering it incorrect.

You will be warned whenever LINGO has to patch a name with the following error message:

```
[Error Code:  179]

The model translator had to patch names to make them compatible:
   var names patched:            1
   row names patched:            0
Name collisions may have occurred.
```

This message displays the number of variable and row names that were patched to get them to conform to LINGO syntax.

If name collisions are a problem, then LINGO has an option that will ensure that all names remain unique. This option involves using *RC format* for names when translating non-native file formats. RC format involves renaming each row (constraint) in a model to be *Rn*, where *n* is the row's index. Similarly, each column (variable) is renamed to *Cn*. In addition, LINGO renames the objective row to be *ROBJ*. To switch to *RC format for MPS names* mode, you will need to use the *SET* command as follows:

```
: SET RCMPSN 1
```

This will cause LINGO to use RC naming conventions for all MPI reads and saves. To cancel the use of RC names, type:

```
: SET RCMPSN 0
```

As an example, we will once again read the same MPI format model we read above, but this time we will switch to RC naming conventions.

```
: set rcmpsn 1

 Parameter         Old Value     New Value
   RCMPSN              0             1

: rmpi c:\sample.mpi
: look all

   1] TITLE  SAMPLE;
   2] [ROBJ] MAX = 20 * C1 + 30 * C2;
   3] [R1] C1 <= 50;
   4] [R2] C2 <= 60;
   5] [R3] C1 + 2 * C2 <= 120;
```

Notice how the variable names now use RC format, guaranteeing that name collisions will not occur. Another potential conflict is that MPI allows variable names to be duplicated as constraint names, and vice versa. LINGO does not allow for this. When you go to solve the model, you will either receive error message 28 (Invalid use of a row name), or error message 37 (Name already in use). However, once again, you can switch to using RC format for names to avoid this conflict.

RLPF Command

The *RLPF* command is used to read *LP formatted* models. The LP file format is an industry standard, and is useful for passing models from one solver or platform to another.

When LINGO reads an LP format file, it converts the formulation to an equivalent LINGO model. As an example, consider the following, simple model:

```
ObjRow) Maximize 20X  + 30Y
Subject To:
  Row1) X < 50
  Row2) Y < 60
  Row3) X + 2Y < 120
```

An equivalent LP format file for this model is:

```
\ LP format example
Maximize
  objrow: 20x + 30y
Subject To
  row1: x <= 50
  row2: y <= 60
  row3: x + 2y <= 120
End
```

In the following session, we read this LP file into LINGO and then display the model with the *LOOK* command. Note how the model is automatically converted from LP format to LINGO format:

```
: rlpf c:\sample.lp
: look all

 MODEL:
 MAX= 20 * X + 30 * Y;
 [ROW1] X <= 50;
 [ROW2] Y <= 60;
 [ROW3] X + 2 * Y <= 120;
 END

:
```

Note: Unlike with MPS and MPI formats, LINGO does not currently support exporting/saving files in LP format.

When it comes to acceptable constraint and variable names, LP format is less restrictive than LINGO. To compensate for this fact, LINGO attempts to patch names when reading an LP file so that all the incoming names are compatible with its syntax. LINGO does this by substituting an underscore for any character in a name that is not admissible. In most cases, this will work out OK. However, there is a chance for name collisions where two or more names get mapped into one. For instance, the variable names *X.1* and *X%1* would both get mapped into the single LINGO name *X_1*. Of course, situations such as this entirely alter the structure of the model rendering it incorrect.

You will be warned whenever LINGO has to patch a name with the following error message:

```
 [Error Code:  179]

 The model translator had to patch names to make them compatible:
    var names patched:            1
    row names patched:            0
 Name collisions may have occurred.
```

This message displays the number of variable and row names that were patched to get them to conform to LINGO syntax.

If name collisions are a problem, then LINGO has an option that will ensure that all names remain unique. This option involves using *RC format* for names when translating non-native file formats. RC format involves renaming each row (constraint) in a model to be *Rn*, where *n* is the row's index. Similarly, each column (variable) is renamed to *Cn*. In addition, LINGO renames the objective row to be *ROBJ*. To switch to the *RC format for MPS names* mode, you will need to use the *SET* command as follows:

```
: SET RCMPSN 1
```

This will cause LINGO to use RC naming conventions for all names in an LP format file. To cancel the use of RC names, type:

```
: SET RCMPSN 0
```

As an example, we will once again read the same LP format format model we read above, but this time we will switch to RC naming conventions.

```
: set rcmpsn 1

 Parameter        Old Value     New Value
   RCMPSN             0             1

: rmps c:\sample.lp
: look all

   1] TITLE  SAMPLE;
   2] [ ROBJ] MAX = 20 * C1 + 30 * C2;
   3] [ R1]  C1 <= 50;
   4] [ R2]  C2 <= 60;
   5] [ R3]  C1 + 2 * C2 <= 120;
```

Notice how the variable names now use RC format, guaranteeing that name collisions will not occur. Another potential conflict is that LP format allows variable names to be duplicated as constraint names, and vice versa. LINGO does not allow for this. When you go to solve the model, you will either receive error message 28 (Invalid use of a row name), or error message 37 (Name already in use). However, once again, you can switch to using RC format for names to avoid this conflict.

TAKE

The *TAKE* command is used to 1) read models saved to disk using the *SAVE* command, and 2) execute command scripts contained in external files. The syntax for the *TAKE* command is:

TAKE [*filename*]

If you omit a filename, LINGO will prompt you for one.

As an example, suppose you used the *SAVE* command to save a model to the file *C:\LINGOMOD\MYMODEL.LNG*. You can read it back into LINGO by giving the command:

```
: TAKE C:\LINGOMOD\MYMODEL.LNG
```

As a second example, we will use the *TAKE* command to execute a LINGO command script. A command script is simply a text file that contains a series of LINGO commands. Suppose we have built the following command script in an editor and have saved it in the text file *D:\LNG\MYSCRIPT.LTF*:

```
MODEL:
!For a given probability P, this
 model returns the value X such
 that the probability that a unit
 normal random variable is less
 than or equal to X is P;

! Here is the probability;
 P = .95;

! Solve for X;
 P = @PSN(X);

END

!Terse output mode;
TERSE

!Solve the model;
GO

!Report X;
SOLU X
```

We can use the *TAKE* command to run the script as follows:

```
: TAKE D:\LNG\MYSCRIPT.LTF
 Feasible solution found at step:     0

               Variable           Value
                      X        1.644854

:
```

3. Display

This category contains commands that display information.

DUAL

The *DUAL* command displays the dual formulation of the current model. Every linear programming model has a corresponding, mirror-image formulation called the *dual*. If the original model has *M* constraints and *N* variables, then its dual will have *N* constraints and *M* variables.

Some interesting properties of the dual are that any feasible solution to the dual model provides a bound on the objective to the original, primal model, while the optimal solution to the dual has the same objective value as the optimal solution to the primal problem. It's also true that the dual of the dual model is, once again, the original primal model. You may wish to refer to any good linear programming text for a further discussion of duality theory.

As an example, consider the following small transportation model:

```
MODEL:
! A 3 Warehouse, 4 Customer
   Transportation Problem;
SETS:
   WAREHOUSE / WH1, WH2, WH3/   : CAPACITY;
   CUSTOMER   / C1, C2, C3, C4/ : DEMAND;
   ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The objective;
 [OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
 @FOR( CUSTOMER( J): [DEM]
  @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
   DEMAND( J));

! The supply constraints;
 @FOR( WAREHOUSE( I): [SUP]
  @SUM( CUSTOMER( J): VOLUME( I, J)) <=
   CAPACITY( I));

! Here are the parameters;
DATA:
   CAPACITY =   30, 25, 21 ;
   DEMAND =   15, 17, 22, 12;
   COST =      6,  2,  6,  7,
               4,  9,  5,  3,
               8,  8,  1,  5;
ENDDATA
END
```

Model: TRAN.LNG

If the sample session below, we load the sample model *TRAN.LNG* and use the *DUAL* command to generate its dual formulation:

```
: take \lingo\samples\tran.lng
: dual

  MODEL:
  MAX = 15 * DEM_C1 + 17 * DEM_C2 + 22 * DEM_C3 + 12 * DEM_C4
  + 30 * SUP_WH1 + 25 * SUP_WH2 + 21 * SUP_WH3;
  [ VOLUME_WH1_C1]  DEM_C1 +  SUP_WH1 <= 6;
  [ VOLUME_WH1_C2]  DEM_C2 +  SUP_WH1 <= 2;
  [ VOLUME_WH1_C3]  DEM_C3 +  SUP_WH1 <= 6;
  [ VOLUME_WH1_C4]  DEM_C4 +  SUP_WH1 <= 7;
  [ VOLUME_WH2_C1]  DEM_C1 +  SUP_WH2 <= 4;
  [ VOLUME_WH2_C2]  DEM_C2 +  SUP_WH2 <= 9;
  [ VOLUME_WH2_C3]  DEM_C3 +  SUP_WH2 <= 5;
  [ VOLUME_WH2_C4]  DEM_C4 +  SUP_WH2 <= 3;
  [ VOLUME_WH3_C1]  DEM_C1 +  SUP_WH3 <= 8;
  [ VOLUME_WH3_C2]  DEM_C2 +  SUP_WH3 <= 8;
  [ VOLUME_WH3_C3]  DEM_C3 +  SUP_WH3 <= 1;
  [ VOLUME_WH3_C4]  DEM_C4 +  SUP_WH3 <= 5;
  @BND( -0.1E+31, SUP_WH1, 0); @BND( -0.1E+31, SUP_WH2, 0);
  @BND( -0.1E+31, SUP_WH3, 0);
  END

:
```

You will notice that in the dual formulation the variables from the primal model become the rows of the dual. Similarly, the rows in the primal become the variables in the dual.

Note: The row names from the primal problem will become the variable names in the dual formulation. For this reason, it is strongly recommended that you name all the rows in the primal model. If a row is unnamed, then a default name will be generated for the corresponding dual variable. The default name will consist of an underscore followed by the row's internal index. These default names will not be very meaningful, and will make the dual formulation difficult to interpret.

GEN

Once you remove all the syntax errors from your LINGO model, there is still one very important step required: *model verification*. LINGO's set-based modeling capabilities are very powerful, and they allow you to generate large, complex models quickly and easily. However, when you first develop a model you will need to verify that the model being generated matches up to the model you actually intended to generate. Many set-based models can be quite complex, and it is highly likely that logic errors may creep into one or more expressions, thereby causing your generated model to be flawed. The *GEN* (short for generate) command is very useful for debugging such errors. It expands all of the model's compact set-based expressions and then writes out the full scalar-based equivalent of the LINGO model. The expanded model report explicitly lists all the generated constraints and variables in your model. You will find that the *Generate* report can be an invaluable tool in tracking down errors.

As an example of the output from the generate command, consider the transportation model developed in Chapter 1:

```
MODEL:
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES: CAPACITY;
   VENDORS: DEMAND;
   LINKS( WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS
DATA:
   !set members;
   WAREHOUSES = WH1 WH2 WH3 WH4 WH5 WH6;
   VENDORS = V1 V2 V3 V4 V5 V6 V7 V8;

   !attribute values;
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
! The objective;
   [OBJECTIVE] MIN = @SUM( LINKS( I, J):
    COST( I, J) * VOLUME( I, J));
! The demand constraints;
   @FOR( VENDORS( J): [DEMAND_ROW]
    @SUM( WAREHOUSES( I): VOLUME( I, J)) =
     DEMAND( J));
! The capacity constraints;
   @FOR( WAREHOUSES( I): [CAPACITY_ROW]
    @SUM( VENDORS( J): VOLUME( I, J)) <=
     CAPACITY( I));
END
```

Model: WIDGETS

The objective will generate one expression; there should be one demand constraint generated for each of the eight vendors and one supply constraint generated for each of the six warehouses, for a grand total of 15 rows in the expanded model. Running the generate command to verify this reveals the following report:

```
MODEL:
   [OBJECTIVE] MIN= 6 * VOLUME_WH1_V1 + 2 * VOLUME_WH1_V2 + 6 *
   VOLUME_WH1_V3 + 7 * VOLUME_WH1_V4 + 4 * VOLUME_WH1_V5 + 2 *
   VOLUME_WH1_V6 + 5 * VOLUME_WH1_V7 + 9 * VOLUME_WH1_V8 + 4 *
   VOLUME_WH2_V1 + 9 * VOLUME_WH2_V2 + 5 * VOLUME_WH2_V3 + 3 *
   VOLUME_WH2_V4 + 8 * VOLUME_WH2_V5 + 5 * VOLUME_WH2_V6 + 8 *
   VOLUME_WH2_V7 + 2 * VOLUME_WH2_V8 + 5 * VOLUME_WH3_V1 + 2 *
   VOLUME_WH3_V2 + VOLUME_WH3_V3 + 9 * VOLUME_WH3_V4 + 7 *
   VOLUME_WH3_V5 + 4 * VOLUME_WH3_V6 + 3 * VOLUME_WH3_V7 + 3 *
   VOLUME_WH3_V8 + 7 * VOLUME_WH4_V1 + 6 * VOLUME_WH4_V2 + 7 *
   VOLUME_WH4_V3 + 3 * VOLUME_WH4_V4 + 9 * VOLUME_WH4_V5 + 2 *
   VOLUME_WH4_V6 + 7 * VOLUME_WH4_V7 + VOLUME_WH4_V8 + 2 *
   VOLUME_WH5_V1 + 3 * VOLUME_WH5_V2 + 9 * VOLUME_WH5_V3 + 5 *
   VOLUME_WH5_V4 + 7 * VOLUME_WH5_V5 + 2 * VOLUME_WH5_V6 + 6 *
   VOLUME_WH5_V7 + 5 * VOLUME_WH5_V8 + 5 * VOLUME_WH6_V1 + 5 *
   VOLUME_WH6_V2 + 2 * VOLUME_WH6_V3 + 2 * VOLUME_WH6_V4 + 8 *
   VOLUME_WH6_V5 + VOLUME_WH6_V6 + 4 * VOLUME_WH6_V7 + 3 *
   VOLUME_WH6_V8 ;
   [DEMAND_ROW_V1] VOLUME_WH1_V1 + VOLUME_WH2_V1 +
   VOLUME_WH3_V1 + VOLUME_WH4_V1 + VOLUME_WH5_V1 +
   VOLUME_WH6_V1 = 35 ;
   [DEMAND_ROW_V2] VOLUME_WH1_V2 + VOLUME_WH2_V2 +
   VOLUME_WH3_V2 + VOLUME_WH4_V2 + VOLUME_WH5_V2 +
   VOLUME_WH6_V2 = 37 ;
   [DEMAND_ROW_V3] VOLUME_WH1_V3 + VOLUME_WH2_V3 +
   VOLUME_WH3_V3 + VOLUME_WH4_V3 + VOLUME_WH5_V3 +
   VOLUME_WH6_V3 = 22 ;
   [DEMAND_ROW_V4] VOLUME_WH1_V4 + VOLUME_WH2_V4 +
   VOLUME_WH3_V4 + VOLUME_WH4_V4 + VOLUME_WH5_V4 +
   VOLUME_WH6_V4 = 32 ;
   [DEMAND_ROW_V5] VOLUME_WH1_V5 + VOLUME_WH2_V5 +
   VOLUME_WH3_V5 + VOLUME_WH4_V5 + VOLUME_WH5_V5 +
   VOLUME_WH6_V5 = 41 ;
   [DEMAND_ROW_V6] VOLUME_WH1_V6 + VOLUME_WH2_V6 +
   VOLUME_WH3_V6 + VOLUME_WH4_V6 + VOLUME_WH5_V6 +
   VOLUME_WH6_V6 = 32 ;
   [DEMAND_ROW_V7] VOLUME_WH1_V7 + VOLUME_WH2_V7 +
   VOLUME_WH3_V7 + VOLUME_WH4_V7 + VOLUME_WH5_V7 +
   VOLUME_WH6_V7 = 43 ;
   [DEMAND_ROW_V8] VOLUME_WH1_V8 + VOLUME_WH2_V8 +
   VOLUME_WH3_V8 + VOLUME_WH4_V8 + VOLUME_WH5_V8 +
   VOLUME_WH6_V8 = 38 ;
   [CAPACITY_ROW_WH1] VOLUME_WH1_V1 + VOLUME_WH1_V2 +
   VOLUME_WH1_V3 + VOLUME_WH1_V4 + VOLUME_WH1_V5 +
   VOLUME_WH1_V6 + VOLUME_WH1_V7 + VOLUME_WH1_V8 <= 60 ;
   [CAPACITY_ROW_WH2] VOLUME_WH2_V1 + VOLUME_WH2_V2 +
   VOLUME_WH2_V3 + VOLUME_WH2_V4 + VOLUME_WH2_V5 +
```

```
   VOLUME_WH2_V6 + VOLUME_WH2_V7 + VOLUME_WH2_V8 <= 55 ;
   [CAPACITY_ROW_WH3] VOLUME_WH3_V1 + VOLUME_WH3_V2 +
   VOLUME_WH3_V3 + VOLUME_WH3_V4 + VOLUME_WH3_V5 +
   VOLUME_WH3_V6 + VOLUME_WH3_V7 + VOLUME_WH3_V8 <= 51 ;
   [CAPACITY_ROW_WH4] VOLUME_WH4_V1 + VOLUME_WH4_V2 +
   VOLUME_WH4_V3 + VOLUME_WH4_V4 + VOLUME_WH4_V5 +
   VOLUME_WH4_V6 + VOLUME_WH4_V7 + VOLUME_WH4_V8 <= 43 ;
   [CAPACITY_ROW_WH5] VOLUME_WH5_V1 + VOLUME_WH5_V2 +
   VOLUME_WH5_V3 + VOLUME_WH5_V4 + VOLUME_WH5_V5 +
   VOLUME_WH5_V6 + VOLUME_WH5_V7 + VOLUME_WH5_V8 <= 41 ;
   [CAPACITY_ROW_WH6] VOLUME_WH6_V1 + VOLUME_WH6_V2 +
   VOLUME_WH6_V3 + VOLUME_WH6_V4 + VOLUME_WH6_V5 +
   VOLUME_WH6_V6 + VOLUME_WH6_V7 + VOLUME_WH6_V8 <= 52 ;
 END
```

Model: WIDGETS

As expected, there are 15 rows in the generated model: [*OBJECTIVE*], [*DEMAND_ROW_V1*] through [*DEMAND_ROW_V8*], and [*CAPACITY_ROW_WH1*] through [*CAPACITY_ROW_WH6*].

As a side note, it's interesting to compare the generated model to the original, set-based model. We think most would agree that the set-based model is much easier to comprehend, thereby illustrating one of the primary benefits of modern algebraic languages over more traditional, scalar-based languages.

In addition to verifying that the correct number of rows is being generated, you should also examine each of the rows to determine that the correct variables are appearing in each row along with their correct coefficients.

Note: The reports generated by the *GEN* command are valid LINGO models. You may load *Generate* reports into LINGO and solve them as you would any other model.

One thing to keep in mind when examining generated model reports is that the LINGO model generator performs *fixed variable reduction*. This means that any variables that are fixed in value are substituted out of the generated model. For example, consider the simple model:

```
MODEL:
  MAX = 200 * WS + 300 * NC;
  WS = 60;
  NC <= 40;
  WS + 2 * NC <= 120;
END
```

If we generate this model we get the following, reduced model:

```
MODEL:
   MAX= 300 * NC + 12000 ;
   NC <= 40 ;
   2 * NC <= 60 ;
 END
```

At first glance, it seems as if both the first constraint and the variable *WS* are missing from the generated model. Note that by the first constraint in the original model (`WS = 60`), *WS* is fixed at a value of 60. The LINGO model generator exploits this fact to reduce the size of the generated model

by substituting *WS* out of the formulation. The final solution report will still contain the values for all the fixed variables; however, the fixed variables will not appear in the generated model report. If you would like to suppress fixed variable reduction so that all variables appear in your generated model, you may do so via the *Fixed Var Reduction option*.

Note:	To capture the results of the *GEN* command in a file, use the *DIVERT* command to open an output file before issuing the *GEN* command.

HIDE

The *HIDE* command hides the text of a model from viewing by the user. This may be useful if you are trying to protect proprietary ideas contained in your model.

When you enter the *HIDE* command, you'll be prompted for a password. You may enter any password with up to eight characters. LINGO will prompt you for this password once more for verification. LINGO is sensitive to the case of the alphabetic characters in the password.

Once a model is hidden, commands allowing the user to view the model text (e.g, *GEN*, *LOOK*, *SMPS*) are disabled. All other commands, however, will function as normal with the exception of *ALTER*. If a model is hidden, *ALTER* will perform modifications, but they will not be echoed to the screen.

When a hidden model is saved to disk, its text will be encrypted. This prevents the user from viewing the model from outside of LINGO as well. You will want to distribute the encrypted version of the model to those using your application. However, you should always keep an unhidden version of the model at your site for safekeeping in the event you forget the password.

A hidden model may be returned to the normal unhidden state by once again issuing the *HIDE* command with the correct password.

A sample session illustrating the use of the *HIDE* command follows:

```
: TAKE TRAN.LNG  !Read in a model
: LOOK 4 6              !Display some rows
   4]   SUPPLY / WH1, WH2, WH3/  : CAP;
   5]   DEST / C1, C2, C3, C4/ : DEM;
   6]   LINKS(SUPPLY, DEST) : COST, VOL;

: HIDE        !Now hide the model
Password?
TIGER
Please reenter password to verify:
TIGER
Model is now hidden.

: ! Model is hidden so LOOK will fail
: LOOK ALL

 [Error Code:  111]
 Command not available when model is hidden.

: ! We can still solve it though
: TERSE
: GO
 Global optimal solution found at step:         6
 Objective value:                        161.0000

: !And get a solution report
: NONZ VOL

    Variable         Value      Reduced Cost
VOL(WH1, C1)      2.000000          0.000000
VOL(WH1, C2)      17.00000          0.000000
VOL(WH1, C3)      1.000000          0.000000
VOL(WH2, C1)      13.00000          0.000000
VOL(WH2, C4)      12.00000          0.000000
VOL(WH3, C3)      21.00000          0.000000

: !Now, unhide the model
: HIDE
Password?
TIGER

Model is no longer hidden.

: !Once again, we can view the model
: LOOK 4 6
   4]   SUPPLY / WH1, WH2, WH3/  : CAP;
   5]   DEST / C1, C2, C3, C4/ : DEM;
   6]   LINKS(SUPPLY, DEST) : COST, VOL;
:
```

LISTFIX

The *LISTFIX* command displays a report on variables that the model generator determined as being fixed in value.

Consider the following small model:

```
[ROW_OBJ] MAX= 20*X + 30*Y + 12*Z;
[ROW_1] X = 2*Y;
[ROW_2] X + Y + Z <= 110;
[ROW_3] Y = 30;
```

Examining the model, we can easily determine that *Y* is fixed in value at 30, while *X* is fixed at 60. Determining the fixed variables in a small model like this is an easy task, however, this would not be the case for larger models. LINGO's Fixed Variable Reduction feature substitutes fixed variables out of the generated model that gets passed to the solver. In which case, you might find it useful when viewing a generated model (*Solver|Generate|Display* command) to know which variables have been substituted out of the model. For our particular model here, running the *LISTFIX* command yields the following report:

```
     Fixed Variable Report
     ---------------------
Variable     Key Row        Value
       Y       ROW_3     30.00000
       X       ROW_1     60.00000
```

As mentioned, the fixed variables *X* and *Y* are listed along with their values. The *Key Row* column indicates the row in which the variable was substituted out in.

Running the *Solver|Generate|Display* Model command on this same model, we get:

```
MODEL:
[ROW_OBJ] MAX= 12 * Z + 2100;
[ROW_2] Z <= 20;
END
```

And we see that both the fixed variables and their key rows have been substituted out of the generated formulation that get passed to the solver.

LOOK

The *LOOK* command displays all or part of the current model. The syntax of the *LOOK* command is:

LOOK *row_index*|*beg_row_index end_row_index*|ALL

Thus, you can specify the index of a single row to view, a range of rows, or *ALL* to view the entire model.

In this next example, we use several forms of the *LOOK* command to view the current model:

```
: LOOK ALL
    1]!For a given probability P, this
    2] model returns the value X such
    3] that the probability that a unit
    4] normal random variable is less
    5] than or equal to X is P;
    6]
    7]! Here is the probability;
    8] P = .95;
    9]
   10]! Solve for X;
   11]P = @PSN(X);
   12]

: LOOK 8
   8] P = .95;

: LOOK 10 11
   10]! Solve for X;
   11]P = @PSN(X);

:
```

PICTURE

The *PICTURE* command displays the model in matrix form. For small to medium sized models, the *PICTURE* command is a useful way to obtain a visual impression of the model and to hunt for formulation errors.

The following letter codes are used to represent the linear coefficients in the *PICTURE* output:

Letter Code	Coefficient Range
Z	(.000000, .000001)
Y	(.000001, .00001)
X	(.00001, .0001)
W	(.0001, .001)
V	(.001, .01)
U	(.01, .1)
T	(.1, 1)
A	(1, 10)
B	(10, 100)
C	(100, 1000)
D	(1000, 10000)
E	(10000, 100000)
F	(100000, 1000000)
G	> 1000000

Single digit integers are shown explicitly rather than being displayed as a code. This is especially handy, because many models have a large number of coefficients of positive or negative 1, which can affect the solution procedure. If a variable appears nonlinearly in a row, then the *PICTURE* command will represent its coefficient with a question mark.

In this example, we read in a copy of the small transportation model supplied with LINGO and use the *PICTURE* command to view the logical structure of the model:

```
: take \lingo\samples\tran.lng
: pic

           V V V V V V V V V V V V
           O O O O O O O O O O O O
           L L L L L L L L L L L L
           U U U U U U U U U U U U
           M M M M M M M M M M M M
           E E E E E E E E E E E E
           ( ( ( ( ( ( ( ( ( ( ( (
           W W W W W W W W W W W W
           H H H H H H H H H H H H
           1 1 1 1 2 2 2 2 3 3 3 3
           , , , , , , , , , , , ,
           C C C C C C C C C C C C
           1 2 3 4 1 2 3 4 1 2 3 4
           ) ) ) ) ) ) ) ) ) ) ) )

      OBJ: 6 2 6 7 4 9 5 3 8 8 1 5  MIN
  DEM(C1): 1     ' 1   '   1 '      > B
  DEM(C2): ' 1'  '  '1 '  '  1  '   > B
  DEM(C3): '   1 '     1     ' 1    > B
  DEM(C4): '     1     ' 1   '   1  > B
 SUP(WH1): 1 1'1 1  '  '  '  '  '   < B
 SUP(WH2): '     ' 1 1 1 1   '      < B
 SUP(WH3): '     '     '   1 1 1 1  < B
```

In this model, all the right-hand side values are in the range [12, 30]. Thus, they are all represented using the letter B. Row names are displayed running down the left-hand side of the matrix, while variable names are displayed along the top. The sense of the objective row and each of the constraints are shown. Spaces stand in for zero coefficients, and single quote marks are inserted to give a grid-like background.

Note: The *PICTURE* command is best used on small models. The amount of output generated for large models can be cumbersome. For larger models, the *Solver|Picture* command in Windows versions of LINGO can compress the matrix picture of large models into a single screen for easier viewing.

SCENE

The ability to generate the deterministic equivalent (DE) of a stochastic programming (SP) model via the *XDETEQ* command can be very useful when initially formulating and debugging an SP model. The only problem is that the DE can become quite unwieldy for all but the simplest SPs. For example, if the core model has 500 variables and there are 1,000 scenarios, then the DE will have 500,000 variables. Tracking down problems in such a large model can be difficult. Fortunately, in most cases when an SP is either infeasible or unbounded, LINGO will report the index of the first scenario that is causing the problem. With such information, we can focus our attention on the model for the isolated scenario that's causing the problem. This is where the *Scenario* command comes in — it allows us to

generate the formulation for an individual scenario, as opposed to the entire DE, which should substantially cut down on the burden of our debugging task.

As an example, we will work again with the *SPGAS* gas buying model. In the following example, we load the SP model and then use the *SCENE* command to generate the algebraic formulation of the second scenario:

```
: take \lingo\samples\spgas.lng
: scene 2

MODEL:
TITLE Scenario 2;
[ _1] MIN =  PURCHASE_COST +  HOLD_COST;
[ _2]  PURCHASE_COST - 5 * PURCHASE_1 - 6 * PURCHASE_2 = 0;
[ _3]  HOLD_COST -  INVENTORY_1 -  INVENTORY_2 = 0;
[ _4]  INVENTORY_1 -  PURCHASE_1 = - 100;
[ _5] -  INVENTORY_1 +  INVENTORY_2 -  PURCHASE_2 = - 150;
END

:
```

You will note that this is the core model, with the period 2 random variable *DEMAND* replaced by its sampled value of 150 in row 5. Nonanticipativity constraints are not generated as part of individual scenario models.

SHOWNL

The *SHOWNL* command will cause LINGO to generate the model and then display only those rows that contain nonlinearities. Please refer to the *Types of Constraints* section for a discussion of linear vs. nonlinear expressions.

In general, one would prefer to always have purely linear models. Linear models solve faster and will always converge to a global solution. Therefore, when developing a model, you will be interested in carefully evaluating nonlinearities to see if they can either be eliminated or rewritten in a linear fashion. The *SHOWNL* command is helpful in tracking down a model's nonlinearities.

STATS

The *STATS* command lists summary statistics for your model. The statistics vary slightly depending on whether the model you're working with is linear or nonlinear. In this next example, we will read in a linear transportation model, run the *STATS* command, and explain some of the details of the report.

```
: take \lingo\samples\tran.lng
: stats

 Rows=   8 Vars=   12 No. integer vars=    0  (all are linear)
 Nonzeros=  43 Constraint nonz= 24( 24 are +- 1) Density=0.413
 Smallest and largest elements in abs value= 1.00000   30.0000
 No. < :   3 No. =:   0 No. > :   4, Obj=MIN, GUBs <=   4
 Single cols=    0
```

The *STATS* report for linear models consists of five lines.

In line one, we see the number of rows (constraints), variables (columns), and integer variables. The *STATS* command lets us know the model is linear by stating that all the variables are linear.

Line two of the report gives a count of the number of nonzero coefficients appearing in the model. The first count is the number of nonzero coefficients in the entire model. The Constraint nonz count is the number of coefficients on the left-hand sides of all the constraints, excluding the nonzero objective and right-hand side coefficients. Next, STATS gives a count of the number of constraint coefficients that are plus or minus one. In general, a linear programming model is easier to solve when the number of unity coefficients increases. Finally, STATS reports a Density figure, defined as:

*(total nonzeros) / [(number of rows) * (number of columns + 1)].*

For large models, densities under .01 are common. High densities can mean that a problem will take longer to solve.

Line three lists the smallest and largest coefficients in the model in absolute value. For stability reasons, the ratio of the largest coefficient to the smallest should, ideally, be close to 1. Also, in absolute terms, it is best to keep coefficient values in the range of 0.0001 to 100,000. Values outside this range can cause numerical difficulties for the linear solver.

Line four lists the number of constraints by type (<, =, and >), the sense of the objective, and an upper bound on the number of *Generalized Upper Bound* (GUB) constraints. A GUB constraint is a constraint that does not intersect with the remainder of the model. Given this, the GUB statistic is a measure of model simplicity. If all the constraints were nonintersecting, the problem could be solved by inspection by considering each constraint as a separate problem.

Line five lists the number of variables that appear in only one row. Such a variable is effectively a slack. If you did not explicitly add slack variables to your model and the single column count is greater than zero, then it suggests a misspelled variable name.

In the next example, we read a nonlinear model, *DNRISK.LG4*, into LINGO and review it's model statistics.

```
: take c:\lingo\samples\dnrisk.lng
: stats
 Rows=     17 Vars=     17 No. integer vars=      0
 Nonlinear rows= 1 Nonlinear vars= 7 Nonlinear constraints=  0
 Nonzeros=     68 Constraint nonz=    52 Density=0.222
 No. < :   0 No. =:   8 No. > :   8, Obj=MIN Single cols=    0
```

The nonlinear *STATS* report drops information about the range of coefficient values, the number of +/-1 coefficients, and the GUB upper bound. A count of the number of nonlinear rows and variables is added in line two. The nonlinear rows count includes the objective, while the nonlinear constraint count does not.

XDETEQ

The XDETEQ command displays the explicit deterministic equivalent (DE) for the current model, assuming the current model is a stochastic program (SP). Viewing the DE can be very helpful in determining if you have properly set up the SP components of your model. You can refer to Chapter 14, *Stochastic Programming*, for more information on SP.

When LINGO solves an SP model, it is really solving the deterministic equivalent of the original SP. The DE contains one instance of the original core model for each scenario, where the random variables in each instance are replaced by their sampled values for the particular scenario. These core instances are tied together by a probabilistically weighted objective of all the objectives form the individual scenarios, plus a set of *nonanticipativity constraints*. The nonanticipativity constraints enforce the condition that all scenarios with the same history must implement the same decisions, otherwise, the DE would have the ability to anticipate future events.

As an example below, we load the SP gas buying sample model, *SPGAS*, and run the *XDETEQ* command:

```
: take \lingo\samples\spgas.lng
: xdeteq

MODEL:
TITLE _DETEQ;
[ _1] MIN = 0.3333333333333333 * PURCHASE_COST_SC1
+ 0.3333333333333333 * HOLD_COST_SC1 + 0.3333333333333333 *
PURCHASE_COST_SC2 + 0.3333333333333333 * HOLD_COST_SC2
+ 0.3333333333333333 * PURCHASE_COST_SC3 +
0.3333333333333333
* HOLD_COST_SC3;
[ _2_SC01]  PURCHASE_COST_SC1 - 5 * PURCHASE_1_SC1 - 5 *
PURCHASE_2_SC1 = 0;
[ _3_SC01]  HOLD_COST_SC1 -  INVENTORY_1_SC1 -
INVENTORY_2_SC1 = 0;
[ _4_SC01]  INVENTORY_1_SC1 -  PURCHASE_1_SC1 = - 100;
[ _5_SC01] -  INVENTORY_1_SC1 +  INVENTORY_2_SC1 -
PURCHASE_2_SC1 = - 100;
[ _2_SC02]  PURCHASE_COST_SC2 - 5 * PURCHASE_1_SC2 - 6 *
PURCHASE_2_SC2 = 0;
[ _3_SC02]  HOLD_COST_SC2 -  INVENTORY_1_SC2 -
INVENTORY_2_SC2 = 0;
[ _4_SC02]  INVENTORY_1_SC2 -  PURCHASE_1_SC2 = - 100;
[ _5_SC02] -  INVENTORY_1_SC2 +  INVENTORY_2_SC2 -
PURCHASE_2_SC2 = - 150;
[ _2_SC03]  PURCHASE_COST_SC3 - 5 * PURCHASE_1_SC3 - 7.5 *
PURCHASE_2_SC3 = 0;
[ _3_SC03]  HOLD_COST_SC3 -  INVENTORY_1_SC3 -
INVENTORY_2_SC3 = 0;
[ _4_SC03]  INVENTORY_1_SC3 -  PURCHASE_1_SC3 = - 100;
[ _5_SC03] -  INVENTORY_1_SC3 +  INVENTORY_2_SC3 -
PURCHASE_2_SC3 = - 180;
[ _NAC01]  INVENTORY_1_SC1 -  INVENTORY_1_SC2 = 0;
[ _NAC02]  PURCHASE_1_SC1 -  PURCHASE_1_SC2 = 0;
[ _NAC03]  INVENTORY_1_SC1 -  INVENTORY_1_SC3 = 0;
[ _NAC04]  PURCHASE_1_SC1 -  PURCHASE_1_SC3 = 0;
END

:
```

The first row of the DE is the probabilistically weighted objective over the three scenarios (Normal, Cold and Very Cold). Note that in the original core model, we had a variable called *PURCHASE_COST*, which was used to compute the cost of the gas purchases. In the DE we now have three *PURCHASE_COST* variables, or one for each of the three scenarios. LINGO appends the strings *_SC1*, *_SC2* or *_SC3* to *PUCHASE_COST* so as to indicate the appropriate scenario. Each of the constraints from the core model are also replicated across each of the scenarios with the appropriate scenario tag appended to each row's name. Finally, at the bottom of the model, there are four constraints beginning with the string: *_NAC*, which are the nonanticipativity constraints.

It's worthwhile pointing out that this report on DE for SPGAS is a valid LINGO model, too. In fact, you can copy this report from into a new model file and solve it directly, in which case, you will get the optimal solution to the original SP model.

4. File Output

The *File Output* category contains commands that output model and session information to a file.

DIVERT

The *DIVERT* command opens a file and causes LINGO to route all subsequent reports (e.g., *SOLUTION*, *RANGE*, and *LOOK* commands) from the screen to the file. This command captures the reports in text format in the file you specify. Since the files created by the *DIVERT* command are in text format, they may be read into other programs, such as word processors and spreadsheets, or they may be queued to your printer.

The syntax for the *DIVERT* command is:

DIVERT *filename*

where *filename* is the name of the file you wish to create.

The *RVRT* command reverses a *DIVERT* command by closing the *DIVERT* file and then rerouting output back to the screen.

In the following example, we create a small model with the *MODEL* command, solve it with the *GO* command, and then use the *DIVERT* command to create a file containing the formulation and solution:

```
: !Enter a small model
: MODEL
? MAX = 20*X + 30*Y;
? X <= 50;
? Y <= 60;
? X + 2*Y <= 120;
? END
: !Solve the model
: TERSE
: GO

 Global optimal solution found at step:         1
 Objective value:                        2050.000

: !Create a DIVERT file with
: !the formulation & solution
: DIVERT MYFILE.TXT  !Opens the file
: LOOK ALL           !Sends model to file
: SOLU               !Sends solution to file
: RVRT               !Closes DIVERT file
:
```

Opening the *DIVERT* file created in this example, we find the following file with the formulation and solution:

```
   1]MAX = 20*X + 30*Y;
   2]X <= 50;
   3]Y <= 60;
   4]X + 2*Y <= 120;

 Variable           Value        Reduced Cost
        X        50.00000            0.000000
        Y        35.00000            0.000000

      Row    Slack or Surplus     Dual Price
        1        2050.000            1.000000
        2        0.000000            5.000000
        3        25.00000            0.000000
        4        0.000000            15.00000
```

Note: Keep in mind that, when a *DIVERT* command is in effect, you will see little or no output on your screen. This is because the majority of output is being routed to the *DIVERT* file rather than to the screen.

Note: Also, be sure you choose a *DIVERT* filename different from your model filename. If not, you will overwrite your model file and will be unable to retrieve it!

RVRT

The *RVRT* command closes an output file opened with the *DIVERT* command. For an example of its use, see the *DIVERT* command immediately above.

SAVE

The *SAVE* command saves the current model to a file. The syntax is:

SAVE *filename*

where *filename* is the name of the file to save your model in. LINGO saves the model in text format. You can read the model back into LINGO with the *TAKE* command. We recommend you use an extension of *.LNG* on your model files, so you can readily identify them.

You may want to use your own text editor to modify your model. If you do, be sure to save the LINGO model in text (ASCII) format. Use the *TAKE* command to reopen the model in LINGO when you are through editing it.

In the following example, we input a small model and save it in the file titled *MYMODEL.LNG*:

```
: !Enter a small model
: MODEL
? MAX = 20*X + 30*Y;
? X <= 50;
? Y <= 60;
? X + 2*Y <= 120;
? END
: !Save model to a file
: SAVE MYMODEL.LNG
:
```

If you open the model file, *MYMODEL.LNG*, in a text editor, you should see the following:

```
MODEL:
   1]MAX = 20*X + 30*Y;
   2]X <= 50;
   3]Y <= 60;
   4]X + 2*Y <= 120;
 END
```

SMPI

The *SMPI* command saves your model in a special format called *Mathematical Programming Interface* (MPI). MPI is a special format developed by LINDO Systems for representing all classes of mathematical programs – linear, integer, and nonlinear. This format is not intended for permanent storage of your models. LINDO API users may be interested in this format for exporting models to the LINDO API.

Note: MPI format is a scalar type format, so all sets-based information is lost. Thus, *it is important that you do not use this format for permanent storage*. Use the *SAVE* command, discussed above, to permanently save your files for later retrieval.

Note: When exporting a stochastic program to MPI format, LINGO will write a total of four SMPI format files, consisting of a header file, the core model, its stochastic declarations and its time structure.

SMPS

The *SMPS* command generates the underlying algebraic formulation for the current model and then writes it to a file in MPS format. MPS format is a common format for representing linear programming models. MPS files can be ported to any solver that reads MPS files—this includes most commercial linear programming packages.

The syntax for the *SMPS* command is:

SMPS *filename*

where *filename* is the name of the file you wish to save the MPS representation of the model under.

In the following example, we input a small model and then save it in an MPS file:

```
: !Enter a small model
: MODEL
? MAX = 20*X + 30*Y;
? X <= 50;
? Y <= 60;
? X + 2*Y <= 120;
? END
: !Save model to an MPS file
: SMPS MYMODEL.MPS
:
```

If you open the MPS file created in a text editor, you should find:

```
NAME    LINGO GENERATED MPS FILE(MAX)
ROWS
  N 1
  L 2
  L 3
  L 4
COLUMNS
    Y         1           30.0000000
    Y         3            1.0000000
    Y         4            2.0000000
    X         1           20.0000000
    X         2            1.0000000
    X         4            1.0000000
RHS
    RHS       2           50.0000000
    RHS       3           60.0000000
    RHS       4          120.0000000
ENDATA
```

Note: Your model must be entirely linear to be able to successfully export it using *SMPS*. If a model is nonlinear, the MPS file will contain question marks in place of numbers for coefficients of nonlinear variables.

Note: *SMPS* truncates all variable names to 8 characters. For instance, the two distinct LINGO names *SHIP(WH1, C1)* and *SHIP(WH1, C2)* would both be truncated to the single 8 character name *SHIPWH1C* under *SMPS* . Either choose names to avoid collisions of truncated names or enable the *RCMPSN* option for converting names to RC format when doing MPS I/O. LINGO will print an error message if potential collisions exist.

Note: The MPS file format is intended primarily for exporting models to other applications or platforms. The MPS format is purely scalar in nature—all set-based information is lost upon converting a LINGO model to MPS format. Thus, when saving copies of a model on your own machine, you should always use the *SAVE* command instead of the *SMPS* command.

Note: When exporting a stochastic program to MPS format, LINGO will write a total of four SMPS format files, consisting of a header file the core model, its stochastic declarations and its time structure.

5. Solution

The *Solution* category contains commands for viewing a model's solution.

DEBUG

In the ideal world, all models would return an optimal solution. Unfortunately, this is not the case. Sooner or later, you are bound to run across either an infeasible or unbounded model. This is particularly true in the development phase of a project when the model will tend to suffer from typographical errors.

Tracking down an error in a large model can prove to be a daunting task. The *DEBUG* command is useful in narrowing the search for problems in both infeasible and unbounded linear programs. A small portion of the original model is isolated as the source of the problem. This allows you to focus your attention on a subsection of the model in search of formulation or data entry errors.

The *DEBUG* command identifies two types of sets: *sufficient* and *necessary*. Removing any sufficient set object from the model is sufficient to fix the entire model. Not all models will have a sufficient set. In which case, they will have a necessary set with the property that removing any object from this set fixes the remaining objects within that set.

As an example, suppose you have an infeasible model. If the complete model would be feasible except for a bug in a single row, that row will be listed as part of the sufficient set. If the model has a necessary set, then, as long as all of them are present, the model will remain infeasible.

The following example illustrates. The coefficient .55 in *ROW4* should have been 5.5:

```
: look all

MODEL:
  1][ROW1] Max = 3*X + 7*Y;
  2][ROW2] X + 2*Y <= 3;
  3][ROW3] 2*X + Y <= 2;
  4][ROW4] 0.55*X + Y >=4;
END
```

When we attempt to solve this formulation, we get the following error:

```
: go

 [Error Code:   81]

 No feasible solution found.

        Variable           Value        Reduced Cost
               X        50.00000            0.000000
               Y       -23.50000            0.000000

             Row    Slack or Surplus      Dual Price
            ROW1        0.000000           -1.000000
            ROW2        0.000000            8.500000
            ROW3       -74.50000            0.000000
            ROW4        0.000000           -10.00000
```

Next, if we run the *DEBUG* command, we are presented with the following report:

```
: debug

  Sufficient Rows:
  ROW4]  .55 X + Y >=   4

  Necessary Rows:
  ROW2]  X + 2 Y <=   3

  Necessary Variable Bounds:
  Y >=  0
```

The *DEBUG* command has correctly identified that the erroneous *ROW4*, when eliminated, is sufficient to make the entire model feasible.

The debug feature operates in a similar manner for unbounded models. In the following example, we introduced an error by placing a minus sign instead of a plus sign in front of variable *Z3* in *ROW3*. A look at *ROW3* reveals that *Z3* can be increased indefinitely, leading to an unbounded objective.

```
: look all

 MODEL:
   1][ROW1] Max = 12*X1 + 13*X2 + 22*Y1 + 23*Z1 +
   2]             28*Z2 + X3 + Y3 + Z3;
   3][ROW2] X1 + X2 + X3 <= 400;
   4][ROW3] Y1 + Y2 + Y3 - Z3 <= 500;
   5][ROW4] Z1 + Z2 <= 500;
 END
```

The resulting model is unbounded and, when issuing the *Solver|Solve* command, we receive the unbounded error message:

```
: go

 [Error Code:   82]

 Unbounded solution.
```

Issuing the *DEBUG* command, we receive the following breakdown:

```
: debug

 Sufficient Variables:
 Z3

 Necessary Variables:
 Y1
```

The *DEBUG* command has successfully determined that bounding *Z3* is sufficient to bound the entire model.

Typically, the *DEBUG* command helps to substantially reduce the search effort. The first version of this feature was implemented in response to a user who had an infeasible model. The user had spent a day searching for a bug in a model with 400 constraints. The debug feature quickly found a necessary set with 55 constraints, as well as one sufficient set constraint. The user immediately noticed that the right-hand side of the sufficient set constraint was incorrect.

GO

The *GO* command compiles and then solves the current model. When LINGO compiles the model, it produces an internally executable version of the model and then runs it to produce the solution.

When LINGO finishes solving the model, it displays a full solution report on your screen. To suppress the full solution report, issue the *TERSE* command before the *GO* command.

To capture the solution report generated by the *GO* command in a file, use the *DIVERT* command before the *GO* command.

To set various parameters pertaining to the operation of LINGO's solver, see the *SET* command later in this chapter.

NONZ

The *NONZ,* or *NONZEROS,* command displays an abbreviated version of the solution for the current model. *NONZ* is identical to the *SOLUTION* command with the exception that *NONZ* displays information only about nonzero variables and binding rows (i.e., the slack or surplus is 0).

The syntax of the *NONZ* command is:

NONZ ['*header_text'*] [*var_or_row_names*]

For a standard *NONZ* solution report, omit the two optional arguments and enter the *NONZ* command by itself. LINGO will print primal and dual values for all nonzero variables and binding rows. LINGO will label all the columns in the report.

The first optional field, *header_text*, will be displayed as a title header in the solution report. If the *header_text* argument is included, LINGO prints primal values only, omitting all labels in the report.

The second optional field, *var_or_row_names*, is a variable and/or row name list that, if included, will limit the report to the given variables or rows. The standard wild card characters (* and %) are supported in the variable and row names.

As an example, in the following session, we load the Chess Snackfoods example from the *Using Sets* section and then generate several solution reports using *NONZ*:

```
: TAKE CHESS.LNG
: TERSE
: GO

 Global optimal solution found at step:         0
 Objective value:                        2692.308

: !Generate a standard NONZ report
: NONZ

                  Variable      Value  Reduced Cost
          SUPPLY( PEANUTS)   750.0000      0.000000
          SUPPLY( CASHEWS)   250.0000      0.000000
              PRICE( PAWN)   2.000000      0.000000
            PRICE( KNIGHT)   3.000000      0.000000
            PRICE( BISHOP)   4.000000      0.000000
              PRICE( KING)   5.000000      0.000000
            PRODUCE( PAWN)   769.2308      0.000000
            PRODUCE( KING)   230.7692      0.000000
   FORMULA( PEANUTS, PAWN)   15.00000      0.000000
 FORMULA( PEANUTS, KNIGHT)   10.00000      0.000000
 FORMULA( PEANUTS, BISHOP)   6.000000      0.000000
   FORMULA( PEANUTS, KING)   2.000000      0.000000
   FORMULA( CASHEWS, PAWN)   1.000000      0.000000
 FORMULA( CASHEWS, KNIGHT)   6.000000      0.000000
 FORMULA( CASHEWS, BISHOP)   10.00000      0.000000
   FORMULA( CASHEWS, KING)   14.00000      0.000000

              Row    Slack or Surplus    Dual Price
                1            2692.308      1.000000
                2            0.000000      1.769231
                3            0.000000      5.461538

: !Generate a NONZ report for PRODUCE
: NONZ PRODUCE

                  Variable      Value  Reduced Cost
            PRODUCE( PAWN)   769.2308      0.000000
            PRODUCE( KING)   230.7692      0.000000

: !Now add a header and use a wildcard in the name
: NONZ 'NONZERO PRODUCTION VALUES:' PROD*

 NONZERO PRODUCTION VALUES:
         769.2308
         230.7692
```

If you would like to capture the solution report in a file, use the *DIVERT* command before the *NONZ* command.

For more information on the interpretation of the various fields in the *NONZEROS* report, see *Getting Started with LINGO*.

Note: If the solution report is scrolling off the screen, you can use the *PAGE*command to set the page length to *n* lines, so LINGO will pause every time *n* lines are printed and wait until you are ready to proceed with the next page.

RANGE

Use the *RANGE* command to generate a range report for the model in the active window. A range report shows over what ranges you can: 1) change a coefficient in the objective without causing any of the optimal values of the decision variables to change, or 2) change a row's constant term (also referred to as the right-hand side coefficient) without causing any of the optimal values of the dual prices or reduced costs to change.

Note: The solver computes range values when you solve a model. Range computations must be enabled in order for the solver to compute range values. Range computations *are not enabled by default*, so you will need to switch them on with the command:

```
SET DUALCO 2
```

Range computations can take a fair amount of computation time. If speed is a concern, you don't want to enable range computations unnecessarily.

The example model below, when solved, yields the range report that follows:

```
[OBJECTIVE] MAX = 20 * A + 30 * C;
[ALIM]      A <= 60;
[CLIM]      C <= 50;
[JOINT]     A + 2 * C  <= 120;
```

Here is the range report:

```
Ranges in which the basis is unchanged:

                Objective Coefficient Ranges
              Current    Allowable    Allowable
Variable  Coefficient     Increase     Decrease
       A     20.00000     INFINITY     5.000000
       C     30.00000     10.00000     30.00000

                    Right-hand side Ranges
     Row      Current    Allowable    Allowable
                  RHS     Increase     Decrease
    ALIM     60.00000     60.00000     40.00000
    CLIM     50.00000     INFINITY     20.00000
   JOINT     120.0000     40.00000     60.00000
```

The first section of the report is titled *Objective Coefficient Ranges*. In the first column, *Variable,* all the optimizable variables are listed by name. The next column, *Current Coefficient*, lists the current coefficient of the variable in the objective row. The third column, *Allowable Increase*, tells us the amount that we could increase the objective coefficient without changing the optimal values for the variables. The final column, *Allowable Decrease*, lists the amount that the objective coefficient of the variable could decrease before the optimal values of the variables would change. Information on the allowable increases and decreases on objective coefficients can be useful when you need answers to questions like, "How much more (less) profitable must this activity be before we should be willing to do more (less) of it?"

Referring to the *Objective Coefficient Ranges* report for our example, we can say, as long as the objective coefficient of *A* is greater-than-or-equal-to 15, the optimal values of the variables will not change. The same may be said for the objective coefficient of variable *C*, as long as it falls within the range of [0-40].

Note: Ranges are valid only if you are planning to alter a single objective or right-hand side coefficient. The range information provided by LINGO cannot be applied in situations where one is simultaneously varying two or more coefficients. Furthermore, ranges are only lower bounds on the amount of change required in a coefficient to actually force a change in the optimal solution. You can change a coefficient by any amount up to the amount that is indicated in the range report without causing a change in the optimal solution. Whether the optimal solution will actually change if you exceed the allowable limit is not certain.

The second section of the range report is titled *Right-hand side Ranges*. The first column, *Row*, lists the names of all the optimizable rows, or constraints, in the model. The second column, *Current RHS*, gives the constant term, or right-hand side value, for the row. The next two columns, *Allowable Increase* and *Allowable Decrease*, tell us how far we can either increase or decrease the right-hand side coefficient of the row without causing a change in the optimal values of the dual prices or reduced costs. If you recall, the dual prices on rows are, effectively, shadow prices, which tell us at what price we should be willing to buy (or sell) our resources for. The dual prices do not, however, tell us what *quantity* we should be willing to buy (or sell) at the dual price. This information is obtained from the allowable increases and decreases on the right-hand side coefficients for the row. So, for our example, the dual prices and reduced costs will remain constant as long as the right-hand side of row *ALIM* falls within the range [20-120], the right-hand side of *CLIM* is greater-than-or-equal-to 30, and the right-hand side of *JOINT* is in [60-160].

Note: We preceded all the rows in our model with a name enclosed in square brackets. This is an important practice if you wish to generate range reports. If you do not name your rows, LINGO assigns them a name that corresponds to the internal index of the row. This internal index *will not* always correspond to the order of the row in the text of the original model. To make the *Right-hand side Ranges* section of range reports meaningful, be sure to name all your rows. For details on assigning names to rows, see page 60.

If a variable is nonlinear in the objective, its value in the *Current Coefficient* column will be displayed as *NONLINEAR*. Similarly, if a row is nonlinear, the value in the *Current RHS* column will be displayed as *NONLINEAR*.

Coefficients that can be increased or decreased indefinitely will display a range of *INFINITY*.

Fixed variables are substituted out of a model and will not appear in a range report. Rows that contain only fixed variables are also substituted out of models and will not appear in range reports. As an example, suppose we changed the following inequality in our sample model from:

```
[ALIM]      A <= 60;
```

to the equality:

```
[ALIM]      A = 60;
```

LINGO can now solve directly for the value of *A*. The variable *A* is considered fixed; as is the row *ALIM* (since it contains no optimizable variables). Given this, the variable *A* will no longer appear in the *Objective Coefficient Ranges* section of the range report, and the row *ALIM* will not appear in the *Right-hand Side Ranges* section. We can verify this by examining the updated range report:

```
Ranges in which the basis is unchanged:

                Objective Coefficient Ranges
              Current    Allowable    Allowable
Variable  Coefficient     Increase     Decrease
       C     30.00000     INFINITY     30.00000

                   Right-hand Side Ranges
     Row      Current    Allowable    Allowable
                  RHS     Increase     Decrease
    CLIM     50.00000     INFINITY     20.00000
   JOINT     60.00000     40.00000     60.00000
```

As a final note, if the range report is scrolling off the screen, you can use the *PAGE n* command to set the page length to *n* lines, so LINGO will pause every time *n* lines are printed and wait until you are ready to proceed with the next page. In addition, if you would like to capture the solution report in a file, use the *DIVERT* command before the *SOLU* command.

SOLU

The *SOLU,* or *SOLUTION,* command displays a solution report for the current model. The syntax of the *SOLU* command is:

SOLU ['*header_text*'] [*var_or_row_names*]

For a standard solution report, omit the two optional arguments and enter the *SOLU* command by itself. LINGO will print primal and dual values for all the variables and rows in the model. LINGO will label all the columns in the report.

The first optional field, *header_text*, will be displayed as a title header in the solution report. If the *header_text* argument is included, LINGO prints primal values only, omitting all labels in the report.

The second optional field, *var_or_row_names*, is a variable and/or row name list that, if included, will limit the report to the given variable or row name. The standard wild card characters (* and %) are supported in the variable and row names.

As an example, in the following session, we load the *Chess Snackfoods* example from the *Using Sets* section and then generate several solution reports using *SOLU*:

```
: TAKE CHESS.LNG
: TERSE
: GO

 Global optimal solution found at step:         0
 Objective value:                        2692.308

: !Generate a standard SOLU report
: SOLU

                  Variable      Value  Reduced Cost
          SUPPLY( PEANUTS)   750.0000     0.0000000
          SUPPLY( CASHEWS)   250.0000     0.0000000
              PRICE( PAWN)   2.000000     0.0000000
            PRICE( KNIGHT)   3.000000     0.0000000
            PRICE( BISHOP)   4.000000     0.0000000
              PRICE( KING)   5.000000     0.0000000
            PRODUCE( PAWN)   769.2308     0.0000000
          PRODUCE( KNIGHT)   0.000000     0.1538461
          PRODUCE( BISHOP)   0.000000     0.7692297E-01
            PRODUCE( KING)   230.7692     0.0000000
   FORMULA( PEANUTS, PAWN)   15.00000     0.0000000
 FORMULA( PEANUTS, KNIGHT)   10.00000     0.0000000
 FORMULA( PEANUTS, BISHOP)   6.000000     0.0000000
   FORMULA( PEANUTS, KING)   2.000000     0.0000000
   FORMULA( CASHEWS, PAWN)   1.000000     0.0000000
 FORMULA( CASHEWS, KNIGHT)   6.000000     0.0000000
 FORMULA( CASHEWS, BISHOP)   10.00000     0.0000000
   FORMULA( CASHEWS, KING)   14.00000     0.0000000

            Row    Slack or Surplus      Dual Price
              1            2692.308        1.000000
              2            0.000000        1.769231
              3            0.000000        5.461538

: !Generate a SOLU report for PRODUCE
: SOLU PRODUCE

                  Variable       Value  Reduced Cost
            PRODUCE( PAWN)    769.2308     0.0000000
          PRODUCE( KNIGHT)    0.000000     0.1538461
          PRODUCE( BISHOP)    0.000000     0.7692297E-01
            PRODUCE( KING)    230.7692     0.0000000

: !Now add a header and use a wildcard
: SOLU 'PRODUCTION QUANTITIES' PROD*

 PRODUCTION QUANTITIES
         769.2308
         0.000000
         0.000000
         230.7692
```

If you would like to capture the solution report in a file, use the *DIVERT* command before the *SOLU* command.

For more information on the interpretation of the various fields in the solution report, see *Getting Started with LINGO*.

If the solution report is scrolling off the screen, you can use the *PAGE* command to set the page length to *n* lines, so LINGO will pause every time *n* lines are printed and wait until you are ready to proceed with the next page.

6. Problem Editing

The *Problem Editing* category contains commands used in editing and modifying models.

ALTER

The *ALTER* command is used to edit the current model. The syntax of *ALTER* is:

ALTER [*line_number*|*line_range*|ALL] '*old_string*'*new_string*'

where,

line_number	is the index of a single line to edit,
line_range	is a range of lines to edit,
ALL	means edit all the lines in the model,
old_string	is the old string to search for and replace, and
new_string	is the string to replace all occurrences of *old_string* with in the specified line range.

In the following sample session, we read in a small knapsack model and perform two *ALTER* commands to modify the model:

```
: TAKE ALTER.LNG
: LOOK ALL
    1]SETS:
    2]   THINGS /1..4/: VALUE, WEIGHT, X;
    3]ENDSETS
    4]DATA:
    5]   VALUE  =  8  6  4  3;
    6]   WEIGHT = 66 44 35 24;
    7]ENDDATA
    8]   MAX = @SUM(THINGS: VALUE * X);
    9]   @SUM(THINGS: WEIGHT * X) >= 100;
   10]   @FOR(THINGS: @BIN(X));
: !Change the direction of the constraint
: ALTER 9 '>='<='
   9]   @SUM(THINGS: WEIGHT * X) <= 100;
: !Change 'THINGS' to 'ITEMS' in ALL rows
: ALTER ALL 'THINGS'ITEMS'
   2] ITEMS /1..4/: VALUE, WEIGHT, X;
   8]   MAX = @SUM(ITEMS: VALUE * X);
   9]   @SUM(ITEMS: WEIGHT * X) <= 100;
   10]   @FOR(ITEMS: @BIN(X));
: LOOK ALL
    1]SETS:
    2] ITEMS /1..4/: VALUE, WEIGHT, X;
    3]ENDSETS
    4]DATA:
    5]   VALUE  =  8  6  4  3;
    6]   WEIGHT = 66 44 35 24;
    7]ENDDATA
    8]   MAX = @SUM(ITEMS: VALUE * X);
    9]   @SUM(ITEMS: WEIGHT * X) <= 100;
   10]   @FOR(ITEMS: @BIN(X));
:
```

Note: In addition to the single quote character ('), LINGO also allows the use of the double quote character (") for delimiting the text fields of the *ALTER* command.

DELETE

The *DELETE* command is used to delete one or more lines of text from the current model. The syntax of *DELETE* is:

DELETE [*line_number*|*line_range*|ALL]

where,

line_number	is the index of a single line to delete,
line_range	is a range of lines to delete, and
ALL	means delete the entire model.

Some examples of the *DELETE* command follow:

Example 1: **`DELETE 3`**
deletes line 3 of the model,

Example 2: **`DEL 2 10`**
deletes lines 2 through 10 of the model, and

Example 3: **`DEL ALL`**
deletes the entire model.

EXTEND

The *EXTEND* command allows you to append lines to the current model. It puts LINGO in model input mode just after the last line of the current model. When you use the *EXTEND* command, you'll see LINGO's question mark prompt. Start entering your new lines of model text. When you're done, enter **`END`** at the prompt.

In the following sample session, we use the *EXTEND* command to append an additional constraint to a small model:

```
: LOOK ALL
   1]MAX 20*X + 30*Y;
   2]X <= 50;
   3]Y <= 60;
   4]X + 2*Y <=120;
: ! Use EXTEND to add another line
: EXTEND
? X >= 30;
? END
: LOOK ALL
   1]MAX 20*X + 30*Y;
   2]X <= 50;
   3]Y <= 60;
   4]X + 2*Y <=120;
   5]X >= 30;
:
```

7. Conversational Parameters

The *Conversational Parameters* category contains commands that control how information is displayed.

PAGE

The *PAGE* command sets the length of the page or screen size in lines. The syntax for *PAGE* is:

PAGE *n*

where *n* is the desired number of lines per page of output. For instance, **PAGE 25** will cause the display to pause after 25 lines and await a carriage return before displaying the next 25 lines. The *PAGE* command is convenient when you wish to page through long reports and not have them scroll off the top of the screen.

When 0 is entered as the argument to *PAGE*, paging is turned off entirely. LINGO will no longer stop output to wait for a carriage return. Entering **PAGE 0** at the top of any command script is helpful in that you generally want command scripts to run uninterrupted.

The *PAGE* command is equivalent to the *SET LENPAG* command and is maintained for backward compatibility

PAUSE

The *PAUSE* command causes screen display to pause until a carriage return is typed. If you enter text on the same line as the *PAUSE* command, the text will be displayed. The *PAUSE* command is useful in command scripts for conveying information to the user.

TERSE

The *TERSE* command causes LINGO to suppress the automatic display of a solution report after a model is solved with the *GO* command. When *TERSE* is enabled, you will need to use the *NONZ* or *SOLU* commands to view the solution.

When LINGO is in terse output mode, *export summary reports* are also suppressed. *Export summary reports* are normally generated each time you export solutions to spreadsheets or databases.

Once you enter the *TERSE* command, LINGO stays in terse output mode until you enter the *VERBOSE* command (see below).

The *TERSE* command is equivalent to the *SET TERSEO 1* command and is maintained for backward compatibility.

VERBOSE

The *VERBOSE* command undoes the effects of the *TERSE* command, and places LINGO in verbose output mode. Verbose output mode is the default mode. It results in the automatic display of solution reports after solving a model. Verbose output mode also results in the automatic display of export summary reports whenever export operations are performed to spreadsheets and databases.

The *VERBOSE* command is equivalent to the *SET TERSEO 0* command and is maintained for backward compatibility.

WIDTH

Use the *WIDTH* command to set the terminal width for input and output. The syntax of the *WIDTH* command is:

WIDTH *n*

where *n* is the desired terminal width. You may set the width between 64 and 200. The default is 76.

When LINGO generates reports, it limits output lines to the terminal width length. In some reports, lines will be wrapped, so they fall within the line limit. In other reports, lines may be truncated. Since LINGO concatenates variable names in performing set operations, a variable name, such as *SHIPMENTS(WAREHOUSE1, CUSTOMER2)*, may result, which may be truncated in a solution report if too narrow a terminal width is used.

The *WIDTH* command is equivalent to the *SET LINLEN* command and is maintained for backward compatibility.

8. Tolerances

The *Tolerances* category contains commands for setting system parameters in LINGO.

APISET

The *APISET* command gives you access to all the parameters in the LINDO API, which is the solver library used by LINGO. LINGO allows access to most of the important solver parameters through the *SET* command and the *Solver|Options* command. However, some of the more advanced parameters may only be accessed through the *APISET* command. The syntax for this command is:

APISET *param_id* {int|double} *param_value*

where *param_id* is the parameter's index and *param_value* is the value you wish to set the parameter to. You will also need to indicate if the parameter is an integer or double precision quantity.
Some examples of the *APISET* command follow:

Example 1: **`APISET 5341 INT 10000`**
sets the MIP branch limit (LS_IPARAM_MIP_BRANCH_LIMIT=5341) to 10000,

Example 2: **`HELP APISET`**
will cause LINGO to display all current *APISET* settings, and

Example 3: **`APISET DEFAULT`**
removes all custom LINDO API settings, returning to the defaults.

You will need to refer to the LINDO API documentation for a list of available parameters and their indices. The LINDO API documentation is available at no charge as part of the LINDO API download on the LINDO Systems Web site. The LINGO installation also comes with a macro definition file, *Lindo.h*, which contains all the parameter indices for the LINDO API.

Parameter values set with the *APISET* command are not stored from one LINGO session to the next. Give the **HELP APISET** command for a listing of parameters that are currently active. To remove all APISET parameter settings type the command: **APISET DEFAULT**.

If there are some LINDO API parameters you wish to permanently set, you may place a series of *APISET* commands in an *AUTOLG.DAT* script file that automatically gets run at the start of each LINGO session.

DBPWD

The *DBPWD* command is used to input a password for accessing databases via the *@ODBC()* function. Any password input with this command will not be permanently stored. Therefore, at the start of each session, you will need to reenter your database password. The syntax for the command is:

DBPWD *my_password*

See the *DBUID* command below for entering any user id required by your database.

DBUID

The *DBUID* command is used to input a user id for accessing databases via the *@ODBC()* function. Any user id input with this command will not be permanently stored. Therefore, at the start of each session, you will need to reenter your database user id. The syntax for the command is:

DBUID *my_user_id*

See the *DBPWD* command above for entering any password required with your user id.

FREEZE

The *FREEZE* command saves your current configuration to LINGO's configuration file, so it may be automatically restored the next time LINGO starts. Any non-default features of the current configuration are saved to the *LINGO.CNF* file in LINGO's main directory. The *LINGO.CNF* configuration file is a text file, and the curious user may examine it by simply opening it in a text editor. All parameters controlled by the *SET* command, see below, are stored by the *FREEZE* command.

Note: Be careful when saving a non-default configuration. The saved configuration will automatically be restored next time you start LINGO. Settings of certain parameters will affect the way models are solved, potentially leading to misleading results when used on a different set of models. To restore the default configuration, use the following command sequence:

```
: SET DEFAULT
: FREEZE
```

SET

The *SET* command allows you to override LINGO's default tolerances and settings. All user configurable options in LINGO are available through the *SET* command. The syntax for the *SET* command is:

SET *parameter_name* | *parameter_index* [*parameter_value*]

where,

parameter_name is the name of the parameter to set,
parameter_index is the index of the parameter to set, and
parameter_value is the new value for the parameter that, if omitted, will cause LINGO to display the current value for the specified parameter.

Use the *FREEZE* command to save any tolerances modified with the *SET* command to the configuration file, so they will be automatically restored the next time LINGO starts. You may also enter **SET DEFAULT** to return all parameters to their default values.

Some examples of the *SET* command follow:

Example 1: **SET MXMEMB 128**
FREEZE
sets the generator memory limit to 128MB and saves parameter settings to the configuration file,

Example 2: **SET 5 1.E-7**
sets the relative integrality tolerance (*RELINT*) to 1.e-7,

Example 3: **SET DEFAULT**
restores all parameters to their default values, and

Example 4: **HELP SET**
causes LINGO to display all parameter settings.

The parameters accessible through the *SET* command are:

Index	Parameter Name	Default	Description
1	ILFTOL	0.3e-5	Initial linear feasibility tolerance
2	FLFTOL	0.1e-6	Final linear feasibility tolerance
3	INFTOL	0.1e-2	Initial nonlinear feasibility tolerance
4	FNFTOL	0.1e-5	Final nonlinear feasibility tolerance

5	RELINT	0.8e-5	Relative integrality tolerance
6	NOPTOL	0.1e-6	NLP optimality tolerance
7	ITRSLW	5	Iteration limit for slow progress
8	DERCMP	0	Derivatives (0:LINGO chooses, 1:backward analytical, 2:forward analytical, 3:central differences, 4:forward differences)
9	ITRLIM	0	Iteration limit (0:no limit)
10	TIMLIM	0	Solver time limit in seconds (0:no limit)
11	OBJCTS	1	Objective cuts (1:yes, 0:no)
12	MXMEMB	32	Memory limit in megabytes for LINGO's model generator (N/A on some machines)
13	CUTAPP	2	Cuts application (0:root, 1:all, 2:solver chooses)
14	ABSINT	.000001	Absolute integrality tolerance
15	HEURIS	3	IP heuristics (0:none, 100:advanced)
16	HURDLE	0	IP hurdle value (Use 'NONE' to disable)
17	IPTOLA	0	IP absolute optimality tolerance
18	IPTOLR	.1e-4	IP relative optimality tolerance
19	TIM2RL	100	Seconds before switching to IP relative optimality tolerance
20	NODESL	0	0:LINGO decides, 1:depth first, 2:worst bound, 3:best bound
21	LENPAG	0	Terminal page length limit (0:none)
22	LINLEN	85	Terminal page width (0:none)
23	TERSEO	0	Output level (0:verbose, 1:terse, 2:errors only, 3:no output)
24	STAWIN	1	Post status window (1:yes, 0:no)
25	SPLASH	1	Display splash screen (1:yes, 0:no)

26	OROUTE	0	Route output to command window (1:yes, 0:no)
27	WNLINE	800	Max command window lines
28	WNTRIM	400	Min command window lines
29	STABAR	1	Display status bar (1:yes, 0:no
30	FILFMT	1	File format (0:lng, 1:lg4, 2:ltx)
31	TOOLBR	1	Display toolbar (1:yes, 0:no)
32	CHKDUP	0	Check for duplicate model names in data (1:yes, 0:no)
33	ECHOIN	0	Echo command input to terminal (1:yes, 0:no)
34	ERRDLG	1	Route error messages to a dialog box (1:yes, 0:no)
35	USEPNM	0	Allow for unrestricted use of primitive set names (1:yes, 0:no)
36	NSTEEP	0	Use steepest edge variable selection in nonlinear solver (1:yes, 0:no)
37	NCRASH	0	Run crash procedure 1 to get an initial starting point in nonlinear models (1:yes, 0:no). See parameter 91, LCRASH, below for alternative procedure.
38	NSLPDR	1	Compute search directions in nonlinear solver using successive linear programming (1:yes, 0:no)
39	SELCON	0	Use selective constraint evaluation in nonlinear solver (1:yes, 0:no)
40	PRBLVL	0	Specify probing level on MILPs (0:LINGO chooses, 1:none, 7:high)
41	SOLVEL	0	Specify linear solver (0:LINGO chooses, 1:primal, 2:dual, 3:barrier)
42	REDUCE	2	Perform model reduction (2:LINGO chooses, 1:yes, 0:no)
43	SCALEM	1	Scale the model (1:yes, 0:no)

44	PRIMPR	0	Select primal pricing method (0:LINGO chooses, 1:partial, 2:devex)
45	DUALPR	0	Select dual pricing method (0:LINGO chooses, 1:Dantzig, 2:steepest edge)
46	DUALCO	1	Specify dual computations (0:none, 1:prices only, 2:prices and ranges, 3:prices only on optimizable rows)
47	RCMPSN	0	Use RC format names for MPS I/O (1:yes, 0:no)
48	MREGEN	2	Select model regeneration (0:only on modifications to model, 1:same as 0 plus whenever model has external references, 2:always)
49	BRANDR	0	Select branch direction (0:both, 1:up, 2:down)
50	BRANPR	0	Select branch priority (0:LINGO decides, 1:binary)
51	CUTOFF	.1e-8	Cutoff solution values smaller than this
52	STRONG	10	Specify strong branch level
53	REOPTB	0	IP warm start LP (0:LINGO, 1:primal, 2:dual, 3:barrier)
54	REOPTX	0	IP cold start LP (0:LINGO, 1:primal, 2:dual, 3:barrier)
55	MAXCTP	100	Max top cuts passes
56	RCTLIM	.75	Relative cuts limit
57	GUBCTS	1	GUB cuts (1:yes, 0:no)
58	FLWCTS	1	Flow cuts (1:yes, 0:no)
59	LFTCTS	1	Lift cuts (1:yes, 0:no)
60	PLOCTS	1	Plant location cuts (1:yes, 0:no)
61	DISCTS	1	Disaggregation cuts (1:yes, 0:no)
62	KNPCTS	1	Knapsack cover cuts (1:yes, 0:no)

63	LATCTS	1	Lattice cuts (1:yes, 0:no)
64	GOMCTS	1	Gomory cuts (1:yes, 0:no)
65	COFCTS	1	Coefficient reduction cuts (1:yes, 0:no)
66	GCDCTS	1	Greatest common divisor cuts (1:yes, 0:no)
67	SCLRLM	1,000	Syntax coloring line limit
68	SCLRDL	0	Syntax coloring delay in seconds
69	PRNCLR	1	Matching parenthesis coloring (1:yes, 0:no)
70	MULTIS	0	NLP multistart attempts (0:LINGO, n:number of attempts)
71	USEQPR	1	Use quadratic recognition (1:yes, 0:no)
72	GLOBAL	0	Use global solver on NLPs (1:yes, 0:no)
73	LNRISE	0	Linearization (0:LINGO, 1:none, 2:low, 3:high)
74	LNBIGM	100,000	Linearization BigM coefficient
75	LNDLTA	.1e-5	Linearization Delta coefficient
76	BASCTS	1	Basis cuts (1:yes, 0:no)
77	MAXCTR	2	Max tree cuts passes
78	HUMNTM	0	Minimum heuristic time limit (seconds)
79	DECOMP	0	Matrix decomposition (1:yes, 0:no)
80	GLBOPT	.1e-4	Global solver optimality tolerance
81	GLBDLT	.1e-6	Global solver delta tolerance
82	GLBVBD	.1e+11	Global solver variable bound limit
83	GLBUBD	2	Global solver bound use (0:no, 1:all, 2:some)
84	GLBBRN	5	Global solver branch selection (see below)
85	GLBBXS	1	Global solver box selection (0:depth first, 1:worst bound)

86	GLBREF	3	Global solver reformulation level (0:none, 3:high)
87	SUBOUT	2	Fixed variable reduction (0:none, 1:max, 2:not when using global or multistart solvers, 3:linear variables only)
88	NLPVER	3	Nonlinear solver version (3:Ver 3.0, 4:Version 4.0)
89	DBGCLD	0	Debugging cold start solver (0:LINGO, 1:primal, 2:dual, 3:barrier)
90	DBGWRM	0	Debug warm start solver (0:LINGO, 1:primal, 2:dual, 3:barrier)
91	LCRASH	0	Run crash procedure 2 to get an initial starting point in nonlinear models (1:yes, 0:no). See parameter 37, NCRASH, above for alternative procedure.
92	BCROSS	1	Perform a basis crossover on LPs when using barrier solver (0:no, 1:yes)
93	LOWMEM	0	Opt for less memory usage (0:no, 1:yes)
94	FILOUT	0	Fill out workbook output ranges (0:no, 1:yes)
95	DBGLVL	15	Debugger output level (1:low, 15:high)
96	UNARYM	1	Unary minus priority (0:low, 1:high)
97	LINEAR	0	Assume model is linear to reduce memory consumption(0:no, 1:yes)
98	LOPTOL	.1e-6	Linear optimality tolerance
99	SECORD	0	Use second order derivatives for NLPs (0:no, 1:yes)
100	NONNEG	1	Variables default to being non-negative (0:no, 1:yes)
101	BIGMVL	1.e8	BigM coefficient threshold value
102	KILLSC	0	Kill scripts on interrupts (0:no, 1:yes)
103	TATSLV	0	@SOLVE time limit in seconds

104	KBESTS	1	Number of K-Best MIP solutions to generate
105	LCORES	1	Number of concurrent LP solvers to run
106	LCORE1	1	LP Solver in Core 1 (1:prm1,2:dual,3:barrier,4:prim2)
107	LCORE2	2	LP Solver in Core 2
108	LCORE3	3	LP Solver in Core 3
109	LCORE4	4	LP Solver in Core 4
110	SCALEW	1.e12	Scaling warning threshold
111	REFRAQ	0	Basis refactor frequency (0:LINGO chooses, iteration count)
112	SPSMSZ	2	Default SP sample size per stage
113	SPSCON	1	Apply SP sampling to continuous random variables only (0:no, 1:yes)
114	SPSEED	1,031	SP sampling random number seed
115	SPMTHD	0	SP solution method (0:LINGO chooses, 1:deterministic equivalent, 2:nested Benders, 3:augmented Lagrangian)
116	SPXVAL	1	Calculate all SP expected value statistics (0:no, 1:yes)
117	SPMAXS	40,000	Max scenarios allowed in an SP before auto sampling takes effect
118	PRECIS	7	Precision in digits for standard solution reports
119	LOOPOP	0	Perform loop optimization during model compiles (0:no, 1:yes)
120	HEUCUT	0	Heuristics cutoff criterion (0:LINGO chooses, 1:time, 2:iterations)
121	NUMBIN	0	Number of bins in histogram charts (0:LINGO chooses, >0: number of bins)
122	SPBIGM	1.E8	Stochastic solver Big M coefficient

123	NSLPSV	0	Use SLP solver for nonlinear models (0:no, 1:yes)
124	FORCEB	0	Enforce variable bounds in calc and data sections (0:no, 1:yes)
125	NTHRDS	1	Max number of executions threads (0:use all cores, 1:single-threaded, *N*>1:use up to *N* threads
126	MTMODE	0	Multithreading mode (-1:LINGO chooses, 0:off in solver,1:prefer parallel, 2:parallel exclusively, 3:prefer concurrent, 4:concurrent exclusively)
127	BNPBLK	2	Branch-and-price (BNP) blocks (0:use row names, 1:user specified, 2:off, >2:max number of blocks)
128	BNPHEU	1	BNP block-finding heuristic (1:GP1, 2:GP2)
129	REPROD	0	Favor reproducibility (0:no, 1:yes)
130	STARTP	1.2345678	Default starting point for variables
131	HRDLMS	NONE	Multistart hurdle value (Use 'NONE' to disable)
132	SOLVLG	NONE	Solver log output level (0:None, >0 to enable log)

1. ILFTOL and 2. FLFTOL

Due to the finite precision available for floating point operations on digital computers, LINGO can't always satisfy each constraint exactly. Given this, LINGO uses these two tolerances as limits on the amount of violation allowed on a constraint while still considering it "satisfied". These two tolerances are referred to as the *initial linear feasibility tolerance* (*ILFTOL*) and the *final linear feasibility tolerance* (*FLFTOL*). The default values for these tolerances are, respectively, 0.000003 and 0.0000001.

ILFTOL is used when the solver first begins iterating. *ILFTOL* should be greater than *FLFTOL*. In the early stages of the solution process, being less concerned with accuracy can boost the performance of the solver. When LINGO thinks it has an optimal solution, it switches to the more restrictive *FLFTOL*. At this stage in the solution process, one wants a relatively high degree of accuracy. Thus, *FLFTOL* should be smaller than *ILFTOL*.

One instance where these tolerances can be of use is when LINGO returns a solution that is almost, but not quite, feasible. You can verify this by checking the values in the *Slack or Surplus* column in the model's solution report. If there are only a few rows with small, negative values in this column, then you have a solution that is close to being feasible. Loosening (i.e., increasing the values of) *ILFTOL* and *FLFTOL* may help you get a feasible solution. This is particularly true in a model where scaling is poor (i.e., very large and very small coefficients are used in the same model), and the units of measurement on some constraints are such that minor violations are insignificant. For instance, suppose you have a budget constraint measured in millions of dollars. In this case, a violation of a few

pennies would be of no consequence. Short of the preferred method of rescaling your model, loosening the feasibility tolerances may be the most expedient way around a problem of this nature.

3. INFTOL and 4. FNFTOL

The *initial nonlinear feasibility tolerance* (*INFTOL*) and the *final nonlinear feasibility tolerance* (*FNFTOL*) are both used by the nonlinear solver in the same manner the initial linear and final linear feasibility tolerances are used by the linear solver. For information on how and why these tolerances are useful, refer to the section immediately above. Default values for these tolerances are, respectively, 0.001 and 0.000001.

5. RELINT

RELINT, the *relative integrality tolerance,* is used by LINGO as a test for integrality in integer programming models. Due to round-off errors, the "integer" variables in a solution may not have values that are *precisely* integral. The relative integrality tolerance specifies the relative amount of violation from integrality that is acceptable. Specifically, if I is the closest integer value to X, X will be considered an integer if:

$$\frac{|X - I|}{|X|} <= \textit{Relative Integrality Tolerance}.$$

The default value for the relative integrality tolerance is .000008. Although one might be tempted to set this tolerance to 0, doing so may result in feasible models being reported as infeasible.

6. NOPTOL

While solving a model, the nonlinear solver is constantly computing a *gradient*. The gradient gives the rate of improvement of the objective function for small changes in the variables. If the gradient's rate of improvement computation for a given variable is less-than-or-equal-to *NOPTOL*, the *nonlinear optimality tolerance*, further adjustments to the variable's value are not considered to be beneficial. The default value for the nonlinear optimality tolerance is .0000001. Decreasing this tolerance towards a limit of 0 will tend to make the solver run longer and may lead to better solutions to poorly formulated or poorly scaled models.

7. ITRSLW

LINGO's nonlinear solver uses the *ITRSLW*, s*low progress iteration limit*, as a means of terminating the solution process if little or no progress is being made in the objective value. Specifically, if the objective function's value has not improved significantly in n iterations, where n is the value of *ITRSLW*, the nonlinear solver will terminate the solution process. Increasing this tolerance's value will tend to force the solver to run longer and may be useful in models that have relatively "flat" objective functions around the optimal solution. The default value for *ITRSLW* is 5 iterations. Refer to the description of *ITRLIM* below for a definition of iterations.

8. DERCMP

Use this parameter to set the style of derivative computation. Set *DERCMP* to 0 (*Solver Decides*) to allow LINGO to select the method, 1 for backward analytical derivatives, 2 for forward analytical derivatives, 3 for numerical derivatives using central differences, and 4 for numerical derivatives using forward differences.

LINGO defaults to the *Solver Decides* setting, which presently involves using backward analytical derivatives. However, we suggest you try the various derivative options to see which works best for your particular models.

9. ITRLIM

Use this tolerance to place an upper limit on the number of iterations the solver will perform. An *iteration* is the fundamental operation performed by the solver. At the risk of oversimplification, it is a process that involves forcing a variable, currently at a zero value, to become nonzero until some other variable is driven to zero, improving the objective as we go. In general, larger models will take longer to perform an iteration, and nonlinear models will take longer than linear models. The default iteration limit is 0, meaning no limit is imposed on the iteration count.

If the solver hits this limit, it returns to normal command mode. If the model contains integer variables, LINGO will restore the best integer solution found so far. You may need to be patient, however, because the solver may have to perform a fair amount of work to reinstall the current best solution after it hits a runtime limit.

Note: Some caution is required when interrupting the solver. There must be an incumbent solution available if you hope to interrupt the solver and have it return a valid solution. You can always tell if an incumbent solution is available by examining the Best Obj field in the Extended Solver Status box of the solver status window. If this field is blank, then an incumbent solution does not exist, and the solution returned after an interrupt will be invalid. If, on the other hand, this field contains a numeric value, then you should be able to interrupt and return to a valid, if not globally optimal, solution.

10. TIMLIM

Use this tolerance to place a limit on the number of seconds the solver runs. If the solver hits this limit, it will stop and return with the best solution found so far. The default limit is 0, meaning no time limit is imposed on the solver.

If the solver hits this limit, it returns to normal command mode. If the model contains integer variables, LINGO will restore the best integer solution found so far. You may need to be patient, however, because the solver may have to perform a fair amount of work to reinstall the current best solution after it hits a runtime limit.

Note: Some caution is required when interrupting the solver. There must be an incumbent solution available if you hope to interrupt the solver and have it return a valid solution. You can always tell if an incumbent solution is available by examining the Best Obj field in the Extended Solver Status box of the solver status window. If this field is blank, then an incumbent solution does not exist, and the solution returned after an interrupt will be invalid. If, on the other hand, this field contains a numeric value, then you should be able to interrupt and return to a valid, if not globally optimal, solution.

11. OBJCTS

LINGO generates twelve different types of constraint cuts when solving mixed integer linear programs. Using the options listed below, these various classes of cuts can be enabled by setting their parameter value to 1, or disabled by setting their parameter value to 0.

The available cut classes are as follows:

Index	Parameter Name	Cut Type
11	OBJCTS	Objective cuts
57	GUBCTS	GUB
58	FLWCTS	Flow
59	LFTCTS	Lift
60	PLOCTS	Plant location
61	DISCTS	Disaggregation
62	KNPCTS	Knapsack cover
63	LATCTS	Lattice
64	GOMCTS	Gomory
65	COFCTS	Coefficient reduction
66	GCDCTS	Greatest common divisor
76	BASCTS	Basis cuts

LINGO's integer programming solver performs extensive evaluation of your model in order to add *constraint cuts*. Constraint cuts are used to "cut" away sections of the feasible region of the continuous model (i.e., the model with integer restrictions dropped) that are not contained in the feasible region to the integer model.

On most integer models, this will accomplish two things. First, solutions to the continuous problem will tend to be more naturally integer. Thus, the branch-and-bound solver will have to branch on fewer variables. Secondly, the bounds derived from intermediate solutions will tend to be tighter, allowing the solver to "fathom" (i.e., drop from consideration) branches higher in the branch-and-bound tree. These improvements should dramatically speed solution times on most integer models. However, on some poorly formulated models, disabling one or more of the cut types may help in finding feasible solutions.

All constraint cut types are enabled by default.

12. MXMEMB

Use this parameter to set an upper limit on the amount of memory, in megabytes, that LINGO allocates as workspace for its model generator. When LINGO starts up, it sets aside a fixed amount of memory to use as a generator workspace. The default workspace size is 32Mb. You can determine the size of the current workspace and the amount of memory allotted in this workspace by issuing the *MEM* command.

Large models may run out of generator memory when attempting to solve them. In this case, you will receive the error message, “The model generator ran out of memory.” To avoid this error, increase the value of *MXMEMB* and issue the *FREEZE* command to preserve the change. You must then restart LINGO.

Note: Changes in LINGO’s generator memory limit are not established until you restart the program.

The model generator is distinct from the actual solver engines. Memory allocated to the generator will not be available to the solver engines. Thus, you shouldn't allocate any more memory to the generator than is required.

If you set *MXMEMB* to 0, LINGO will allocate all available memory when it starts up. This is not a recommended practice.

Note: Setting LINGO's generator memory limit abnormally high can result in poor performance of LINGO and the operating system. By setting aside excessive amounts of memory for the model generator, both LINGO and the operating system may have to resort to swapping of virtual memory to and from the hard drive. Accessing the hard drive for memory swaps can slow down your machine dramatically.

13. CUTAPP

Use this parameter to control the nodes in the solution tree where the branch-and-bound solver adds constraint cuts in linear integer models. You have the following three options:

CUTAPP Setting	Cuts Application at ...
0	Root only
1	All nodes
2	Solver decides

Under the *Root Only* option, the solver appends cuts only at the first node, or root node, in the solution tree. With the *All Nodes* option, cuts are appended at each node of the tree. Under the *Solver Decides* option, the solver dynamically decides when it is best to append cuts at a node.

The default is to let the solver decide when to append cuts. In general, this will offer superior performance. There may be instances, however, where one of the other two options prevails.

14. ABSINT

Use this parameter to specify an *absolute integrality* tolerance. This tolerance is used by LINGO as a test for integrality in integer programming models. Due to round-off errors, the "integer" variables in a solution may not have values that are *precisely* integer. The absolute integrality tolerance specifies the absolute amount of violation from integrality that is acceptable. Specifically, if X is an "integer" variable and I is the closest integer to X, then X would be accepted as being integer valued if:

$$|X - I| <= \textit{Absolute Integrality Tolerance}.$$

The default value for the absolute integrality tolerance is .000001. Although one might be tempted to set this tolerance to 0, this may result in feasible models being reported as infeasible.

15. HEURIS

Use this parameter to control the level of integer programming heuristics used by the integer solver. These heuristics use the continuous solution at each node in the branch-and-bound tree to attempt to quickly find a good integer solution. If an integer solution better than the incumbent is found, then it is used to fix or tighten global and local variable bounds. Heuristics are only applied to linear models. Requesting heuristics on nonlinear models will result in no benefits.

HEURIS may be set anywhere from 0 (none) to 100 (highest level), with 3 being the default.

16. HURDLE

If you know the objective value of a solution to a model, you can enter it as a *hurdle* tolerance. This value is used in the branch-and-bound solver to narrow the search for the optimum. More specifically, LINGO will only search for integer solutions where the objective is better than the hurdle value. This comes into play when LINGO is searching for an initial integer solution. LINGO can ignore branches in the search tree with objective values worse than the hurdle value, because a better solution exists (i.e., the hurdle) on some alternate branch. Depending on the problem, a good hurdle value can greatly reduce solution time. Once LINGO finds an initial integer solution, however, the Hurdle tolerance no longer has an effect.

Note: Be sure when entering a hurdle value that a solution exists that is at least as good or better than your hurdle. If such a solution does not exist, LINGO will not be able to find a feasible solution to the model.

The default hurdle value is *None*. In other words, a hurdle value is not used by the solver. To clear an existing hurdle value, type **`SET HURDLE NONE`**.

17. IPTOLA

Use this parameter to specify the *absolute optimality tolerance.* This tolerance is a positive value *r*, indicating to the branch-and-bound solver that it should only search for integer solutions with objective values at least *r* units better than the best integer solution found so far. In many integer programming models, there are huge numbers of branches with roughly equivalent potential. This tolerance can help to keep the branch-and-bound solver from being distracted by branches that can't offer a solution significantly better than the incumbent solution.

In general, you shouldn't have to set this tolerance. Occasionally, particularly on poorly formulated models, you might need to increase this tolerance slightly from zero to improve performance. In most cases, you should experiment with the *relative optimality tolerance* rather than the absolute optimality tolerance in order to improve performance.

The default value for the absolute optimality tolerance is 0.

18. IPTOLR

Use this parameter to specify the *relative optimality tolerance*. This tolerance is a value *r*, ranging from 0 to 1, indicating to the branch-and-bound solver that it should only search for integer solutions with objective values at least 100**r*% better than the best integer solution found so far.

The end results of modifying the search procedure in this way are twofold. First, on the positive side, solution times can be improved tremendously. Second, on the negative side, the final solution obtained by LINGO may not be the true optimal solution. You will, however, be guaranteed the solution is within 100**r*% of the true optimum.

Typical values for the relative optimality tolerance would be in the range .01 to .05. In other words, you would be happy to get a solution within 1% to 5% of the true optimal value. On larger integer models, the alternative of getting a solution within a few percentage points of the true optimum after several *minutes* of runtime, as opposed to the true optimum after several *days*, makes the use of an optimality tolerance quite attractive.

Note: Generally speaking, the relative integrality tolerance is the tolerance that will most likely improve runtimes on integer models. You should be sure to set this tolerance whenever possible.

The default for the relative optimality tolerance is 1e-5.

19. TIM2RL

If an integer programming model is relatively easy to solve, then we would like to have the solver press on to the true optimal solution without immediately resorting to a relative optimality tolerance (discussed above). On the other hand, if, after running for a while, it becomes apparent that the optimal solution won't be immediately forthcoming, then you might want the solver to switch to using a relative optimality tolerance. *TIM2RL,* the *time to relative* tolerance, can be used in this manner. This tolerance is the number of seconds before the branch-and-bound solver begins using the relative optimality tolerance. For the first *n* seconds, where *n* is the value of the time to relative tolerance, the branch-and-bound solver will not use the relative optimality tolerance and will attempt to find the true optimal solution to the model. Thereafter, the solver will use the relative optimality tolerance in its search.

The default value for the time to relative tolerance is 100 seconds.

20. NODESL

The branch-and-bound solver has a great deal of freedom in deciding how to span the branch-and-bound solution tree. *NODESL, t*he *node selection* option, allows you to control the order in which the solver selects branch nodes in the tree.

The four choices available for *NODESL* are as follows:

NODESL Setting	Branch Selection
0	*LINGO Decides* –This is the default option. LINGO makes an educated guess as to the best node to branch on.
1	*Depth First* – LINGO spans the branch-and-bound tree using a depth first strategy.
2	*Worst Bound* – LINGO picks the node with the worst bound.
3	*Best Bound* – LINGO picks the node with the best bound.

In general, *LINGO Decides* will offer the best results. Experimentation with the other options may be beneficial with some classes of models.

21. LENPAG

The *LENPAG* parameter sets the length of the page or screen size in lines. For instance, setting *LENPAG* to 25 will cause the display to pause after 25 lines and await a carriage return before displaying the next 25 lines. This is convenient when you wish to page through long reports and not have them scroll off the top of the screen.

When *LENPAG* is set to 0, paging is turned off entirely. LINGO will no longer stop output to wait for a carriage return. Entering *SET LENPAGE* 0 at the top of any command script is helpful in that you generally want command scripts to run uninterrupted.

22. LINLEN

When LINGO generates reports, it limits output lines to a certain width. In some reports, lines will be wrapped so that they fall within the line length limit. In other reports, lines may be truncated. Since LINGO concatenates variable names in performing set operations, a variable name such as *SHIPMENTS(WAREHOUSE1, CUSTOMER2)* may result, which may be truncated in a solution report if too narrow an output width is used. You can control this line width limit through the *LINLEN* parameter. You may set it anywhere between 64 and 200, with the default being 85.

23. TERSEO

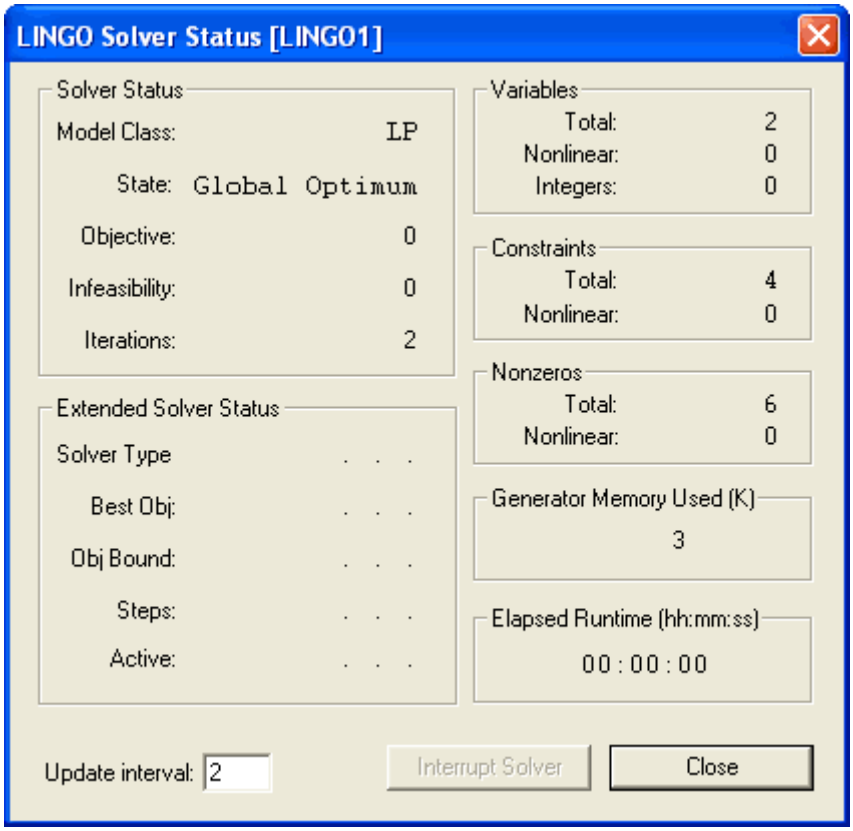
You can use the *TERSEO* parameter to control the amount of output LINGO generates. There are four settings available:

TERSEO	Description
0	*Verbose*—Causes LINGO to display the maximum amount of output, including full solution reports.
1	*Terse*—Less output than *Verbose,* with full solution reports suppressed. This is a good output level if you tend to solve large models. LINGO also suppresses Export Summary Reports generated when exporting data to spreadsheets or databases.
2	*Errors Only*—All output is suppressed, with the exception of error messages
3	*Nothing*—LINGO suppresses all output. This level may be useful when taking advantage of the programming capabilities in LINGO, in which case, you will add statements to your model to generate all required output.

The default setting for *TERSEO* is 0, or verbose mode.

24. STAWIN

If the *STAWIN* parameter is set to 1, LINGO displays a *solver status window* whenever you issue the *GO* command. This window resembles the following:



The solver status window is useful for monitoring the progress of the solver and the dimensions of your model. It is updated every *n* seconds, where *n* is the value in the *Update interval* field in the lower right corner of the window. LINGO defaults to displaying the solver status window.

For a detailed description of the various fields in the solver status window, see Chapter 1, *Getting Started with LINGO*.

25. SPLASH
If the *SPLASH* parameter is set to 1, LINGO will display its splash screen each time it starts up. The splash screen lists the release number of LINGO and the software's copyright notice. Setting *SPLASH* to 0 disables the splash screen. The default is for the splash screen to be displayed.

26. OROUTE
Set this parameter to 1 to send reports generated by LINGO to the command window, or 0 to send them to individual report windows. Since you can log all output to the command window in a log file, routing reports to the command window can be a useful way of logging all reports to disk. This may also be a desirable option when you are using LINGO as part of an automated system where you need LINGO to run without user input. The default is for LINGO to display reports in individual windows.

27. WNLINE and 28. WNTRIM
When LINGO sends output to the command window, it places new lines at the bottom of the window. All previous output is scrolled up to make way for the new output. The total number of output lines that can be stored in the command window is limited. When LINGO hits this limit, it begins deleting lines from the top of the command window. You can control this feature by setting the *WNLINE* and *WNTRIM* parameters.

The *WNLINE* parameter sets the maximum number of lines allowed in the command window. When LINGO removes lines from the top of the command window, it stops once there are *n* lines left in the command window, where *n* is the value of the *WNTRIM* parameter. In general, output to the command window will become slower as you increase the maximum and minimum line counts.

The default values for *WNLINE* and *WNTRIM* are, respectively, 800 and 400. Minimum values are 200 and 100, while there are no upper limits.

29. STABAR
If the *STABAR* parameter is set to 1, LINGO for Windows displays a status bar along the bottom of the main frame window. Among other things, the status bar displays the time of day, location of the cursor, menu tips, and the current status of the program.

To remove the status bar from the screen, set *STABAR* to 0.

The default is for LINGO to display the status bar.

30. FILFMT
Use *FILFMT* to set the default file format LINGO uses when opening a new document. The options are:

FILFMT	File Type	Description
0	LNG	LINGO text
1	LG4	LINGO binary
2	LTX	LINDO text

The LG4 format is the default file format for Windows versions of LINGO. This is a binary format that is readable only by LINGO. This format enables you to have custom formatting and fonts in your models, and allows you to use LINGO as an OLE server and container. Files written in LG4 format are useful only on Windows hardware.

The LNG and LTX formats are text based. Given this, LNG and LTX files may be read into other applications. However, these formats don't support custom formatting and embedded objects. In general, LNG files use LINGO syntax, while LTX files use LINDO syntax.

31. TOOLBR

In Windows versions, LINGO can display a row of buttons that act as shortcuts to various commands contained in the LINGO menu. This row of buttons is known as the *toolbar*. Set *TOOLBR* to 1 to display the toolbar or 0 to remove it. The default is for LINGO to display the toolbar.

32. CHKDUP

Prior to release 4.0, LINGO allowed you to use primitive set names in the equations of a model. Primitive set names in a model's equations returned the index of the set member. Starting with release 4.0, LINGO required you to use the *@INDEX* function (see Chapter 7, *LINGO's Operators and Functions*) to get the index of a primitive set member. If you would like to test your LINGO models from releases prior to 4.0 for instances where primitive set members appear in the model's equations, set *CHKDUP* to 1. Whenever you run a model, LINGO will issue an error message if duplicate names appear as set members and as variables in the model.

33. ECHOIN

When you run a LINGO command script with the *TAKE* command, the commands LINGO processes are normally not displayed. If you would like the commands echoed to your screen, set the *ECHOIN* parameter to 1. This can be a useful feature when you are trying to develop and debug a LINGO command script.

34. ERRDLG

Set the *ERRDLG* parameter to 1 and LINGO will display error messages issued by the solver in a modal dialog box. This dialog box must be cleared before LINGO proceeds with any other operation. In some instances, you may have LINGO embedded in other applications, where it may not be desirable, or possible, to have users clearing the error dialog boxes. By setting *ERRDLG* to 0, LINGO will route the solver's error messages to the report window, where they will be displayed and no user intervention will be required to clear the messages. Note that this option allows you to route only those error messages generated by LINGO's solver to the report window. Error messages displayed by LINGO's interactive front-end (error codes 1000 and above) will always be posted in dialog boxes. The default is for solver errors to be displayed in dialog boxes.

35. USEPNM

In many instances, you will need to get the index of a primitive set member within its set. Prior to release 4 of LINGO, you could do this by using the primitive set member's name directly in the model's equations. This can create problems when you are importing set members from an external source. In this case, you will not necessarily know the names of the set members beforehand. When one of the imported primitive set members happens to have the same name as a variable in your model, unintended results can occur. More specifically, LINGO would not treat the variable as optimizable. In fact, LINGO would treat it as if it were a constant equal to the value of the index of the primitive set member! In short, different primitive set names could potentially lead to different results. Therefore, starting with release 4.0 of LINGO, models such as the following are no longer permitted:

```
MODEL:
SETS:
   DAYS /MO TU WE TH FR SA SU/;
ENDSETS

   INDEX_OF_FRIDAY = FR;
END
```

If you want the index of *FR* in the *DAYS* set, you should use the *@INDEX* function (see Chapter 7, *LINGO's Operators and Functions*):

```
INDEX_OF_FRIDAY = @INDEX(DAYS, FR);
```

If you are unable to update your models for some reason and you would like to allow for the direct use of primitive set names, you can enable the *USEPNM* parameter by setting it to 1. The default is for LINGO to disable *USEPNM*.

36. NSTEEP

Setting the *NSTEEP* parameter to 1 causes LINGO's nonlinear solver to use steepest-edge variable selection. When LINGO is not in steepest-edge mode, the nonlinear solver will tend to select variables that offer the highest *absolute* rate of improvement to the objective, regardless of how far other variables may have to move per unit of movement in the newly introduced variable. The problem with this strategy is that other variables may quickly hit a bound, resulting in little gain to the objective. With the steepest-edge option, the nonlinear solver spends a little more time in selecting variables by looking at what rate the objective will improve *relative* to movements in the other nonzero variables. Thus, on average, each iteration will lead to larger gains in the objective. In general, the steepest-edge option will result in fewer iterations. However, each iteration will take longer. LINGO defaults to not using the steepest-edge option.

37. NCRASH

LINGO has two solution "crashing" methods that use different heuristics for generating starting points for nonlinear models: *NCRASH* and *LCRASH*. If at least one of the two crash parameters is set to 1, LINGO's nonlinear solver will invoke a heuristic for generating a "good" starting point when you solve a model. If this initial point is relatively good, subsequent solver iterations should be reduced along with overall runtimes. It's possible, however, that runtimes may slow if the selected point turns out to be poor. It is difficult to know beforehand if a particular crashing technique, if any, will help performance. You will need to experiment with your model to determine the best settings for the two crash procedures.

LINGO defaults to not crashing an initial solution.

38. NSLPDR

If you set *NSLPDR* to 1, LINGO's nonlinear solver will use successive linear programming (SLP) to compute new search directions. This technique uses a linear approximation in search computations in order to speed iteration times. In general, the number of total iterations will tend to rise when SLP directions are used, but on some models overall runtimes may improve. LINGO defaults to using SLP directions.

39. SELCON

If you set *SELCON* to 1, LINGO's nonlinear solver will only evaluate constraints on an as needed basis. Thus, not every constraint will be evaluated at each iteration. This generally leads to faster solution times, but can also lead to problems in models with undefined functions in certain regions. LINGO may not evaluate a constraint for many iterations only to find that it has moved into a region where the constraint is no longer defined. In this case, there may not be a valid point for the solver to retreat to and the solution process terminates with an error. Turning off selective constraint evaluation eliminates these errors. LINGO defaults to not using selective constraint evaluation.

40. PRBLVL

On a mixed-integer linear program, LINGO can perform an operation known as *probing*. Probing involves taking a close look at the integer variables in a model and deducing tighter variable bounds and right-hand side values. In many cases, probing can tighten an integer model sufficiently, thereby speed overall solution times. In other cases, however, probing may not be able to do much tightening and the overall solution time will increase due to the extra time spent probing. You can choose from seven successive levels of probing ranging from 1 to 7. Level 1 disables probing completely, while level 7 involves the highest degree of probing. Setting this option to 0 lets LINGO select the level of probing. LINGO defaults to 0.

41. SOLVEL

This option allows you to choose the type of algorithm invoked by LINGO's linear solver. At present, LINGO offers the following four options:

SOLVEL Value	Linear Solver Algorithm
0	LINGO chooses
1	Primal simplex
2	Dual simplex
3	Barrier (only available as an option)

In general, it is difficult to say what algorithm will be fastest for a particular model. A rough guideline is that primal simplex tends to do better on sparse models with fewer rows than columns; the dual does well on sparse models with fewer columns than rows; and the barrier works best on densely structured models or very large models.

The barrier solver is available only as an additional option to the LINGO package.

LINGO defaults to 0, *LINGO chooses*.

42. REDUCE
When this parameter is set to 1, LINGO's linear solver tries to identify and remove extraneous variables and constraints from the formulation before solving. In certain cases, this can greatly reduce the size of the final model to be solved. Setting *REDUCE* to 1 enables reduction, while 0 disables it. Setting *REDUCE* to 2 allows LINGO to choose whether or not to enable reduction. LINGO defaults to this last option.

43. SCALEM
Setting *SCALEM* to 1 enables the scaling option in LINGO's linear solver. This option rescales the coefficients in the model's matrix, causing the ratio of the largest to smallest coefficients to be reduced. By doing this, LINGO reduces the chances of round-off error, which leads to greater numerical stability and accuracy in the linear solver.

LINGO defaults to using scaling.

44. PRIMPR
Setting this parameter to 2 causes LINGO's primal simplex linear solver to use devex pricing techniques. If this parameter is set to 1, the primal simplex solver will use partial pricing. If this parameter is set to 0, LINGO chooses the primal simplex pricing method.

LINGO defaults to choosing the primal pricing method.

45. DUALPR
If *DUALPR* is set to 2, LINGO's dual simplex solver will use steepest edge pricing. If *DUALPR* is 1, the dual solver will use Dantzig pricing methods. If *DUALPR* is 0, LINGO chooses the most appropriate pricing method.

In Dantzig pricing mode, the dual simplex solver will tend to select variables that offer the highest *absolute* rate of improvement to the objective, regardless of how far other variables may have to move per unit of movement in the newly introduced variable. The problem with this strategy is that other variables may quickly hit a bound, resulting in little gain to the objective. With the steepest-edge option, the solver spends a little more time selecting variables by looking at the total improvement in the objective by adjusting a particular variable. Thus, on average, each iteration will lead to larger gains in the objective. In general, the steepest-edge option will result in fewer iterations. However, each iteration will take longer.

LINGO defaults to choosing the pricing method for the dual solver.

46. DUALCO
The *DUALCO* parameter is used to set the level of dual computations performed by the solver. Setting *DUALCO* to 0 will cause LINGO to *not* compute dual values and ranges. This is the fastest option, but is suitable only if you don't need this information. In fact, the *RANGE* command will not execute when *DUALCO* is 0. When *DUALCO* is 1, LINGO will compute dual values, but not ranges. When

DUALCO is 2, LINGO computes both dual prices and ranges. Setting *DUALCO* to 3 causes LINGO to compute the dual values on optimizable rows only (i.e., fixed rows are excluded) and forgo range computations, LINGO defaults to a *DUALCO* value of 1.

Note: Range computations can take some time, so, if speed is a concern, you don't want to enable range computations unnecessarily.

Note: The barrier crossover option must be enabled if you plan to do range analysis. Range computations cannot be performed if the final solution is not a basic solution.

47. RCMPSN

Setting *RCMPSN* to 1 causes LINGO to convert all variable and row names to RC notation when performing MPS file format I/O. Refer to the *RMPS* command on page 330 for a discussion of why this option is useful. By default, LINGO disables the use of RC format names.

48. MREGEN

The *MREGEN* parameter controls the frequency with which LINGO regenerates a model. With *MREGEN* set to 0, LINGO regenerates a model only when a change has been made to the model's text since the last generation took place. When *MREGEN* is 1, LINGO regenerates whenever a change is made to the model text or if it contains references to external data sources (e.g., text files, databases, or spreadsheets). If *MREGEN* is 2, then LINGO *always* regenerates the model each time information regarding the generated model is needed. Commands that will trigger a model generation are *GO*, *GEN*, *GENL*, *STATS*, *RMPS*, *FRMPS*, *SMPS*, and *PICTURE*. LINGO defaults to a *MREGEN* value of 2.

49. BRANDR

LINGO uses a branch-and-bound solution procedure when solving integer programming models. One of the fundamental operations involved in the branch-and-bound algorithm is *branching* on variables. Branching involves forcing an integer variable that is currently fractional to either the next greatest integer value or to the next lowest integer value. As an example, suppose there is a general integer variable that currently has a value of 5.6. If LINGO were to branch on this variable, it would have to choose whether to set the variable first to 6 or 5. The *BRANDR* parameter controls how LINGO makes this branching decision.

There are three possible settings for *BRANDR*:

BRANDR Value	Preferred Branching Direction
0	Both up and down
1	Up
2	Down

The default option, *Both up and down*, involves LINGO making an intelligent guess as to whether it should branch up or down first on each individual variable. If the *Up* option is selected, LINGO will always branch up to the next highest integer first. If *Down* is selected, LINGO will always branch down first. In most cases, the *Both up and down* option will result in the best performance. Occasionally, models will benefit from use of one of the other two options.

50. BRANPR

When branching on variables, the branch-and-bound procedure can give priority to branching on the binary variables first, or it can make an intelligent guess as to the next best variable to branch on, regardless of whether it is binary or general.

There are two possible settings for *BRANPR*:

BRANPR Value	Branching Priority
0	LINGO decides
1	Binary variables first

Select the *Binary variables first* option to have LINGO give branching priority to the binary variables. Select *LINGO Decides* to have LINGO select the next integer variable for branching based on an intelligent guess regardless of whether it is binary or general. The default for this option is *LINGO Decides,* which should generally give the best results. However, on occasion, the *Binary* option may prevail.

51. CUTOFF

On occasion, due to round-off error, some of the values returned by LINGO's solver will be very small (less than 1e-10). In reality, the true values of these variables are either zero or so small as to be of no consequence. These tiny values can be distracting when interpreting a solution report. The *CUTOFF* parameter can be used to suppress small solution values. Any solution value less-than-or-equal-to *CUTOFF* will be reported as being zero. The default value for *CUTOFF* is 1e-9.

52. STRONG

The *strong branch* option uses a more intensive branching strategy during the first *n* levels of the branch-and-bound tree, where *n* is the value of the *STRONG* parameter. During these initial levels, LINGO picks a subset of the fractional variables as branching candidates. LINGO then performs a tentative branch on each variable in the subset, selecting as the final candidate the variable that offers the greatest improvement in the bound on the objective. Although strong branching is useful in tightening the bound quickly, it does take additional computation time. So, you may want to try different settings to determine what works best for your model.

The default setting is 10 levels.

53. REOPTB

The *warm start* option controls the linear solver that is used by the branch-and-bound solver at each node of the solution tree when a previous solution is present to use as a "warm start". The *cold start* option, discussed below, determines the solver to use when a previous solution *does not exist.*

There are four possible settings for *REOPTB*:

REOPTB Value	Warm Start Solver
0	*LINGO Decides* – LINGO chooses the most appropriate solver.
1	*Primal* – The primal solver will be used exclusively.
2	*Dual* – The dual solver will be used exclusively.
3	*Barrier* – LINGO uses the barrier method, assuming you have purchased a license for the barrier solver. Otherwise, the dual solver will be used.

In general, *LINGO Decides* will yield the best results. The barrier solver can't make use of a pre-existing solution, so *Barrier* usually won't give good results. In general, *Dual* will be faster than *Primal* for reoptimization in branch-and-bound.

54. REOPTX

The *cold start* option controls the linear solver that is used by the branch-and-bound solver at each node of the solution tree when a previous solution is *not* present to use as a "warm start". The *warm start* option, discussed above, determines the solver to use when a previous solution *does exist.*

There are four possible settings for *REOPTX*:

REOPTX Value	Warm Start Solver
0	*LINGO Decides* – LINGO chooses the most appropriate solver.
1	*Primal* – The primal solver will be used exclusively.
2	*Dual* – The dual solver will be used exclusively.
3	*Barrier* – LINGO uses the barrier method, assuming you have purchased a license for the barrier solver. Otherwise, the dual solver will be used.

In general, *LINGO Decides* will yield the best results. However, experimentation with the other options may be fruitful.

55. MAXCTP

The integer pre-solver makes iterative passes through a model determining appropriate constraint cuts to append to the formulation. In general, the marginal benefits of each additional pass declines. At some point, additional passes will only add to total solution times. Thus, LINGO imposes a limit on the maximum number of passes.

LINGO applies constraint cuts at both the top, or root, node of the branch-and-bound tree, and at all subsequent nodes within the tree. The *MAXCTP* parameter limits the maximum number of cuts at the top node, while the *MAXCTR* parameter sets the cut limit on all subsequent nodes in the tree. The default limit is 100 passes.

56. RCTLIM
Most integer programming models benefit from the addition of some constraint cuts. However, at some point, additional cuts take more time to generate than they save in solution time. For this reason, LINGO imposes a relative limit on the number of constraint cuts that are generated. The default limit is set to .75 times the number of true constraints in the original formulation. You may override this relative limit by changing the setting of *RCTLIM*.

Constraint Cut Types
LINGO generates twelve different types of constraint cuts when solving mixed integer linear programs. Using options listed below, these various classes of cuts can be enabled by setting their parameter value to 1, or disabled by setting their parameter value to 0.

The available cut classes are as follows:

Index	Parameter Name	Cut Type
11	OBJCTS	Objective cuts
57	GUBCTS	GUB
58	FLWCTS	Flow
59	LFTCTS	Lift
60	PLOCTS	Plant location
61	DISCTS	Disaggregation
62	KNPCTS	Knapsack cover
63	LATCTS	Lattice
64	GOMCTS	Gomory
65	COFCTS	Coefficient reduction
66	GCDCTS	Greatest common divisor
76	BASCTS	Basis cuts

LINGO's integer programming solver performs extensive evaluation of your model in order to add *constraint cuts*. Constraint cuts are used to "cut" away sections of the feasible region of the continuous model (i.e., the model with integer restrictions dropped) that are not contained in the feasible region to the integer model.

On most integer models, this will accomplish two things. First, solutions to the continuous problem will tend to be more naturally integer. Thus, the branch-and-bound solver will have to branch on fewer variables. Secondly, the bounds derived from intermediate solutions will tend to be tighter, allowing the solver to "fathom" (i.e., drop from consideration) branches higher in the branch-and-bound tree. These improvements should dramatically speed solution times on most integer models. However, on some poorly formulated models, disabling one or more of the cut types may help in finding feasible solutions.

All constraint cut types are enabled by default.

67. SCLRLM
The LINGO editor in Windows is "syntax aware." In other words, when it encounters LINGO keywords, it displays them in blue. Comments are displayed in green, and all remaining text is displayed in black. Syntax coloring can take a long time if you have very large files. The *SCLRLM* parameter sets the maximum acceptable file size for syntax coloring. Files with line counts exceeding this parameter will not be syntax colored. Setting this parameter to 0 will disable the syntax coloring feature. The default limit is 1000 lines.

68. SCLRDL
The LINGO editor in Windows is "syntax aware". In other words, when it encounters LINGO keywords it displays them in blue. Comments are displayed in green, and all remaining text is displayed in black. The *SCLRDL* parameter sets the number of seconds LINGO waits after the last keystroke was typed before recoloring modified text. Users on slower machines may want to set this higher to avoid having syntax coloring interfere with typing. Users on faster machines may want to decrease this value, so text is recolored more quickly. The default is 0 seconds (i.e., LINGO recolors modified text immediately).

69. PRNCLR
The LINGO editor in Windows displays matching parentheses in red when you place the cursor immediately following a parenthesis. The *PRNCLR* parameter allows you to disable this feature. Setting *PRNCLR* to 0 will disable parenthesis matching, while setting it to 1 will enable it.

70. MULTIS
LINGO exploits the convex nature of linear models to find globally optimal solutions. However, we aren't as fortunate with nonlinear models. With nonlinear programming (NLP) models, LINGO's default NLP solver uses a local search procedure. This can lead to LINGO stopping at locally optimal points, perhaps missing a global point lying elsewhere. Refer to Chapter 15, *On Mathematical Modeling*, for more information on how and why this can happen.

A strategy that has proven successful in overcoming this problem is to restart the NLP solver several times from different initial points. It is not uncommon for a different starting point to lead to a different local solution point. The idea is that, if we restart from enough unique points, saving the best local solution as we go, then we have a much better chance of finding the true global solution.

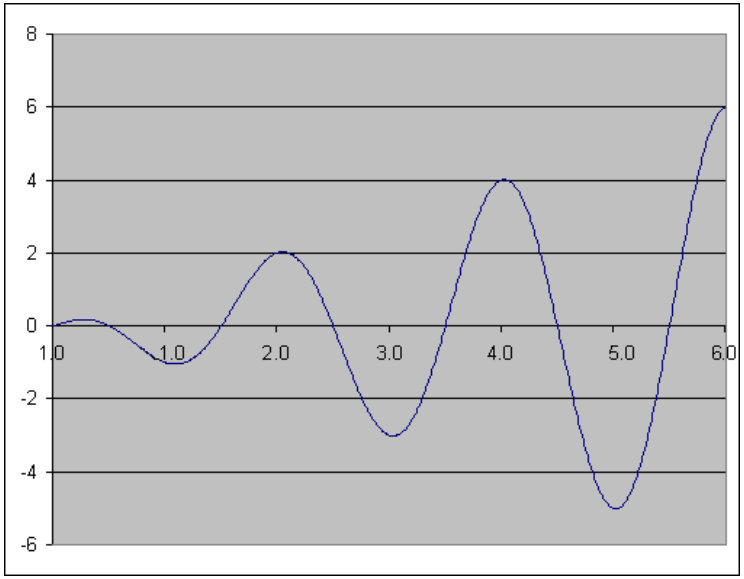
The *MULTIS* parameter allows you to set the number of times you would like the NLP solver to re-solve your model, starting each time from an intelligently generated, new starting point. We refer to this feature as *multistart.* The default value for *MULTIS*, 0, entails restarting 5 times on small NLPs and disabling multistart on larger models. Setting *MULTIS* to 1 disables multistart on all NLPs. Setting *MULTIS* to any value greater than 1 will cause the NLP solver to restart that number of times on all NLPs. We have found that setting *MULTIS* around 5 tends to be adequate for most models. Highly nonlinear models may require a larger setting.

Keep in mind, however, that multistart will dramatically increase runtimes. Thus, one should avoid using it unnecessarily on convex models that will converge to a global point in a single pass without any additional prodding.

The following example illustrates the usefulness of multistart. Consider the simple, yet highly nonlinear, model:

```
MODEL:
   MIN = X * @COS( 3.1416 * X);
   @BND( 0, X, 6);
END
```

The graph of the objective function is as follows:



The objective function has three local, minimal points over the feasible range. These points are summarized in the following table:

Point	X	Objective
1	1.09	-1.05
2	3.03	-3.02
3	5.02	-5.01

Clearly, the third local point is also the globally best point, and we would like the NLP solver to converge to this point. Below, we attempt this by loading the model, turning off the multistart option, and then solving:

```
: take wavy.lng
: look all

 MODEL:
   1]   MIN = X * @COS( 3.1416 * X);
   2]   @BND( 0, X, 6);
 END

: set multis 1 !set solver attempts to 1 only (i.e., disable ms)

  Parameter        Old Value     New Value
   MULTIS             0             1

: go
   Local optimal solution found at step:             11
   Objective value:                           -1.046719

           Variable           Value        Reduced Cost
                  X        1.090405           0.1181082E-07

                Row    Slack or Surplus      Dual Price
                  1       -1.046719           -1.000000
```

Unfortunately, as you can see, we converged to the least preferable of the local minimums. Below, we will do the same as in the previous run. However, this time, we will set the number of multistarts to five:

```
: take wavy.lng
: look all

 MODEL:
   1]   MIN = X * @COS( 3.1416 * X);
   2]   @BND( 0, X, 6);
 END

: set multis 5

  Parameter        Old Value     New Value
   MULTIS             0             5

: go
   Local optimal solution found at step:             39
   Objective value:                           -5.010083

           Variable           Value        Reduced Cost
                  X        5.020143          -0.7076917E-08

                Row    Slack or Surplus      Dual Price
                  1       -5.010083           -1.000000
```

The extra four restarts allowed LINGO to find the global optimal point.

71. USEQPR

The *USEQPR* parameter controls the *Quadratic Recognition* option. This option consists of an algebraic preprocessor that automatically determines if an arbitrary nonlinear model is actually a quadratic programming (QP) model. If a model is found to be a convex QP, then it can be passed to the faster quadratic solver. Note that the QP solver is not included with the base version of LINGO, but comes as part of the Barrier option.

LINGO defaults to using quadratic recognition

72. GLOBAL

Many nonlinear models are non-convex and/or non-smooth (for more information see Chapter 15, *On Mathematical Modeling*). Nonlinear solvers that rely on local search procedures, as does LINGO's default nonlinear solver, will tend to do poorly on these types of models. Typically, they will converge to a local, sub-optimal point that may be quite distant from the true, global optimal point. Global solvers overcome this weakness through methods of range bounding (e.g., interval analysis and convex analysis) and range reduction techniques (e.g., linear programming and constraint propagation) within a branch-and-bound framework to find the global solutions to non-convex models. LINGO has a global solver capability that is enabled through the *GLOBAL* parameter. Setting *GLOBAL* to 1 will enable the global solver on nonlinear models, while setting it to 0 (the default) will not.

The following example illustrates the power of the global solver on a non-smooth model. Consider the following model:

```
model:

sets:
   projects: baths, sqft, beds, cost, est;
endsets

data:
projects,     beds,  baths, sqft,         cost =
p1            5      4      6200          559608
p2            2      1      820           151826
p3            1      1      710           125943
p4            4      3      4300          420801
p5            4      2      3800          374751
p6            3      1      2200          251674
p7            3      2      3400          332426
;
enddata

min = @max( projects: @abs( cost - est));

@for( projects:
 est = a0 + a1 * beds + a2 * baths + a3 * sqft
);

end
```

Model: COSTING

This model estimates the cost of home construction jobs based on historical data on the number of bedrooms, bathrooms, and square footage. The objective minimizes the maximum error over the sample project set. Both the *@MAX()* and *@ABS()* functions in the objective are non-smooth, and, as a result, can present problems for LINGO's default, local search NLP solver. Running the model under the default settings with the global solver disabled, we get the following result:

```
Local optimal solution found at step:              91
Objective value:                              3997.347

      Variable           Value        Reduced Cost
            A0        37441.55            0.000000
            A1        27234.51            0.000000
            A2        23416.53            0.000000
            A3        47.77956            0.000000
```

Enabling the global solver with the **SET GLOBAL 1** command and re-optimizing yields the substantially better solution:

```
Global optimal solution found at step:            186
Objective value:                              1426.660

      Variable           Value        Reduced Cost
            A0        46814.64            0.000000
            A1        22824.18            0.000000
            A2        16717.33            0.000000
            A3        53.74674            0.000000
```

Note that the maximum error has been reduced from 3,997 to 1,426!

This example illustrates the power of the global solver. Unfortunately, there is a drawback. You will find the global solver runs considerably slower than the default local solver, and may be swamped when trying to solve larger models. Therefore, the preferred option is to always try to create smooth, convex models, so that the faster, default local solver can successfully solve them.

Keep in mind that the global solver supports most, but not all, of the functions available in the LINGO language. The following is a list of the nonlinear functions ***not*** currently supported by the global solver:

- *All probability distributions* — cumulative, inverse and pdf, with the exception of the normal distribution, which is fully supported
- *@PFS()* — Poisson finite source
- *@PPL()* — Poisson linear loss
- *@USER( )*— User supplied function

Note: The global solver will not operate on models containing one or more unsupported nonlinear operations that reference optimizable quantities; the default NLP solver will be called in this case.

The global solver is disabled by default.

73-75. LNRISE, LNBIGM, LNDLTA

The *LNRISE, LNBIGM*, and *LNDLTA* parameters control the *linearization* option in LINGO. Many nonlinear operations can be replaced by linear operations that are mathematically equivalent. The ultimate goal is to replace all the nonlinear operations in a model with equivalent linear ones, thereby allowing use of the faster and more robust linear solvers. We refer to this process as *linearization*. The *LNRISE* parameter determines the extent to which LINGO will attempt to linearize models. The available options are:

LNRISE Setting	Linearization Level
0	Solver Decides
1	None
2	Math
3	Math and Logic

Under the *None* option, no linearization occurs. With the *Math* option, LINGO linearizes the mathematical functions: *@ABS()*, *@MAX()*, *@MIN(), @SMAX()*, and *@SMIN()* along with any products of binary and continuous variables. The *Math and Logic*option is equivalent to the *Math* option, plus LINGO will linearize all logical operators (*#LT#, #LE#, #EQ#, #GT#, #GE#*, and *#NE#*). Under the *Solver Decides* option, LINGO will do maximum linearization if the number of variables doesn't exceed 12. Otherwise, LINGO will not perform any linearization. LINGO defaults to the *Solver Decides* setting.

The *LNDLTA* parameter controls the *Delta Coefficient*, which is a tolerance indicating how closely you want the additional constraints added as part of linearization to be satisfied. Most models won't require any changes to this parameter. However, some numerically challenging formulations may benefit from increasing *Delta* slightly. LINGO defaults to a *Delta* of 1.e-6.

When LINGO linearizes a model, it adds *forcing constraints* to the mathematical program generated to optimize your model. These forcing constraints are of the form:

$$f(x) \leq M \bullet y$$

where M is the *BigM Coefficient* and y is a 0/1 variable. The idea is that, if some activity in the variables is occurring, then the forcing constraint will drive y to take on the value of 1. Given this, if we set the *BigM* value to be too small, we may end up with an infeasible model. Therefore, the astute reader might conclude that it would be smart to make BigM quite large, thereby minimizing the chance of an infeasible model. Unfortunately, setting BigM to a large number can lead to numerical stability problems in the solver resulting in infeasible or sub-optimal solutions. So, getting a good value for the BigM Coefficient may take some experimentation. The default value for BigM is 100,000.

As an example of linearization, consider the following model:

```
model:

sets:
   projects: baths, sqft, beds, cost, est;
endsets

data:
projects,     beds,  baths, sqft,         cost =
p1            5      4      6200          559608
p2            2      1      820           151826
p3            1      1      710           125943
p4            4      3      4300          420801
p5            4      2      3800          374751
p6            3      1      2200          251674
p7            3      2      3400          332426
;
enddata

min = @max( projects: @abs( cost - est));

@for( projects:
 est = a0 + a1 * beds + a2 * baths + a3 * sqft
);

end
```

Model: COSTING

This model estimates the cost of home construction jobs based on historical data on the number of bedrooms, bathrooms, and square footage. The objective minimizes the maximum error over the sample project set. Both the *@MAX()* and *@ABS()* functions in the objective are non-smooth nonlinear functions, and, as a result, can present problems for LINGO's default, local search NLP solver. Running the model under the default settings with linearization disabled, we get the following result:

```
   Local optimal solution found at step:              91
   Objective value:                             3997.347

         Variable           Value        Reduced Cost
               A0        37441.55            0.000000
               A1        27234.51            0.000000
               A2        23416.53            0.000000
               A3        47.77956            0.000000
```

Enabling linearization and re-optimizing yields the substantially better solution:

```
   Global optimal solution found at step:            186
   Objective value:                             1426.660

         Variable           Value        Reduced Cost
               A0        46814.64            0.000000
               A1        22824.18            0.000000
               A2        16717.33            0.000000
               A3        53.74674            0.000000
```

Note that the maximum error has been reduced from 3,997 to 1,426!

Linearization will substantially increase the size of your model. The sample model above, in un-linearized form, has a mere 8 rows and 11 continuous variables. On the other hand, the linearized version has *51 rows*, *33 continuous variables*, and *14 binary variables*! Although linearization will cause your model to grow in size, you will tend to get much better solution results if the model can be converted entirely to an equivalent linear form.

Note: Linearization will be of most use when a nonlinear model can be 100% linearized. If LINGO can only linearize a portion of your model, then you may actually end up with a *more difficult* nonlinear model.

The linearization option is set to *Solver Decides* by default.

76. BASCTS
Please refer to the *Constraint Cut Types* section above for information on this parameter.

77. MAXCTR
This parameter controls the number of passes the branch-and-bound solver makes at each node of the tree for cut generation. There is one exception in that *MAXCTR* does not control the number of passes at the root node of the tree. You must use *MAXCTP*, see above, to control the number of passes at the root node. The default value for *MAXCTR* is 2 passes.

78. HUMNTM
This parameter sets the minimum amount of time spent in heuristics at each node of the branch-and-bound tree. The default value for HUMNTM is 0 seconds.

79. DECOMP
Many large scale linear and mixed integer problems have constraint matrices that are totally decomposable into a series of block structures. If total decomposition is possible, LINGO can solve the independent problems sequentially and report a solution for the original model, resulting in dramatic speed improvements. Setting *DECOMP* to 1 enables the decomposition feature.

LINGO defaults to *not* using matrix decomposition.

80. GLBOPT
The *GLBOPT* tolerance specifies by how much a new solution must beat the objective value of the incumbent solution in order to become the new incumbent in the global solver. The default value for *GLBOPT* is 1. e-5.

81. GLBDLT
The *GLBDLT* tolerance specifies how closely the additional constraints, added as part of the global solver's convexification process, must be satisfied. The default value for *GLBDLT* is 1. e-7.

82. GLBVBD
The *GLBVBD* tolerance sets the default variable bounds while the global solver is running. If this parameter is set to *d*, then variables will not be permitted to assume values outside the range of [-*d*, *d*]. Setting this parameter as tightly as possible in the *Value Field* restricts the global solver from straying into uninteresting regions and will reduce run times. You may also need to set the *GLBUBD* tolerance (see below) to control how the global solver uses the bound. The default value for *GLBVBD* is 1. e +10.

83. GLBUBD
The *GLBUBD* tolerance controls how the global solver's variable bound tolerance, *GLBVBD* (see above), is applied. There are three choices available: 0:*None*, 1:*All*, and 2:*Selected*. Selecting *None* removes the variable bound entirely and is not recommended. The *All* setting applies the bound to all variables. Finally, the *Selected* setting causes the global solver to apply the bound after an initial solver pass to find the first local solution. The bound will only be applied to a variable if it does not cut off the initial local solution. LINGO defaults to the *Selected* setting.

84. GLBBRN
The *GLBBRN* tolerance specifies the branching direction for variables when the global solver initially branches on them. Six options are available:

GLBBRN Setting	Branching Direction
0	*Absolute Width*
1	*Local Width*
2	*Global Width*
3	*Global Distance*
4	*Absolute Violation*
5	*Relative Violation*

The default setting for branching is 5, or *Relative Violation*.

85. GLBBXS
The *GLBBXS* parameter specifies the strategy to use for choosing between all active nodes in the global solver's branch-and-bound tree. The choices are: 0:*Depth First* and 1:*Worst Bound*. The default is 1, or *Worst Bound*.

86. GLBREF
The *GLBREF* option sets the degree of algebraic reformulation performed by the global solver. Algebraic reformulation is critical for construction of tight, convex sub-regions to enclose the nonlinear and nonconvex functions. The available settings are: 0:*None*, 1:*Low*, 2:*Medium*, and 3:*High*. The default is 3, or *High*.

87. SUBOUT
The *SUBOUT* option is used to control the degree to which fixed variables are substituted out of the ultimate math program passed to the solver engines.

For example, consider the model:

```
MAX= 20*X + 30*Y + 12*Z;
X = 2*Y;
X + Y + Z <= 110;
Y = 30;
```

If we run the *GEN* command, we see that LINGO is able to reduce this model down to the equivalent, but smaller model:

```
MAX= 12 * Z + 2100;
Z <= 20;
```

From the third constraint of the original model it is obvious that *Y* is fixed at the value 30. Plugging this value for *Y* into the first constraint, we can conclude that *X* has a value of 60. Substituting these two fixed variables out of the original formulation yields the reduced formulation above.

In most cases, substituting out fixed variables yields a smaller, more manageable model. In some cases, however, you may wish to avoid this substitution. An instance in which you might want to avoid substitution would be when equations have more than one root. When m multiple roots are present, reduction may select a suboptimal root for a particular equation. On the other hand, the global and multistart solvers are adept at handling equations containing multiple roots. Thus, when using these solvers one may wish to forgo fixed variable reduction.

The available options are:

SUBOUT Setting	Reduction Degree
0	*None*
1	*Always*
2	*Not with global and multistart*
3	*Linear only*

Selecting *None* disables all fixed variable reduction. Selecting *Always* enables reduction. When *Not with global and multistart* is selected, LINGO disables reduction whenever either the global or multistart solvers are selected, otherwise reduction is performed. With the *Linear Only* option, LINGO will not substitute a variable out unless it is a linear variable.

Note: You should be careful when turning off fixed variable reduction. If the model generator is unable to substitute out fixed variables, you may end up turning a linear model into a more difficult nonlinear model.

LINGO defaults to the *Linear Only* setting for fixed variable reduction.

88. NLPVER

The *Nonlinear Solver Version* option on the *Nonlinear Solver* may be used to select the version number of the nonlinear solver. Presently, there are two options: *Version 3.0* and *Version 4.0.* LINGO defaults to using Version 3.0, but you may find that on some models Version 4.0 can perform better.

89. DBGCLD and 90. DBGWRM

These two parameters give you control over the linear solver that is used by the *DEBUG* command for model debugging. The available choices are:

DBGCLD/DBGWRM	Debug Linear Solver
0	*Solver Decides* — LINGO selects the solver it believes is the most appropriate,
1	*Primal* — the primal simplex solver will be used,
2	*Dual* — the dual simplex solver will be used, and
3	*Barrier* — the barrier solver will be used (requires a barrier solver license).

DBGCLD selects the solver for cold starts (starting without an existing basis in memory) and *DBGWRM* selects the solver for warm starts (restarting from an existing basis).

LINGO defaults to *Solver Decides* for both the cold and warm debug solver.

91. LCRASH

LINGO has two solution "crashing" methods that use different heuristics for generating starting points for nonlinear models: *NCRASH* and *LCRASH*. If at least one of the two crash parameters is set to 1, LINGO's nonlinear solver will invoke a heuristic for generating a "good" starting point when you solve a model. If this initial point is relatively good, subsequent solver iterations should be reduced along with overall runtimes. It's possible, however, that runtimes may slow if the selected point turns out to be poor. It is difficult to know beforehand if a particular crashing technique, if any, will help performance. You will need to experiment with your model to determine the best settings for the two crash procedures.

LINGO defaults to not crashing an initial solution.

92. BCROSS

The *BCROSS* parameter controls whether or not the barrier solver performs a *basis crossover* on linear programs. Barrier solvers do not normally return basic solutions. For example, if alternate optima exist, the barrier method will return a solution that is, loosely speaking, the "average" of all alternate optima. The basis crossover process converts a non-basic barrier solver solution to a basic (i.e., corner point) solution. The available choices are: 0 for no crossover and 1 (the default) to perform a crossover.

93. LOWMEM

The *LOWMEM* option may be used to guide LINGO's memory usage. Enabling this option (**`SET LOWMEM 1`**) causes LINGO to opt for less memory usage when solving a model. The downside is that opting for less memory may result in longer runtimes.

LINGO defaults to disabling the LOWMEM option.

94. FILOUT
LINGO can export a model's solution to databases using the ODBC (Open DataBase Connectivity) standard. Solutions exported to a database are sent to tables within the database. The tables may contain more space for values than you are actually exporting. In other words, there may be records at the end of a table that will not be receiving exported values from LINGO. The *Fill Out ODBC Tables* option determines how these extra cells and records are treated.
When the *Fill Out Ranges and Tables* option is enabled (**`SET FILOUT 1`**), LINGO overwrites the extra values with null entries. Conversely, when the option is not enabled, LINGO leaves any extra values untouched.

The *Fill Out ODBC Tables* option is disabled by default.

95. DBGLVL
The *DBGLVL* option gives you control over the output level of the model debugging command, *DEBUG*. The debugger is very useful in tracking down problems in models that are either infeasible or unbounded. Possible output levels range from 1 (minimum output) to 15 (maximum output). In general, you will want to generate as much output as possible. The only reason to restrict the amount of output would be to speed debugging times on large models.

The default setting for the debugger output level is 15.

96. UNARYM
The *UNARYM* option is used to set the priority of the unary minus operator. The two available options are *High* (**`SET UNARYM 1`**) are *Low* (**`SET UNARYM 0`**).

There are two theories as to the priority that should be assigned to the unary minus (i.e., negation) operator in mathematical expressions. On the one hand, there is the Excel practice that the unary minus operator should have the highest priority, in which case, the expression 3^2 would evaluate to +9. On the other hand, there is the mathematicians' preference for assigning a lower priority to unary minus than is assigned to exponentiation, in which case, 3^2 evaluates to 9. Note that regardless which relative priority is used, one can force the desired result through the use of parenthesis.

LINGO defaults to the Excel approach of setting a higher priority (*High*) on negation than on exponentiation.

97. LINEAR
The *LINEAR* option can be enabled (**`SET LINEAR 1`**) to minimize memory usage on models that are entirely linear. When this option is in effect, the model generator can take steps to dramatically reduce overall memory consumption without sacrificing performance. In fact, if all your models are linear, we recommend that you enable this option permanently as the default for your installation. The one restriction is that models must prove to be *entirely linear*. If a single nonlinearity is detected, you will receive an error message stating that the model is nonlinear and model generation will cease. At which point, you should clear this option and attempt to solve the model again.

By default, the *LINEAR* option is disabled.

98. LOPTOL

The *LOPTOL* parameter allows you to control the setting for the *linear optimality tolerance*. This tolerance is used to determine whether a reduced cost on a variable is significantly different from zero. You may wish to loosen this tolerance (make it larger) on poorly scaled and/or large models to improve performance.

The default setting for the *LOPTOL* parameter is 1.e-7.

99. SECORD

The *SECORD* option determines if the nonlinear solver will use second order derivates. If used (**`SET SECORD 1`**), second order derivatives will always be computed analytically, as opposed to using numerical differences. Computing second order derivatives will take more time, but the additional information they provide may lead to faster runtimes and/or more accurate solutions.

LINGO defaults to not using second order derivatives.

100. NONNEG

When enabled (**`SET NONNEG 1`**), the *NONNEG* option tells LINGO to place a default lower bound of 0 on all variables. In other words, unless otherwise specified, variables will not be allowed to go negative. Should you want a variable to take on a negative value, you may always override the default lower bound of 0 using the *@BND()* function. If this option is disabled, then LINGO's default assumption is that variables are unconstrained and may take on any value, positive or negative. Unconstrained variables are also referred to as be being *free*.

By default, LINGO enables the non-negative option, thereby setting a default lower bound of 0 on all variables.

101. BIGMVL

Many integer programming models have constraints of the form:

$$f(x) \leq M * z$$

where $f(x)$ is some function of the decision variables, M is a large constant term, and z is a binary variable. These types of constraints are called *forcing constraints* and are used to force the binary variable, z, to 1 when $f(x)$ is nonzero. In many instances, the binary variable is multiplied by a fixed cost term in the objective; a fixed cost that is incurred when a particular activity, represented by $f(x)$, occurs. The large constant tem, M, Is frequently referred to as being a *BigM* coefficient.

Setting BigM too small can lead to infeasible or suboptimal models. Therefore, the BigM value will typically have to be rather large in order to exceed the largest activity level of $f(x)$. When BigM is large, the solver may discover that by setting z slightly positive (within normal integrality tolerances), it can increase $f(x)$ to a significant level and thereby improve the objective. Although such solutions are technically feasible to tolerances, they are invalid in that the activity is occurring without incurring its associated fixed cost.

The *BIGMVL* parameter, or *BigM threshold*, is designed to avoid this problem by allowing LINGO to identify the binary variables that are being set by forcing constraints. Any binary variable with a coefficient larger than the BigM threshold will be subject to a much tighter integrality tolerance.

The default value for the *BigM Threshold* is 1.e8.

102. KILLSC
LINGO allows the input of scripts in the calc section. These scripts are useful for running multiple models, where the outputs of one model feed into subsequent models as input. Models are solved in calc sections with the *@SOLVE* command. Time limits can be placed on *@SOLVE's* via the *TATSLV* parameter (see below). If a time limit is hit while *@SOLVE* is running, LINGO will interrupt the solve and either continue executing the script with the next command, or terminate all processing. When the *KILLSC* option is set to 0 (default), processing continues with the next statement. Setting *KILLSC* to 1 causes LINGO to terminate all processing whenever the *@SOLVE* time limit is hit, and LINGO will subsequently return to command-prompt level.

103. TATSLV
LINGO allows the input of scripts in the calc section. These scripts are useful for running multiple models, where the outputs of one model feed into subsequent models as input. Models are solved in calc sections with the *@SOLVE* command. Time limits can be placed on *@SOLVE's* via the *TATSLV* parameter. If a time limit is hit while *@SOLVE* is running, LINGO will interrupt the solver and either continue executing the script with the next command, or terminate all processing based on the setting for the *KILLSC* parameter (see above). LINGO defaults to placing no time limit on *@SOLVE* commands.

104. KBESTS
The *KBESTS* parameter is used to set the number of solutions desired as part of the *K-Best solutions feature* of LINGO's mixed integer solver. Whenever this value is greater than 1, say *K,* LINGO will return up to K unique solutions to the model. These solutions will have the property that they are the next best solutions available in terms of their objective values. Less than K solutions may be returned if a sufficient number of feasible solutions do not exist. Please refer to section *K-Best Solutions Example* for an example of the use of the K-Best feature. The default value for this parameter is 1, meaning that LINGO will find only one solution to integer models, i.e, the K-Best feature is disabled by default.

105. LCORES
The *LCORES* parameter may be used to perform parallel solves of linear programs on multiple-cored machines. One of four different linear solvers is chosen for each core. Assignment of solvers to cores is controlled by the *LCORE1 - LCORE4* parameters (see below). LINGO will take the solution from the solver that finishes first and then interrupt the remaining solver threads.
The idea behind this approach is that different linear solvers will have relatively better or worse performance on different classes of models. However, it may be difficult to predict beforehand the solver that is most likely to outperform. So, by enabling multi-core solves, you guarantee that you will always get top performance, even without knowledge beforehand of which solver is likely to run the fastest.

Note: The multi-core feature requires that your machine have at least one core free for each solver you wish to run. Using this feature with an inadequate number of cores will tend to *decrease overall performance*.

For the *LCORES* parameter, you have the following choices: 1*,* 2, 3*,* or 4. When the default 1 option is selected, the multi-core feature is disabled, and LINGO will run only one solver on linear programs, namely the one specified as part of the *SOLVEL* option detailed above. When either option 2, 3, or 4 is

selected, LINGO will run linear solvers in the requested number of cores. The choice of the actual solvers used is controlled by the *LCORE1 - LCORE4* parameters (see below).

106-109. LCORE1 - LCORE4

The *LCORE1, LCORE2, LCORE3* and *LCORE4* parameters are used in conjunction with the *LCORES* parameter to perform parallel solves of linear programs on multiple-cored machines. One of four different linear solvers is chosen for each core, with assignments controlled by the *LCORE1 - LCORE4* parameters. LINGO will take the solution from the solver that finishes first and then interrupt the remaining solver threads.

The *LCORES* parameter gives the number of parallel solves that are to be performed on linear programs, while *LCORE1 - LCORE4* control the selection of the actual LP solver to use in each core. Parameters *LCORE1 - LCORE4* are meaningful only when *LCORES* is greater than 1. In addition, if *LCORES=<n>*, then only the parameters *LCORE1 - LCORE<n>* are meaningful. When the default 1 option is selected for *LCORES*, the multi-core feature is disabled, and LINGO will run only one solver on linear programs, namely the one specified as part of the *SOLVEL* option detailed above. When either option 2, 3, or 4 is selected for *LCORES*, LINGO will run linear solvers in the requested number of cores. The choice of the actual solvers used is controlled by the *LCORE1 - LCORE4* parameters.

The idea behind this approach is that different linear solvers will have relatively better or worse performance on different classes of models. However, it may be difficult to predict beforehand the solver that is most likely to outperform. So, by enabling multi-core solves, you guarantee that you will always get top performance, even without knowledge beforehand of which solver is likely to run the fastest.

Note: The multi-core feature requires that your machine have at least one core free for each solver you wish to run. Using this feature with an inadequate number of cores will tend to *decrease overall performance*.

For each of the *LCORE1-4* parameters, you have the following choices:

LCORE(i) Setting	LP Solver Used in Core i
1	*Primal1* — Primal simplex algorithm 1
2	*Dual* — Dual simplex algorithm
3	*Barrier* — Barrier/Interior point solver (available as a option)
4	*Primal2* — Primal simplex algorithm 2, installed as part of the Barrier option

As an example, the following session runs an LP model in two cores (*LCORES*=2), with the barrier solver in core 1 (*LCORE1*=3) and the dual simplex solver in core2 (*LCORE2*=2):

```
: set lcores 2   !run in 2 cores

   Parameter        Old Value     New Value
    LCORES             1             2

: set lcore1 3   !barrier in core 1

   Parameter        Old Value     New Value
    LCORE1             1             3

: set lcore2 2   !dual simplex in core 2

   Parameter        Old Value     New Value
    LCORE2             2             2

: take lp.lng    !load the model
: set terseo 1   !minimal output

   Parameter        Old Value     New Value
    TERSEO             0             1

: go             !solve the model
  First returning solver: BARRIER
  Global optimal solution found.
  Objective value:                      1272282.
  Infeasibilities:                      0.000000
  Total solver iterations:                    27

:
```

Once optimization is complete, LINGO will display a line indicating the solver that finished first. In the solution report excerpt above, we see that the barrier solver was the first to completion.

110. SCALEW

After LINGO generates a model, it checks all the nonzero coefficients in the model and computes the ratio of the largest to smallest coefficients. This ratio is an indicator of how well the model is scaled.

When the ratio gets to be too high, scaling is considered to be poor, and numerical difficulties may result during the solution phase. If the scaling ratio exceeds the value of the *SCALEW* parameter, LINGO will display error message 205. The default value for *SCALEW* is 1e12. Instead of simply increasing the *SCALEW* setting to eliminate error 205, we strongly suggest that you attempt to rescale the units of your model so as to reduce the largest-to-smallest coefficient ratio.

111. REFRAQ

The *REFRAQ* parameter allows you to control how frequently the linear solver refactors the basis matrix. The options are either to set *REFRAQ* to 0, thereby letting LINGO determine the frequency, or to set *REFRAQ* to some positive integer quantity. If an integer value, *N*, is selected, then the linear solver will refactor every *N* iterations. Numerically tough and/or poorly scaled models may benefit from more frequent refactoring. However, refactoring too frequently will cause the solver to slow down.

The default setting for the *REFRAQ* is 0, which will typically result in refactoring about once every 100 iterations.

112. SPSMSZ

The *SPSMSZ* parameter is used to control the default sample size for random variables in stochastic programming (SP) whose outcomes are determined via sampling.

In many SP models, LINGO will generate a set of sample values for the some or all of the random variables. This is particularly true when you have one or more continuous random variables, which have an infinite number of possible outcomes. In such a case, sampling is the only viable alternative.

One way to specify the desired sample size for each stage is via the *@SPSAMPSIZE* function used directly in the model's text. If, on the other hand, all or most stages should have the same sample size, then you can use the *SPSMSZ* parameter to control the default sample size. Any stage which has not had its sample size specified with *@SPSAMPSIZE* will default to a sample size equal to the *SPSMSZ* parameter.

Note: In general, we prefer larger sample sizes to smaller ones in order to reduce sampling error. However, SP models can become quite large if sample sizes aren't kept reasonably small. This is particularly true for multiperiod models. For example, suppose we have a model with just one random variable and ten periods/stages. If the default sample size is set to 3, then there will be 3^10=59,049 possible scenarios. With this many scenarios, it would only take a handful of decision variables to end up with an underlying deterministic equivalent model approaching one million variables.

The *SPSMSZ* parameter defaults to a value of 2.

113. SPSCON

The *SPSCON* parameter is used to control whether LINGO samples continuous distributions only for the random variables in stochastic programs, or if it samples both continuous *and* discrete random variables. Obviously, continuous random variables must be subject to sampling, given the infinite number of possible outcomes. On the other hand, we have a choice when it comes to discretely distributed random variables. If this option is enabled, then LINGO will generate one outcome for each density point in the discrete distribution of the random variable. These outcomes will be weighted by their corresponding probabilities.

Note: If there are many density points in the distributions for the discrete random variables, the deterministic equivalent generated to solve the SP may become exceedingly large, in which case, disabling this option may be required. Once this option is disabled, discrete random variables will be sampled in the same way as the continuous random variables.

The *SPSCON* option defaults to being on.

114. SPSEED

The *SPSEED* parameter is used to establish the seed for the random number generator used in generating samples for random variables in stochastic programming (SP) models.

In many SP models, LINGO will generate a set of sample values for the random variables. This is particularly true when you have one or more continuous random variables, which leads to an infinite number of possible outcomes. In such a case, sampling is the only viable alternative.

The *SPSEED* parameter defaults to a value of 1031.

Note: The seed parameter has no effect when running demo versions of LINGO. Demo versions always use the default seed value, regardless of this parameter's setting.

115. SPMTHD

The *SPMTHD*, or SP Solver Method, option is used to select the solution method for a stochastic programming (SP) model. Presently, the available choices are:

SPMTHD Setting	SP Solver Method
0	*Solver Decides* — LINGO decides the most appropriate method for solving the SP model
1	*Deterministic Equivalent* — LINGO generates and directly solves the deterministic equivalent (DE) of the SP model
2	*Nested Benders Decomposition* — The DE for realistic SP models can get to be quite large, in that the core model is replicated once for every scenario. Fortunately, the DE displays strong block-angular structure, which makes it adaptable to decomposition techniques, such as nested Bender decomposition (NBD). Note that the model must be entirely linear to successfully use the NBD option.

The default setting for the SP Solver Method option is 0, or Solver Decides.

116. SPXVAL

The *SPXVAL*, or *Calculate All Expected Value Statistics,* option controls whether LINGO displays information regarding the expected values for a number of statistics when solving stochastic programming (SP) models. To illustrate, when solving the *SPGAS.LG4* model when this option is enabled, you will see the following expected values at the top of the solution report:

```
Expected value of:
   Objective (EV):  1400.000
   Wait-and-see model's objective (WS): 1326.667
   Perfect information (EVPI = |EV - WS|): 73.33333
   Policy based on mean outcome (EM): 1479.444
   Modeling uncertainty (EVMU = |EM - EV|): 79.44444
```

These values are a guide as to how the stochastic nature of the model is impacting the objective value. The following is a brief description of these expected values:

Expected Value of Objective (EV) - is the expected value for the model's objective over all the scenarios, and is the same as the reported objective value for the model.

Expected Value of Wait-and-See Model's Objective (WS) - reports the expected value of the objective if we could wait and see the outcomes of all the random variables before making our decisions. Such a policy would allow us to always make the best decision regardless of the outcomes for the random variables, and, of course, is not possible in practice. For a minimization, it's true that WS <= EV, with the converse holding for a maximization. Technically speaking, WS is a relaxation of the true SP model, obtained by dropping the nonanticipativity constraints.

Expected Value of Perfect Information (EVPI) - is the absolute value of the difference between EV and WS. This corresponds to the expected improvement to the objective were we to obtain perfect information about the random outcomes. As such, this is a expected measure of how much we should be willing to pay to obtain perfect information regarding the outcomes of the random variables.

Expected Value of Policy Based On Mean Outcome (EM) - is the expected true objective value if we (mistakenly) assume that all random variables will always take on exactly their mean values. EM is computed using a two-step process. First, the values of all random variables are fixed at their means, and the resulting deterministic model is solved to yield the optimal values for the stage 0 decision variables. Next, a) the stage 0 variables are fixed at their optimal values from the previous step, b) the random variables are freed up, c) the nonanticipativity constraints are dropped, and d) this wait-and-see model is solved. EM is the objective value from this WS model.

Expected Value of Modeling Uncertainty (EVMU) - is the absolute value of the difference EV - EM. It is a measure of what we can expect to gain by taking into account uncertainty in our modeling analysis, as opposed to mistakenly assuming that random variables always take on their mean outcomes.

Note: The above approach for computing EM and EVMU makes unambiguous sense only for models with a stage 0 and a stage 1. If there are later random variables in stages 2, 3, etc., then there are complications. For example, for decisions in later stages, we have seen the outcomes from the random variables in earlier stages, so considering these random variables to take on their mean value does not make sense. For models with additional stages beyond 0 and 1, EVMU will merely be an approximation of the true expected value of modeling uncertainty.

Note: Computing these expected value statistics can be very time consuming for large models. If speed is an issue, you may wish to disable this feature on the *Solver|Options|SP Solver* tab.

The *SPXVAL* option is enabled by default.

117. SPMAXS

The *SPMAXS,* or *Max Scenarios Limit,* parameter is used to establish a limit on the maximum number of scenarios in a stochastic programming (SP) model before forcing automatic sampling of the random variables.

The *SPMAXS* parameter defaults to a value of 40,000 scenarios.

118. PRECIS

LINGO defaults to displaying seven significant digits for values in standard solution reports. In some cases, you may desire more or less than seven digits of precision, in which case, you will need to adjust the *PRECIS* parameter.

For example, suppose we have a very simple model consisting of the single expression:

```
X = 1/3;
```

The default solution report for this model using seven digits of precision would contain:

```
Variable           Value
       X       0.3333333

     Row    Slack or Surplus
       1        0.000000
```

Increasing the *Precision* parameter to 16 and resolving yields the following solution report with 16 significant digits:

```
Variable                     Value
       X       0.3333333333333333

     Row              Slack or Surplus
       1        0.000000000000000
```

The default value for the *PRECIS* parameter is seven significant digits.

119. LOOPOP

The *LOOPOP*, or *Loop optimization,* parameter is used to either enable or disable LINGO's loop optimization step. Loop optimization reformulates expressions containing set looping functions in order to make them more efficient, while maintaining mathematical equivalency. The end goal of loop optimization is to minimize the number of passes through the inner loop of any nested loops in an expression.

As an example, consider the following transportation model fragment, that just contains constraints for satisfying customer demand:

```
MODEL:
! A transportation problem fragment;
SETS:
   WAREHOUSE /1..50/  : CAPACITY;
   CUSTOMER /1..5000/ : DEMAND;
   ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The demand constraints;
 @FOR( CUSTOMER( J):
    @SUM( ROUTES( I, J): VOLUME( I, J)) >= DEMAND( J)
 );

END
```

Transportation Model Fragment with Inefficient Constraints

In the demand constraints expression there are two loops — an outer *@FOR()* over the *CUSTOMER* set and an inner *@SUM()* over the *ROUTES* set. As written, the inner loop must be executed 50*50*5000=1.25 million times. Note that a valid reformulation of these demand constraints would be:

```
! The demand constraints;
 @FOR( CUSTOMER( J):
    @SUM( WAREHOUSE( I): VOLUME( I, J)) >= DEMAND( J)
 );
```

With the expression rewritten in this manner, the inner loop will now only be executed 50*5000 times, for a 98% reduction in total passes. LINGO's loop optimizer seeks out such inefficient loops and, invisible to the user, rewrites them before the model gets passed to the generator. Of course, the end result of such reformulations are faster generation times. In fact, the speedup from reformulation can be quite dramatic in many cases. In this particular example, the reformulated version ran over 400 times faster in the model generator.

LINGO defaults to disabling the *LOOPOP* feature.

120. HEUCUT
The *HEUCUT, or heuristics cutoff criterion,* option controls the criterion for terminating integer programming heuristics. The possible settings for *HEUCUT* are:

HEUCUT Value	Cutoff Criterion
0	LINGO decides
1	Time
2	Iterations

Under the *Time* setting, LINGO terminates heuristics after a certain amount of elapsed time. The *Iterations* option terminates heuristics after a certain number of iterations. In general, the *Time* setting results in the fastest performance. However, due to shifting computational loads on a machine, solution paths may change under the *Time* setting from one solve to the next, potentially resulting in non-reproducible solutions. If reproducibility of a runs is a concern, then the *Iterations* option should be selected. Under the *Solver Decides* setting, LINGO chooses the most appropriate strategy.

The default setting for *HEUCUT* is 1, *Solver Decides.*

121. NUMBIN - Number of Histogram Bins
The *NUMBIN* parameter determines the number of bins used in histogram charts. Set this parameter to 0 to have LINGO automatically determine a sensible bin count.

The default setting for *NUMBIN* is 0.

122. SPBIGM - SP Big M Coefficient
The *SPBIGM* parameter is used by the Stochastic Programming (SP) solver in constructing forcing constraints that may be required in the deterministic models generated by the solver. Forcing constraints are generally added to force binary variables to 1 when some activity occurs. For example, suppose x is a continuous variable and z is a binary variable. A forcing constraint that would drive z to 1 whenever x is positive would be:

$$x \leq M * z$$

where M is some large number. M in this case is the *SPBIGM* parameter.

There are two things to note about M. First, if M is too small, a forcing constraint can become infeasible, making the entire model infeasible. In general, M should be at least as large as the largest possible value for x so as not to introduce an infeasibility. However, from an algorithmic point of view, M should not be too large. If M is unrealistically large, it will be tougher for the integer solver to converge, plus it also introduces the potential round off error.

The default setting for M should be sufficient for most models. However, if your SP model is infeasible for no known reason, you may want to try increasing M. On the other hand, if your SP is running slow, you may want to try reducing M.

The *SPBIGM* parameter defaults to a value of 1.e8.

123. NSLPSV - SLP Solver
If the *NSLPSV* parameter is enabled, LINGO uses a successive linear programming (SLP) algorithm for its nonlinear solver. This technique uses a linear approximation of the true nonlinear model within successive, small regions. This then allows the use of the fast linear solver for optimizing each linear subregion. In general, the SLP solver will not be well suited for highly nonlinear models. However, it may offer better performance than the standard nonlinear solver when a model has few nonlinear variables, such that the model is "mostly linear".

LINGO defaults to not enabling the *SLP Solver* option.

124. FORCEB - Enforce Bounds in CALC and DATA
If the Enforce Bounds In *CALC* and *DATA* option is enabled, it causes LINGO to check the values of any variables input in calc and data sections against their bounds. Normally, variable bounds apply only to optimizable variables that appear in the model's constraints and not to variables fixed to constant values in calc and data sections. If you would also like LINGO to check variables fixed in calc and data sections against their bounds, then you will need to enable this option.

By default, this option is disabled.

125. NTHRDS - Number of Execution Threads
The *NTHRDS* option controls the maximum number of threads, or processors, to utilize at any given time. Interesting settings for the thread count are:

- *0* — LINGO determines the number of processors on the system and sets the thread limit to this number.
- *1* — Only one processor will be used, causing LINGO to run in single-threaded mode.
- *N* — Here, N is some positive integer greater than 1 and less-than-or-equal-to 32, in which case, LINGO will set the maximum number of threads to N.

LINGO defaults to running in single-threaded mode with a thread limit of 1.

By default, this option is disabled.

Note: Setting the maximum thread count parameter, *NTHRDS*, to *N* does not guarantee than *N* cores will be fully utilized throughout the entire solve. There may be serial sections inside parallel code that get executed from time to time, causing only one core to be utilized. Concurrent algorithms may also have fewer than N different serial algorithms to launch, meaning fewer than *N* cores will be occupied.

126. MTMODE - Multithread Mode

The *MTMODE* parameter controls the multithreading mode that LINGO operates in. LINGO offers multicore extensions to its model generator and solvers. The multicore extensions are of two types: *concurrent* and *parallel*. Concurrent algorithms run two or more different serial algorithms on multiple copies of the same model, using a separate thread for each algorithm, terminating as soon as the winner thread finishes. These "different algorithms" may in fact be the same algorithm type, but using different strategies and/or parameters. Parallel algorithms, on the other hand, parallelize computationally intensive portions of the serial algorithm to distribute the workload across multiple threads.

The following multicore extensions are currently available in LINGO:

LINGO Component	Model Class	Parallel Option	Concurrent Option
Barrier Solver	Linear Programs	Yes	No
BNP Solver	Mixed Integer Programs	Yes	No
Global Solver	Nonlinear Programs	Yes	No
Integer Solver	Mixed Integer Programs	Yes	Yes
Linear Solver	Linear Programs	No	Yes
Model Generator	All	Yes	No
Multistart Solver	Nonlinear Programs	Yes	No
Stochastic Solver	Stochastic Programs	Yes	No

The available settings for the Mode, or multithreading mode, parameter are as follows:

- *-1 — Solver Decides* — The best available multithreading strategy, either parallel or concurrent, will be selected for each step of the solution process.
- *0 — Off in Solver* — Multithreading will be disabled in the solvers, but allowed in the model generator if the number of threads is greater than 1.
- *1 — Prefer Parallel* — If a parallel option is available for a particular solution step, then it will be selected, otherwise, a concurrent option will be selected when available.
- *2 — Parallel Only* - If a parallel option is available for a particular solution step, then it will be selected, otherwise, the step will be executed in single-thread mode.
- *3 — Prefer Concurrent* - If a concurrent option is available for a particular solution step, then it will be selected, otherwise, a parallel option will be selected when available.
- *4 — Concurrent Only* - If a concurrent option is available for a particular solution step, then it will be selected, otherwise, the step will be executed in single-thread mode.

Note: If the maximum thread count parameter, *NTHRDS*, is set to 1, then the multithreading mode setting will be ignored, and LINGO will execute in single-threaded mode.

Note: Setting the maximum thread count parameter, *NTHRDS*, to *N* does not guarantee than *N* cores will be fully utilized throughout the entire solve. There may be serial sections inside parallel code that get executed from time to time, causing only one core to be utilized. Concurrent algorithms may also have fewer than *N* different serial algorithms to launch, meaning fewer than *N* cores will be occupied.

127. BNPBLK - BNP Block Determination

The *BNPBLK* parameter controls how the the *branch-and-price* (BNP) solver determines the block structure of the model. The BNP solver is a mixed integer programming solver for solving models with block structures like the following:

$$\text{Minimize: } \sum c(k) * x(k)$$
$$\text{Subject To:}$$
$$\sum A(k) * x(k) = d \qquad \text{(linking constraints)}$$
$$x(k) \text{ in } X(k), \text{ for all } k \qquad \text{(decomposition structure)}$$

where *d*, *c(k)* and *x(k)* are vectors and *A(k)* is a matrix with appropriate dimensions. *x(k)* contains decision variables and X(k) denotes a linear feasible domain for x(k).

The BNP solver is a hybrid of branch-and-bound, column generation, and Lagrangean relaxation methods. It can help to find either the optimal solution or a better lower bound (the Lagrangean bound) for a minimization problem. Based on the decomposition structure, the solver divides the original problem into several subproblems, or blocks, and solves them (almost) independently, exploiting parallel processing if multiple cores are available.

BNP may perform better than the default MIP solver if: a) the number of linking constraints is small, b) the number of blocks is large and they are of approximately the same size, and c) the number of available processors (or cores) is large, e.g., 4 or more. Also, there may be some models for which BNP finds a good solution and good bound more quickly than the default MIP algorithm, although it may take longer to prove optimality.

The *Blocks* option for the BNP solver controls the number of subproblems, or blocks, that the model will be partitioned into. Possible setting for the *Blocks* parameter are:

- *-1 - Row Names* - Row names are constructed in such a way as to specify each row's block (an example is given below).
- *0 - Off* - This will disable the BNP solver, in which case, the standard MIP solver will be used to solve all mixed integer linear programs.
- *1 - Specified* - The user explicitly specifies each row's block using the *@BLOCKROW* function.
- *N* - A positive integer, greater-than-or-equal-to 2, indicating the number of independent blocks to try and partition the model into via one of the graph partitioning algorithms provided by LINGO. The actual heuristic used is chosen with the Heuristic parameter.

The default setting for *Blocks* is 0, or *Off*, i.e., the BNP solver will not be used on integer programming models.

Note:	The BNP solver can run the independent subproblems on separate threads to improve performance. So, if your machine has multiple cores, be sure to set the thread limit to allow for multithreading. Refer to the *NTHRDS* parameter above.

For more information on block determination, the BNP solver and the *BNPBLK* parameter, refer to the section BNP Solver in *Chapter 5*.

128. BNPHEU - BNP Block Finding Heuristic
The *Block Heuristic* parameter controls the heuristic used to partition the model into blocks. You may currently select from two graph partitioning algorithms named simply *GP1* and *GP2*; simple set *BNPHEU* to 1 for GP1 and 2 for GP2.

For more information on block finding heuristics, the BNP solver and the *BNPHEU* parameter, refer to the section *BNP Solver* in Chapter 5.

The default setting for *BNPHEU* is 1, i.e., the *GP1* graph partitioning algorithm.

129. REPROD - Favor Reproducibility
The *REPROD* parameter allows you to indicate if your preference is for reproducible solutions across multiple runs. In general, being able to reproduce results is a desirable outcome. However, some models may run faster if we opt to not favor reproducible runs.

A number of solver steps have been found to perform better when using elapsed time as a measure of work performed. The problem with this approach is that the actual amount of work that gets performed over a fixed time interval will vary slightly across runs depending on a machine's load factor. Many models have alternate optimal solutions, with equal objectives but different variable values. LINGO is indifferent as to which solution is selected; it just wants to find a feasible solution with the best objective value, regardless of the variables values. The end result of this variability in work performed is that a different solution path may be selected from one run to the next, in which case, you may end up with a different alternate optimum from a previous run.

On the other hand, enabling the *REPROD* option causes LINGO to use fixed measures of work (such as iteration counts), allowing solutions to be reproducible across runs when using the same machine.

By default, LINGO enables the *REPROD* option.

130. STARTP - Default Starting Point
The *STARTP* parameter is used to set the default starting value for variables. Choosing a different starting point may be useful for some nonlinear programs, where you want the solver to start in a particular neighborhood close to good solutions to improve runtimes.

131. HRDLMS - Multistart Hurdle
The multistart solver solves nonlinear programs by successively restarting the NLP solver from different starting points. This can be very useful in finding good solutions to non-convex models. Sometimes, however, the multistart solver may take longer to solve than you'd desire. If, in this case, you would be happy with any solution with an objective value better than, say, 100, then you can input this as the *HRDLMS* value. The multistart solver will interrupt and return when it finds any solution better than your multistart hurdle value.

Note: If the iteration limit for the multistart solver, MULTIS, is hit before the HRDLMS value, then the solver will also interrupt.

The default hurdle value is None. In other words, a hurdle value is not used by the multistart solver. To clear an existing hurdle value, type SET HRDLMS NONE.

132. SOLVLG - Solver Log Level

The *SOLVLG* parameter can be used to control the amount of output the solver engines display while running. By default, LINGO does not display much solver log output. However, some technical users may be curious to see this information. Below is a selection of some log output from a small integer program:

```
Number of constraints:       2    le:       1, ge:       0, eq:
         1, rn:       0 (ne:0)
Number of variables  :       9    lb:       1, ub:       0, fr:
   0, bx:       8 (fx:0)
Number of nonzeroes  :      12    density:  0.0067(%)     , sb:
   0

Abs. Ranges     :         Min.          Max.    Condition.
Matrix Coef. (A):      1.00000       7.00000       7.00000
Obj. Vector  (c):      1.00000      10.00000      10.00000
RHS Vector   (b):     15.00000      15.00000       1.00000
Lower Bounds (l):  1.0000e-100   1.0000e-100       1.00000
Upper Bounds (u):      1.00000       1.00000       1.00000
BadScale Measure: 0

Binary variables     :       8 (in 2 constraints)
Integer variables    :       0 (in 0 constraints)

Startpoint info (not feasible):
 Objvalue                        : 3.500000e+01  (startpoint)
 Infeasibility of solution       : 1.8e+01
 Integer infeasibility of solution: 0.0e+00


Feasible obj value found in MIP prerelax heu=0.000000
```

Setting the log level to higher integer values may result in a higher level of output. In general, a level of 1 is perhaps the most useful and meaningful. Setting the log level to 0 disables solver logs, which is the default setting.

9. Miscellaneous

The *Miscellaneous* category contains various LINGO commands that don't fall into one of the other eight command categories.

!

Place an exclamation mark in a command and LINGO ignores the remainder of the line following the exclamation mark.

QUIT

Issue the *QUIT* command to close the LINGO application. Be sure to save any changes made to your model before quitting.

TIME

Displays the current elapsed time since the start of the current LINGO session as illustrated in the following example:

```
: TIME
 Cumulative HR:MIN:SEC =   2:22:39.54
:
```

7 LINGO's Operators and Functions

LINGO provides the mathematical modeler with a number of functions and operators. For our purposes, we have broken them down into the following categories:

- *Standard Operators* - Arithmetic, logical, and relational operators such as +, -, =, and <=.
- *Mathematical* - Trigonometric and general mathematical functions.
- *Financial* - Common financial functions used to determine present values.
- *Probability* - Functions used to determine a wide range of probability and statistical answers. Poisson and Erlang queuing functions are among those provided.
- *Variable Domain* - Functions used to define the range of values (domain) a variable can take on (e.g., lower and upper bounds or integer restrictions).
- *Set Handling* - Functions useful for manipulating sets.
- *Set Looping* - Looping functions used to perform an operation over a set (e.g., to compute the sum, maximum, or minimum of a set of numbers).
- *Interface* - Functions used to create links to external data sources.
- *Distributions* - Probability distributions, including cumulative, inverse and pdf functions.
- *Miscellaneous* - Miscellaneous functions are listed under this heading.

Select a link above for an in-depth description of the operators and functions available in LINGO.

Standard Operators

LINGO has three types of standard operators:

1. *Arithmetic*,
2. *Logical*, and
3. *Relational*.

Arithmetic Operators

Arithmetic operators work with numeric operands. LINGO has five binary (two-operand) arithmetic operators, shown here:

Operator	Interpretation
^	Exponentiation
*	Multiplication
/	Division
+	Addition
-	Subtraction

Since these are binary operators, they require two arguments—one immediately to the left of the operator and one immediately to the right.

The only unary (one-operand) arithmetic operator in LINGO is negation (-). In this case, the operator applies to the operand immediately to the right of the negation sign.

These operators should be familiar to all readers. The priority of the operators is given in the following:

Priority Level	Operator(s)
Highest	- (negation)
	^
	* /
Lowest	+ -

Operators with the highest priority are evaluated first, in order from left to right. As an example, consider the expression:

4 + 6 / 2

The division operator (/) has higher priority than the addition operator (+). Thus, it is evaluated first, leaving: 4 + 3. Evaluating the remaining addition gives a final result of 7.

The order of evaluation of the operators can be controlled with parentheses. LINGO evaluates the equation in the innermost parentheses first and works out from there. If we recast the expression from above as:

(4 + 6) / 2

we will now get a final result of 5, instead of 7. The 4 and 6 are added first because they appear in parentheses. The resulting sum of 10 is divided by 2, giving the final result of 5.

Note: LINGO follows the Excel convention of assigning the highest priority to the negation operator. Given this, LINGO evaluates -3^2 as positive 9. Some users may prefer to give the unary minus operator a lower priority so that -3^2 evaluates to minus 9. You can do this by setting the *Unary Minus Priority* option to *Low* via the *Model Generator* tab of the *Solver|Options* command. Once you set the unary minus operator's priority is set to low its priority will be lower than multiplication and division, but higher than addition and subtraction.

Logical Operators

Logical operators were used in Chapter 2, *Using Sets*, when we introduced set looping functions. In LINGO, logical operators are primarily used in conditional expressions on set looping functions to control which members of a set are to be included or excluded in the function. They also play a role in building set membership conditions.

Logical operators return either TRUE or FALSE as a result. LINGO uses the value 1 to represent TRUE, and the value 0 to represent FALSE. *LINGO considers an argument to be FALSE if, and only if, it is equal to 0.* Thus, for example, arguments of 1, 7, -1, and .1234 would all be considered TRUE.

LINGO has nine logical operators, which are all binary with the single exception of the *#NOT#* operator, which is unary. LINGO's logical operators and their return values are listed below:

Logical Operator	Return Value
#NOT#	TRUE if the operand immediately to the right is FALSE, else FALSE.
#EQ#	TRUE if both operands are equal, else FALSE.
#NE#	TRUE if both operands are not equal, else FALSE.
#GT#	TRUE if the left operand is strictly greater than the right operand, else FALSE.
#GE#	TRUE if the left operand is greater-than-or-equal-to the right operand, else FALSE.
#LT#	TRUE if the left operand is strictly less than the right operand, else FALSE.
#LE#	TRUE if the left operand is less-than-or-equal-to the right operand, else FALSE.
#AND#	TRUE only if both arguments are TRUE, else FALSE.
#OR#	FALSE only if both its arguments are FALSE, else TRUE.

The priority ranking of the logical operators is:

Priority Level	Operator(s)
Highest	#NOT#
	#EQ# #NE# #GT# #GE# #LT# #LE#
Lowest	#AND# #OR#

Relational Operators

In LINGO, relational operators are used in a model to specify whether the left-hand side of an expression should be equal to, less-than-or-equal-to, or greater-than-or-equal-to the right-hand side. Relational operators are used to form the constraints of a model. Relational operators are distinct from the logical operators *#EQ#*, *#LE#*, and *#GE#*, in that they tell LINGO the optimal solution of the model *must* satisfy the direction of the relational operator. Logical operators, on the other hand, merely report whether or not a condition is satisfied.

Relational operators have the lowest priority of all the operators.

The three relational operators are described below:

Relational Operator	Interpretation
=	The expression to the left must equal the one on the right.
<=	The expression to the left must be less-than-or-equal-to the expression on the right
>=	The expression to the left must be greater-than-or-equal-to the expression on the right

LINGO will also accept "<" for less-than-or-equal-to, and ">" for greater-than-or-equal-to.

Note: LINGO does not directly support *strictly* less than and *strictly* greater than relational operators. In general, it would be unusual to find a good formulation that requires such a feature. However, if you want A to be *strictly* less than B:

$$A < B,$$

then convert this expression to an equivalent less-than-or-equal-to expression as follows:

$$A + e \leq B,$$

where e is a small constant term whose value is dependent upon how much A must be "less than" B in order for you to consider them to be "not equal".

Operator Priority Table

The following table combines all three types of operators—arithmetic, logical, and relational—into a single table showing their relative priority rankings.

Priority Level	Operator(s)
Highest	#NOT# -(negation)
	^
	* /
	+ -
	#EQ# #NE# #GT# #GE# #LT# #LE#
	#AND# #OR#
Lowest	<= = >=

Note: LINGO follows the Excel convention of assigning the highest priority to the negation operator. Given this, LINGO evaluates -3^2 as positive 9. Some users may prefer to give the unary minus operator a lower priority so that -3^2 evaluates to minus 9. You can do this by setting the *Unary Minus Priority* option to *Low* via the *Model Generator* tab of the *Solver|Options* command. Once you set the unary minus operator's priority is set to low its priority will be lower than multiplication and division, but higher than addition and subtraction.

Note: In the absence of parentheses, all operators of the same priority are processed from left to right. Thus, 4^3^2 evaluates to 4096. Be forewarned that for the exponentiation operator, "^", this differs from the convention that some mathematicians follow, namely, to have the exponentiation operator evaluated from right to left, i.e., 4^3^2 would evaluate to 262144. When in doubt, use parentheses to enforce your intentions, e.g., 4^(3^2) unambiguously evaluates to 262144.

Mathematical Functions

LINGO offers a number of standard, mathematical functions. These functions return a single result based on one or more scalar arguments. These functions are listed below:

Note: Not all mathematical functions are defined over all argument values.
For example, *@LOG*(*X*-5) is not defined for values of X less-than-or-equal-to 5. In such cases, it may be helpful to bound variables so that all functions are defined, otherwise the solver may return a numeric error. For our *@LOG* example here, we would want to add a bound statement like *@BND*(5.001, *X*, 100), to force X to lie within the range 5.001 and 100, forcing the *@LOG* function to be defined over the entire range for *X*.

@ABS(X)
Returns the absolute value of *X*.

@ACOS(X)
Returns the inverse cosine, or arccosine, of *X,* where *X* is an angle in radians.

@ACOSH(X)
Returns the inverse hyperbolic cosine of *X*, where *X* is an angle in radians.

@ASIN(X)
Returns the inverse sine, or arcsine, of *X,* where *X* is an angle in radians.

@ASINH(X)
Returns the inverse hyperbolic sine of *X*, where *X* is an angle in radians.

@ATAN(X)
Returns the inverse tangent, or arctangent, of *X,* where *X* is an angle in radians.

@ATAN2(X, Y)
Returns the inverse tangent of *Y*/*X.*

@ATANH(X)
Returns the inverse hyperbolic tangent of X, where X is an angle in radians.

@COS(X)
Returns the cosine of *X*, where *X* is an angle in radians.

@COSH(X)
Returns the hyperbolic cosine of *X*, where *X* is an angle in radians.

@EXP(X)
Returns *e* (i.e., 2.718281 ...) raised to the power *X*.

@FLOOR(X)
Returns the integer part of *X*. To be specific, if $X \geq 0$, *@FLOOR* returns the largest integer, *I*, such that $I \leq X$. If *X* is negative, @FLOOR returns the most negative integer, *I*, such that $I \geq X$.

@INT(X)
This returns the integer part of *X*. To be specific, if $X \geq 0$, *@INT* returns the largest integer, *I*, such that $I \leq X$. If *X* is negative, *@INT* returns the largest negative integer, *I*, such that $X \geq I$.

@INTEGRAL(PROCEDURE, X, XL, XU, Y)
This function performs numeric integration using a variant of Simpson's Rule. Argument *PROCEDURE* is the name of a procedure which calculates the function to be integrated. The function's value should be placed in *Y* in the specified procedure. The integral will be taken with respect to *X* over the interval [*XL,XU*]. At present, *@INTEGRAL* may only be used in calc sections.

Example 1:
In this example, we illustrate how to compute the integral of the standard normal distribution over the interval [-5,1.645].

```
MODEL:
! Takes the integral of the standard normal
distribution over the interval [-5,1.645];
PROCEDURE NORMAL:
Y = @EXP( -X * X / 2) / (( 2 * @PI()) ^ 0.5);
ENDPROCEDURE
CALC:
PROB = @INTEGRAL( NORMAL, X, -5, 1.645, Y);
ENDCALC
END
```

@LGM(X)
Returns the natural (base *e*) logarithm of the gamma function of *X* (i.e., log of (*X* - 1)!). It is extended to noninteger values of *X* by linear interpolation.

@LMTD(X, Y)
Returns the Log Mean Temperature Difference, (X - Y) / LN(X /Y).

@LOG(X)
Returns the natural logarithm of *X*.

@LOG10(X)
Returns the base-10 logarithm of *X*.

@LOGB(X, B)
Returns the base *B* logarithm of *X*, e.g., *@LOGB(8, 2)* = 3.

@MOD(X, Y)
Returns the value of *X* modulo *Y*, or, in other words, the remainder of an integer divide of *X* by *Y*.

@PI()
Returns the value of PI, i.e., 3.14159265....

@POW(X, Y)
Returns the value of *X* raised to the *Y* power.

@RLMTD(X, Y)
Returns the Reciprocal of Log Mean Temperature Difference, LN(X /Y) / (X - Y) .

@ROUND(X, N)
The *@ROUND* function rounds *X* to the closest number to *X* having *N* digits. For example, if *X* is equal to 2.576 and *N* is 2, then *@ROUND(X, N)* will return 2.58. If *X* is equal to -2.576 and *N* is 2, then *@ROUND(X, N)* will return -2.58.

@ROUNDDOWN(X, N)
The *@ROUNDDOWN* function rounds *X* down (towards 0) to the closest number to *X* having *N* digits. For example, if *X* is equal to 2.576 and *N* is 2, then *@ROUNDDOWN(X, N)* will return 2.57. If *X* is equal to -2.576 and *N* is 2, then *@ROUNDDOWN(X, N)* will return -2.57.

@ROUNDUP(X, N)
The *@ROUNDUP* function rounds *X* up (away from 0) to the closest number to *X* having *N* digits. For example, if *X* is equal to 2.576 and *N* is 2, then *@ROUNDUP(X, N)* will return 2.58. If *X* is equal to -2.576 and *N* is 2, then *@ROUNDUP(X, N)* will return -2.58.

@SIGN(X)
Returns -1 if $X < 0$, 0 if $X = 0$ and +1 if $X > 0$.

@SIGNPOWER(X, Y)
Returns *X* raised to the power *Y* if $X \geq 0$, otherwise returns *ABS(X) ^ Y*.

@SIN(X)
Returns the sine of *X*, where *X* is the angle in radians.

@SINH(X)
Returns the hyperbolic sine of *X*, where *X* is an angle in radians.

@SMAX (X1, X2, ..., XN)
Returns the maximum value of *X1*, *X2*, ..., and *XN*.

@SMIN(X1, X2, ..., XN)
Returns the minimum value of *X1*, *X2*, ..., and *XN*.

@SQR(X)
Returns the value of *X* squared.

@SQRT(X)
Returns the square root of *X*.

@TAN(X)
Returns the tangent of *X*, where *X* is the angle in radians.

@TANH(X)
Returns the hyperbolic tangent of *X*, where *X* is an angle in radians.

Financial Functions

LINGO currently offers two financial functions. One computes the present value of an annuity. The other returns the present value of a lump sum.

@FPA(I, N)
This returns the present value of an annuity. That is, a stream of $1 payments per period at an interest rate of *I* for *N* periods starting one period from now. *I* is not a percentage, but a fraction representing the interest rate (e.g., you would use .1 to represent 10%). To get the present value of an annuity stream of $*X* payments, multiply the result by *X*.

@FPL(I, N)
This returns the present value of a lump sum of $1 *N* periods from now if the interest rate is *I* per period. *I* is not a percentage, but a fraction representing the interest rate (e.g., you would use .1 to represent 10%). To get the present value of a lump sum of $*X*, multiply the result by *X*.

Probability Functions

LINGO has a number of probability related functions. There are examples that make use of most of these functions in Chapter 12, *Developing More Advanced Models*, and in Appendix A*, Additional Examples of LINGO Modeling*.

@NORMINV(P, MU, SIGMA)
Inverse of the normal cumulative distribution. Given a probability, *P*, the mean of the of the normal distribution, *MU*, and its standard deviation, *SIGMA*, this function returns the value *Z* such that the probability of a normally distributed random variable being less-than-or-equal to *Z* is *P*. This function is being replaced by the *@PNORMINV* function, documented below in the *Distributions* section.

@NORMSINV(P)
Inverse of the standard normal cumulative distribution. Given a probability, *P*, this function returns the value *Z* such that the probability of a normally distributed random variable with a mean of 0 and a standard deviation of 1 being less-than-or-equal to *Z* is *P*.

@PBN(P, N, X)
Cumulative binomial probability. It returns the probability that a sample of *N* items, from a universe with a fraction of *P* of those items defective, has *X* or less defective items. It is extended to noninteger values of *X* and *N* by linear interpolation. This function is being replaced by the *@PBINOCDF* function, documented below in the *Distributions* section.

@PCX(N, X)
Cumulative distribution function for the Chi-squared distribution with *N* degrees of freedom. It returns the probability that an observation from this distribution is less-than-or-equal-to *X*. This function is being replaced by the *@PCHISCDF* function, documented below in the *Distributions* section.

***@PEB*(*A*, *X*)**
Erlang's busy probability for a service system with *X* servers and an arriving load of *A*, *with infinite queue allowed*. The result of *@PEB* can be interpreted as either the fraction of time all servers are busy or the fraction of customers that must wait in the queue. It is extended to noninteger values of *X* by linear interpolation. The arriving load, *A*, is the expected number of customers arriving per unit of time multiplied by the expected time to process one customer.

***@PEL*(*A*, *X*)**
Erlang's loss probability for a service system with *X* servers and an arriving load of *A*, *no queue allowed*. The result of *@PEL* can be interpreted as either the fraction of time all servers are busy or the fraction of customers lost due to all servers being busy when they arrive. It is extended to noninteger values of *X* by linear interpolation. The arriving load, *A*, is the expected number of customers arriving per unit of time multiplied by the expected time to process one customer.

***@PFD*(*N*, *D*, *X*)**
Cumulative distribution function for the F distribution with *N* degrees of freedom in the numerator and *D* degrees of freedom in the denominator. It returns the probability that an observation from this distribution is less-than-or-equal-to *X*. This function is being replaced by the *@PFDSTCDF* function, documented below in the *Distributions* section.

***@PFS*(*A*, *X*, *C*)**
This returns the expected number of customers waiting for or under repair in a finite source Poisson service system with *X* servers in parallel, *C* customers, and a limiting load *A*. It is extended to noninteger values of *X* and *C* by linear interpolation. *A*, the limiting load, is the number of customers multiplied by the mean service time divided by the mean repair time.

***@PHG*(*POP*, *G*, *N*, *X*)**
Cumulative hypergeometric probability. It returns the probability that *X* or fewer items in the sample are good, given a sample without replacement of *N* items from a population of size *POP* where *G* items in the population are good. It is extended to noninteger values of *POP*, *G*, *N*, and *X* by linear interpolation. This function is being replaced by the *@PHYPGCDF* function, documented below in the *Distributions* section.

***@PPL*(*A*, *X*)**
Linear loss function for the Poisson distribution. It returns the expected value of *MAX(0, Z-X)*, where *Z* is a Poisson random variable with mean value *A*.

***@PPS*(*A*, *X*)**
Cumulative Poisson probability distribution. It returns the probability that a Poisson random variable, with mean value *A*, is less-than-or-equal-to *X*. It is extended to noninteger values of *X* by linear interpolation. This function is being replaced by the *@PPOISCDF* function, documented below in the *Distributions* section.

***@PSL*(*X*)**
Unit normal linear loss function. It returns the expected value of *MAX(0, Z-X)*, where *Z* is a standard normal random variable. In inventory modeling, *@PSL(X)* is the expected amount that demand exceeds a level *X*, if demand has a standard normal distribution.

***@PSN*(*X*)**
Cumulative standard normal probability distribution. A standard normal random variable has mean 0.0 and standard deviation 1.0 (the bell curve, centered on the origin). The value returned by *@PSN* is the area under the curve to the left of the point on the ordinate indicated by *X*. This function is being replaced by the *@PNORMCDF* function, documented below in the *Distributions* section.

***@PTD*(*N*, *X*)**
Cumulative distribution function for the t distribution with *N* degrees of freedom. It returns the probability that an observation from this distribution is less-than-or-equal-to *X*. This function is being replaced by the *@PSTUTCDF* function, documented below in the *Distributions* section.

***@QRAND*(*SEED*)**
@QRAND produces a sequence of "quasi-random" uniform numbers in the *n*-cube (0,1)^*n*. *@QRAND* is only permitted in a data and calc sections. It will fill an entire attribute with quasi-random numbers. Generally, you will be filling two-dimensional tables with, say, *m* rows and *n* variables. *m* represents the number of scenarios, or experiments, you want to run. *n* represents the number of random variables you need for each scenario or experiment. Within a row, the numbers are independently distributed. Among rows, the numbers are "super uniformly" distributed. That is, the numbers are more uniformly distributed than you would expect by chance. These numbers are generated by a form of "stratified sampling".

For example, suppose *m* = 4 and *n* = 2. Even though the numbers are random, you will find that there will be exactly one row in which both numbers are in the interval (0, .5), exactly one row in which both numbers are in (.5, 1), and two rows in which one number is less than .5 and the other is greater than .5. Using *@QRAND* allows you to get much more accurate results for a given number of random numbers in a Monte Carlo model. If you want 8 ordinary random numbers, then use *@QRAND(1,8)* rather than *@QRAND(4,2).* An example of *@QRAND* follows:

```
MODEL:
   DATA:
      M = 4;
      N = 2;
      SEED = 1234567;
   ENDDATA
   SETS:
      ROWS /1..M/;
      COLS /1..N/;
      TABLE( ROWS, COLS): X;
   ENDSETS
   DATA:
      X = @QRAND( SEED);
      @TEXT() = @TABLE( X);
   ENDDATA
END
```

Example of @QRAND function

This example produces the following table of quasi-random numbers:

	1	2
1	0.2085788	0.1381721
2	0.6283858	0.2530084
3	0.3767461	0.7546936
4	0.9335576	0.5737700

Variable Domain Functions

The default assumption for a variable is that it is continuous with a lower bound of 0. Variable domain functions place additional restrictions on the values that variables can assume. The functions and their effects are as follows:

@BIN(variable)
Restricts *variable* to being a binary (0/1) integer value.

@BND(lower_bound, variable, upper_bound)
Limits *variable* to being greater-than-or-equal-to *lower_bound* and less-than-or-equal-to *upper_bound*.

@CARD('card_set_name', variable|N)
Use *@CARD* to restrict a a set of variables to have a cardinality of *N*. See section *Cardinality* for more information.

@FREE(variable)
Removes the default lower bound of zero on *variable*, allowing it to take any positive or negative value.

@GIN(variable)
Restricts *variable* to integer values (e.g., 0,1,2, ...).

@PRIORITY(variable, priority)
Sets the branching priority for integer variable *variable* to the value *priority*. Variables with higher priorities will tend to be branched on sooner than variables with lower priorities.

@SEMIC(lower_bound, variable, upper_bound)
Restricts *variable* to being either 0 o in the range of [*lower_bound, upperbound*]. Refer to section *Semicontinuous Variables* for more information.

@SOS{1|2|3}('sos_set_name', variable)
Use the *@SOS1, @SOS2* and *@SOS3* functions to set up special ordered sets of variables. Refer to section *SOS Variables* for more information.

@POSD(matrix)
Use the *@POSD* functions to force a matrix of variables to be positive semi-definite. Refer to section *Positive Semi-Definite Matrices* for more information.

You may use the *@FOR* function to apply variable domain functions to all the elements of an attribute.

Variable domain functions are discussed in detail in *Using Variable Domain Functions*.

@ALLDIFF('set_name', variable_name)
Requires a specified set of variables to taking on different integer values.

Set Handling Functions

LINGO offers several functions that assist with the manipulation of sets:

- ***@IN(set_name, set_member)*** — Returns true if a set member is contained in set *set_name*, else false.
- ***@INSERT(set_name, set_member)*** — Inserts a set member into the set *set_name*.— Returns the index of *set_member* in set *set_name*. Returns 0 if the set member is not contained in the specified set.
- ***@NULLSET(set_name)*** — Deletes all set members from the derived set, *set_name*.
- **@SIZE(set_name)** — Returns the number of set members in set *set_name*.
- ***@WRAP(index, limit)*** — Used to "wrap" a set index from one end of a time horizon to another in multiperiod planning models.

Each of these functions are discussed in more detail below.

@IN(set_name, primitive_1_index [, primitive_2_index ...])
This returns TRUE if the set member referenced by the primitive set member index tuple (*primitive_1_index*, *primitive*_2_index, ...) is contained in the *set_name* set. As the following example shows, the *@IN* operator is useful for generating complements of subsets in set membership conditions.

Example 1:
For example, to derive a set of open plants based on a subset of closed plants, your sets section might resemble the following:

```
SETS:
   PLANTS / SEATTLE, DENVER,
    CHICAGO, ATLANTA/:;
   CLOSED( PLANTS) /DENVER/:;
   OPEN( PLANTS) |
    #NOT# @IN( CLOSED, &1):;
ENDSETS
```

The *OPEN* set is derived from the *PLANTS* set. We use a membership condition containing the *@IN* function to allow only those plants not contained in the *CLOSED* set to be in the *OPEN* set.

Example 2:
In this example, we illustrate how to determine if the set element (*B*, *Y*) belongs to the derived *S3* set. In this case, (*B*, *Y*) is indeed a member of *S3*, so *X* will be set to 1. Note that, in order to get the index of the primitive set elements *B* and *Y*, we made use of the *@INDEX* function, which is discussed next.

```
SETS:
   S1 / A B C/:;
   S2 / X Y Z/:;
   S3( S1, S2) / A,X A,Z B,Y C,X/:;
ENDSETS
   X = @IN( S3, @INDEX( S1, B), @INDEX( S2, Y));
```

@INDEX(set_name, set_member)
This returns the index of a set member *set_member* in the set *set_name.* If the specified set member does not belong to the set *@INDEX* will return 0. Unlike *@IN*, which requires you to specify the indices for the set elements, *@INDEX* allows you to refer to set member names directly.

Example 1:
In this example, we illustrate how to get the index of set member (*R1, C3*) in the derived *S3* set. In this case, (*R1*, *C3*) is the third member *SRXC*, so *NDX* will be set to 3.

```
SETS:
  ROWS /R1..R27/;
  COLS /C1..C3/;
  RXC( ROWS, COLS): XRNG;
ENDSETS
! return the index of (r1,c3) in the rxc set;
NDX = @INDEX( RXC, R1, C3);
```

LINGO allows you to omit the set name argument if the set member belongs to a primitive set. This is to maintain compatibility with earlier releases of LINGO. As the following example illustrates, it's good practice to always specify a set name in the *@INDEX* function:

Example 2:
A model's set elements can come from external sources that the modeler may have little control over. This can potentially lead to confusion when using the *@INDEX* function. Consider the sets section:

```
SETS:
   GIRLS /DEBBIE, SUE, ALICE/;
   BOYS /BOB, JOE, SUE, FRED/;
ENDSETS
```

Now, suppose you want to get the index of the boy named Sue within the *BOYS* set. The value of this index should be 3. Simply using *@INDEX(SUE)* would return 2 instead of 3, because LINGO finds *SUE* in the *GIRLS* set first. In this case, to get the desired result, you *must* specify the *BOYS* set as an argument and enter *@INDEX(BOYS, SUE)*.

Note: The set member argument to @INDEX is considered to be a text literal. The following example illustrates how this can potentially lead to unexpected results.

Example 3:
In the model below, we loop over the *GIRLS* set using a set index variable named *G*. Given that *@INDEX* considers the set member argument (*G* in this case) to be a text literal, each element of *XINDEX* will be set to 0; this is because the *GIRLS* set does not contain a set member called *G*. On the other hand, the elements of *XIN* will all be set to 1, because *@IN* treats *G* as a set index variable, as opposed to a text literal.

```
SETS:
GIRLS /DEBBIE, SUE, ALICE/: XINDEX, XIN;
BOYS /BOB, JOE, SUE, FRED/;
ENDSETS
!XINDEX will be 0 because GIRLS does not contain the member 'G';
!XIN will be 1 because G is treated as a set index variable as
opposed to the text literal 'G';
@FOR( GIRLS( G):
XINDEX( G) = @INDEX( GIRLS, G);
XIN( G) = @IN( GIRLS, G);
);
```

@INSERT(set_name, primitive_1_index [, primitive_2_index ...])
This function may be used to dynamically add members to derived sets. Each of the primitive set members forming the new derived set member must have been included in their respective primitive sets.

The following example illustrates:

```
MODEL:

SETS:
  PRIMITIVE;
  LATTICE( PRIMITIVE, PRIMITIVE);
ENDSETS

DATA:
  PRIMITIVE = 1..10;
ENDDATA

CALC:
! Starting at 1,1, generate the lattice of points reachable
  by the two integer vectors;
  X1 = 1; Y1 = 3;
  X2 = 7; Y2 = 5;

  @INSERT( LATTICE, 1,1); ! insert the seed;

  @SET( 'TERSEO', 3); !minimal output;

! Now generate all the points reachable, directly or
  indirectly from the seed via the two vectors within
  a finite region;
  @FOR( LATTICE( I, J) | I + I #LE# @SIZE( PRIMITIVE):

    I1 = I + X1;
    J1 = J + Y1;
    @IFC( #NOT# @IN( LATTICE, I1, J1):
      @IFC( @IN( PRIMITIVE, I1) #AND# @IN( PRIMITIVE, J1):
         @INSERT( LATTICE, I1, J1);
       );
    );

    I1 = I + X2;
    J1 = J + Y2;
    @IFC( #NOT# @IN( LATTICE, I1, I1):
      @IFC( @IN( PRIMITIVE, I1) #AND# @IN( PRIMITIVE, J1):
        @INSERT( LATTICE, I1, J1);
       );
    );
  );

ENDCALC

DATA:
  !display the lattice set;
  @TEXT() = @TABLE( LATTICE);
ENDDATA
```

```
END
```

Model: LATTICE

In this model, we want to find all the points on a 10-by-10 grid that are reachable, either directly or indirectly, from point (1,1) using combinations of the two vectors (1,3) and (7,5). The grid is represented by the 2-dimensional set *LATTICE*. Initially, *LATTICE* is empty, but we will add members using *@INSERT* if we discover they are reachable from the seed (1,1).

First, we place the seed into the *LATTICE* with the following insert statement:

```
@INSERT( LATTICE, 1, 1); ! insert the seed;
```

The following doubly-nested loop iterates through the entire 10x10 grid:

```
@FOR( LATTICE( I, J) | I #LE# @SIZE( PRIMITIVE):
  @FOR( VECTORS( I2, J2):
    I3 = I + I2;
    J3 = J + J2;
    @IFC( #NOT# @IN( LATTICE, I3, J3):
      @IFC( @IN( PRIMITIVE, I3) #AND# @IN( PRIMITIVE, J3):
        @INSERT( LATTICE, I3, J3);
      );
    );
  );
);
```

Each of the two vectors are then added individually to the current point, with the resulting point tested to see if a) if it has not already been added to *LATTICE,* and b) if it lies within the 10x10 grid. If the new point passes both these tests, it gets added to the lattice using an *@INSERT* statement:

```
@INSERT( LATTICE, I3, J3);
```

At the end of the model we use the *@TABLE* output function to display the lattice we found:

	1	2	3	4	5	6	7	8	9	10
1	X									
2				X						
3							X			
4										X
5										
6										
7										
8						X				
9									X	
10										

Here we see that, in addition to the seed, the following points are members of the lattice: (2,4), (3,7), (4,10), (8,6) and (9,9).

@NULLSET(set_name)
This function deletes all members of the derived set set_name.

Example 1:
In this example, we create a 3x3 derived set, *S2*, with 9 members. We then delete all of *S2*'s set members with *@NULLSET* and verify *S2* is empty by showing its member count becomes 0:

```
MODEL:

SETS:
 S1;
 S2(S1,S1);
ENDSETS

DATA:
 S1 = A B C;
ENDDATA

CALC:

 ! Print out the current size of set S2;
 @WRITE( 'Set S2 size = ', @SIZE( S2), @NEWLINE( 1));

 ! Delete all members from S2;
 @NULLSET( S2);

 ! Verify that S2 now has no members;
 @WRITE( 'Set S2 size after @NULLSET() = ', @SIZE( S2),
     @NEWLINE( 2));

ENDCALC

END
```

Running this model, we get the following output, showing that all members of *S2* were removed by *@NULLSET*:

```
Set S2 size = 9
Set S2 size after @NULLSET() = 0
```

Note that *@NULLSET* does not currently work on primitive sets. But you can workaround this by creating a derived set directly from a primitive set, and then null out this derived set. For example:

```
SETS:
  MYPRIMITIVESET1 /1..3/;
  MYDERIVEDSET( MYSET1);
ENDSETS
CALC:
  ! The following nulling of a primitive set is not allowed;
  @NULLSET( MYPRIMITIVESET);

  ! The next statement is allowed, given that we are nulling a
   derived set;
  @NULLSET( MYDERIVEDSET);
ENDCALC
```

Once a set is nulled, you will typically add selected members back using the @INSERT function, described above.

@SIZE(set_name)
This returns the number of elements in the *set_name* set. Using the *@SIZE* function is preferred to explicitly listing the size of a set in a model. This serves to make your models more data independent and, therefore, easier to maintain should the size of your sets change.

To view an example of the *@SIZE* function, refer to the PERT/CPM example in *the Sparse Derived Set Example - Explicit List* section of *Using Sets*.

@WRAP(INDEX, LIMIT)
This allows you to "wrap" an index around the end of a set and continue indexing at the other end of the set. That is, when the last (first) member of a set is reached in a set looping function, use of *@WRAP* will allow you to wrap the set index to the first (last) member of the set. This is a particularly useful function in cyclical, multiperiod planning models.

Formally, *J = @WRAP(INDEX, LIMIT)* returns *J* such that *J = INDEX - K * LIMIT*, where *K* is an integer such that 1 <= *J* < *LIMIT*+1. Informally, *@WRAP* will subtract or add *LIMIT* to *INDEX* until it falls in the range 1 to *LIMIT + 0.99999*.

For an example on the use of the *@WRAP* function in a staff scheduling model, refer to the *Primitive Set Example* section in *Using Sets*.

Set Looping Functions

Set looping functions operate over an entire set and, with the exception of the *@FOR* function, produce a single result. The syntax for a set looping function is:

@function(*setname* [(*set_index_list*) [| *conditional_qualifier*]] : *expression_list*);

@function corresponds to one of the set looping functions listed below. *setname* is the name of the set you want to loop over. The *set_index_list* is optional, and is used to create a list of indices, which correspond to the parent primitive sets that form the set *setname*. As LINGO loops through the members of the set *setname*, it will set the values of the indices in the *set_index_list* to correspond to the current member of the set *setname*.

The *conditional_qualifier* is optional and may be used to limit the scope of the set looping function. When LINGO is looping over each member of the set *setname*, it evaluates the *conditional_qualifier*. If the *conditional_qualifier* evaluates to true, then *@function* is performed for the set member. Otherwise, it is skipped.

The *expression_list* is a list of expressions to be applied to each member of the set *setname*. When using the *@FOR* function, the *expression_list* may contain multiple expressions, separated by semicolons. These expressions will be added as constraints to the model. When using the remaining three set looping functions (*@SUM*, *@MAX*, and *@MIN*), the *expression_list* must contain one expression only. If the *set_index_list* is omitted, all attributes referenced in the *expression_list* must be defined on the set *setname*.

The available set looping functions are listed below:

***@FOR* (*setname* [(*set_index_list*) [| *cond_qualifier*]]: *exp_list*)**
This generates the expressions contained in *exp_list* for all members of the set *setname*.

***@MAX*(*setname* [(*set_index_list*) [| *cond_qualifier*]]: *expression*)**
This returns the maximum value of *expression* taken over the set *setname*.

***@MIN*(*setname* [(*set_index_list*) [| *cond_qualifier*]]: *expression*)**
This returns the minimum value of *expression* taken over the set *setname*.

***@PROD*(*setname* [(*set_index_list*) [| *cond_qualifier*]]: *expression*)**
This returns the product of an expression over the *setname* set.

@S***UM*(*setname* [(*set_index_list*) [| *cond_qualifier*]]: *expression*)**
This returns the sum of expression over the set setname.

Set looping functions are discussed in more detail in Chapter 2, *Using Sets*.

Interface Functions

Interface functions allow you to link your model to external data sources such as text files, databases, spreadsheets and external applications. *With the exception of @FILE, interface functions are valid only in sets and data sections, and may not be used in calc and model sections.* The interface functions currently available in LINGO are listed below.

@FILE ('filename')
The *@FILE* function allows you to include data from external text files anywhere in your model, where *filename* is the name of the file to include text form. This is particularly useful for incorporating data stored in text files in your sets and data sections.

When this function is encountered in a model, LINGO will continue to take text from this file until it encounters either the end-of-file or a LINGO end-of-record mark (~). For subsequent *@FILE* references in the same model that use the same file name, LINGO resumes taking input from the file at the point where it left off. Nesting of *@FILE* function calls (embedding an *@FILE* in a file which is itself called by *@FILE*) is not allowed.

For more information on use of the *@FILE* function, refer to *Interfacing with External Files.*

@ODBC (['data_source'[, 'table_name'[, 'col_1'[, 'col_2' ...]]]])
The *@ODBC* function is used to open ODBC links between LINGO and databases. You can use *@ODBC* in the sets section to retrieve set members from a database, or in the data section to import data and/or export solutions.

The *data_source* is the name of the ODBC data source you registered with the ODBC Administrator. The *table_name* is the name of the table in the *data_source* you want to open a link to. Finally, *col_i* is the column in the table *table_name* that you wish to link to.

The *@ODBC* function is discussed in detail in *Interfacing with Databases.*

@OLE ('workbook_file'[, range_name_list])
The *@OLE* function is used to move data and solutions back and forth from Excel using OLE based transfers. You can use *@OLE* in the sets section to retrieve set members from Excel, or in the data section to import data and/or export solutions.

OLE transfers are direct memory transfers and do not make use of intermediate files. When using *@OLE* for exports, LINGO loads Excel, tells Excel to load the desired spreadsheet, and sends ranges of data containing solution values to the sheet. You must have Excel 5, or later, to use the *@OLE* function. The *@OLE* function is valid only in data and sets sections. *@OLE* can export two-dimensional ranges (rectangular ranges that lie on a single worksheet in Excel), but cannot export three-dimensional ranges (ranges which traverse more than one worksheet in Excel) or discontinuous ranges.

The *workbook_file* argument is the name of the workbook to link to. The *range_name_list* is the list of named ranges in the sheet to link to.

For more information on use of the *@OLE* function, refer to *Interfacing with Spreadsheets.*

@POINTER (N)
This function is strictly for use with the LINGO callable API. *@POINTER* allows you to transfer data directly through shared memory locations. For more information on the use of the *@POINTER* function, refer to *Interfacing with Other Applications.*

@TEXT ([['filename'], 'a'])
The *@TEXT* function is used in the data section of a model to export solutions to text files, where *filename* is the name of the file you want to export the solution to. If *filename* is omitted, the solution data will be sent to the standard output device (in most cases this corresponds to the screen). If you specify a file name and you wish to append output to the file instead of overwriting the file, include a second argument of 'a'.

For additional documentation on the *@TEXT* function, see *Interfacing with External Files.*

Distributions

Distribution functions are available for an extensive number of probability distributions. LINGO supports the probability density functions (PDF) for each distribution, as well as their cumulative (CDF) and inverse functions. Supported distributions are listed below:

Continuous Distribution Functions	Description	Parameters and Domain
@PBETACDF(A, B, X)	Cumulative Beta	A = alpha > 0 B = beta > 0 $X \in (0,1)$
@PBETAINV(A, B, X)	Inverse Beta	A = alpha > 0 B = beta > 0 $X \in [0,1]$
@PBETAPDF(A, B, X)	Beta PDF	A = alpha > 0 B = beta > 0 $X \in (0,1)$
@PCACYCDF(L, S, X)	Cumulative Cauchy	L = location S = scale > 0 X a real
@PCACYINV(L, S, X)	Inverse Cauchy	L = location S = scale > 0 $X \in [0,1]$

@PCACYPDF(L, S, X)	Cauchy PDF	*L* = location *S* = scale > 0 *X* a real
@PCHISCDF(DF, X)	Cumulative Chi-Square	*DF* = degrees of freedom = a positive integer *X* ≥ 0
@PCHISINV(DF, X)	Inverse Chi-Square	*DF* = degrees of freedom = a positive integer *X* ∈ [0,1]
@PCHISPDF(DF, X)	Chi-Square PDF	*DF* = degrees of freedom = a positive integer *X* ≥ 0
@PEXPOCDF(L, X)	Cumulative Exponential	*L* = lambda > 0 *X* ≥ 0
@PEXPOINV(L, X)	Inverse Exponential	*L* = lambda > 0 *X* ∈ [0,1]
@PEXPOPDF(L, X)	Exponential PDF	*L* = lambda > 0 *X* ≥ 0
@PFDSTCDF(DF1, DF2, X)	Cumulative F-Distribution	*DF1,DF2* = degrees of freedom = a positive integer *X* ≥ 0
@PFDSTINV(DF1, DF2, X)	Inverse F-Distribution	*DF1,DF2* = degrees of freedom = a positive integer *X* ∈ [0,1]
@PFDSTPDF(DF1, DF2, X)	F-Distribution PDF	*DF1,DF2* = degrees of freedom = a positive integer *X* ≥ 0
@PGAMMCDF(SC, SH, X)	Cumulative Gamma	*SC* = scale > 0 *SH* = shape > 0 X ≥ 0
@PGAMMINV(SC, SH, X)	Inverse Gamma	*SC* = scale > 0 *SH* = shape > 0 *X* ∈ [0,1]
@PGAMMPDF(SC, SH, X)	Gamma PDF	*SC* = scale > 0 *SH* = shape > 0 X ≥ 0
@PGMBLCDF(L, S, X)	Cumulative Gumbel	*L* = location

		S = scale > 0 X a real
@PGMBLINV(L, S, X)	Inverse Gumbel	L = location S = scale > 0 $X \in [0,1]$
@PGMBLPDF(L, S, X)	Gumbel PDF	L = location S = scale > 0 X a real
@PLAPLCDF(L, S, X)	Cumulative Laplace	L = location S = scale > 0 X a real
@PLAPLINV(L, S, X)	Inverse Laplace	L = location S = scale > 0 $X \in [0,1]$
@PLAPLPDF(L, S, X)	Laplace PDF	L = location S = scale > 0 X a real
@PLGSTCDF(L, S, X)	Cumulative Logistic	L = location S = scale > 0 X a real
@PLGSTINV(L, S, X)	Inverse Logistic	L = location S = scale > 0 $X \in [0,1]$
@PLGSTPDF(L, S, X)	Logistic PDF	L = location S = scale > 0 X a real
@PLOGNCDF(M, S, X)	Cumulative Lognormal	M = mu S = sigma > 0 $X > 0$
@PLOGNINV(M, S, X)	Inverse Lognormal	M = mu S = sigma > 0 $X \in [0,1]$
@PLOGNPDF(M, S, X)	Lognormal PDF	M = mu S = sigma > 0 $X > 0$
@PNORMCDF(M, S, X)	Cumulative Normal	M = mu S = sigma > 0 X a real

@PNORMINV(M, S, X)	Inverse Normal	M = mu S = sigma > 0 $X \in [0,1]$
@PNORMPDF(M, S, X)	Normal PDF	M = mu S = sigma > 0 X a real
@PPRTOCDF(SC, SH, X)	Cumulative Pareto	SC = scale > 0 SH = shape > 0 $X \geq SC$
@PPRTOINV(SC, SH, X)	Inverse Pareto	SC = scale > 0 SH = shape > 0 $X \in [0,1]$
@PPRTOPDF(SC, SH, X)	Pareto PDF	SC = scale > 0 SH = shape > 0 $X \geq SC$
@PSMSTCDF(A, X)	Cumulative Symmetric Stable	A = alpha $\in [0.2,2]$ X a real
@PSMSTINV(A, X)	Inverse Symmetric Stable	A = alpha $\in [0.2,2]$ $X \in [0,1]$
@PSMSTPDF(A, X)	Symmetric Stable PDF	A = alpha $\in [0.2,2]$ X a real
@PSTUTCDF(DF, X)	Cumulative Student's t	DF = degrees of freedom = a positive integer X a real
@PSTUTINV(DF, X)	Inverse Student's t	DF = degrees of freedom = a positive integer $X \in [0,1]$
@PSTUTPDF(DF, X)	Student's t PDF	DF = degrees of freedom = a positive integer X a real
@PTRIACDF(L, U, M, X)	Cumulative Triangular	L = lower limit U = upper limit M = mode $X \in [L,U]$
@PTRIAINV(L, U, M, X)	Inverse Triangular	L = lower limit U = upper limit M = mode

		$X \in [0,1]$
@PTRIAPDF(L, U, M, X)	Triangular PDF	L = lower limit U = upper limit M = mode $X \in [L,U]$
@PUNIFCDF(L, U, X)	Cumulative Uniform	L = lower limit U = upper limit $X \in [L,U]$
@PUNIFINV(L, U, X)	Inverse Uniform	L = lower limit U = upper limit $X \in [0,1]$
@PUNIFPDF(L, U, X)	Uniform PDF	L = lower limit U = upper limit $X \in [L,U]$
@PWEIBCDF(SC, SH, X)	Cumulative Weibull	SC = scale > 0 SH = shape > 0 $X \geq 0$
@PWEIBINV(SC, SH, X)	Inverse Weibull	SC = scale > 0 SH = shape > 0 $X \in [0,1]$
@PWEIBPDF(SC, SH, X)	Weibull PDF	SC = scale > 0 SH = shape > 0 $X \geq 0$

Discrete Distribution Functions	**Description**	**Parameters**
@PBTBNCDF(N, A, B, X)	Cumulative Beta Binomial	N = trials $\in \{0,1,...\}$ A = alpha $\in$ (0,+inf) B = beta $\in$ (0,+inf) $X \in$ [0, +inf)
@PBTBNINV(N, A, B, X)	Beta Binomial Inverse	N = trials $\in \{0,1,...\}$ A = alpha $\in$ (0,+inf) B = beta $\in$ (0,+inf) $X \in$ [0,+inf)
@PBTBNPDF(N, A, B, X)	Beta Binomial PDF	N = trials $\in \{0,1,...\}$ A = alpha $\in$ (0,+inf)

		B = beta ∈ (0,+inf) X ∈ [0,+inf)
@PBINOCDF(N, P, X)	Cumulative Binomial	N = trials ∈ {0,1,...} P = probability of success ∈ [0,1] X ∈ {0,1,...,N}
@PBINOINV(N, P, X)	Inverse Binomial	N = trials ∈ {0,1,...} P = probability of success ∈ [0,1] X ∈ [0,1]
@PBINOPDF(N, P, X)	Binomial PDF	N = trials ∈ {0,1,...} P = probability of success ∈ [0,1] X ∈ {0,1,...,N}
@PGEOMCDF(P, X)	Cumulative Geometric	P = probability of success ∈ (0,1] X ∈ {0,1,...}
@PGEOMINV(P, X)	Inverse Geometric	P = probability of success ∈ (0,1] X ∈ [0,1]
@PGEOMPDF(P, X)	Geometric PDF	P = probability of success∈ (0,1] X ∈ {0,1,...}
@PHYPGCDF(N, D, K, X)	Cumulative Hypergeometric	N = population ∈ {0,1,...} D = number defective ∈ {0,1,...,N} K = sample size ∈ {0,1,...,N} X ∈ {max(0,D+K-N),...,min(D,K)}
@PHYPGINV(N, D, K, X)	Inverse Hypergeometric	N = population ∈ {0,1,...} D = number defective ∈ {0,1,...,N} K = sample size ∈ {0,1,...,N} X ∈ [0,1]
@PHYPGPDF(N, D, K, X)	Hypergeometric PDF	N = population ∈ {0,1,...} D = number defective ∈ {0,1,...,N} K = sample size ∈ {0,1,...,N} X ∈ {max(0,D+K-N),...,min(D,K)}

@PLOGRCDF(P, X)	Cumulative Logarithmic	P = p-factor ∈ (0,1) X ∈ {0,1,...,N}
@PLOGRINV(P, X)	Inverse Logarithmic	P = p-factor ∈ (0,1) X ∈ [0,1]
@PLOGRPDF(P, X)	Logarithmic PDF	P = p-factor ∈ (0,1) X ∈ {0,1,...,N}
@PNEGBCDF(R, P, X)	Cumulative Negative Binomial	R = number of failures ∈ (0,+inf) P = probability of success ∈ (0,1) X ∈ {0,1,...}
@PNEGBINV(R, P, X)	Inverse Negative Binomial	R = number of failures ∈ (0,+inf) P = probability of success ∈ (0,1) X ∈ [0,1]
@PNEGBPDF(R, P, X)	Negative Binomial PDF	R = number of failures ∈ (0,+inf) P = probability of success ∈ (0,1) X ∈ {0,1,...}
@PPOISCDF(L, X)	Cumulative Poisson	L = lambda ∈ (0,+inf) X ∈ {0,1,...,N}
@PPOISINV(L, X)	Inverse Poisson	L = lambda ∈ (0,+inf) X ∈ [0,1]
@PPOISPDF(L, X)	Poisson PDF	L = lambda ∈ (0,+inf) X ∈ {0,1,...,N}

Report Functions

Report functions are used to construct reports based on a model's results, and are valid on both calc and data sections. Combining report functions with interface functions in a data section allows you to export the reports to text files, spreadsheets, databases, or your own calling application.

Note: The interested reader will find an exhaustive example of the use of report functions in the sample model *TRANSOL.LG4* in the *Samples* subfolder. This model makes extensive use of many of the report functions to mimic the standard LINGO solution report.

@DUAL (variable_or_row_name)
The *@DUAL* function outputs the dual value of a variable or a row. For example, consider a model with the following data section:

```
DATA:
   @TEXT( 'C:\RESULTS\OUTPUT.TXT') =
    @WRITEFOR( SET1( I): X( I), @DUAL( X( I)), @NEWLINE(1));
ENDDATA
```

When this model is solved, the values of attribute *X* and their reduced costs will be written to the file *C:\RESULTS\OUTPUT.TXT*. Output may be routed to a file, spreadsheet, database or memory location. The exact destination will depend on the export function used on the left-hand side of the output statement.

If the argument to the *@DUAL* function is a row name, then the dual price on the generated row will be output.

@FORMAT (value, format_descriptor)
@FORMAT may be used in *@WRITE* and *@WRITEFOR* statements to format a numeric or string value for output as text, where *value* is the numeric or string value to be formatted, and *format_descriptor* is a string detailing how the number is to be formatted. The format descriptor is interpreted using C programming conventions. For instance, a format descriptor of '12.2f' would cause a numeric value to be printed in a field of 12 characters with 2 digits after the decimal point. For a string values, such as a set member name, a format descriptor of '12s' would cause the string to be right justified in a field of 12 characters, while '-12s' would cause the string to be left justified in a field of 12. You can refer to a C reference manual for more details on the available formatting options.

The following example uses the *@FORMAT* function to place a shipping quantity into a field of eight characters with no trailing decimal value:

```
DATA:
  @TEXT() = @WRITE( ' From    To       Quantity', @NEWLINE(1));
  @TEXT() = @WRITE( '-------------------------', @NEWLINE(1));
  @TEXT() = @WRITEFOR( ROUTES( I, J) | X( I, J) #GT# 0:
     3*' ', WAREHOUSE( I), 4*' ',
    CUSTOMER( J), 4*' ', @FORMAT( X( I, J), '8.0f'),
    @NEWLINE( 1));
ENDDATA
```

The report will resemble the following:

```
From    To       Quantity
-------------------------
  WH1   C1           2
  WH1   C2          17
  WH1   C3           1
  WH2   C1          13
  WH2   C4          12
  WH3   C3          21
```

This next example, *GRAPHPSN.LG4,* graphs the standard normal function, *@PSN,* over the interval [–2.4, 2.4]. The *@FORMAT* function is used to print the *X* coordinates with one trailing decimal point.

```
! Graphs @PSN() over a specified
  interval around 0;
DATA:
   ! height of graph;
   H = 49;
   ! width of graph;
   W = 56;
   ! interval around 0;
   R = 2.4;
ENDDATA
SETS:
  S1 /1..H/: X, FX;
ENDSETS
@FOR( S1( I):
   ! X can be negative;
   @FREE( X);
   ! Compute x coordinate;
   X( I) = -R +  ( I - 1)* 2 * R / ( H - 1);
   ! Compute y coordinate = @psn( x);
   FX( I) = @PSN( X( I));
);
DATA:
  ! Print the header;
  @TEXT() = @WRITE(
   'Graph of @PSN() on the interval [-',
   R,',+',R,']:',@NEWLINE(1));
  @TEXT() = @WRITE( '| 0 ',(W/2-5)*'-',
   ' 0.5 ',(W/2-5)*'-', '1.0   X(i)',@NEWLINE(1));
  ! Loop to print the graph over;
  @TEXT() = @WRITEFOR( S1( I): '| ',
   ( W * FX( I) + 1/2) * '*',
   @IF( X( I) #LT# 0, '', ' '), ( W -
   ( W * FX( I) + 1/2) + 3)*' ',
   @FORMAT( X(I), '.1f'),@NEWLINE(1));
  !Trailer;
  @TEXT() = @WRITE( '| 0 ',(W/2-5)*'-',
   ' 0.5 ',(W/2-5)*'-', '1.0',@NEWLINE(1));
ENDDATA
```

Model: GRAPHPSN

Here is how the graph will appear when the model is solved:

```
Graph of @PSN() on the interval [-2.4,+2.4]:
| 0 ----------------------- 0.5 -----------------------1.0   X(i)
|                                                           -2.4
| *                                                         -2.3
| *                                                         -2.2
| *                                                         -2.1
| *                                                         -2.0
| **                                                        -1.9
| **                                                        -1.8
| **                                                        -1.7
| ***                                                       -1.6
| ****                                                      -1.5
| *****                                                     -1.4
| *****                                                     -1.3
| ******                                                    -1.2
| ********                                                  -1.1
| *********                                                 -1.0
| **********                                                -0.9
| ************                                              -0.8
| **************                                            -0.7
| ***************                                           -0.6
| *****************                                         -0.5
| *******************                                       -0.4
| **********************                                    -0.3
| *************************                                 -0.2
| ***************************                               -0.1
| ******************************                             0.0
| ********************************                           0.1
| **********************************                         0.2
| ************************************                       0.3
| **************************************                     0.4
| ****************************************                   0.5
| ******************************************                 0.6
| ********************************************               0.7
| **********************************************             0.8
| ************************************************           0.9
| **************************************************         1.0
| ****************************************************       1.1
| ******************************************************     1.2
| ********************************************************   1.3
| **********************************************************  1.4
| ************************************************************ 1.5
| ************************************************************** 1.6
| **************************************************************** 1.7
| ****************************************************************  1.8
| ******************************************************************  1.9
| ********************************************************************  2.0
| **********************************************************************  2.1
| ************************************************************************  2.2
| **************************************************************************  2.3
| ****************************************************************************  2.4
| 0 ----------------------- 0.5 -----------------------1.0
```

@ITERS ()
The *@ITERS* function returns the total number of iterations required to solve the model. *@ITERS* is available only in the data and calc sections, and is not allowed in the constraints of a model. For example, the following output statement writes the iteration count to the standard output device:

```
DATA:
   @TEXT() = @WRITE('Iterations= ', @ITERS());
ENDDATA
```

@NAME (var_or_row_reference)
Use *@NAME* to return the name of a variable or row as text. *@NAME* is available only in the data and calc sections, and is not allowed in the constraints of a model. The following example prints a variable name and its value:

```
DATA:
   @TEXT() = @WRITEFOR( ROUTES( I, J) |
    X( I, J) #GT# 0: @NAME( X), '  ', X( I, J), @NEWLINE(1));
ENDDATA
```

The report will resemble the following:

```
X( WH1, C1)       2
X( WH1, C2)      17
X( WH1, C3)       1
X( WH2, C1)      13
X( WH2, C4)      12
X( WH3, C3)      21
```

@NEWLINE (n)
Use *@NEWLINE* to write *n* new lines to the output device. *@NEWLINE* is available only in the data and calc sections, and is not allowed in the constraints of a model. See the example immediately below in the *@RANGED* section.

@OBJBND ()
@OBJBND returns the bound on the objective value.

@RANGED (variable_or_row_name)
@RANGED outputs the allowable decrease on a specified variable's objective coefficient or on a specified row's right-hand side. *@RANGED* is available only in the data and calc sections, and is not allowed in the constraints of a model. For example, consider a model with the following data section:

```
DATA:
   @TEXT( 'C:\RESULTS\OUTPUT.TXT') =
    @WRITEFOR( SET( I): X( I), @RANGED( X( I), @NEWLINE(1));
ENDDATA
```

When this model is solved, the values of attribute *X* and the allowable decreases on its objective coefficients will be written to the file *C:\RESULTS\OUTPUT.TXT*. If *@RANGED* is passed a row name it will output the allowable decrease on the right-hand side value for the row. Output may be routed to a file, spreadsheet, database, or memory location. The exact destination will depend on the export function used on the left-hand side of the output statement. Range computations must be enabled in order for *@RANGED* to function properly. For more information on the interpretation of allowable decreases, refer to the *Solver|Range* command.

@RANGEU (variable_or_row_name)
@RANGEU outputs the allowable increase on a specified variable's objective coefficient or on a specified row's right-hand side. For example, consider a model with the following data section:

```
DATA:
   @TEXT( 'C:\RESULTS\OUTPUT.TXT') = @WRITEFOR( SET( I):
    X, @RANGEU(X), @NEWLINE(1));
ENDDATA
```

When this model is solved, the values of *X* and the allowable increases on its objective coefficients will be written to the file *C:\RESULTS\OUTPUT.TXT*. If *@RANGEU* is passed a row name it will output the allowable increase on the right-hand side value for the row. Output may be routed to a file, spreadsheet, database, or memory location. The exact destination will depend on the export function used on the left-hand side of the output statement. Range computations must be enabled in order for *@RANGED* to function properly. For more information on the interpretation of allowable increases, refer to the *Solver|Range* command.

@STATUS ()

This returns the final status of the solution process using the following codes:

@STATUS() Code	Interpretation
0	*Global Optimum* — The optimal solution has been found.
1	*Infeasible* — No solution exists that satisfies all constraints.
2	*Unbounded* — The objective can be improved without bound.
3	*Undetermined* — The solution process failed.
4	*Feasible* — A feasible solution was found that may, or may not, be the optimal solution.
5	*Infeasible or Unbounded* — The preprocessor determined the model is either infeasible or unbounded. If you need to narrow the result down to either infeasible or unbounded, then you will need to turn off presolving and run the model again.
6	*Local Optimum* — Although a better solution may exist, a locally optimal solution has been found.
7	*Locally Infeasible* — Although feasible solutions may exist, LINGO was not able to find one.
8	*Cutoff* — The objective cutoff level was achieved.
9	*Numeric Error* — The solver stopped due to an undefined arithmetic operation in one of the constraints.

In general, if *@STATUS* does not return a code of 0, 4, 6 or 8, the solution is of little use and should not be trusted. The *@STATUS* function is available only in the data and calc sections. The *@STATUS* function is not allowed in the constraints of a model.

For example, the following output statement uses *@STATUS* to print a message to let the user know if the solution is globally optimal, or not:

```
DATA:
   @TEXT() = @WRITE( @IF( @STATUS() #EQ# 0,
    'Global solution found',
     'WARNING: Solution *not* globally optimal!');
ENDDATA
```

For additional examples of the use of the *@STATUS* function, refer to *Interfacing with Other Applications*.

@STRLEN(string)
Use *@STRLEN* to get the length of a specified string. This can be a useful feature when formatting reports. As an example, *@STRLEN('123')* would return the value 3. *@STRLEN* is available only in the data and calc sections, and is not allowed in the constraints of a model.

@TABLE('attr|set')
The *@TABLE* function is used to display either an attribute's values or a set's members in tabular format. The *@TABLE* function is available only in the data and calc sections of a model. You can refer to either *QUEENS8.LG4* or *PERT.LG4* for examples of *@TABLE*. These models can be found in the *SAMPLES* folder off the main LINGO folder.

For instance, *QUEENS8.LG4* is a model for positioning eight queens on a chessboard so that no one queen can attack another. At the end of this model you will find the following data section:

```
DATA:
   @TEXT() = '  The final chessboard:';
   @TEXT() = @TABLE( X);
ENDDATA
```

Here we are using the *@TABLE* function to display the *X* attribute in the standard output window via the *@TEXT* interface function (see below for more on *@TEXT*). The *X* attribute is an 8-by-8 table of 0's and 1's indicating if a queen is positioned in a given square of the chessboard, or not. The output generated by *@TABLE* in this instance follows:

```
The final chessboard:
     E1  E2  E3  E4  E5  E6  E7  E8
E1    0   1   0   0   0   0   0   0
E2    0   0   0   0   1   0   0   0
E3    0   0   0   0   0   0   1   0
E4    0   0   0   1   0   0   0   0
E5    1   0   0   0   0   0   0   0
E6    0   0   0   0   0   0   0   1
E7    0   0   0   0   0   1   0   0
E8    0   0   1   0   0   0   0   0
```

Note that all eight queens (indicated by the 1's on the board) are safe from attack.

In addition to displaying attributes, *@TABLE* can also display sets. When displaying a set, *@TABLE* will print the letter X in the table cell if the corresponding set member exists, otherwise it will leave the table cell blank.

The *PERT.LG4* sample model is a project scheduling model. A project consists of many tasks, and some tasks must be completed before others can begin. The list of all the tasks that must precede certain other tasks is called the precedence relations. At the end of *PERT4.LG4* there is the following data section which uses *@TABLE* to display the precedence relations set, *PRED*:

```
DATA:
!Use @TABLE() to display the precedence relations set,
PRED;
   @TEXT() = @TABLE( PRED);
ENDDATA
```

When we run the model, *@TABLE* displays the following table:

```
         DESIGN FORECAST SURVEY PRICE SCHEDULE COSTOUT
TRAIN
DESIGN                X      X
FORECAST                            X        X
SURVEY                              X
PRICE
X
SCHEDULE                                            X
COSTOUT
X
TRAIN
```

Whenever one task must precede another, an ‘X’ appears in the particular cell of the table. So, for instance, the *DESIGN* task must precede the *FORECAST* and *SURVEY* tasks.

If a line of a table exceeds the page width setting in Lingo, it simply gets wrapped around. So, if you want to display wide tables without line wraps, you may need to increase the page width.

Note: Currently, *@TABLE* can only send tables to the standard solution window or to text files. In other words, it is not possible to send tables to Excel, databases or to applications calling the LINGO DLL.

@TABLE can also display sets and attributes of more than two dimensions. In fact, *@TABLE* allows you great control over how multidimensional objects get displayed. Specifically, four forms of *@TABLE* are supported:

- ***@TABLE(attr/set)*** – This is the simplest form of *@TABLE*. If the object is of one dimension, it will be displayed as a column. Otherwise, the first *n*-1 dimensions will be displayed on the vertical axis, while the *n*-th dimension will be displayed on the horizontal axis of the table. An example follows:

```
@TABLE( X)
```

- ***@TABLE(attr/set, num_horz_indices)*** – In this form, a second argument, *num_horz_indices*, is supplied. This argument is an integer quantity equal to the number of the object's dimensions to display along the horizontal axis. In which case, dimensions (*n – num_horz_indices)* to *n* will be displayed along the horizontal axis of the table. The following example displays dimension 2 and 3 of a 3-dimensional set along the horizontal axis:

```
@TABLE( MY_3_DIM_SET, 2)
```

- ***@TABLE(attr/set, prim_set1,...,prim_setn)*** – Here we specify the exact ordering of the object's *n* dimensions. In which case, the first *n*-1 dimensions specified will be displayed along the vertical axis, while the last dimension specified will be displayed along the horizontal axis. Here's an example that displays a 4-dimensional attribute with dimensions 3, 4 and 1 on the vertical, and dimension 2 on the horizontal:

```
@TABLE( MY_4_DIM_ATTRIBUTE, 3, 4, 1, 2)
```

- ***@TABLE(attr/set, prim_set1,...,prim_setn, num_horz_indices)*** – In this final form, we again specify the ordering of all the indices, but a final integer argument is added to specify the number of dimensions to be displayed on the horizontal axis. The following example displays a 4-dimensional attribute with dimensions 3 and 4 on the vertical, and dimensions 1 and 2 on the horizontal:

```
@TABLE( MY_4_DIM_ATTRIBUTE, 3, 4, 1, 2, 2)
```

@VERSION()(

Use *@VERSION* to retrieve LINGO's version number. *@VERSION* is available only in calc sections, and is not allowed in the constraints of a model.

@WRITE(obj1[, …, objn])

Use *@WRITE* to output one or more objects. *@WRITE* is available only in the data and calc sections, and is not allowed in the constraints of a model. In a data section, output from *@WRITE* may be routed to a file, spreadsheet, or database. The exact destination will depend on the interface function used on the left-hand side of the output statement.

@WRITE may also be used to display computations that are a function of variable values. As an example, the output statement below prints the ratio of the two variables *X* and *Y*:

```
DATA:
   @TEXT() = @WRITE( 'The ratio of X to Y is: ', X / Y);
ENDDATA
```

@WRITEFOR (setname[(set_index_list) [| cond_qualifier]]: obj1[, …, objn])

Use *@WRITEFOR* to output one or more objects across a set. *@WRITEFOR* is available only in the data and calc sections, and is not allowed in the constraints of a model

@WRITEFOR operates like the other set looping functions in that you may, or may not, specify an index list and a conditional qualifier. The ability to use a conditional qualifier allows great flexibility in specifying exactly what you wish to display in your reports. *@WRITEFOR* may also be used to display computations that are a function of variable values.

Using *@WRITEFOR* in conjunction with export functions in data sections allows you to route output to a file, spreadsheet, or database. The exact destination will depend on the export function used on the left-hand side of the output statement.

As an example, the output statement below prints the number of units to ship, the warehouse name, and the customer name for each route with nonzero volume:

```
DATA:
   @TEXT() = @WRITEFOR( ROUTES( I, J) | X( I, J) #GT# 0:
    'Ship ', X( I, J), ' units from warehouse ', WAREHOUSE( I),
     ' to customer ', CUSTOMER( J), @NEWLINE( 1));
ENDDATA
```

The resulting report would appear as follows:

```
Ship 2 units from warehouse WH1 to customer C1
Ship 17 units from warehouse WH1 to customer C2
Ship 1 units from warehouse WH1 to customer C3
Ship 13 units from warehouse WH2 to customer C1
Ship 12 units from warehouse WH2 to customer C4
Ship 21 units from warehouse WH3 to customer C3
```

Text Replication Operator (*)

The *text replication operator* (*) may be used inside either the *@WRITE* or *@WRITEFFOR* functions to repeat a string a specified number of times. The operator should be preceded by a numeric value and then followed by a string (e.g., 3*’text’), which will cause the string to be printed *n* times, where *n* is the numeric value.

In the following example, the text replication operator is used twice to produce a simple graph of on-duty staff during the seven days of the week:

```
DATA:
   LEAD = 3;
   @TEXT() = 'Staff on duty graph:';
   @TEXT() = @WRITEFOR( DAY( D): LEAD*' ',
    DAY( D), ' ', ON_DUTY( D), ' ', ON_DUTY( D)*'+',
    @NEWLINE(1)
   );
ENDDATA
```

The graph would appear as follows, with one plus sign displayed for each staff member on duty:

```
Staff on duty graph:
   MON 20 +++++++++++++++++++++
   TUE 16 ++++++++++++++++
   WED 12 ++++++++++++
   THU 16 ++++++++++++++++
   FRI 19 +++++++++++++++++++++
   SAT 14 ++++++++++++++
   SUN 13 +++++++++++++
```

Date, Time and Calendar Functions

@TIME()

The *@TIME* function returns the total runtime, in seconds, required so far to generate and solve the model. *@TIME* is available only in the data and calc sections, and is not allowed in the constraints of a model. For example, the following output statement writes the solution time to the standard output device:

```
CALC:
  @WRITE('Solve time in seconds =', @TIME());
ENDCALC
```

@YMD2STM(year, month, day, hour, minute, second)

The *@YMD2STM* function takes a date and time as input and returns the equivalent *scalar time*, where scalar time is the number of seconds since the start of the base date of 1 Jan 1900. Note that dates prior to 1 Jan 1900 will return negative scalar time values. The first five arguments must be integral values, while the number of seconds can be fractional. If the seconds value is fractional, it will be rounded to the nearest 10 microseconds (or .00001 seconds). Hours are reported using military time and can range from 0 to 23. An example follows:

```
CALC:
 !Convert two dates to scalar time;
  STM1 = @YMD2STM( 2013, 1, 24, 13, 30, 10.5);
  STM2 = @YMD2STM( 1787, 9, 17, 0, 0, 0);
ENDCALC
```

Here, we compute the scalar time for the two dates and times: 1/23/2013 13:30:10.5 and 9/17/1787 00:00:00. The solution report lists the following values:

```
Variable            Value
 STM1          412349410.50
 STM2         -6699196800.0
```

There are two things to note: First, *STM1* has a fractional value of .5, given that its time includes .5 fractional seconds. Second, *STM2* is negative reflecting the fact that its date if prior to the base date to 1 Jan 1900.

YEAR, MON, DAY, DAYOFWEEK, HOUR, MIN, SEC = @STM2YMD(stm)

The *@STM2YMD* function takes a standard time value and returns the year, month, day of month, day of week (Sunday being day 1), hour, minute and second. Any of the left-hand side arguments may be omitted if they are not of interest. Below, we pass the current standard time via *@STMNOW* (see below) to *@STM2YMD* and then display the results:

```
MODEL:

SETS:
 MONTHS / JAN..DEC/;
 DAYS / SUN..SAT/;
ENDSETS

CALC:

 ! Convert the current standard time to human
   readable form;
 YR, MN, DAY, DWK, HR, MI, SE = @STM2YMD( @STMNOW());

 @WRITE(
  '       Year: ', YR, @NEWLINE( 1),
  '      Month: ', MONTHS( MN), @NEWLINE( 1),
  '        Day: ', DAY, @NEWLINE( 1),
  'Day of week: ', DAYS( DWK), @NEWLINE( 1),
  '       Hour: ', HR, @NEWLINE( 1),
  '     Minute: ', MI, @NEWLINE( 1),
  '     Second: ', SE, @NEWLINE( 2)
 );
ENDCALC

END
```

The output from the model shows the current date and time:

```
      Year: 2016
     Month: FEB
       Day: 11
Day of week: THU
      Hour: 9
    Minute: 14
    Second: 36.427
```

The *@STM2YMD* function may be used in calc sections only and may not be used in the optimizable sections of a model.

@STM2YR(stm) / @STM2MON(stm) / @STM2DAY(stm)
@STM2HR(stm) / @STM2MIN(stm) / @STM2SEC(stm)
These six functions are the inverses of the *@STM2YR* function presented above. Each function takes a scalar time value as input and returns, respectively, the year, month, day, hour, minute or second of the point in time represented by the scalar time value. Note that *@STM2SEC* can return fractional seconds. The following example takes a date and time, converts it to scalar time, and then converts the scalar time back to the original date and time:

```
SETS:
  MONTHS /JAN..DEC/;
  DAYS /SUN..SAT/;
ENDSETS

CALC:
!Convert a date to scalar time;
STM = @YMD2STM( 1787, 9, 17, 12, 0, 0);

!Convert the scalar time back to the original date and time;
YR = @STM2YR( STM);
MO = @STM2MON( STM);
DA = @STM2DAY( STM);

HR = @STM2HR( STM);
MN = @STM2MIN( STM);
SC = @STM2SEC( STM);

DOW = @STM2DWK( STM);

!Write out the original date;
@WRITE( 'Date: ',
  DA, ' ', MONTHS( MO), ' ', YR,
  @NEWLINE( 1);
);

!And the original date;
@WRITE( 'Time: ',
  HR, ':',
  @FORMAT( MN, '%02g'), ':',
  @FORMAT( SC, '%02g'),
  @NEWLINE( 1);
);

!And the day of the week;
@WRITE( 'Day: ',
  DAYS( DOW),
  @NEWLINE( 2);
);
```

```
ENDCALC
```

The output from this sample follows:

```
Date: 17 SEP 1787
Time: 12:00:00
Day: MON
```

@STMNOW()

The *@STMNOW* returns the current date and time as a *scalar time* value, where scalar time is the number of seconds since the base date of 1 Jan 2000. The returned may be fractional to indicate fractions of a second. The returned value can be broken down into the corresponding year, month, day, hour, minute and second by calling the *@STM2** inverse functions described above. An example for displaying the current date and time follows:

```
MODEL:

SETS:
  MONTHS /JAN..DEC/;
  DAYS /SUN..SAT/;
ENDSETS

CALC:

  T = @STMNOW();
  YR = @STM2YR( T);
  MO = @STM2MON( T);
  DA = @STM2DAY( T);
  HR = @STM2HR( T);
  MN = @STM2MIN( T);
  SC = @STM2SEC( T);
  DWK = @STM2DWK( T);

@SET( 'TERSEO', 1);

@WRITE( ' The current date and time is:',
' ', DAYS( DWK),
' ', DA,
' ', MONTHS( MO),
' ', YR,
', ', HR,
':', @FORMAT( MN, '%02g'),
':', @FORMAT( SC, '%04.1f'),
@NEWLINE( 2)
);

ENDCALC

END
```

Here, we retrieve the current date and time as a scalar time value using *@STMNOW*. We then convert the scalar time to a calendar date and time and then write out the result. The following is a sample of output from the model:

```
The current date and time is: MON 11 FEB 2013, 15:54:56.7
```

@STM2DWK(stm)
The *@STM2DWK* function takes a scalar time and returns the index of its day of the week, with Sunday returning 1 and Saturday returning 7. An example of *@STM2DWK* can be found immediately above.

Matrix Functions

Matrix functions take one or more matrices (or attributes) as arguments. Unlike most other LINGO functions, which primarily return scalar (or single-value) results, some matrix functions can return multiple results, with some of the results being matrices or vectors. Matrix functions supported are:

- Cholesky factorization,
- Matrix determinants,
- Eigenvalues and eigenvectors,
- Matrix inversion,
- Multiple linear regression,
- Matrix transpose, and
- QR factorization.

Note :	All matrix functions are only supported in calc sections of models. At present, matrix functions are not valid inside model (optimizable) sections.

Note:	All matrix functions work entirely with dense matrices. Matrices defined on sparse sets are not supported.

L[, err] = @CHOLESKY(A)
The *@CHOLESKY* function performs a Cholesky factorization of the symmetric, positive-definite matrix A, where $LL' = A$, and L is a lower-triangular matrix with real and positive diagonal elements. One can think of L as the "square root" of matrix A. Both A and L must be square and of the same dimension. The upper triangle of L will be filled with zeros.

The optional argument, *err*, is an error flag that will be 0 if the factorization was successful, otherwise it will be 1. If err is present, any error will be ignored by LINGO and will be left for your model to handle. If *err* is not present, then, on any error, LINGO will display an error message and terminate the run.

In this example below, we take a small 3x3 matrix, *A*, compute its Cholesky factor, *L*, then show that *LL'* is equal to the original matrix *A*.

```
MODEL:

! Compute the Cholesky factorization of matrix A.
! Back check by taking the Cholesky factor, L, and
! multiplying it with its transpose, L', to get
! the original matrix A;

SETS:
 S1;
 S2(S1,S1): A, A2, L, LT;
ENDSET

DATA:
  S1 = 1..3;
  A = 10 11 12
      11 54 51
      12 51 86;
ENDDATA

CALC:
  ! Get the Cholesky factor;
  L = @CHOLESKY( A);

  ! L transpose;
  LT = @TRANSPOSE( L);

  ! Compute L * L';
  @FOR( S2( I, J):
      A2( I, J) = @SUM( S1( J2): L( I, J2) * LT( J2, J));
  );
ENDCALC

CALC:
  @SET( 'TERSEO', 2)  ; ! Just display errors;
  @WRITE( "Original A matrix:", @NEWLINE( 1));
  @TABLE( A);
  @WRITE( @NEWLINE( 1), "Cholesky factor L:", @NEWLINE( 1));
  @TABLE( L);
  @WRITE( @NEWLINE( 1), "L transpose:", @NEWLINE( 1));
  @TABLE( LT);
  @WRITE( @NEWLINE( 1), "L * L' = A:", @NEWLINE( 1));
  @TABLE( A2);
ENDCALC
END
```

Model: CHOLESKY

The output from this model is reproduced below, where we can see, as per definition, $LL' = A$:

```
Original A matrix
            1          2          3
  1  10.00000  11.00000  12.00000
  2  11.00000  54.00000  51.00000
  3  12.00000  51.00000  86.00000

Cholesky factor L:
            1          2          3
  1  3.162278  0.000000  0.000000
  2  3.478505  6.473021  0.000000
  3  3.794733  5.839623  6.123627

L transpose:
            1          2          3
  1  3.162278  3.478505  3.794733
  2  0.000000  6.473021  5.839623
  3  0.000000  0.000000  6.123627

L * L' = A:
            1          2          3
  1  10.00000  11.00000  12.00000
  2  11.00000  54.00000  51.00000
  3  12.00000  51.00000  86.00000
```

@DETERMINANT(A)

The *@DETERMINANT* function returns the determinant value for matrix *A*.

In the example below, we make use of Cramer's Rule for finding the solution to a system of linear equations of the form $Ax=b$. According to Cramer's Rule, the solution for *x(i)* may be calculated by computing the determinant of the *A* matrix with column *i* replaced by the right-hand side values *b*. This determinant value is then divided by the determinant of the original *A* matrix to get the final value for *x(i)*.

```
MODEL:
! Use Cramer's Rule to find the solution
 to a system of linear equations, Ax=b;

SETS:
  DIM: X, B, B2;
  DXD( DIM, DIM): A, A2;
ENDSETS

DATA:
  DIM = 1..3;
  B = 12 0 -3;
  A =
     1  -2   5
     4   1  -2
    -2   1  -1
  ;
ENDDATA

CALC:
  ! Determinant of the original matrix;
  D = @DETERMINANT( A);

  ! Determine if a unique solution exists;

  @IFC( D #EQ# 0:
     @STOP( '  No unique solution exists!!!');
  @ELSE
     @WRITE( '  A unique solution exists...');
  );

  @WRITE( @NEWLINE( 1));

  ! Copy matrix;
  @FOR( DXD( I, J): A2( I, J) = A( I, J));

  @FOR( DIM( J):
     ! Replace column J with the B values;
     @FOR( DIM( I):
        A2( I, J) = B( I);
     );

     ! Calculate solution for X( J);
     X( J) = @DETERMINANT( A2) / D;

     ! Restore column J in the original matrix;
     @FOR( DIM( I):
        A2( I, J) = A( I, J);
     );
  );

  ! Back check the solution;
```

```
  @FOR( DIM( I):
     B2( I) = @SUM( DIM( J): A( I, J) * X( J));
     @IFC( @ABS( B2( I) - B( I)) #GT# 1.E-6:
        @STOP( 'Solution fails!!!!');
     );

  );

  @WRITE( '  Solution verified...', @NEWLINE( 1),
   '  Solution = ');

  @FOR( DIM( I): @WRITE( X( I), '   '));

  @WRITE( @NEWLINE( 2));
ENDCALC

END
```

Model: CRAMER

We first checked to see if a unique solution to the system exists by computing the determinant of the original matrix and checking that it was not 0. We then used Cramer's Rule to compute the solution vector, *x*. Finally, we took the *x* values and verified that they solve the equations. The output from the model is:

```
A unique solution exists...
Solution verified...
Solution = 1   2   3
```

LAMDAR, VR, LAMBDAI, VI, ERR = @EIGEN(A)

The *@EIGEN* function returns eigenvectors and eigenvalues for the matrix *A*, where:

- ***A*** a *nxn* matrix,
- ***LAMBDAR*** a vector of length *n* for returning the real parts of the eigenvalues,
- ***VR*** a *nxn* matrix for returning the real parts of the eigenvectors (one vector per column),
- ***LAMBDAI*** a vector of length *n* for returning the imaginary parts of the eigenvalues,
- ***VI*** a *nxn* matrix for returning the imaginary parts of the eigenvectors, and
- ***err*** an error flag that will be 1 if a numeric problem occurs, otherwise 0.

As long as at least one left-hand side argument is present, all other arguments may be omitted. If *err* is present, then LINGO will not halt the run if a numeric error occurs; it will be up to your model to handle the error. If *err* is not present and a numeric error occurs, then LINGO will halt model execution.

In the next example, we compute the eigenvalues and eigenvectors for a 3x3 symmetric matrix.

```
MODEL:

SETS:
  DIM: LAMBDA;
  DXD( DIM, DIM): A, V, AV, LV;
  DXDL( DXD) | &1 #GT# &2: PROD;
ENDSETS

DATA:
  TOL = 1.E-6;
  DIM = 1..3;
  A =  3 4 1
       4 0 9
       1 9 5;
ENDDATA

CALC:
  ! Compute the eigenvalues and eigen vectors;
  ! Note: The matrix is symmetric, so the vectors
  ! and values will not have imaginary parts, so
  ! we can omit the imaginary arguments.;
  LAMBDA, V,,, ERR = @EIGEN( A);

  ! Numeric error?;
  @IFC( ERR #NE# 0: @STOP( 'Numeric error'));

  ! Display them with the @TABLE function.;
  @WRITE( @NEWLINE( 1), ' Eigenvalues:', @NEWLINE( 2));
  @TABLE( LAMBDA);

  @WRITE( @NEWLINE( 1), ' Eigenvectors:', @NEWLINE( 1));
  @TABLE( V);

  ! Check that A * v = Lambda * v holds.;
  NZ = 0;

  @FOR( DXD( I, J):
     LV( I, J) = LAMBDA( J) * V( I, J);
     AV( I, J) = @SUM( DIM( K): A( I, K) * V( K, J));
     @IFC( @ABS( LV( I, J) - AV( I, J)) #GT# TOL: NZ = NZ + 1);

  );

  @WRITE( @NEWLINE( 1));
  @IFC( NZ #GT# 0:
     @WRITE( ' Lambda * v = A * v is not verified.');
  @ELSE
     @WRITE( ' Lambda * v = A * v is verified.');
  );

  ! Symmetric matrices have orthogonal eigenvectors.
  ! Verify this by computing the inner products.;
```

```
  NZ = 0;
  @FOR( DXDL( I, J):
     PROD( I, J) = @sum( DIM( K): V( K, I) * V( K, J));
     @IFC( @ABS( PROD( I, J)) #GT# TOL: NZ = NZ + 1);
  );

  @WRITE( @NEWLINE( 1));
  @IFC( NZ #GT# 0:
     @WRITE( ' Eigenvectors are NOT orthogonal.', @NEWLINE( 2));
  @ELSE
     @WRITE( ' Eigenvectors are orthogonal.', @NEWLINE( 2));
  );
ENDCALC
END
```

Model: EIGENEX1

Symmetric matrices have the property that their eigenvalues and eigenvectors are all real valued, allowing us to ignore the imaginary components. Looking at the call to *@EIGEN*:

```
LAMBDA, V,,, ERR = @EIGEN( A);
```

we see that two left-hand side arguments are omitted, corresponding to the imaginary parts. Note, we could have passed these two arguments, however, they would have simply been filled with all zeros.

By the definition of eigenvalues and eigenvectors, it's true that $Av = Lambda * v$, where A is the original nxn matrix, v is an nx1 eigenvector of A, and Lambda is the eigenvalue corresponding to v. The model verifies this relationship and displays the message:

```
Lambda * v = A * v is verified.
```

Another nice property of symmetric matrices is that the eigenvectors are all orthogonal. The model verifies this property by calculating all the inner products between the eigenvectors and comparing these products to 0.

The output from the model follows:

```
Eigenvalues:

 1   12.92039
 2   2.587500
 3  -7.507889

Eigenvectors:
                 1               2               3
 1      -0.3179176      -0.9145159       0.2501781
 2      -0.6062189  -0.6813624E-02      -0.7952686
 3      -0.7289904       0.4044926       0.5522307

Lambda * v = A * v is verified.
Eigenvectors are orthogonal.
```

AINV[, err] = *@INVERSE(A)*
The *@INVERSE* function returns the inverse of matrix *A*. Both *AINV* and *A* must be square, *nxn* matrices.

The optional argument, *err*, is an error flag that will be 0 if the invert operation was successful, otherwise it will be 1. A situation where *err* would be nonzero is when *A* is a singular matrix. If *err* is present, any error will be ignored by LINGO and will be left for your model to handle. If *err* is not present, then, on any error, LINGO will display an error message and terminate the run.

The example below may be familiar from the *@DETERMINANT* example above, where we solve a 3x3 system of linear equations. In this example, however, we will not use Cramer's Rule to solve the system. Instead, we will compute the inverse of the *A* matrix, and then multiply the inverse by the right-hand side vector *b* to get the solution vector *x*. We also verify that a unique solution exists by computing the determinant of *A*, as well as verify that our solution does indeed solve the original set of equations.

```
MODEL:
SETS:
  DIM: X, B, B2;
  DXD( DIM, DIM): A, AINV;
ENDSETS

DATA:
  DIM = 1..3;
  B = 12 0 -3;
  A =
     1  -2   5
     4   1  -2
    -2   1  -1
  ;
ENDDATA

CALC:
  ! Determinant of the original matrix;
  D = @DETERMINANT( A);

  ! Determine if a unique solution exists;
  @IFC( D #EQ# 0:
     @STOP( '  No unique solution exists!!!');
  @ELSE
     @WRITE( '  A unique solution exists...');
  );

  @WRITE( @NEWLINE( 1));

  ! Get the matrix inverse;
  AINV = @INVERSE( A);

  ! Use the inverse to compute the solution vector: x = AINV * b;
  @FOR( DIM( I): X( I) = @SUM( DIM( J): AINV( I, J) * B( J)));

  ! Back check the solution;
  @FOR( DIM( I):
     B2( I) = @SUM( DIM( J): A( I, J) * X( J));
     @IFC( @ABS( B2( I) - B( I)) #GT# 1.E-6:
        @STOP( 'Solution fails!!!!');
     );
  );
  @WRITE( '  Solution verified...', @NEWLINE( 1),
   '  Solution = ');

  @FOR( DIM( I): @WRITE( @FORMAT( X( I), '10.5g')));

  @WRITE( @NEWLINE( 2));
ENDCALC
END
```

Model: INVERSE

The output from the model follows:

```
A unique solution exists...
Solution verified...
Solution =          1         2         3
```

Again, as with the *@DETERMINANT* example, the solution is [1,2,3].

B, b0, RES, rsq, f, p, var = @REGRESS(Y, X)
The *@REGRESS* function is used to perform multiple linear regression (MLR), a technique that models the linear relationship, $Y = b0 + B*X$, between one dependent variable and one or more independent variables. MLR is frequently used in forecasting, e.g., by economists in macroeconomics when modeling of the economy.

Using the syntax above, we have the following definitions:

- ♦ ***B*** The vector of coefficient terms for the regression. If there are p independent variables, then *B* is a vector of length *p*.
- ♦ ***b0*** The *constant* term of the regression. If a *constant term* is not desired, then you can omit this argument and LINGO will force the constant term to 0.
- ♦ ***RES*** The residuals, or error terms. These are the differences between the predicted and actual observed values for the dependent variable.
- ♦ ***rsq*** The *R*-squared statistic, a measure of strength of the relationship between the model and dependent variable. The fraction of the original variance removed by the forecast formula versus using just ***b0*** alone.
- ♦ ***f*** The *F*-value, a measure of the overall significance of the model.
- ♦ ***P*** The *p*-value for the regression, a measure of the significance of the *F*-test.
- ♦ ***var*** Estimate of variance of the residual terms.
- ♦ ***Y*** The observed values for the dependent variable. If there are n observations, then *Y* should be a vector of length *n*.
- ♦ ***X*** The independent variable values. If there are *n* observations and *p* independent variables, then *X* should be an *nxp* matrix.

Note, that any of the left-hand side arguments may be omitted if they are not of interest. The only requirement is that at least one left-hand side argument must be present.

The goal of MLR is to fit a linear function to the observed data, *X*, for predicting the independent observations, *Y*. The form of the function is:

$$Y = b0 + \sum i\ B(i) * X(i) + RES$$

The fit is determined by the set of coefficients that minimizes the sum of the squared residual values. For more information, see: https://en.wikipedia.org/wiki/Line.

In the LINGO sample models folder, there is a regression example called *REGRLONG*, shown below. This model makes use of the Longley dataset:

http://www.itl.nist.gov/div898/strd/lls/data/LINKS/DATA/Longley.dat

This is a series of seven macroeconomic statistics, with one being total employment, the dependent variable. The Longley dataset is commonly used for testing statistical software, and the link above contains the certified results for regressions based on the data.

```
MODEL:

! Multiple linear regression;
! The output is the set of regression coefficients, B,
for the model:
   Y(i) = B(0) + B(1)*X(i,1) + B(2)*X(i,2)+... + ERROR(i)

We use the Longley macroeconomic data set for:
http://www.itl.nist.gov/div898/strd/lls/data/LINKS/DATA/Longley.dat

Certified Regression Statistics for Longley data:

                                          Standard Deviation
 Parameter         Estimate                     of Estimate
  B(0)          -3482258.63459582          890420.383607373
  B(1)           15.0618722713733          84.9149257747669
  B(2)          -0.358191792925910E-01     0.334910077722432E-01
  B(3)          -2.02022980381683          0.488399681651699
  B(4)          -1.03322686717359          0.214274163161675
  B(5)          -0.511041056535807E-01     0.226073200069370
  B(6)           1829.15146461355          455.478499142212

Residual
Standard Deviation   304.854073561965

R-Squared            0.995479004577296

Certified Analysis of Variance Table:

Source of   Degrees of    Sums of              Mean
Variation    Freedom          Squares              Squares
Regression      6      184172401.944494    30695400.3240823
Residual        9      836424.055505915    92936.0061673238

  F Statistic
330.285339234588
;

SETS:
 OBS : Y, RES;
 VARS;
 DEPVAR( VARS);    ! The dependent variable;
 EXPVAR( VARS): B; ! The explanatory variables;
 OXV( OBS, VARS): DATATAB; ! The data table;
 OXE( OBS, EXPVAR): X;
ENDSETS

DATA:

! Names of the variables;
 VARS = EMPLD PRICE GNP__ JOBLS MILIT POPLN  YEAR_;
```

```
! Dependent variable. Must be exactly 1;
 DEPVAR = EMPLD;

! Explanatory variables. Should not include
  dependent variable;
 EXPVAR = PRICE GNP__ JOBLS MILIT POPLN YEAR_;

! The dataset;
 DATATAB =
   60323   83  234289  2356  1590  107608  1947
   61122  88.5 259426  2325  1456  108632  1948
   60171  88.2 258054  3682  1616  109773  1949
   61187  89.5 284599  3351  1650  110929  1950
   63221  96.2 328975  2099  3099  112075  1951
   63639  98.1 346999  1932  3594  113270  1952
   64989  99   365385  1870  3547  115094  1953
   63761 100   363112  3578  3350  116219  1954
   66019 101.2 397469  2904  3048  117388  1955
   67857 104.6 419180  2822  2857  118734  1956
   68169 108.4 442769  2936  2798  120445  1957
   66513 110.8 444546  4681  2637  121950  1958
   68655 112.6 482704  3813  2552  123366  1959
   69564 114.2 502601  3931  2514  125368  1960
   69331 115.7 518173  4806  2572  127852  1961
   70551 116.9 554894  4007  2827  130081  1962;

ENDDATA

CALC:
   !Suppress some of Lingo's output;
   @SET( 'TERSEO', 2);

   !Set up data for the @REGRESS function
    by moving the dependent variable into
    Y and the independents into X;
   @FOR( DEPVAR( K):
      @FOR( OBS( I):
         Y( I) = DATATAB( I, K);
         @FOR( OXE( I, J): X( I, J) = DATATAB( I, J))
      );
   );

   ! Do the regression;
   B, B0, RES, RSQUARE, F, P, VAR = @REGRESS( Y, X);

   ! Print a report;
   @WRITE( '   Explained error/R-Square: ',
    @FORMAT( RSQUARE, '16.8g'), @NEWLINE(1));
   @WRITE( '                F statistic: ',
    @FORMAT( F, '16.8g'), @NEWLINE(1));
   @WRITE( '                    p value: ',
    @FORMAT( P, '16.8g'), @NEWLINE(1));
   @WRITE( '          Residual variance: ',
```

```
    @FORMAT( VAR, '16.8g'), @NEWLINE(1));
   @WRITE( '         Residual std error: ',
    @FORMAT( VAR ^ .5, '16.8g'), @NEWLINE(2));
   @WRITE( '   B( 0):     ', @FORMAT( B0, '16.8g'),
    @NEWLINE( 1));
   @FOR( EXPVAR( I):
    @WRITE( '   B( ', EXPVAR( I), '): ',
    @FORMAT( B( I), '16.8g'), @NEWLINE( 1));
   );
ENDCALC
END
```

Model: REGRLONG

The interesting parts of this model are the following two expressions:

```
   !Set up data for the @REGRESS function
    by moving the dependent variable into
    Y and the independents into X;
   @FOR( OBS( I):
      Y( I) = DATATAB( I, @INDEX( EMPLD));
      @FOR( OXE( I, J): X( I, J) = DATATAB( I, J))
   );

   ! Do the regression;
   B, B0, RES, RSQUARE, F, P, VAR = @REGRESS( Y, X);
```

Note, that the data is provided as a single table, containing both the dependent and independent observations. The first expression partitions the raw data into a vector *Y* of dependent employment observations, and a 16x6 *X* matrix of independent observations. At this point, the data is in the correct format to submit to *@REGRESS*, and we perform the regression with the statement:

```
    ! Do the regression;
    B, B0, RES, RSQUARE, F, P, VAR = @REGRESS( Y, X);
```

Results from the regression are then displayed:

```
  Explained error/R-Square:         0.995479
              F statistic:        330.28534
                  p value:    4.984031e-010
        Residual variance:        92936.006
       Residual std error:        304.85407

  B( 0):           -3482258.6
  B( PRICE):        15.061872
  B( GNP__):     -0.035819179
  B( JOBLS):       -2.0202298
  B( MILIT):       -1.0332269
  B( POPLN):     -0.051104106
  B( YEAR_):        1829.1515
```

T = @TRANSPOSE(A)
The *@TRANSPOSE* function returns the transpose of matrix *A*, where *T(i,j) = A(j,i)* for every *(i,j)*. Matrix *A* does not have to be square, but if *A* is an *mxn* matrix, then *T* must be of dimension *nxm*. For an example of *@TRANSPOSE*, refer to the section above on *@CHOLESKY*, where we use *@TRANSPOSE* to get the transpose of the Cholesky factor. This transpose is then multiplied by the original Cholesky factor to get the original matrix back.

Q,R[, err] = @QRFACTOR(A)
The *@QRFACTOR* function returns a QR factorization of matrix *A*, where *A = QR*, with *R* being upper triangular and *Q* being orthogonal (i.e., *Q'Q* is the identity matrix). The matrices must all be defined on dense sets, with *A* and *Q* being *m* x *n* matrices and *R* an *n* x *n* matrix.

The optional argument, *err*, is an error flag that will be 0 if the factorization operation was successful, otherwise it will be 1. If *err* is present, any error will be ignored by LINGO and will be left for your model to handle. If err is not present, then, on any error, LINGO will display an error message and terminate the run.

The sample model *QRFACTOR* contained in the LINGO Samples folder illustrates the use of the *@QRFACTOR* function.

Miscellaneous Functions

@BLOCKROW(block_index, row_name)
The *@BLOCKROW* function is used in conjunction with the branch-and-price (BNP) solver. Models suited for BNP have 2 or more mostly independent subproblems known as blocks. *@BLOCKROW* is used to assign rows to their respective blocks, where *block_index* is the index of the block and *row_name* is the name of the row to assign to block block_index. Block indices may be any non-negative, integer value, with 0 corresponding to the block of linking rows. Refer to the *BNP Solver* section in Chapter 5 for more information on the use of *@BLOCKROW*.

@IF (logical_condition, true_result, false_result)
The *@IF* function evaluates *logical_condition* and, if true, returns *true_result*. Otherwise, it returns *false_result*. For example, consider the following simple model that uses *@IF* to compute fixed production costs:

```
MIN = COST;
COST = XCOST +YCOST;
XCOST = @IF( X #GT# 0, 100, 0) + 2 * X;
YCOST = @IF( Y #GT# 0,  60, 0) + 3 * Y;
X + Y >= 30;
```

Model: IFCOST

We produce two products—*X* and *Y*. We want to minimize total cost, subject to producing at least 30 total units of *X* and *Y*. If we produce *X*, there is a fixed charge of 100 along with a variable cost of 2. Similarly, for *Y*, these respective values are 60 and 3. We use the *@IF* function to determine if either of the products are being produced in order to apply the relevant fixed cost. This is accomplished by

testing to see if their production levels are greater than 0. If so, we return the fixed cost value. Otherwise, we return zero.

Experienced modelers know that, without the benefit of an *@IF* function, modeling fixed costs requires invoking some "tricks" using binary integer variables. The resulting models are not as intuitive as models constructed using *@IF*. However, the caveat is that the *@IF* function is not a linear function. At best, the graph of an *@IF* function will be piecewise linear. In our current example, the *@IF* functions are piecewise linear with a discontinuous break at the origin.

It is always best to try and keep a model linear (see Chapter 15, *On Mathematical Modeling*). Barring this, it is best for all functions in a nonlinear model to be continuous. The *@IF* function violates both these conditions. Thus, models containing *@IF* functions may be tough to solve to global optimality. Fortunately, LINGO has two options that can help overcome the difficult nature of models containing *@IF* functions—*linearization* and *global optimization*.

To illustrate the difficulty in solving models with discontinuous functions such as *@IF*, we will solve our example model with both linearization and global optimization disabled. When we do this, we get the following solution:

```
Local optimal solution found at iteration:  42
Objective value:  160.0000

       Variable           Value
           COST        160.0000
          XCOST        160.0000
          YCOST        0.000000
              X        30.00000
              Y        0.000000
```

This solution involves producing only *X* at a total cost of 160. Given that producing only *Y* and not *X* will result in a lower total cost of 150, this is clearly a locally optimal point. In order to find the globally optimal point, we must resort to either the linearization or global optimization features in LINGO.

Briefly, linearization seeks to reformulate a nonlinear model into a mathematically equivalent linear model. This is desirable for two reasons. First, linear models can always be solved to global optimality. Secondly, linear models will tend to solve much faster than equivalent nonlinear models. Unfortunately, linearization can not always transform a model into an equivalent linear state. In which case, it may be of no benefit. Fortunately, our sample model can be entirely linearized. To enable the linearization option, run the *Solver|Options* command and set the *Linearization Degree* to *High* on the *General Solver* tab.

Global optimization breaks a model down into a series of smaller, local models. Once this series of local models has been solved, a globally optimal solution can be determined. To enable global optimization, run the *Solver|Options* command, select the *Global Solver* tab, then click on the *Global Solver* checkbox. Note that the global solver is an add-on option to LINGO. The global solver feature will not be enabled for some installations. Run the *Help|About LINGO* command to determine if your installation has the global solver capability enabled.

Whether using the linearization option or the global solver, LINGO obtains the true, global solution:

```
Global optimal solution found at iteration:  6
Objective value:   150.0000

       Variable           Value
           COST        150.0000
          XCOST        0.000000
          YCOST        150.0000
              X        0.000000
              Y        30.00000
```

Note: Starting with release 9.0, the false branch of the *@IF* function may contain arithmetic errors without causing the solver to trigger an error. This makes the *@IF* function useful in avoiding problems when the solver strays into areas where certain functions become undefined. For instance, if your model involves division by a variable, you might use *@IF* as follows: *@IF(X #GT# 1.E-10, 1/X, 1.E10).*

***@WARN* (‘*text*’, *logical_condition*)**

This displays the message *‘text’* if the *logical_condition* is met. This feature is useful for verifying the validity of a model’s data. In the following example, if the user has entered a negative interest rate, the message “INVALID INTEREST RATE” is displayed:

```
! A model of a home mortgage;
DATA:
! Prompt the user for the interest
  rate, years, and value of mortgage.
  We will compute the monthly payment;
   YRATE  = ?;
   YEARS  = ?;
   LUMP   = ?;
ENDDATA

! Number of monthly payment;
   MONTHS = YEARS * 12;

! Monthly interest rate;
   (1 + MRATE) ^ 12  =  1 + YRATE;

! Solve next line for monthly payment;
   LUMP = PAYMENT * @FPA(MRATE, MONTHS);

! Warn them if interest rate is negative
   @WARN('INVALID INTEREST RATE',
    YRATE #LT# 0);
```

***@USER* (*user_determined_arguments*)**

The user can supply this in an external DLL or object code file. For a detailed example on the use of *@USER*, please see the *User Defined Functions* section in Chapter 11, *Interfacing with Other Applications.*

8 Interfacing with External Files

It can be cumbersome and impractical to try to maintain your data in a LINGO model file. In most cases, your model's data will reside externally in text files, spreadsheets, and databases. Also, a solution generated by LINGO is of little use if you can't export it to other applications. For these reasons, LINGO has many methods to assist you in moving information in and out of the application. The primary focus of this chapter is to illustrate how to move data in and out of LINGO through the use of text based ASCII files. In Chapter 9, *Interfacing with Spreadsheets*, we will look at using spreadsheets. In Chapter 10, *Interfacing with Databases*, we will illustrate the use of databases for maintaining your model's data.

Cut and Paste Transfers

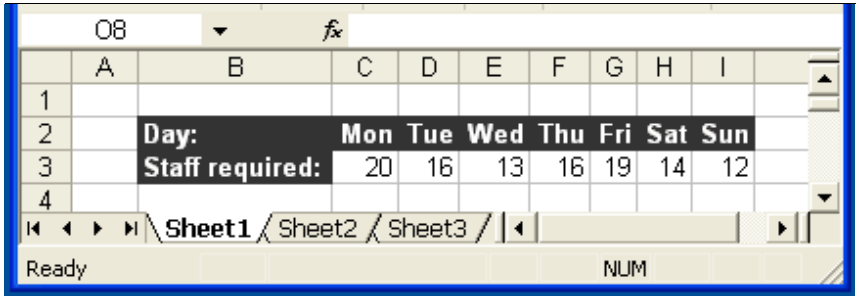
Perhaps the simplest and most straightforward way to move data in and out of an application in Windows is by using *cut* and *paste* commands. Windows maintains an information buffer called the *clipboard*. Applications that support the cut command can move information into the clipboard. Applications that support the paste command can move information from the clipboard into their memory. Thus, cut and paste offers a simple, but effective, technique for moving small amounts of data from one application to another.

Pasting in Data from Excel

You should be able to paste data into LINGO from any application that supports a cut command. For illustration purposes, we will show how to paste data from an Excel worksheet into a LINGO model. Recall our staff-scheduling model from Chapter 2, *Using Sets*, which is reproduced here with the data for the *REQUIRED* attribute omitted from the data section:

```
SETS:
   DAYS / MON TUE WED THU FRI SAT SUN/:
    REQUIRED, START;
ENDSETS
DATA:
   REQUIRED = <data omitted>;
ENDDATA
MIN = @SUM(DAYS(I): START(I));
@FOR(DAYS(J):
   @SUM(DAYS(I) | I #LE# 5:
      START(@WRAP(J - I + 1, 7)))
         >= REQUIRED(J)
);
```

Suppose your staffing requirements data is maintained in an Excel worksheet resembling the following:



To paste the staffing requirements data from Excel into the LINGO model above, follow these steps:

1. Select the range containing the data (*C3:I3*) by placing the cursor on the *C3* cell, press and hold down the left mouse button, drag the mouse to cell *I3*, then release the mouse button.
2. Select the *Copy* command from Excel's *Edit* Menu.
3. Click once on the LINGO model window.
4. Place the LINGO cursor directly to the right of the data statement: *REQUIRED =*.
5. Select the *Paste* command from LINGO's *Edit* menu.

The data should now appear in the LINGO model as follows:

```
DATA:
   REQUIRED = 20 16 13 16 19 14 12;
ENDDATA
```

You may need to adjust the font of the data to your liking. You can use the *Edit|Select Font* command in LINGO to accomplish this. Your model now has the required data and is ready to be solved. Note that LINGO also has features that allow you to import data directly from Excel. See Chapter 9, *Interfacing with Spreadsheets*, for more information.

Pasting Data Out to Microsoft Word

Suppose you went ahead and solved the previous staffing model. LINGO will present you with a new Window containing the solution to your model. Now, suppose you would like to get a copy of the solution into MS Word for a report you are writing. You can do this by following these steps:

1. Select the solution report window in LINGO by clicking on it once.
2. Select all the text in the window by issuing the *Edit|Select All* command in LINGO.
3. Place the solution into the clipboard by selecting the *Edit|Copy* command in LINGO.
4. Activate MS Word by clicking once on the window containing the report you are writing.
5. Paste the solution from the clipboard into the report by issuing the *Edit|Paste* command in MS Word.

Text File Interface Functions

LINGO has several *interface functions* that perform input and output operations. There are interface functions for dealing with text files, spreadsheets, and databases. There is even an interface function that lets you pass data back and forth from other applications. In this chapter, we are focusing on interfacing with text files, so we will investigate the *@FILE* function for importing the contents of external text files and the *@TEXT* function for exporting solutions to text files.

Including External Files with @FILE

The *@FILE* interface function in LINGO allows you to include data from external text files anywhere in your model. This is particularly useful for incorporating data stored in text files into your sets and data sections.

The syntax for the *@FILE* function is:

@FILE(‘*filename*’)

where *filename* is the name of the file to include text from. When this function is encountered in a model, LINGO will continue to take text from this file until it encounters either the end-of-file mark or a LINGO end-of-record mark (~). For subsequent *@FILE* references in the same model that use the same file name, LINGO resumes taking input from the file at the point where it left off. Nesting of *@FILE* function calls (embedding an *@FILE* in a file that is itself called by *@FILE*) is not allowed.

Using @FILE in a Transportation Model

As an example, we will use the Wireless Widgets transportation model developed in Chapter 1, *Getting Started with LINGO*. It is reproduced in its original form below:

```
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES /WH1 WH2 WH3 WH4 WH5 WH6/: CAPACITY;
   VENDORS /V1 V2 V3 V4 V5 V6 V7 V8/   : DEMAND;
   LINKS(WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS

! The objective;
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));

! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I): VOLUME(I, J)) =
     DEMAND(J));

! The capacity constraints;
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J)) <=
     CAPACITY(I));

! Here is the data;
DATA:
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
```

Model: WIDGETS

Note that data appears two places in the model. First, there are the lists of warehouses and vendors in the sets section. Second, there is data on capacity, demand, and shipping costs in the data section.

In order to completely isolate the data from our model, we would like to move it to an external text file, and modify the model so it will draw the data from the text file using the *@FILE* function. The following modified version of the model has all the data removed. Changes are represented in bold type:

```
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES / @FILE('WIDGETS2.LDT')/: CAPACITY;
   VENDORS / @FILE('WIDGETS2.LDT')/   : DEMAND;
   LINKS(WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS

! The objective;
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));

! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I): VOLUME(I, J)) =
     DEMAND(J));

! The capacity constraints;
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J)) <=
     CAPACITY(I));

! Here is the data;
DATA:
   CAPACITY = @FILE('WIDGETS2.LDT');
   DEMAND = @FILE('WIDGETS2.LDT');
   COST = @FILE('WIDGETS2.LDT');
ENDDATA
```

Model: WIDGETS2

The model is now set to draw all data from the file *WIDGETS2.LDT*. The contents of this data file appear below:

```
!List of warehouses;
WH1 WH2 WH3 WH4 WH5 WH6 ~

!List of vendors;
V1 V2 V3 V4 V5 V6 V7 V8 ~

!Warehouse capacities;
60 55 51 43 41 52 ~

!Vendor requirements;
35 37 22 32 41 32 43 38 ~

!Unit shipping costs;
6 2 6 7 4 2 5 9
4 9 5 3 8 5 8 2
5 2 1 9 7 4 3 3
7 6 7 3 9 2 7 1
2 3 9 5 7 2 6 5
5 5 2 2 8 1 4 3
```

File: WIDGETS2.LDT

Note: We use the convention of placing the extension of *.LDT* on all LINGO data files.

Sections of the data file between end-of-record marks (~) are called records. If an included file has no end-of-record marks, LINGO reads the whole file as a single record. Notice that, with the exception of the end-of-record marks, the model text and data appear just as they would if they were in the model itself.

Also, notice how the end-of-record marks in the include file work along with the *@FILE* function calls in the model. The first call to *@FILE* opens *WIDGETS2.LDT* and includes the first record. The second call includes the second record, and so on.

The last record in the file does not need an end-of-record mark. When LINGO encounters an end-of-file, it includes the last record and closes the file. If you end the last record in an include file with an end-of-record mark, LINGO will not close the file until it is done solving the current model. This could cause problems if multiple data files are opened in the model—files that remain open can cause the limit on open files to be exceeded.

When using the *@FILE* function, think of the contents of the record (except for any end-of-record mark) as replacing the text *@FILE('filename')* in the model. This way, you can include a whole statement, part of a statement, or a whole series of statements in a record. For example, the first two records of the *WIDGETS2.LDT* file in the above example:

```
!List of warehouses;
WH1 WH2 WH3 WH4 WH5 WH6 ~

!List of vendors;
V1 V2 V3 V4 V5 V6 V7 V8 ~
```

are included in the model in the sets section as follows:

```
WAREHOUSES / @FILE('WIDGETS2.LDT')/: CAPACITY;
VENDORS / @FILE('WIDGETS2.LDT')/   : DEMAND;
```

The net effect of these *@FILE* calls is to turn the model statements into:

```
WAREHOUSES / WH1 WH2 WH3 WH4 WH5 WH6/: CAPACITY;
VENDORS / V1 V2 V3 V4 V5 V6 V7 V8/   : DEMAND;
```

Comments in the include file are ignored. The maximum number of include files a model can simultaneously reference is 16.

Writing to Files Using @TEXT

The *@TEXT* interface function is used for exporting solutions to text files. The *@TEXT* function can export both set members and attribute values. The syntax of the *@TEXT* function is:

@TEXT(['*filename*', [‘a’]])

where *filename* is the name of the file you want to export the solution to. If *filename* is omitted, the solution data will be sent to the standard output device (this is typically the screen). If the second argument of ‘a’ is present, then LINGO will append output to the file, otherwise it will create a new file for subsequent output, erasing any existing file. The *@TEXT* function may only appear on the left-hand side of a data statement in the data section of a model.

We refer to data statements that use interface functions to generate output as *output operations*. Output operations are only performed when the solver finishes running a model. The operations are run in the sequence that they were encountered in the model.

Here are some examples of using *@TEXT*:

Example 1: `@TEXT('RESULTS.TXT') = X;`
Sends the value(s) for *X* to the file *RESULTS.TXT.* Any existing version of the file is overwritten

Example 2: `@TEXT() = DAYS, START;`
In this example, we are exporting the *DAYS* set and the *START* attribute. We routed the output to the screen by omitting the *filename* argument.

Example 3: `@TEXT() = @WRITEFOR( DAYS( D) | START( D) #GT# 0: DAYS( D), '  ', START( D));`
In this example, we use the *@WRITEFOR* reporting function to loop over the members of the DAYS set. Contrary to the previous example, we only print information for those days where *START(D)* is greater than 0.

Now, let’s turn to a more detailed example of the use of *@TEXT* in our staff scheduling model.

Example - Using @TEXT for Staff-Scheduling

Let's once again make use of the staff scheduling model from Chapter 2, *Using Sets*. However, this time we will modify it to use the *@TEXT* function and write the solution to a file. The model follows with the critical change listed in bold type:

```
SETS:
   DAYS / MON TUE WED THU FRI SAT SUN/:
    REQUIRED, START;
ENDSETS

DATA:
   REQUIRED = 20 16 13 16 19 14 12;
   @TEXT('OUT.TXT') = DAYS, START;
ENDDATA

MIN = @SUM(DAYS(I): START(I));

@FOR(DAYS(J):
   @SUM(DAYS(I) | I #LE# 5:
      START(@WRAP(J - I + 1, 7)))
         >= REQUIRED(J)
);
```

We have added the one output operation:

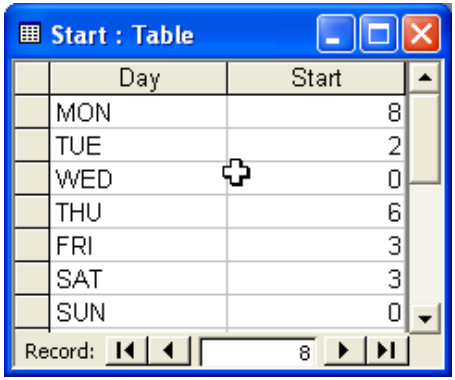
```
@TEXT('OUT.TXT') = DAYS, START;
```

which writes the values of the *DAYS* set and the values of the *START* attribute to the file *OUT.TXT*. Once you solve the model, LINGO will run this output operation, the file *OUT.TXT* will be generated, and it will contain the members of the *DAYS* set and the optimal values for the *START* attribute:

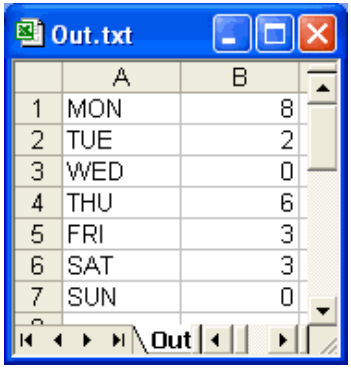
```
MON    8.0000000
TUE    2.0000000
WED    0.0000000
THU    6.0000000
FRI    3.0000000
SAT    3.0000000
SUN    0.0000000
```

File: OUT.TXT

You may now import the data from *OUT.TXT* into other applications. For instance, if you want to import the data into MS Access, you could use the *File|Get External Data|Import* command in Access to read the data into a table. We defined a small table in Access called *Start*, and imported the data in this manner yielding the following:



To import the data into an Excel sheet, you must first use the *File|Open* command on the *OUT.TXT* file to get the data into a spreadsheet by itself, as we have done here:



Once the results are imported into a spreadsheet, you may cut and paste them to any other sheet you desire.

Before we move on, suppose you are not interested in all the output generated by the standard LINGO solution report. Suppose, in the case of this example, all you want to see is the objective value and the values for the *DAYS* set and the *START* attribute. Here's how you can do it. First, add the additional output operation, shown here in bold, to your data section, so it looks like:

```
DATA:
   @TEXT('OUT.TXT') = DAYS, START;
   @TEXT() = DAYS, START;
ENDDATA
```

The new output operation causes the values of *DAYS* and *START* to be sent to the screen (since we omitted a file name). Next, you will need to suppress the normal LINGO solution report. In Windows versions of LINGO, select the *Solver|Options* command, click on the *Interface* tab in the *Options* dialog box, check the *Terse output* checkbox, then press the *OK* button (on platforms other than

Windows, enter the *TERSE* command). Now, solve your model and you will be presented with the following, abbreviated report:

```
Global optimal solution found at step:         8
Objective value:                        22.00000

MON    8.0000000
TUE    2.0000000
WED    0.0000000
THU    6.0000000
FRI    3.0000000
SAT    3.0000000
SUN    0.0000000
```

In the example above, we simply listed the names of set *DAYS* and attribute *START* on the right-hand side of our output operation. This causes LINGO to display *all* values for *DAYS* and *START*. Suppose we'd like more control over what values do and do not get displayed. In particular, suppose we are only interested in viewing those days in which the value for *START* is nonzero. We can do this by using the *@WRITEFOR* report function, which allows us to provide a condition to test before printing output:

```
DATA:
   @TEXT( ) = @WRITEFOR( DAYS( D) | START( D) #GT# 0:
    DAYS( D), @FORMAT( START( D), '6.1f'));
ENDDATA
```

Note how we now only display the days where *START* > 0 in our new report:

```
Global optimal solution found at iteration:           15
Objective value:                                 22.00000

MON   8.0
TUE   2.0
THU   6.0
FRI   3.0
SAT   3.0
```

Another feature of this last example to note is the use of the *@FORMAT* function, which we used to display the nonzero start values in a field of six columns with one trailing decimal point.

@WRITEFOR also allows us to form arithmetic expressions of the variable values. Here's an example that we could add to our staff model to compute the number of staff on duty each day:

```
DATA:
   @TEXT( ) = @WRITE( 'Day    On-Duty');
   @TEXT( ) = @WRITE( 14*'-');
   @TEXT( ) = @WRITEFOR( DAYS( D): DAYS(D),
   @FORMAT( @SUM( DAYS( D2) | D2 #LE# 5:
   START( @WRAP( D - D2 + 1, 7))), '11.1f'));
ENDDATA
```

Here's the report generated by these output operations:

```
Day    On-Duty
--------------
MON       20.0
TUE       16.0
WED       13.0
THU       16.0
FRI       19.0
SAT       14.0
SUN       12.0
```

This previous example also illustrates the use of the *@WRITE* function. The *@WRITE* function is similar to the *@WRITEFOR* function with the exception that it does not accept a set to loop on, and, therefore, is used to write single occurrences of text output. As with *@WRITEFOR*, *@WRITE* accepts expressions of variables. Here's an example that calculates the maximum number of employees starting on a particular day.

```
DATA:
   @TEXT() = @WRITE( 'Max start = ', @MAX( DAYS: START));
ENDDATA
```

which yields the following output: `Max start = 8.`

LINGO Command Scripts

A LINGO *command script* is any text file containing a series of LINGO commands. In addition to understanding the syntax of LINGO models, using command scripts requires some understanding of LINGO's command language (covered in Chapter 6, *Command-line Commands*). You can think of these commands as a macro language that allows you to automate the running of frequently used commands and/or models.

To run a command script in Windows versions of LINGO, use the *File|Take Commands* command. In other versions of LINGO, use the *TAKE* command. In both cases, you will be prompted for the name of a file that contains your command script. Once you have input the file name, LINGO will begin to execute the commands in this file. Execution will continue until either a *QUIT* command is encountered, causing LINGO to terminate, or an end-of-file is encountered, causing LINGO to return to normal input mode.

A Command Script Example

Once again, we will make use of the staff-scheduling model introduced on page 84 to illustrate the use of a command script. Suppose, instead of one hot dog stand, our operations have expanded and we now have three hot dog stands: Pluto Dogs, Mars Dogs, and Saturn Dogs. Our staffing requirements at the three sites are:

Site	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Pluto	20	16	13	16	19	14	12
Mars	10	12	10	11	14	16	8
Saturn	8	12	16	16	18	22	19

Running staffing models for all three sites is cumbersome and prone to error. We would like to automate the process by constructing a script file that runs all three staffing models automatically. To do this, we construct the following script file:

```
! Have LINGO echo input to the screen;
SET ECHOIN 1

! Suppresses the standard solution report;
SET TERSEO 1

! Begins input of a new model;
MODEL:
SETS:
   DAYS / MON TUE WED THU FRI SAT SUN/:
    REQUIRED, START;
ENDSETS

DATA:
   REQUIRED = @FILE('PLUTO.LDT');
   @TEXT('PLUTO.TXT') = START;
ENDDATA

MIN = @SUM(DAYS(I): START(I));

@FOR(DAYS(J):
 @SUM(DAYS(I) | I #LE# 5:
  START(@WRAP(J - I + 1, 7)))
   >= REQUIRED(J)
);

@FOR(DAYS: @GIN(START));
END

! Solve Pluto Dogs model;
GO
! Alter model for Mars;
ALTER ALL 'PLUTO'MARS'
! Solve Mars model;
GO
! Alter model for Saturn;
ALTER ALL 'MARS'SATURN'
! Solve Saturn model;
GO
```

```
! Restore parameters;
SET TERSEO 0
SET ECHOIN 0
```

Command Script: DOGS.LTF

We use two *SET* commands to set two of LINGO's parameters. First, we set *ECHOIN* to 1, which causes LINGO to echo all command script input to the screen. This can be useful when you are trying to debug a script file. Next, we set *TERSEO* to 1. This causes LINGO to go into terse output mode, which suppresses the default solution report each time we solve a model.

Next, we include the *MODEL*: command to put LINGO into model input mode. It is important here to remember the *MODEL:* statement is a command. When LINGO encounters this command in a script file, it reads all subsequent text in the file as model text until it encounters the *END* command. This model then becomes the current model in memory.

The key feature to note in our model is the data section:

```
DATA:
   REQUIRED = @FILE('PLUTO.LDT');
   @TEXT('PLUTO.TXT') = START;
ENDDATA
```

We use the *@FILE* function to include the staffing requirements from an external file and we use the *@TEXT* function to send the values of the *START* attribute to a file.

After the *END* statement, we have a *GO* command to solve the model for the Pluto stand. We then include an *ALTER* command to change all occurrences of *PLUTO* with *MARS*. This command will change the data section to (changes in bold):

```
DATA:
   REQUIRED = @FILE('MARS.LDT');
   @TEXT('MARS.TXT') = START;
ENDDATA
```

Assuming we have the staffing requirements for the Mars stand in the file *MARS.LDT*, our model is then ready to run again. However, this time it will solve for the *START* values for the Mars hot dog stand. We include commands to do the same for the Saturn location as well. Finally, we have two *SET* commands to restore the modified parameters.

You can run this command script by issuing the *File|Take Commands* command in Windows versions of LINGO, or you can use the *TAKE* command in other versions. Once the command script has been executed, you will find the three solution files: *PLUTO.TXT*, *MARS.TXT,* and *SATURN.TXT*. These files will contain the optimal values for the *START* attribute for the three locations.

The AUTOLG.DAT Script File

LINGO has an option that allows you to automatically execute a command script each time LINGO starts. To do this, simply name the command script *AUTOLG.DAT* and place it in LINGO's working directory. Each time LINGO starts, it will automatically execute the commands in this script file.

Specifying Files in the Command-line

When a Windows version of LINGO starts up, it checks the command-line for the presence of the following three commands:

Command	Action at Runtime
-T*filename*	LINGO executes a *File|Take Commands* command on the script file *filename*. If an *AUTOLG.DAT* file is present in LINGO's working directory, it will be queued for execution before *filename*.
-O*filename*	LINGO performs a *File|Open* command on *filename*, reading the file into a standard window.
-L*filename*	LINGO executes a *File|Log Output* command, causing all output that would normally have been sent to the command window to be routed to *filename*.

As an example, suppose we have the following command script:

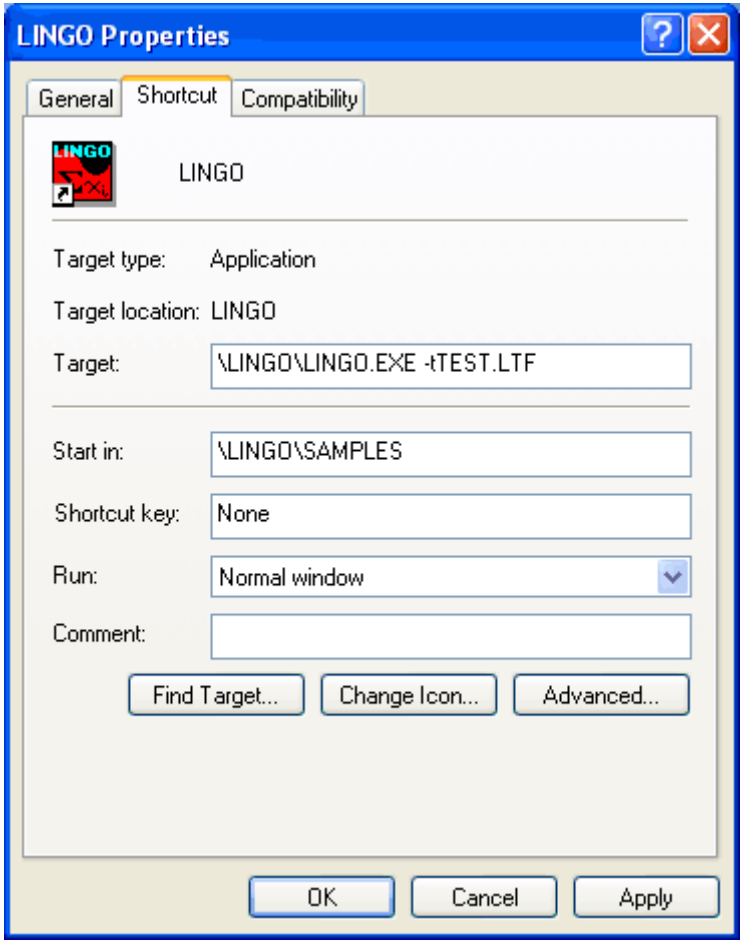
```
! Input a small model
MODEL:
MAX = 20 * X + 30 * Y;
X <= 50;
Y  <= 60;
X + 2 * Y <= 120;
END

! Terse output mode
SET TERSEO 1

! Solve the model
GO

! Open a file
DIVERT SOLU.TXT

! Send solution to the file
SOLUTION

! Close solution file
RVRT

! Quit LINGO
QUIT
```

Command Script: TEST.LTF

This script file inputs a small model, solves it, and then writes a solution report out to the file *SOLU.TXT*. Let's suppose the script file is titled *TEST.LTF*. We can instruct LINGO to automatically execute this file by adding the following command to LINGO's command-line: *-tTEST.LTF*. To do this under Windows, you will first need to create a shortcut icon for LINGO. Click the right mouse button on your desktop, and then select the *New* command followed by the *Shortcut* command. Press

the *Browse* button and then select the LINGO application file, which is found under the name *LINGO.EXE* in your main LINGO directory. You should now have a LINGO shortcut icon on your desktop that looks like:

To edit the command-line, you must right click on this icon and then select the *Properties* command. You will then see the dialog box:



In the *Target* edit box, add the command *-tTEST.LTF*. If you want LINGO to run without opening up a window, you can also select the *Minimized* option from the *Run* list box. Now, click the *Apply* button followed by the *OK* button.

You can now run LINGO and have it execute the script file by double clicking the shortcut icon on the desktop. Once you have done this, the solution file, *SOLU.TXT*, should contain:

```
 Variable        Value          Reduced Cost
        X       50.00000          0.000000
        Y       35.00000          0.000000

      Row   Slack or Surplus   Dual Price
        1       2050.000          1.000000
        2       0.000000          5.000000
        3       25.00000          0.000000
        4       0.000000          15.00000
```

File: SOLU.TXT

RunLingo

Windows and Linux versions of LINGO include a utility program called *RunLingo.* RunLingo can be used to process LINGO script files, however, RunLingo does not include the front-end graphical interface found in the interactive version of LINGO. RunLingo is invoked from the command line, where you may enter a script file name. RunLingo writes its output to the standard output device. RunLingo can be useful in a production environment, where you want LINGO to operate quietly in the background as part of a larger planning system.

The following example illustrates RunLingo being invoked from the command line to process a small transportation model contained in the script file *TRAN.LTF* (also included in the main LINGO folder).

```
C:\LINGO19>runlingo tran.ltf
 LINGO/WIN32 19.0.1.10 (04 Jun 20)

 LINDO API 13.0.1905.126  (10 Mar 2020 23:27:41)

 Copyright (C) 2011-2020 LINDO Systems Inc. Licensed material,
 all rights reserved. Copying except as authorized in license
 agreement is prohibited.

 License location: C:\LINGO19\lndlng19.lic
 Config  location: C:\LINGO19\LINGO.CNF

 Licensed for commercial use.
 Branch-and-bound solver enabled.
 Nonlinear solver enabled.
 Barrier solver enabled.
 Global solver enabled.
 Integer solver enabled.
 Stochastic solver enabled.
 Conic solver enabled.


 Default parameter values restored.


  Parameter        Old Value     New Value
   ECHOIN             0             1

:  MODEL:
? ! A 3 Warehouse, 4 Customer
?    Transportation Problem;
? SETS:
?    WAREHOUSE / WH1, WH2, WH3/   : CAPACITY;
?    CUSTOMER   / C1, C2, C3, C4/ : DEMAND;
?    ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
? ENDSETS
? ! The objective;
?  [OBJ] min = @SUM( ROUTES: COST * VOLUME);
? ! The demand constraints;
?  @FOR( CUSTOMER( J): [DEM]
?   @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
?    DEMAND( J));
? ! The supply constraints;
?  @FOR( WAREHOUSE( I): [SUP]
?   @SUM( CUSTOMER( J): VOLUME( I, J)) <=
?    CAPACITY( I));
? ! Here are the parameters;
? DATA:
?    CAPACITY =   30, 25, 21 ;
?    DEMAND =   15, 17, 22, 12;
?    COST =      6,  2,  6,  7,
?                4,  9,  5,  3,
?                8,  8,  1,  5;
? ENDDATA
? END
: set terseo 1
```

```
  Parameter        Old Value     New Value

   TERSEO             0             1


: go
 Compiling model ...
 Structural analysis, pass 1 ...
 Scalarizing model ...
 Generating nonzero matrix ...
 Solving...

 Global optimal solution found.
 Objective value:                              161.0000
 Infeasibilities:                              0.000000
 Total solver iterations:                             6


: nonz volume
 Global optimal solution found.
 Objective value:                              161.0000
 Infeasibilities:                              0.000000
 Total solver iterations:                             6


                        Variable           Value        Reduced Cost

               VOLUME( WH1, C1)        2.000000            0.000000
               VOLUME( WH1, C2)        17.00000            0.000000
               VOLUME( WH1, C3)        1.000000            0.000000

               VOLUME( WH2, C1)        13.00000            0.000000
               VOLUME( WH2, C4)        12.00000            0.000000
               VOLUME( WH3, C3)        21.00000            0.000000

: quit

C:\LINGO19>
```

Redirecting Input and Output

In most Unix environments, it is possible to redirect all screen output from LINGO to a text file. You can also redirect all input from the keyboard to an input file. This is accomplished by specifying the files in the command-line using the following syntax:

LINGO < *input_file* > *output_file*

Upon execution of this command, LINGO will take its input from *input_file*, and a text file will be created called *output_file*, which contains everything that would have appeared on the screen had LINGO been started normally. Path names may be included in the names of the input and output files.

Exploiting this capability allows you to use LINGO as a "black box" in larger turnkey applications. If done properly, the user will not be aware LINGO is running in the background.

As an example, we could run the script file from the previous section, *TEST.LTF*, generating the same solution file, *SOLU.TXT*, with the command:

```
LINGO < TEST.LTF > CAPTURE.TXT
```

The file *CAPTURE.TXT* is used to capture all screen output.

Managing LINGO Files

In order to help keep track of your files, you may want to adopt a file naming standard, at least as far as the file name extensions are concerned. Here are some suggestions for file name extensions:

Extension	Description
.LG4	model files (Windows only)
.LNG	model files in text format
.LTF	script files
.LDT	included data files
.LGR	report files

A Note to Windows Users:
If you use the default LINGO format (.LG4) for a model, the file will be saved in a special binary format. Files in this format will only be readable by Windows versions of LINGO, and will appear garbled if read into other applications. This binary format allows LINGO models to function as both Object Linking and Embedding (OLE) containers and servers, allows for embedding of objects (e.g., bitmaps), and permits custom formatting of model text. Thus, you will generally want to use the .LG4 format. Cases where you *must* use a text format are:

1. a model that must be transferred to another platform or application,
2. a LINGO script file, or
3. a LINGO data file to be included with *@FILE*.

LINGO anticipates this by saving any file with an extension other than .LG4 as text.

You can force LINGO to always use the .LNG text format for models by checking the .LNG box on the *Interface* tab of the *Solver|Options* dialog box. Any special formatting of the text (e.g., bold fonts) will be lost when a model is saved in this format. Also, models saved as text may not function as OLE containers or servers.

On platforms other than Windows, LINGO *always* saves files in text format.

9 Interfacing With Spreadsheets

As we have mentioned, it can be cumbersome and impractical to try to maintain your data in a LINGO model file. This is particularly true if you have more than just a small handful of data—as is the case with most practical models. Spreadsheets are adept at handling small to moderate amounts of data. Spreadsheets are also *very* useful tools for manipulating and presenting the results generated by your model. For these reasons, LINGO has a number of features that allow the user to import data from spreadsheets and export solutions back out to spreadsheets. These features include real-time Object Linking and Embedding (OLE) links to Excel, OLE automation links that can be used to drive LINGO from Excel macros, and embedded OLE links that allow you to import the functionality of LINGO into Excel. ***At present, all of these features are supported only under Windows versions of LINGO.***

Importing Data from Spreadsheets

LINGO provides the *@OLE* function for importing data from spreadsheets. This function is only available under Windows versions of LINGO. *@OLE* performs direct OLE transfers of data between LINGO and Excel.

Using @OLE to Import Data from Excel

@OLE is an interface function for moving data back and forth from Excel using OLE based transfers. OLE transfers are direct memory transfers and do not make use of intermediate files. When using *@OLE*, LINGO loads Excel, tells Excel to load the desired spreadsheet, and requests ranges of data from the sheet. You must have Excel 5, or later, to be able to use the *@OLE* function. The *@OLE* function may be used in the data and init sections to import data.

@OLE can read both set members and set attributes—set members are expected in text format, while set attributes are expected in numeric format. Primitive sets require one cell of data for each set member (i.e., one set member per cell). You will need *n* cells of values to initialize each *n*-dimensional derived set member, where the first *n* cells contain the first set member, the second *n* cells contain the second set member, and so on.

@OLE can read one or two-dimensional ranges (ranges on a single worksheet in Excel), but cannot read discontinuous ranges or three-dimensional ranges (ranges that traverse more than one worksheet in Excel).

Ranges are read left-to-right, top-to-bottom.

Importing in the Data and Init Sections with @OLE

The syntax for using *@OLE* to import data in both the data and init sections is:

object_list = *@OLE*([‘*spreadsheet_file*’] [, *range_name_list*]);

The *object_list* is a list of model objects, optionally separated by commas, which are to be initialized from the spreadsheet. *Object_list* may contain any combination of set names, set attributes, and scalar variables.

The *spreadsheet_file* is the name of the Excel spreadsheet file to retrieve the data from. If the name is omitted, LINGO defaults to using whatever workbook is currently open in Excel.

The *range_name_list* is the list of named ranges in the sheet to retrieve the data from. The ranges must contain exactly one element for each member in the *object_list*. There are three options available in how you specify the ranges. First, you can omit the range arguments entirely. In which case, LINGO defaults to using a list of range names identical to the list of object names in *object_list.* Second, you can specify a single range name to retrieve all the data from. In which case, all the objects in *object_list* must be defined on the same set, and LINGO reads the data as if it was a table. When specifying a single range name for multiple objects, all the objects must be of the same data type. Furthermore, you can’t mix set members (text) with set attributes (numeric). Finally, you can specify one range name for each object in *object_list*. In which case, the objects do not have to be defined on the same set and can be of differing data types. Examples of these three methods for using *@OLE* follow:

Example 1: `COST, CAPACITY = @OLE();`

In this example we specify no arguments to the *@OLE()* function. In which case, LINGO will supply the default arguments. Given that no workbook name was specified, LINGO will use whatever workbook is currently open and active in Excel. In addition, no range names were specified, so LINGO defaults to using the names of the model objects. Thus, *COST* and *CAPACITY* are initialized to the values found, respectively, in the ranges *COST* and *CAPACITY* in the currently open workbook in Excel.

Note: If you do not specify a workbook name in *@OLE()* function references, LINGO defaults to using whatever workbook is currently open in Excel. Therfore, you will need to open Excel and load the relevant workbook prior to solving your model.

Example 2: `COST, CAPACITY = @OLE( 'SPECS.XLS', 'DATATABLE');`

In this example, we are specifying a single range to initialize both *COST* and *CAPACITY*. Assuming the range *DATATABLE* has two columns, LINGO initializes *COST* to the data in column 1 and *CAPACITY* to the data in column 2. Note, in order for this method to work, both *COST* and *CAPACITY* must be defined on the same set. Furthermore, they must both be either sets or set attributes—mixed types aren’t allowed using this form.

Example 3: `COST, CAPACITY = @OLE( 'SPECS.XLS', 'COST01', 'CAP01');`

In this example, we are specifying individual ranges to initialize both *COST* and *CAPACITY*. *COST* will be initialized with the data in the *COST01* range and *CAPACITY* will receive the values in the *CAP01* range.

As a final note, it is important to know that derived sets may be imported from either a single range or from *n* ranges, where *n* is the dimension of the derived set. Here are two examples to illustrate:

Example 4: `ARCS, COST = @OLE( 'TRAN.XLS', 'ARCS', 'COST');`

You might find something similar to this example in a transportation model. *ARCS* is the set of shipping arcs and *COST* is an attribute for storing the cost of shipping one unit down each arc. Typically, *ARCS* would be a 2-dimensional derived set of from-to coordinate pairs. In this example, we've chosen to bring the list of arcs in from a single workbook range. Suppose there are 10 shipping arcs in our model, then the *ARCS* range would have 20 cells, with the first column of 10 cells being the ship-from points and the second column of 10 cells being the ship-to points. Since we are using the single range method to input a derived set, the two columns are adjacent and are both contained in the single range call ARCS. This is in contrast to the following example…

Example 5: `ARCS, COST = @OLE( 'TRAN.XLS', 'FROM', 'TO', 'COST');`

…where we use two ranges to store the two-dimensional *ARCS* set. The first range, *FROM*, would contain 10 ship-from points, while the second range, *TO*, would contain 10 ship-to points. These two ranges may lie in different areas of the workbook and need not be adjacent.

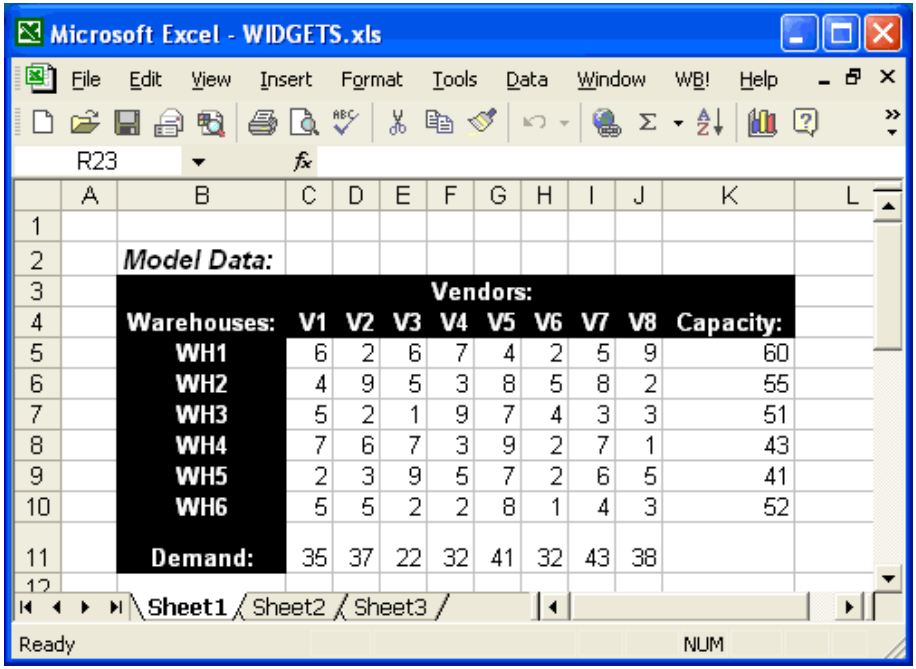
Importing in a Transportation Model with @OLE

We will now make use of the Wireless Widgets transportation model introduced in Chapter 1, *Getting Started with LINGO*, to illustrate in more detail the use of the *@OLE* function. The model is reproduced below with changes listed in bold type:

```
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
! Import warehouses and vendors from Excel;
   WAREHOUSES: CAPACITY;
   VENDORS   : DEMAND;
   LINKS(WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS

! The objective;
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));

! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I):
     VOLUME(I, J)) = DEMAND(J));

! The capacity constraints;
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J))
     <= CAPACITY(I));

DATA:
! Import the data from Excel;
   WAREHOUSES, VENDORS, CAPACITY, DEMAND, COST =
    @OLE('\LINGO\SAMPLES\WIDGETS.XLS',
     'WAREHOUSES', 'VENDORS', 'CAPACITY',
      'DEMAND', 'COST');
ENDDATA
```

Model: WIDGETS3

Instead of explicitly listing the data in the text of the model, we are now importing it entirely from the *WIDGETS.XLS* spreadsheet. Below is an illustration of the *WIDGETS.XLS*:



In addition to inputting the data into this sheet, we also had to define range names for the cost, capacity, demand, vendor name, and warehouse name regions. Specifically, we defined the following range names:

Name	Range
Capacity	`K5:K10`
Cost	`C5:J10`
Demand	`C11:J11`
Vendors	`C4:J4`
Warehouses	`B5:B10`

To define a range name in Excel:

1) select the range by dragging over it with the mouse with the left button down,
2) release the mouse button,
3) select the *Insert|Name|Define* command,
4) enter the desired name, and
5) click the OK button.

We use the following instance of the *@OLE* function in the data section of our model to import the data from Excel:

```
WAREHOUSES, VENDORS, CAPACITY, DEMAND, COST =
 @OLE('\LINGO\SAMPLES\WIDGETS.XLS',
  'WAREHOUSES', 'VENDORS', 'CAPACITY',
   'DEMAND', 'COST');
```

Note that because the model objects are all either primitive sets or set attributes, and they have the same names as their corresponding spreadsheet ranges, we could have dropped the range name arguments and used the equivalent, but shorter, version:

```
WAREHOUSES, VENDORS, CAPACITY, DEMAND, COST =
 @OLE('\LINGO\SAMPLES\WIDGETS.XLS');
```

As an aside, note that we used a single *@OLE* function call to read all the data for this model. This is not a requirement, however. For clarity, you may choose to use multiple *@OLE* function calls—perhaps one for each model object.

When we solve this model, LINGO will load Excel (assuming it isn't already running), load the *WIDGETS* worksheet, and then pull the values for the *CAPACITY*, *COST*, and *DEMAND* attributes from the worksheet, along with the members of the *WAREHOUSES* and *VENDORS* sets. Excerpts from the solution appear below:

```
Global optimal solution found.
Objective value:                              664.0000
Total solver iterations:                            15

       Variable      Value   Reduced Cost
VOLUME(WH1, V2)   19.00000      0.0000000
VOLUME(WH1, V5)   41.00000      0.0000000
VOLUME(WH2, V4)   32.00000      0.0000000
VOLUME(WH2, V8)   1.000000      0.0000000
VOLUME(WH3, V2)   12.00000      0.0000000
VOLUME(WH3, V3)   22.00000      0.0000000
VOLUME(WH3, V7)   17.00000      0.0000000
VOLUME(WH4, V6)   6.000000      0.0000000
VOLUME(WH4, V8)   37.00000      0.0000000
VOLUME(WH5, V1)   35.00000      0.0000000
VOLUME(WH5, V2)   6.000000      0.0000000
VOLUME(WH6, V6)   26.00000      0.0000000
VOLUME(WH6, V7)   26.00000      0.0000000
```

Exporting Solutions to Spreadsheets

LINGO allows you to place interface functions in a model's data section to automatically perform exports to Excel each time your model is solved. In this chapter, we will focus on how to place interface functions in your model to export your model's results to Excel.

In the previous section, we saw how the *@OLE* interface function can be used to import data from Excel. As we will demonstrate here, this function may also be used to send solutions back out to Excel. As with imports, *@OLE* uses OLE technology to create real time links with Excel.

Using @OLE to Export Solutions to Excel

@OLE is an interface function for moving data back and forth from Excel using OLE based transfers. OLE transfers are direct memory transfers and do not make use of intermediate files. When using *@OLE* for exports, LINGO loads Excel, tells Excel to load the desired spreadsheet, and sends ranges of data containing solution values to the sheet. *@OLE* can export one and two-dimensional ranges (rectangular ranges that lie on a single spreadsheet in Excel), but cannot export discontinuous or three-dimensional ranges (ranges that traverse more than one spreadsheet in Excel). In order to export solutions with *@OLE*, you place calls to *@OLE* in the data section of your model. These *@OLE* export instructions are executed each time your model is solved.

Syntax Form 1

The first form of syntax for using *@OLE* to export data is:

@OLE(['*spreadsheet_file'*] [, *range_name_list*]) = *object_list*;

The *object_list* is a comma delimited list of sets, set attributes, and/or scalar variables to be exported.

The '*spreadsheet_file'*is the name of the workbook file to export the values to. If the name is omitted, LINGO defaults to using whatever workbook is currently open in Excel.

The *range_name_list* is the list of named ranges in the sheet to export solution values to. The ranges must contain exactly one cell for each exported value for each object in the *object_list*. Primitive sets and set attributes export one value per element. Derived sets, on the other hand, export one value for each *dimension* of the set. Thus, a two-dimensional set exports two values per set member.

As an example, consider the following model and its solution:

```
SETS:
   S1: X;
   S2(S1, S1): Y;
ENDSETS
DATA:
   S1,X = M1,1  M2,2  M3,3;
   S2,Y = M1,M2,4  M3,M1,5;
ENDDATA

    Variable           Value
       X(M1)        1.000000
       X(M2)        2.000000
       X(M3)        3.000000
   Y(M1, M2)        4.000000
   Y(M3, M1)        5.000000
```

X and *Y*, both set attributes, export one numeric value per element. More specifically, *X* exports 1,2, and 3; while *Y* exports 4 and 5. *S1*, a primitive set, exports one text value per element, or the values: *M1*, *M2*, and *M3*. *S2*, on the other hand, is a two-dimensional derived set. Thus, it exports two text values per element. In this case, *S2* has two members, so it exports the four values *M1*, *M2, M3, and M1*.

There are three options available for how you specify the range names. First, you can explicitly specify one receiving range for each model object in *object_list*. Secondly, you can omit the range arguments entirely. In that case, LINGO supplies the range names with default values that are the same as the names of the model objects. Finally, you can specify a single range name to export all the solution values to. In this final case, all the variables in *object_list* must be defined on the same set, and LINGO exports the data in a tabular format. Examples of these three methods for using *@OLE* to export solutions follow:

Example 1:

```
@OLE('\XLS\DEVELOP.XLS', 'BUILD_IT', 'HOW_BIG') =
 BUILD, SQ_FEET;
```

Here, an individual range for receiving each model object is specified. Thus, the values of *BUILD* will be placed in the range *BUILD_IT* and *SQ_FEET* in the range *HOW_BIG*. When specifying individual ranges, model objects are not required to be defined on the same set.

Example 2:

```
@OLE('\XLS\DEVELOP.XLS') = BUILD, SQ_FEET;
```

In this case, we omitted the range name argument. Thus, LINGO defaults to using the model object names for the range names. So, LINGO exports *BUILD* and *SQ_FEET* to ranges of the same name in the *DEVELOP.XLS* Excel sheet.

Example 3:

```
@OLE('\XLS\DEVELOP.XLS', 'SOLUTION') = BUILD, SQ_FEET;
```

Here we have specified a single range, *SOLUTION*, to receive both model objects. Assuming that the receiving range has two columns and our model objects are both one-dimensional, the values of *BUILD* will be placed in column 1 and *SQ_FEET* in column 2. In order for this method to work, *BUILD* and *SQ_FEET* must be defined on the same set.

Note: The major difference to notice between using *@OLE* for exports, as opposed to imports, is the side of the statement the *@OLE* function appears on. When the *@OLE* function appears on the left of the equals sign, you are exporting. When it appears on the right, you are importing. So, always remember:

@*OLE*(…) = object_list; ↔ Export, and
object_list = @*OLE*(…); ↔ Import.

Another way to remember this convention is that the left-hand side of the expression is receiving the data, while the right-hand side is the source. For those familiar with computer programming languages, this follows the convention used in assignment statements.

Syntax Form 2

As with *@TEXT*, you may also use the *@WRITEFOR* function in conjunction with *@OLE* to give you more control over the values that are exported, which brings us to our second form of syntax for exporting to Excel:

@OLE(['*spreadsheet_file*'], *range_name_list*) = @WRITEFOR(*setname*
[(*set_index_list*) [| *conditional_qualifier*]] : *output_obj_1*[,…,*output_obj_n*]);

One thing to note that differs from the previous syntax is that the range name list is now required when exporting via the *@WRITEFOR* function. The range name list can be a single-cell range, a single multiple-cell range, or a list of multiple-cell ranges.

In the case of a single cell range, the *i*-th output object will be written to the *(i-1)*-th column to the right of the named cell. Note that single-cell ranges act dynamically in that *all* values will be written to the workbook even though, of course, they lie outside the single-cell range. When all output is written, the original single-cell range will be at the upper left corner of the table of output values.

In the case of a single multiple-cell range, LINGO creates a table of all the output values, where output object *i* forms the *i*-th column of the table. This table is then written to the output range. Items are written from upper-left to lower-right. In general, your output range will have one column for each output object. If not, the columns will get scrambled on output.

If a list of multiple cell ranges is specified, then you must specify one range name for each output object. Each output object will be written to its output range in upper-left to lower-right direction.

@WRITEFOR functions like any other set looping function in that, as a minimum, you will need to specify the set to loop over. Optionally, you may also specify an explicit set index list and a conditional qualifier. If a conditional qualifier is used, it is tested for each member of the looping set and output will not occur for any members that don't pass the test. It's this feature of being able to base output on the results of a condition that distinguish this second style of syntax.

The list of output objects, of course, specifies what it is you want to output. As with the first form of syntax, the output objects may be labels, set members and variable values. However, you have additional latitude in that the output objects may now consist of complex expressions of the variable values (e.g., you could compute the ratio of two variables). This is a useful feature when you need to report statistics and quantities derived from the variable values. By placing these calculations in the data section, as opposed to the model section, you avoid adding unnecessary complications to the constraints of the model.

In general, you can do everything in the second form of syntax that you can do in the first, and more. However, the first form has an advantage in that it can be very concise.

Some examples follow:

Example 1:

```
@OLE( 'RESULTS.XLS', 'A1') =
 @WRITEFOR( DAYS( D) | START( DAYS) #GT# 0:
  DAYS( D), START( D));
```

Here, our target is the single cell *A1*. Starting at *A1* we will write two columns. The first column will contain the names of the *DAYS* set for which the attribute *START* is nonzero. The second column will contain the *START* values. Assuming that there are five days that

have nonzero values, then range *A1:A5* will contain the names of the days and *B1:B5* will contain the start values.

Example 2:

```
@OLE( 'RESULTS.XLS', 'SKED') =
 @WRITEFOR( DAYS( D) | START( DAYS) #GT# 0:
  DAYS( D), START( D));
```

Here, our target is the multiple-cell range *SKED*. Assuming *SKED* is a two-column range, column one will receive the *DAYS* set members and column 2 will receive the *START values.*

Example 3:

```
@OLE( 'RESULTS.XLS', 'DAYS', 'START') =
 @WRITEFOR( DAYS( D) | START( DAYS) #GT# 0:
  DAYS( D), START( D));
```

In this example, we specify one named range for each output object. In which case, each output object will be written to its corresponding range.

Note: When exporting to workbooks, receiving ranges that are larger than the number of exported values can be filled out by either erasing the contents of the extra cells or leaving the extra cells untouched. The default is to leave the extra cells untouched. If you would like to erase the contents of the extra cells, you'll need to enable the *Fill Out Ranges and Tables* option.

Exporting in a Transportation Model with @OLE

In a previous section of this chapter, *@OLE Importing in a Transportation Model*, we used the Wireless Widgets transportation model to demonstrate the use of the *@OLE* function for importing data from Excel. At the time, we did not use *@OLE* to export the solution back to the spreadsheet file. We will now extend the model in order to have it also export the solution back to the spreadsheet. The model is reproduced below with changes in the data section listed in bold type:

```
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
! Import warehouses and vendors from Excel;
   WAREHOUSES: CAPACITY;
   VENDORS   : DEMAND;
   LINKS(WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS

! The objective;
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));

! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I):
     VOLUME(I, J)) = DEMAND(J));

! The capacity constraints;
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J))
     <= CAPACITY(I));

DATA:
! Import the data from Excel;
   WAREHOUSES, VENDORS, CAPACITY, DEMAND, COST =
    @OLE('\LINGO\SAMPLES\WIDGETS.XLS',
     'WAREHOUSES', ' VENDORS', 'CAPACITY',
      'DEMAND', 'COST');

! Export the solution back to Excel;
   @OLE('\LINGO\SAMPLES\WIDGETS.XLS',
    'VOLUME') = VOLUME;
ENDDATA
```

Model: WIDGETS5

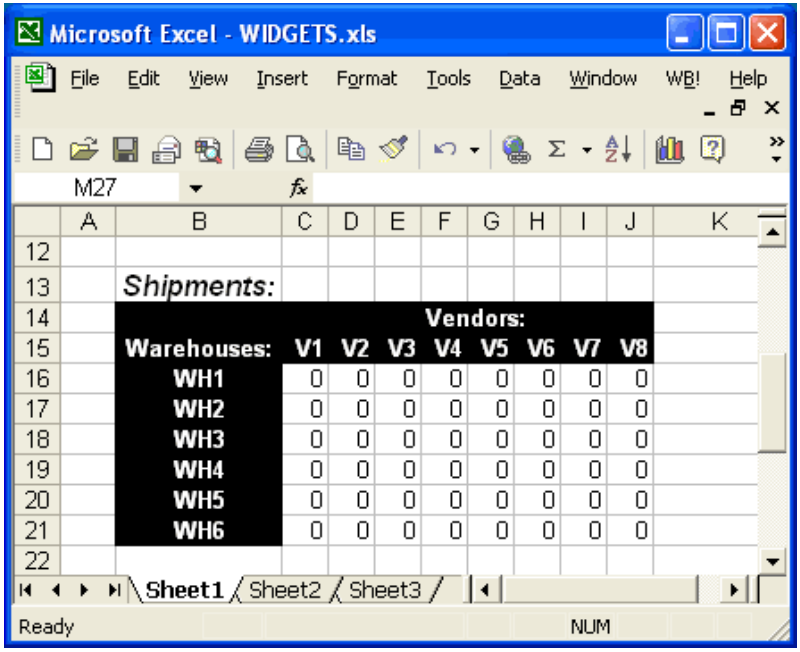
We now use the *@OLE* function to send the decision variables contained in the *VOLUME* attribute back to the Excel file *WIDGETS.XLS* with the statement:

```
@OLE('\LINGO\SAMPLES\WIDGETS.XLS', 'VOLUME') = VOLUME;
```

Note, since the attribute name is identical to the range name, we could have omitted the range name in the *@OLE* function and simply used the following:

```
@OLE('\LINGO\SAMPLES\WIDGETS.XLS') = VOLUME;
```

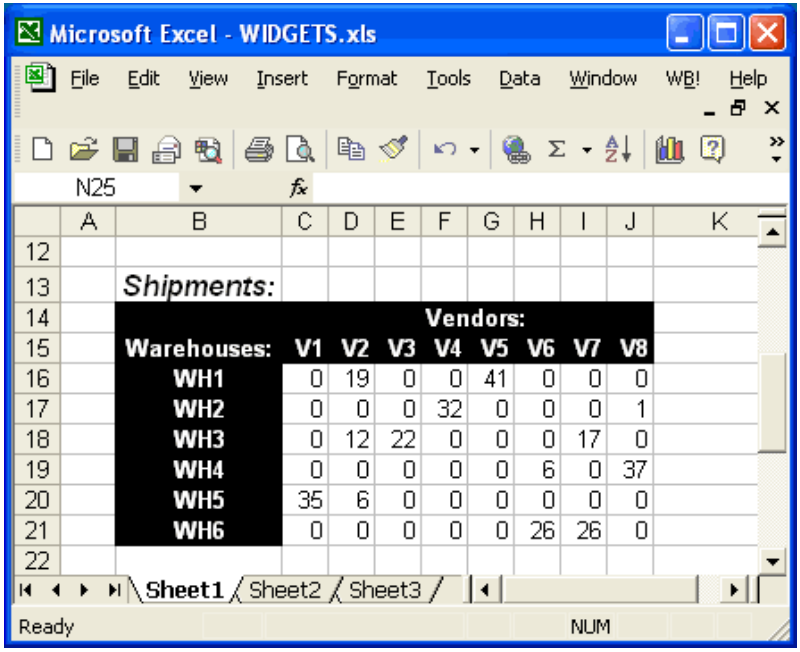
We will now need to add a range titled *VOLUME* for receiving the solution values in the *WIDGETS* spreadsheet. Here is the range as it appears after adding it to the sheet:



We have also used the *Insert|Name|Define* command in Excel to assign the range name *VOLUME* to the receiving range of *C16:J21*. To define a range name in Excel:

1. select the range by dragging over it with the mouse with the left button down,
2. release the mouse button,
3. select the *Insert|Name|Define* command,
4. enter the desired name (*VOLUME* in this case), and
5. click the *OK* button.

When we solve this model, LINGO will load Excel (assuming it isn't already running), load the *WIDGETS* worksheet, and then pull the data for *WAREHOUSES*, *VENDORS*, *CAPACITY*, *COST*, and *DEMAND* from the worksheet. Once the solver has found the optimal solution, LINGO will send the values for the *VOLUME* attribute back to the worksheet storing them in the range of the same name and the updated range will appear as follows:



Export Summary Reports

Whenever you use the *@OLE* function to export solutions to spreadsheets, you will receive a summary of the results of the export process. This summary is called the *export summary report*. These reports will be found at the top of the solution report window. There will be one export summary report for each *@OLE* function in the model used for exporting solutions. Here is the export summary report obtained when we exported the solution of the Wireless Widgets transportation model to Excel using *@OLE*:

```
Export Summary Report
---------------------
Transfer Method:  OLE BASED
Workbook:         \LINGO\SAMPLES\WIDGETS.XLS
Ranges Specified:                1
   VOLUME
Ranges Found:                    1
Range Size Mismatches:           0
Values Transferred:             48
```

The *Transfer Method* field lists the type of transfer used. *@OLE* transfers will be listed as “OLE BASED”.

The *Workbook* field lists the name of the workbook the export was performed on.

The *Ranges Specified* field lists the total number of ranges specified in the export function followed by a listing of the range names.

The *Ranges Found* figure lists the number of ranges actually found in the sheet from the total number specified.

In general, the spreadsheet range should have one cell for each data value being exported. If a range has too few or too many cells, it gets counted in the *Range Size Mismatches* field.

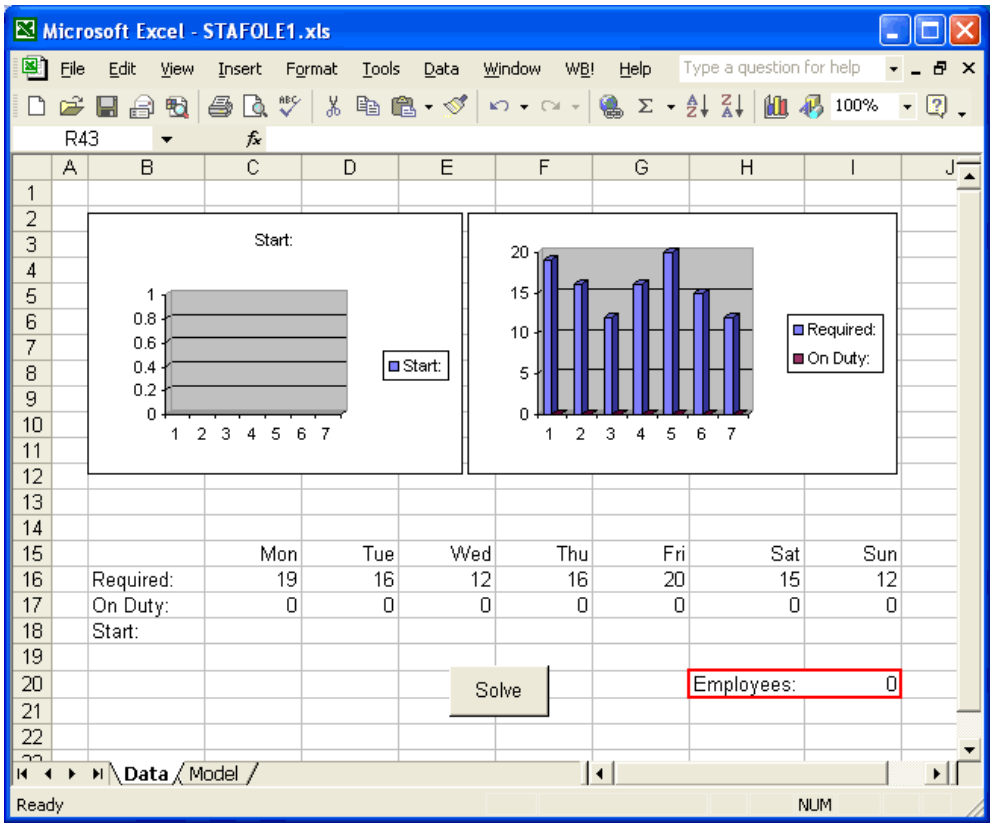
The *Values Transferred* field lists the total number of data values actually transferred into all the specified ranges.

OLE Automation Links to Excel

LINGO allows you to place a LINGO command script in a range in an Excel spreadsheet and then pass the script to LINGO by means of OLE Automation. This allows you to setup a client-server relationship between Excel and LINGO.

To illustrate this feature, we will once again make use of the staff-scheduling model introduced in the *Primitive Set Example — Staff-Scheduling* section in Chapter 2, *Using Sets*. This illustration assumes the reader is moderately familiar with the use of Excel Visual Basic macros. If needed, you can refer to the Excel documentation for more background.

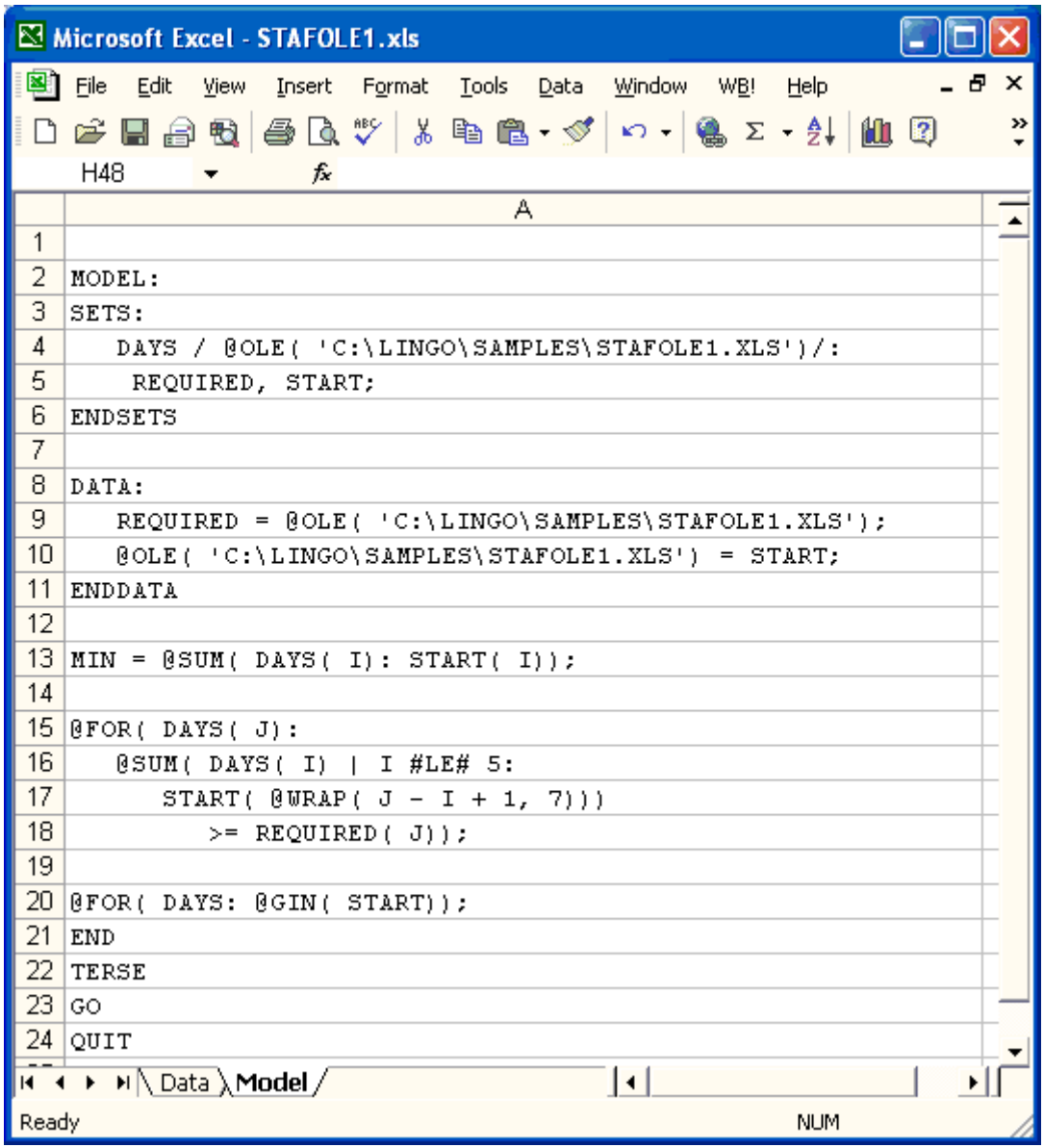
Consider the following Excel spreadsheet:



Spreadsheet: STAFOLE1.XLS

We have placed the staffing requirements in the range *C16:I16* and assigned the name *REQUIREMENTS* to this range. We have also assigned the name *START* to the range *C18:I18*. LINGO will be sending the solution to the *START* range. We have also included two graphs in the sheet to help visualize the solution. The graph on the left shows how many employees to start on each day of the week, while the graph on the right compares the number on duty to the number required for each day.

Note that our spreadsheet has a second tab at the bottom titled *Model*. Select this tab and you will find the following:



Spreadsheet: STAFOLE1.XLS

This page contains the command script we will use to solve the staffing model. For more information on command scripts, refer to page 495. In line 1, we turn on terminal echoing, so LINGO will echo the command script to the command window as it is read. Lines 2 through 21 contain the text of the model, which should be familiar by now. Note, in the data section, we are using two *@OLE* functions—the first to import the data from the spreadsheet and the second to export the solution back to the spreadsheet. The data is read from the range named *REQUIRED*, and the solution is written to the *START* range on the first tab of the sheet. In line 22, we use the *GO* command to solve the model. We have also assigned the range name *MODEL* to the range that contains this script (*Model!A1:A23*).

Given that we have our LINGO command script contained in our spreadsheet, the next question is how we pass it to LINGO to run it. This is where OLE Automation comes in. If you recall, the first tab of our sheet (the tab labeled *Data*) had a *Solve* button. We added this button to the sheet and attached the following Excel Visual Basic macro to it:

```
Sub LINGOSolve()
  Dim iErr As Integer
  iErr = LINGO.RunScriptRange("MODEL")
  If (iErr > 0) Then
    MsgBox ("Unable to solve model")
  End If
End Sub
```

We use OLE Automation to call the LINGO exported method *RunScriptRange*, passing it the range name *MODEL*. This, of course, is the name of the range that contains the command script. The *RunScriptRange* routine calls Excel to obtain the contents of the range and begins processing the commands contained therein. Processing continues until either a *QUIT* command is encountered or there are no further commands remaining in the range.

RunScriptRange will return a value of 0 if it was successfully able to queue the script for processing. If *RunScriptRange* was not successful, it will return one of the following error codes:

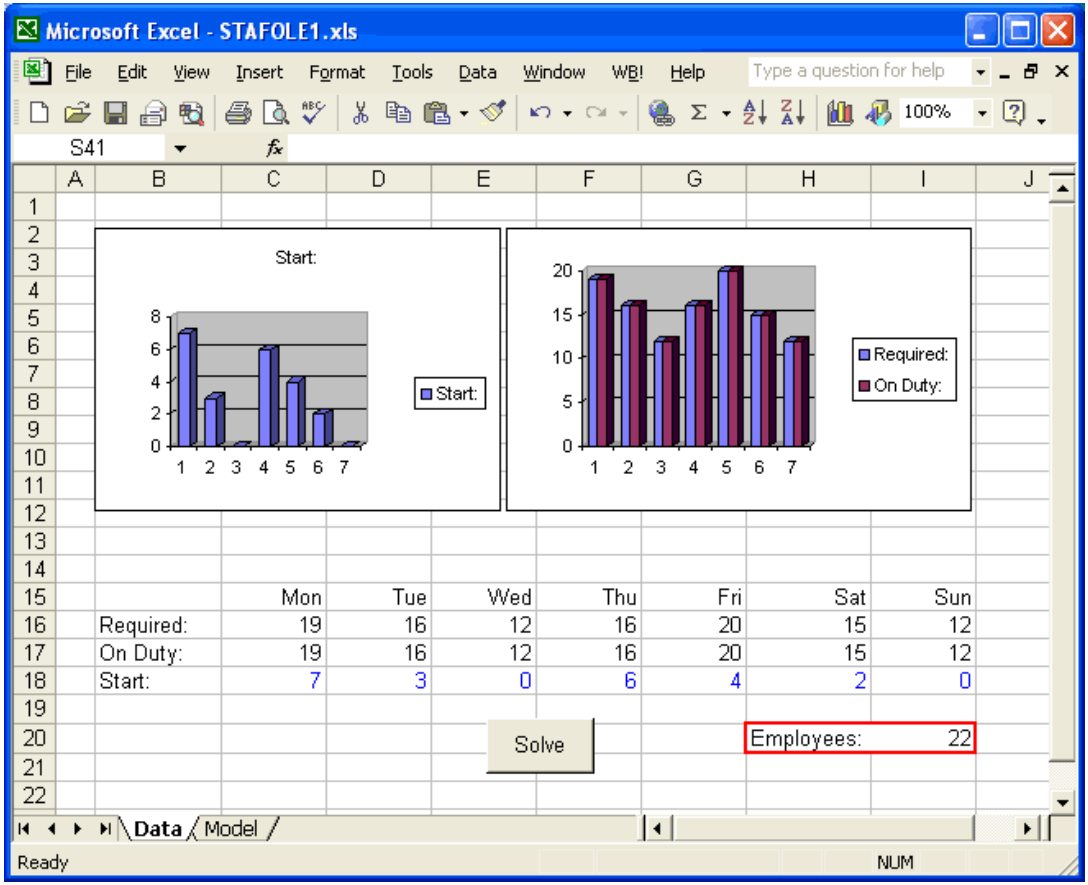
Error Code	Description
1	Invalid argument
2	<Reserved>
3	Unable to open log file
4	Null script
5	Invalid array format
6	Invalid array dimension
7	Invalid array bounds
8	Unable to lock data
9	Unable to allocate memory
10	Unable to configure script reader
11	LINGO is busy
12	OLE exception
13	Unable to initialize Excel
14	Unable to read Excel range
15	Unable to find Excel range

We have also added the following *Auto_Open* macro to the sheet:

```
Dim LINGO As Object
Sub Auto_Open()
   Set LINGO = CreateObject("LINGO.Document.4")
End Sub
```

An *Auto_Open* macro is automatically executed each time a sheet is opened. We declare LINGO as an object and attach the LINGO object to the LINGO application with the *CreateObject* function.

Now, go back to the first tab on the workbook and press the *Solve* button. After a brief pause, you should see the optimal solution installed, so the sheet resembles:



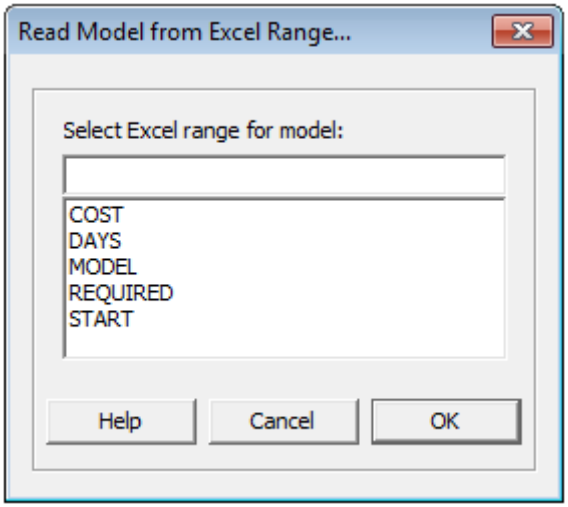
Spreadsheet: STAFOLE1.XLS

The optimal number of employees to start on each day of the week is now contained in the *START* range (*C18:I18*), and the graphs have been updated to reflect this solution.

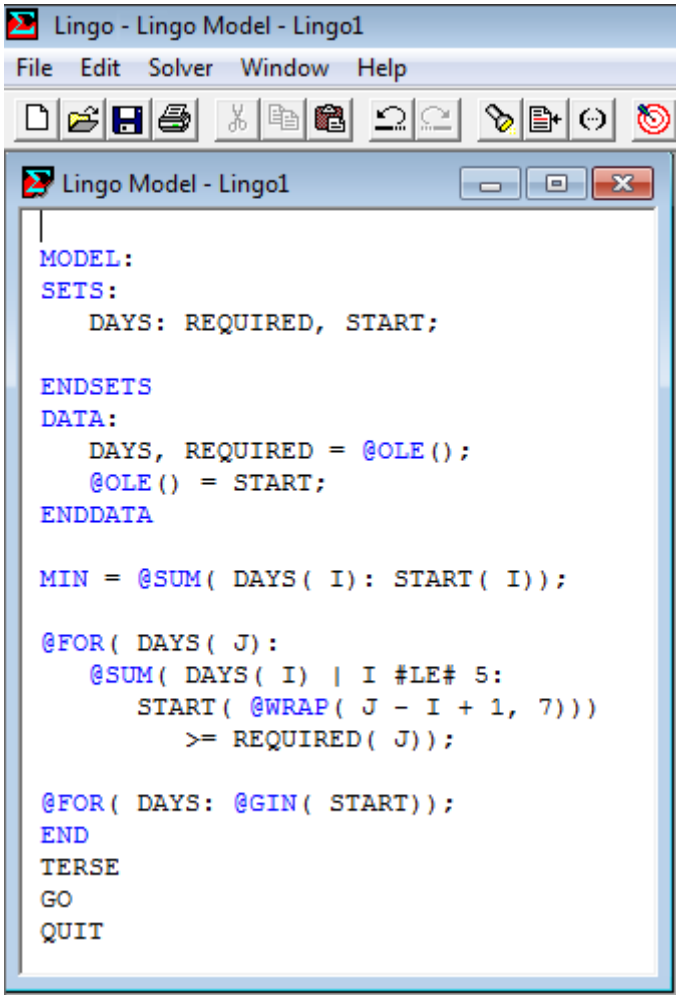
Editing Models Stored in Excel

In the example above, we have placed the LINGO model on a tab in an Excel workbook. There will be times when you may wish to load this model back into LINGO for testing and development purposes. You could certainly do this by cutting-and-pasting the model text back and forth from LINGO to Excel, but LINGO has a two commands to simplify the process.

To temporarily load a model from Excel into LINGO, you can use the *File|Excel|Read Model from Range* command. This command will prompt you for the workbook name that you wish to retrieve the model from. It will then display a list of range names found in the model, as seen here:



In this example, the model is stored in the range titled MODEL. Selecting MODEL from the list and then clicking on the OK button, you will then be presented with a copy of the model in the LINGO GUI, ready for testing and editing:

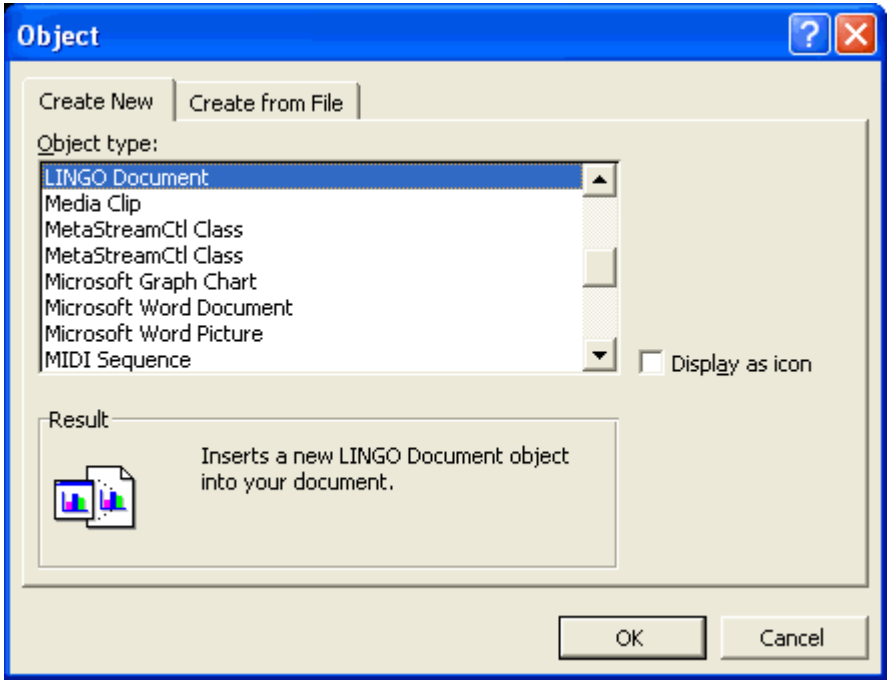


Once you have the model ready to return Excel, you may use the *File|Excel|Write Model to Range* command to return the model to the original Excel range. Note that if the range is too small for the entire model, you will receive an error message. On the other hand, if the range is larger than required, LINGO will erase the content of the range from the end of the model text to the end of the workbook range, so that no spurious text remains after the end of the new model text. Given this, it's a good idea to always use ranges that are actually larger than required by the model text.

Embedding LINGO Models in Excel

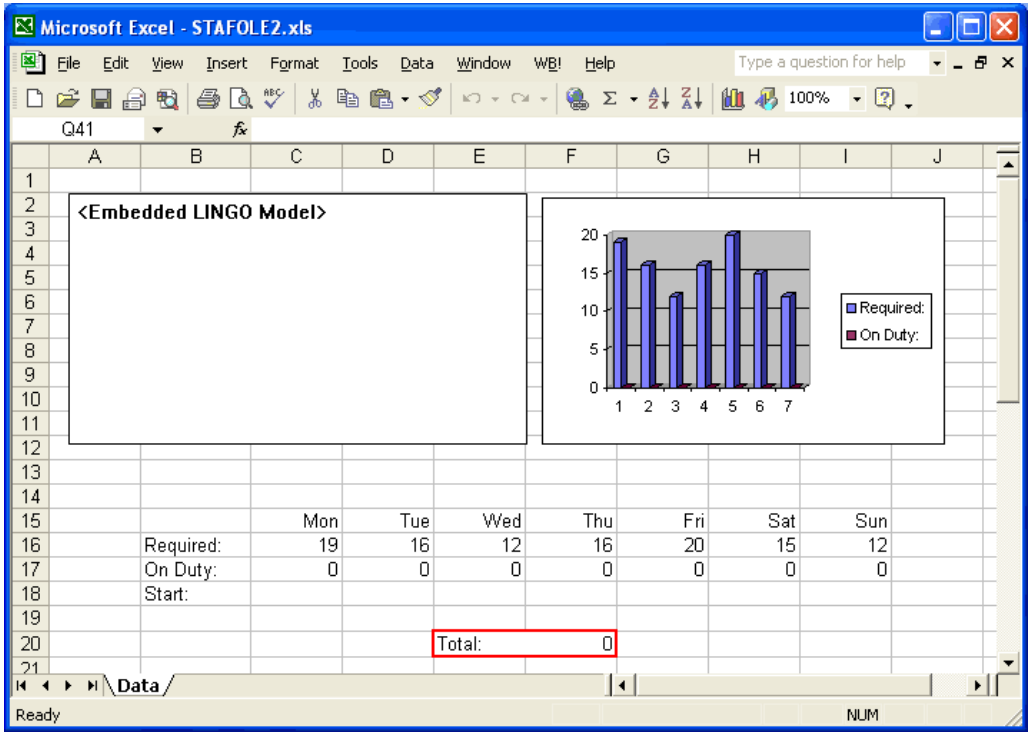
LINGO is capable of functioning as an OLE server. This means you can embed a LINGO model in any application that can function as an OLE container. Excel is one such application. Embedding a LINGO model into Excel is convenient in that the LINGO model is always immediately available once the spreadsheet is opened. You don't have to worry about also starting LINGO and finding the correct LINGO model that corresponds to the spreadsheet.

To embed a LINGO document in an Excel file, select the Excel command *Insert|Object*. You will be presented with a list of embeddable objects available on your system. Select the *LINGO Document* object from this list as shown here:



Click the *OK* button and a blank LINGO model window will be embedded within the spreadsheet. You can enter text directly into this window just as you would in LINGO, or you can paste it in from another application. When you save the Excel sheet, the embedded LINGO model will automatically be saved with it. Similarly, whenever you read the sheet back into Excel, the embedded LINGO model will be restored, as well.

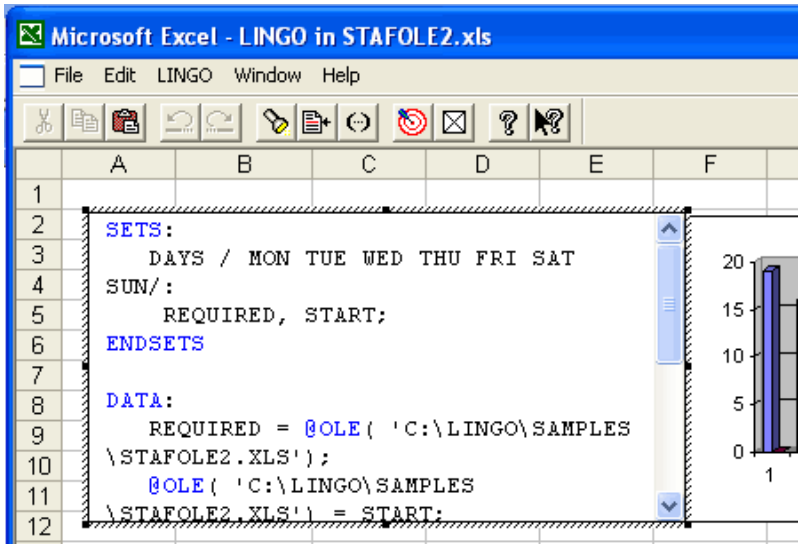
To illustrate this feature, we will continue with the staffing model introduced in Chapter 2, *Using Sets*. The spreadsheet will contain the data for the model, and it will also contain an embedded LINGO model to perform the optimization and install the solution back into the spreadsheet. This example may be found in the spreadsheet file *SAMPLES\STAFOLE2.XLS*. If you load this sheet into Excel, you should see the following:



Spreadsheet: STAFOLE2.XLS

As in the previous example, the staffing requirements are in the range *C16:I16*. This range has been titled *REQUIRED*. The range *C18:I18* has been assigned the name *START* and will receive the solution after the model is solved. In the upper right-hand corner of the sheet, we have defined a graph to help visualize the solution.

In the upper left corner, there is a region labeled <*Embedded LINGO Model*>. This region contains a LINGO model that will solve the staffing model and place the solution values into the spreadsheet. If you double-click on this region, you will be able to see the model:



Spreadsheet: STAFOLE2.XLS

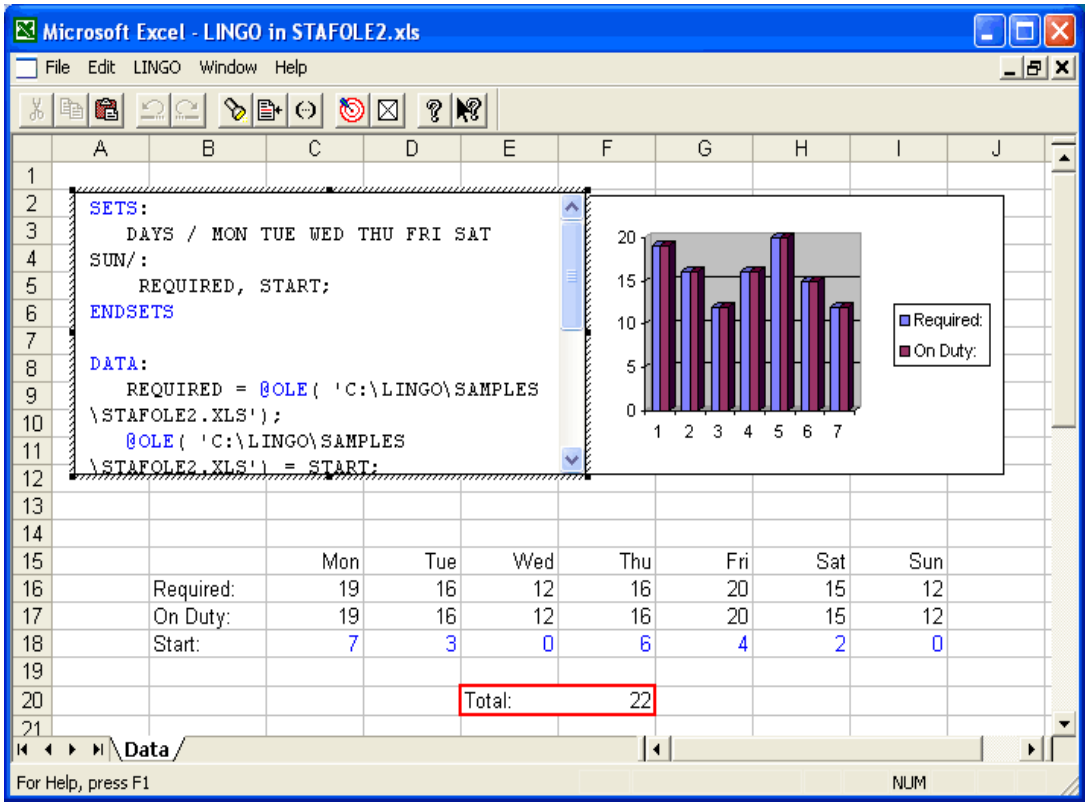
Note, when this LINGO model is active, the LINGO menus and toolbar replace the Excel menus and toolbar. Thus, when working with an embedded LINGO model in Excel, you have all the functionality of LINGO available to you. When you deselect the LINGO model, the Excel menus and toolbar will automatically become available once again. This begins to illustrate the power of embedded OLE—it allows the user to seamlessly combine the features of two or more applications together as if they were a single, integrated application.

You can drag the lower right-hand corner of the LINGO model region to expose the contents of the entire model:

```
SETS:
   DAYS / MON TUE WED THU FRI SAT SUN/:
    REQUIRED, START;
ENDSETS

DATA:
 REQUIRED =
  @OLE('C:\LINGO\SAMPLES\STAFOLE2.XLS');
 @OLE('C:\LINGO\SAMPLES\STAFOLE2.XLS') = START;
ENDDATA

MIN = @SUM(DAYS: START);

@FOR(DAYS(J):
   @SUM(DAYS(I) | I #LE# 5:
      START(@WRAP(J - I + 1, 7)))
         >= REQUIRED(J));

@FOR(DAYS: @GIN(START));
```

Once again, we are making use of our familiar staff-scheduling model. The main feature to note is that we are using two instances of the *@OLE* function. The first instance gets the staffing requirements from the spreadsheet. The second sends the solution back to the *START* range.

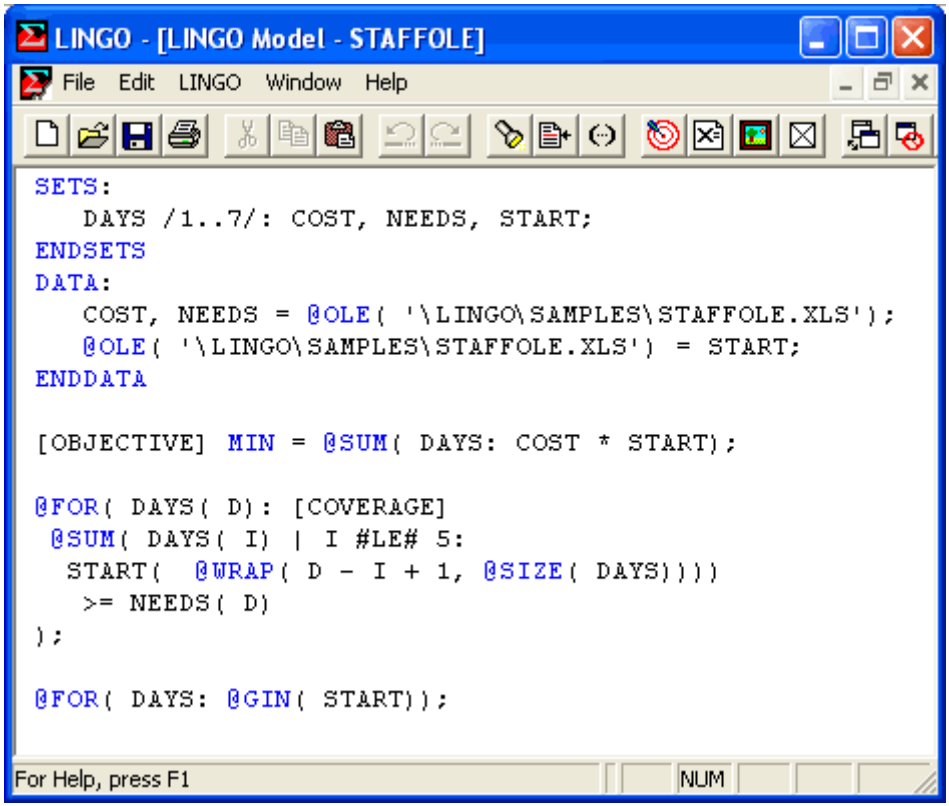
To solve the model, double-click on the region containing the LINGO model. The LINGO command menus will become visible along the top of the screen. Select the *Solver|Solve* command. After LINGO finishes optimizing the model, it will return the solution to the sheet and we will have:



Spreadsheet: STAFOLE2.XLS

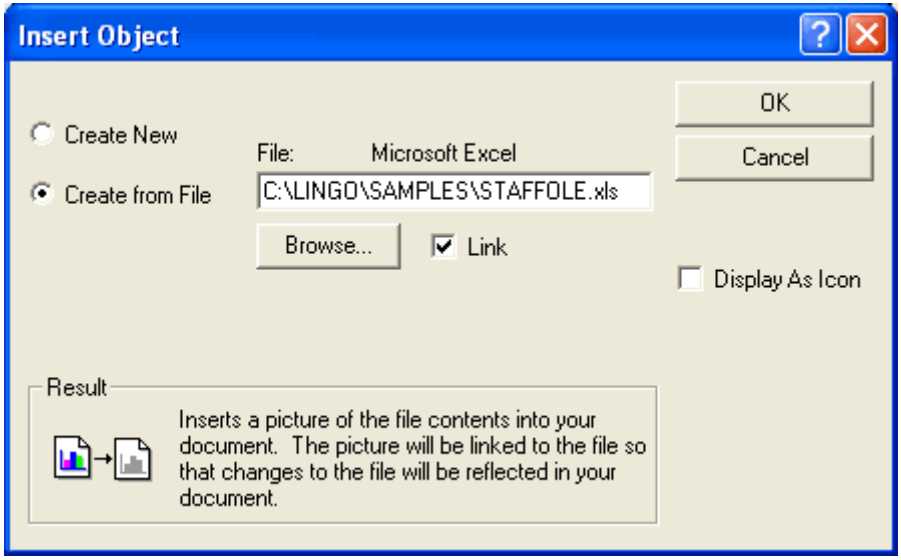
Embedding Excel Sheets in LINGO

Just as you can embed a LINGO model into Excel, you can reverse the process and embed an Excel sheet in a LINGO model. To illustrate this, load the *STAFFOLE* model from the *SAMPLES* subdirectory into LINGO. Once again, this is our familiar staff-scheduling model and it appears as follows:



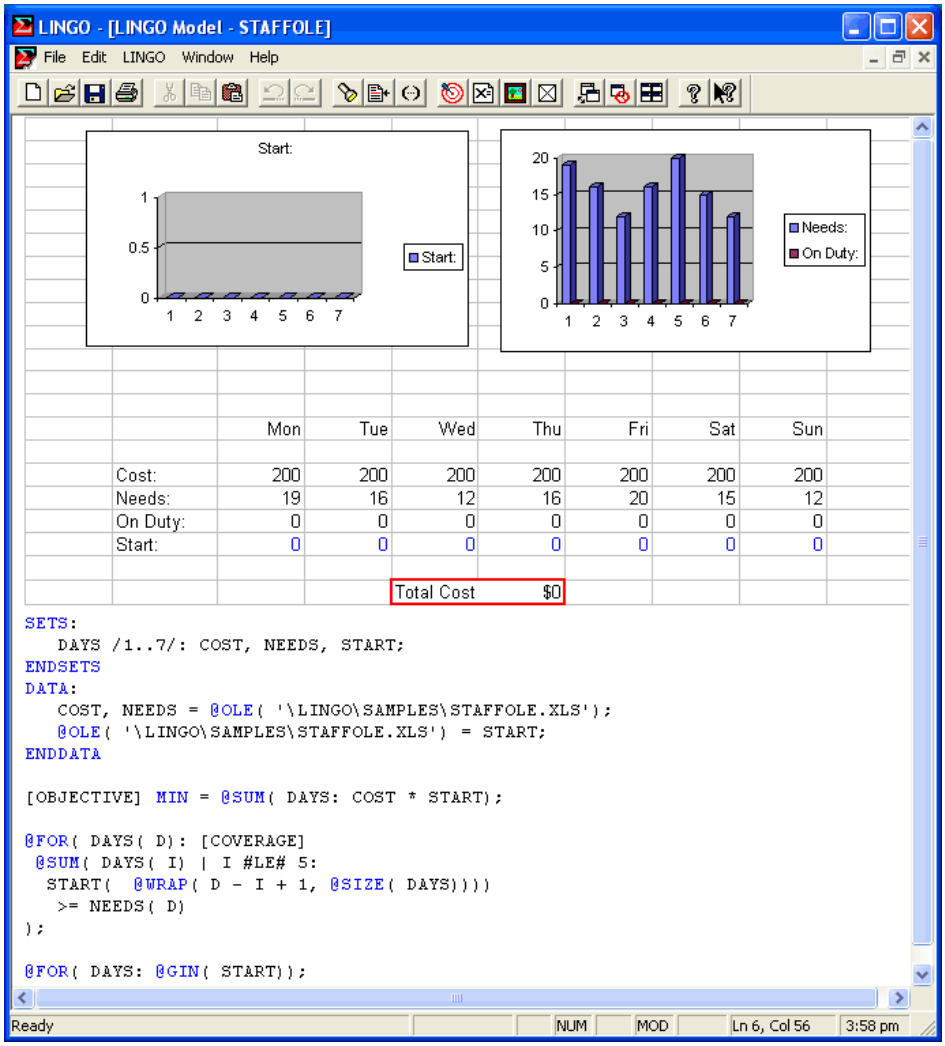
Model: STAFFOLE

This model reads data from and writes the solution to the Excel file *STAFFOLE.XLS*. To make our lives easier, it would be nice to embed this spreadsheet in the LINGO model to avoid having to load it into Excel each time we need to work with our model. To do this, select the *Edit|Insert New Object* command in LINGO. You will be presented with the dialog box:



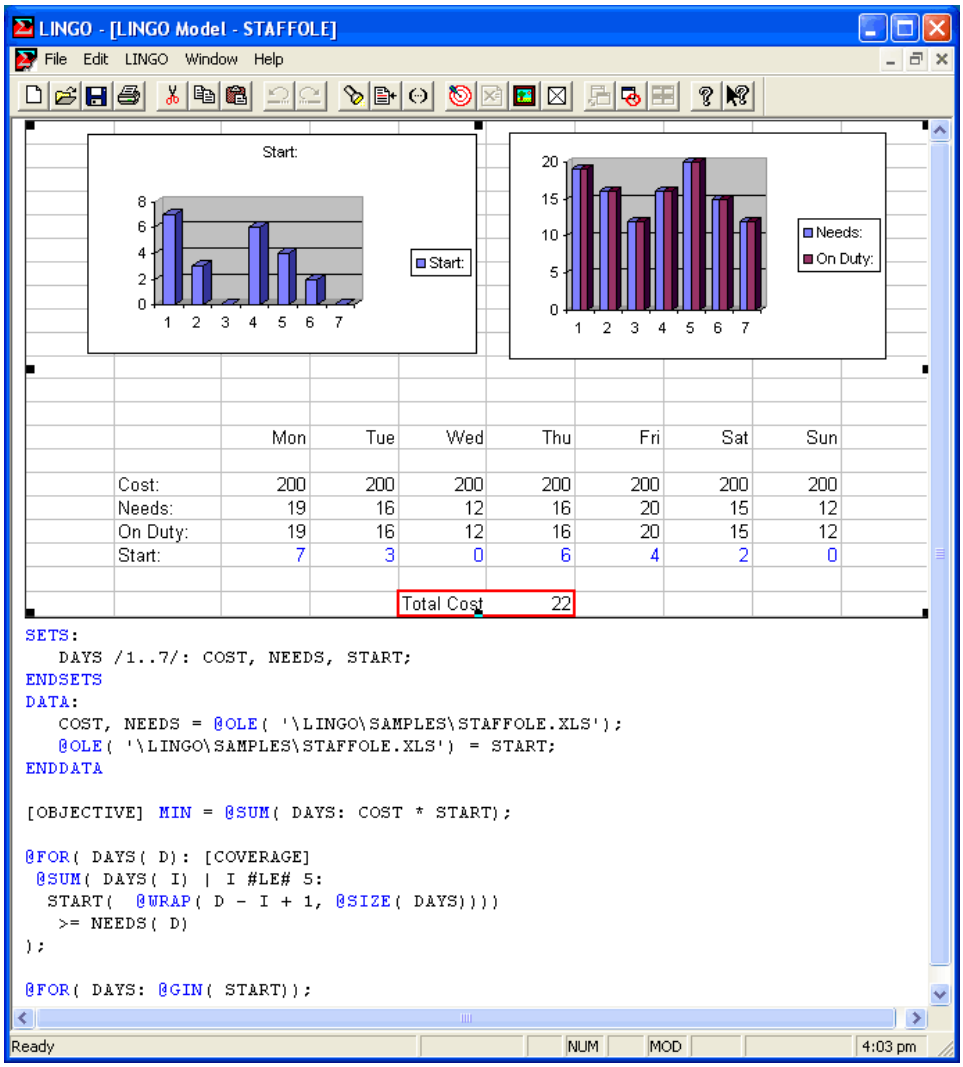
Click on the *Create from File* button, enter the spreadsheet file name in the *File* field, click on the *Link* checkbox, and then press the *OK* button.

Your LINGO model should now appear as:



The data spreadsheet is now embedded at the top of the LINGO document. You can easily edit the spreadsheet by double-clicking on it. When you save the LINGO model, the link to the Excel file will be saved as well.

At this point, go ahead and optimize the model by selecting the *Solver|Solve* command. When LINGO has finished solving the model, you should see the following:



The optimal solution now appears in the embedded spreadsheet and the graphs of the solution have been updated.

Summary

We have demonstrated a number of intriguing methods for combining the modeling features of LINGO with the data management features of spreadsheets. The concise, structured nature of LINGO's modeling language makes reading and understanding even large models relatively easy. Whereas, it can often be a challenge to discern the underlying mathematical relationships of large, spreadsheet models with formulas spread throughout numerous cells on multiple tabbed sheets. LINGO models are also easier to scale than spreadsheet models. As the dimensions of your problem change, spreadsheet models can require the insertion or deletion of rows or columns and the editing of cell formulas. Whereas, if your data is separate from the model, your LINGO formulation will generally require little or no modification. Combining the power of model expression with LINGO and the ease of data handling in spreadsheets gives the mathematical modeler the best of both worlds.

10 Interfacing with Databases

Spreadsheets are good at managing small to moderate amounts of data. Once your models start dealing with large amounts of data, database management systems (DBMSs) are, unquestionably, the tool of choice. Also, in many business modeling situations, you will find most, if not all, of the data is contained in one or more databases. For these reasons, LINGO supports links to any DBMS that has an Open DataBase Connectivity (ODBC) driver (effectively all popular DBMSs). ODBC defines a standardized interface to DBMSs. Given this standardized interface, LINGO can access any database that supports ODBC.

LINGO has one interface function for accessing databases. This function's name is *@ODBC*. The *@ODBC* function is used to import data from and export data to any ODBC data source. *@ODBC* is currently available only in Windows versions of LINGO.

ODBC Data Sources

Windows versions of LINGO include a variation of the standard transportation model that retrieves all data from a database, and writes a solution back to the same database. This file can be found in the file *SAMPLES\TRANDB.LG4*. The contents of this model are displayed below:

```
MODEL:
   ! A 3 Plant, 4 Customer Transportation Problem;

   ! Data is retrieved from an either an Access database or
     an Oracle database an ODBC link.  You *MUST* use the
     ODBC Administrator to register one of the supplied
     databases under the name "Transportation" in order
     to get this model to run.  Refer to Chapter 10 for
     more details.;

   TITLE Transportation;

   SETS:
      PLANTS: CAPACITY;
      CUSTOMERS: DEMAND;
      ARCS(PLANTS, CUSTOMERS): COST, VOLUME;
   ENDSETS

   ! The objective;
     [OBJ] MIN = @SUM(ARCS: COST * VOLUME);

   ! The demand constraints;
     @FOR(CUSTOMERS(C):
        @SUM(PLANTS(P): VOLUME(P, C)) >= DEMAND(C));

   ! The supply constraints;
     @FOR(PLANTS(P):
        @SUM(CUSTOMERS(C): VOLUME(P, C)) <= CAPACITY(P));

   DATA:

      ! Import the data via ODBC;
      PLANTS, CAPACITY    = @ODBC();
      CUSTOMERS, DEMAND   = @ODBC();
      ARCS, COST          = @ODBC();

      ! Export the solution via ODBC;
      @ODBC()  = VOLUME;

   ENDDATA

END
```

Model: TRANDB

You will note that in the data section of this model, we use the *@ODBC* function to establish a link to an ODBC data source to retrieve all the data and to export the final solution. The technical details of the *@ODBC* function are discussed below. Right now, we will focus on how you set up an ODBC data source that can be accessed by LINGO.

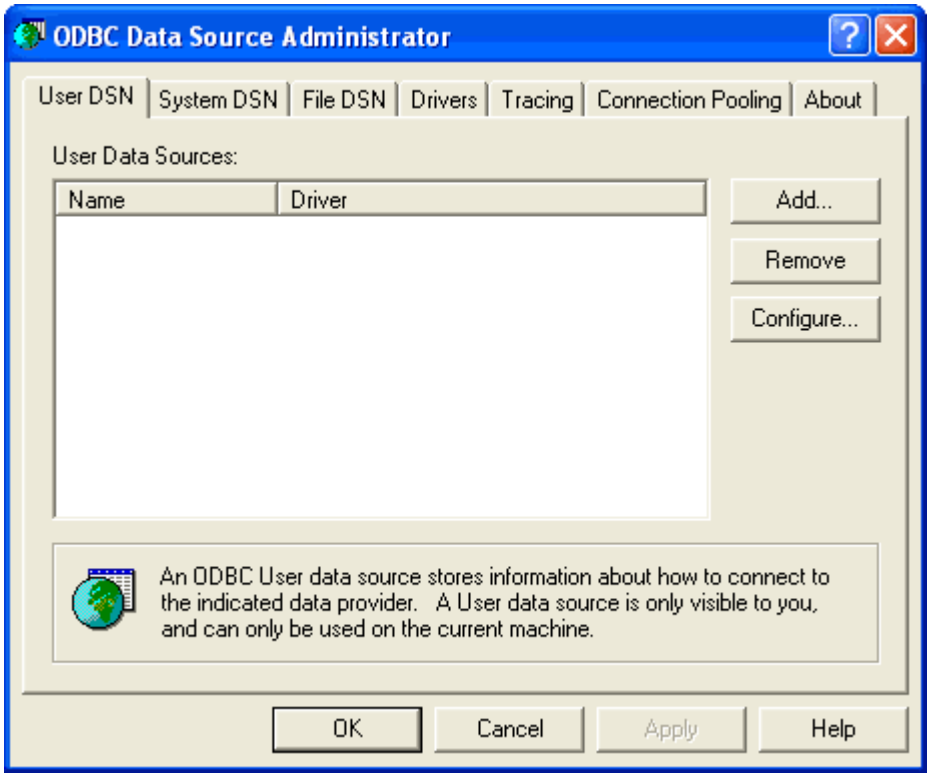
An ODBC data source is a database that 1) resides in a DBMS for which you have an ODBC driver, and 2) has been registered with the ODBC Administrator. Databases in an ODBC enabled DBMS do not qualify as an ODBC data source until they have been registered with the ODBC Administrator. The ODBC Administrator is a Windows Control Panel utility. Registering a database with the ODBC Administrator is a straightforward process. In the following two sections, we will illustrate the registration process for a Microsoft Access database and for an Oracle database.

Creating an ODBC Data Source from an Access Database

When you installed LINGO, a Microsoft Access database for the transportation model above was included as part of the installation. This file is contained in the directory *SAMPLES* under the name *TRANDB.MDB*. To register this database as an ODBC data source for our transportation model, you must start the ODBC Administrator by doing the following:

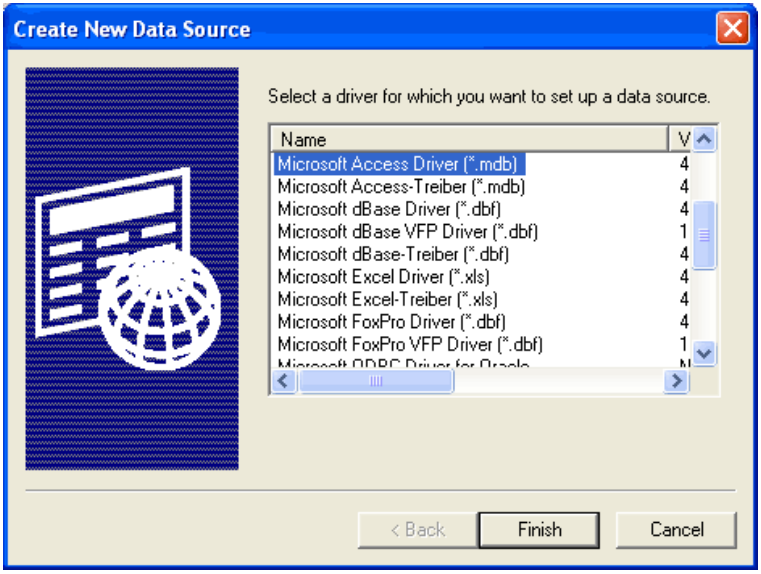
1) double-click on the *My Computer* icon on your desktop,
2) find the *Control Panel* icon and double-click on it,
3) double-click on the *Administrative Tools* icon, and
4) search for the *Data Sources (ODBC)* icon and double-click on it.

You should now see the ODBC Administrator dialog box shown below:



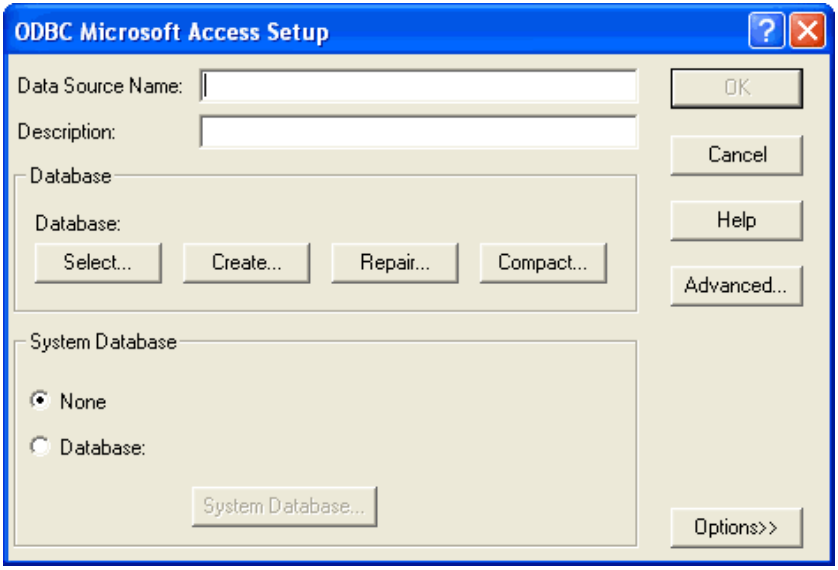
To install the *TRANDB.MDB* database as a data source, do the following:

1. Click the *Add* button in the ODBC Administrator dialog box to reveal the dialog box below:

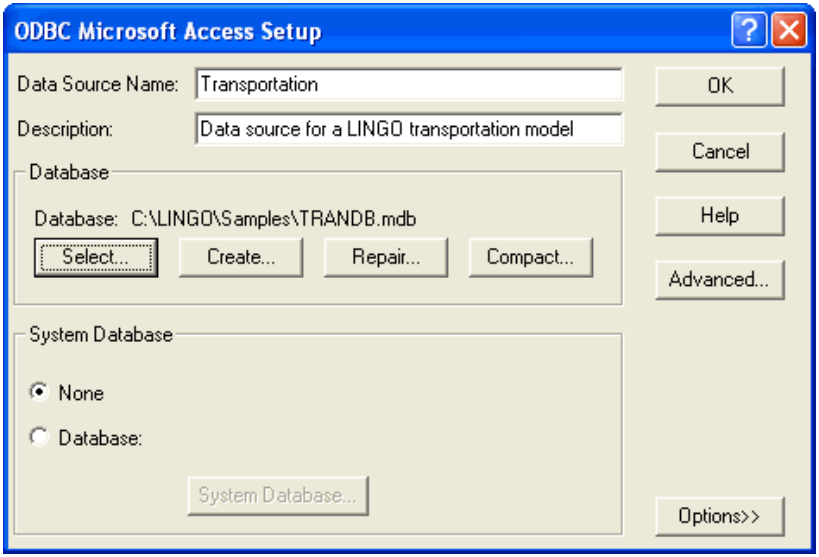


2. We are installing an Access data source, so select the *Microsoft Access Driver* option and press the *Finish* button.

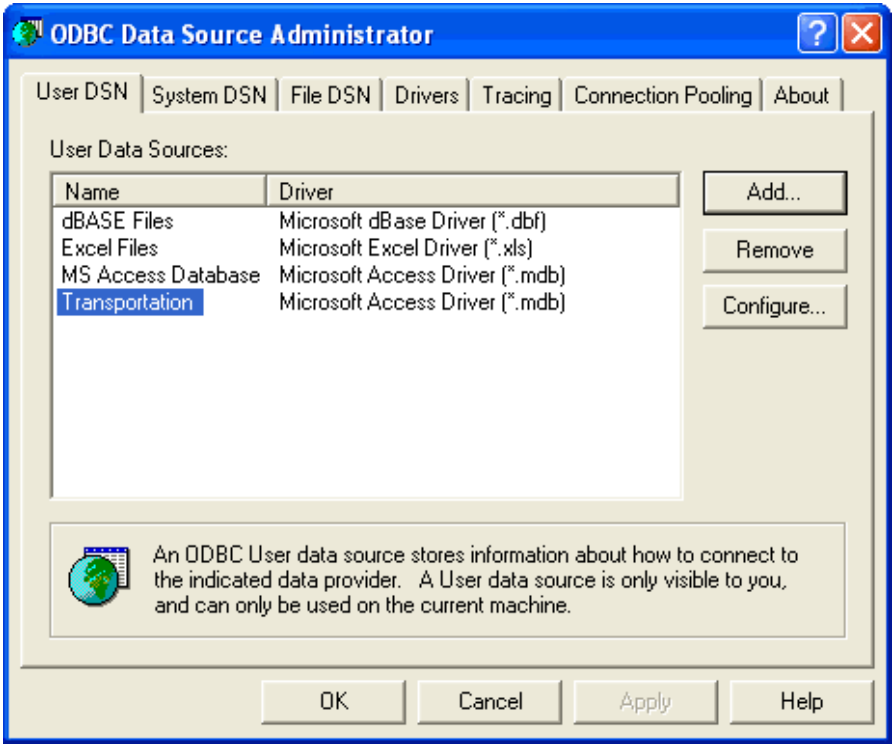
3. In the next dialog box:



assign the data source the name *Transportation* in the *Data Source Name* field. In the *Description* field, enter "Datasource for a LINGO transportation model". Press the *Select* button and enter the name of the database "LINGO\SAMPLES\TRANDB.MDB" (this assumes LINGO has been installed in the LINGO directory—your installation may differ). The dialog box should now resemble the one below:



4. Press the *OK* button and you should see the *Transportation* data source has been added to the list of ODBC data sources:



5. Click the *OK* button to close the ODBC Administrator.

You should now be able to start LINGO and solve the *TRANDB.LG4* model. LINGO knows to go to the Transportation data source for data because the model's title (input with the *TITLE* statement) is Transportation. If you solve this model, you should see the following results:

```
Global optimal solution found at step:              6
Objective value:                              161.0000
Model Title: Transportation
              Variable           Value        Reduced Cost
      CAPACITY(PLANT1)        30.00000           0.0000000
      CAPACITY(PLANT2)        25.00000           0.0000000
      CAPACITY(PLANT3)        21.00000           0.0000000
         DEMAND(CUST1)        15.00000           0.0000000
         DEMAND(CUST2)        17.00000           0.0000000
         DEMAND(CUST3)        22.00000           0.0000000
         DEMAND(CUST4)        12.00000           0.0000000
   COST(PLANT1, CUST1)        6.000000           0.0000000
   COST(PLANT1, CUST2)        2.000000           0.0000000
   COST(PLANT1, CUST3)        6.000000           0.0000000
   COST(PLANT1, CUST4)        7.000000           0.0000000
   COST(PLANT2, CUST1)        4.000000           0.0000000
   COST(PLANT2, CUST2)        9.000000           0.0000000
   COST(PLANT2, CUST3)        5.000000           0.0000000
   COST(PLANT2, CUST4)        3.000000           0.0000000
   COST(PLANT3, CUST1)        8.000000           0.0000000
   COST(PLANT3, CUST2)        8.000000           0.0000000
   COST(PLANT3, CUST3)        1.000000           0.0000000
   COST(PLANT3, CUST4)        5.000000           0.0000000
 VOLUME(PLANT1, CUST1)        2.000000           0.0000000
 VOLUME(PLANT1, CUST2)        17.00000           0.0000000
 VOLUME(PLANT1, CUST3)        1.000000           0.0000000
 VOLUME(PLANT1, CUST4)       0.0000000            2.000000
 VOLUME(PLANT2, CUST1)        13.00000           0.0000000
 VOLUME(PLANT2, CUST2)       0.0000000            9.000000
 VOLUME(PLANT2, CUST3)       0.0000000            1.000000
 VOLUME(PLANT2, CUST4)        12.00000           0.0000000
 VOLUME(PLANT3, CUST1)       0.0000000            7.000000
 VOLUME(PLANT3, CUST2)       0.0000000            11.00000
 VOLUME(PLANT3, CUST3)        21.00000           0.0000000
 VOLUME(PLANT3, CUST4)       0.0000000            5.000000

                    Row    Slack or Surplus      Dual Price
                    OBJ        161.0000            1.000000
                      2       0.0000000           -6.000000
                      3       0.0000000           -2.000000
                      4       0.0000000           -6.000000
                      5       0.0000000           -5.000000
                      6        10.00000           0.0000000
                      7       0.0000000            2.000000
                      8       0.0000000            5.000000
```

TRANDB Solution

As an interesting exercise, you may wish to redirect *TRANDB.LG4* to use a second database that was provided as part of your installation. The second database is called *TRANDB2.MDB* and is also located in the *SAMPLES* directory. The main difference between the files is the dimension of data. *TRANDB.MDB* involves only 3 plants and 4 customers, while *TRANDB2.MDB* contains 50 plants and 200 customers. If you return to the ODBC Administrator and register *TRANDB2. MDB* under the name "Transportation2", then you can redirect LINGO to use the new data source by changing the model title from "Transportation" to "Transportation2" in the following line in the LINGO model:

```
TITLE Transportation2;
```

The new model will have 10,000 variables as opposed to the 12 variables generated with the smaller data source. This ability to easily run different size data sets illustrates the usefulness of writing data independent, set-based models.

Creating an ODBC Data Source from an Oracle Database

LINGO installs an SQL script that can be run by the SQL Plus utility in Oracle to build a small database for the *TRANDB.LG4* transportation model. The script file is named *TRANDB.SQL* and may be found in the *SAMPLES* folder off the main LINGO directory. Here is the procedure you will need to follow to set up this data source:

1. Start up the SQL Plus utility provided with Oracle by going to the Start menu in Windows, select the Run command, type "SQLPLUSW", and then click on the *OK* button.
2. Enter your Oracle User ID and password. If you have a default installation of Oracle, the User ID "sys" and the password "change_on_install" should be valid. A host name is required only if Oracle is running remotely.
3. Run the *TRANDB.SQL* script by typing "@\LINGO\SAMPLES\TRANDB.SQL" to the SQL Plus system prompt.
4. Exit SQL Plus by entering "EXIT" to the prompt.
5. Start up the ODBC Administrator as described in the previous section.
6. When selecting an ODBC driver, be sure to use the "Microsoft ODBC for Oracle" driver. Do not use the "Oracle ODBC Driver", because it does not provide all the necessary functionality.
7. Assign the data source the name "Transportation".
8. Load *TRANDB.LG4* into LINGO and solve it.
9. Enter your Oracle User ID and password as prompted.

If you would like to avoid entering your Oracle User ID and password each time LINGO accesses a database, you may set them once at the start of your session with the *File|Database User Info* command. For more information, see the *File|Database User Info* section above in Chapter 5, *Windows Commands*.

Creating an ODBC Data Source from an SQL Server Database

LINGO installs an SQL script that can be run by the SQL Analyzer utility in SQL Server to build a small database for the *TRANDB.LG4* transportation model. The script file is named *TRANDB_SQL.SQL* and may be found in the *SAMPLES* folder off the main LINGO directory. Here is the procedure you will need to follow to set up this data source:

1. Start up the SQL Analyzer utility provided with SQL Server by going to the Start menu in Windows, selecting the *Programs* command, selecting the *Microsoft SQL Server* program group, and then clicking on *Query Analyzer*.
2. Point the Query Analyzer to the machine running SQL Server.
3. Run the *File|Open* command and select the script file *TRANDB_SQL.SQL* from the LINGO samples folder.
4. Run the script by issuing the *Query|Execute* command.
5. Exit the Query Analyzer.
6. Start up the ODBC Administrator as described in the section above: *Creating an ODBC Data Source from an Access Database*.
7. Press the *Add* button to add a new ODBC data source: Select the SQL Server ODBC drive, assign the data source the name "Transportation", and select the appropriate server machine.
8. You should now be able to load *TRANDB.LG4* into LINGO and solve it.

Note: If you plan to use LINGO's ODBC interface to write solutions to a table in SQL Server, then the table must contain a field that is declared as a unique identifier. Failure to do so will result in an error message claiming that the table is read-only.

Importing Data from Databases with @ODBC

To import a model's data from an ODBC data source, we use the *@ODBC* function in the model's data section. *@ODBC* allows us to import both text formatted set members and numerical set attribute values.

The syntax for using *@ODBC* to import data inside a data section is:

object_list = *@ODBC*([‘*data_source*’[, ‘*table_name*’
[,‘*column_name_1*’[, ‘*column_name_2*’ …]]]]);

The *object_list* is a list, optionally separated by commas, containing model objects (i.e., attributes, sets, or variables) that are to be initialized from the ODBC data source. *Object_list* may contain up to one set and/or multiple set attributes. All set attributes in *object_list* must be defined on the same set. If *object_list* contains a set, then all attributes in *object_list* must be defined on this set. The *data_source* argument is the name of the ODBC data source that contains the data table. The *table_name* argument is the name of the data table within the data source that contains the data. Finally, the *column_name* arguments are the names of the data columns, or fields, in the data table *table_name* to retrieve the initialization data from. Set attributes and primitive sets require one column name each to retrieve their data from. Derived sets require one column name for each dimension of the set. Thus, a two-dimensional derived set would require two columns of data to initialize its members.

If the *data_source* argument is omitted, the model's title is used in its place (see the discussion of the *TITLE* statement in Chapter 1, *Getting Started with LINGO*). If *table_name* is omitted, the name of any set in the *object_list* is used in its place. If there is no set in *object_list*, then the name of the set that the attributes in *object_list* are defined on is used.

If the *column_name* arguments are omitted, LINGO will choose default names based on whether the corresponding object in *object_list* is a set attribute, a primitive set, or a derived set. When the object to be initialized is a set attribute or a primitive set, LINGO will use the name of the object as the default column name. When the object is a derived set, LINGO will generate one default column name for each dimension of the derived set, with each name being the same as the parent set that the given dimension is derived from. As an example, a two-dimensional set named *LINKS* derived from the two primitive sets *SOURCE* and *DESTINATION* would default to being initialized from the two columns titled *SOURCE* and *DESTINATION*.

Keep in mind that LINGO expects to find set members in text format in the database, while set attributes are expected to be in numeric format.

Some examples of using *@ODBC* to import data in a model's data section are:

Example 1:

```
SHIPPING_COST =
 @ODBC( 'TRANSPORTATION',
  'LINKS','COST');
```

LINGO initializes the attribute *SHIPPING_COST* from the column *COST* contained in the data table *LINKS* found in the ODBC data source *TRANSPORTATION*.

Example 2:

```
VOLUME, WEIGHT =
 @ODBC( 'TRUCKS', 'CAPACITY');
```

The database column names are omitted, so, assuming *VOLUME* and *WEIGHT* are set attributes, LINGO defaults to using the attribute names (*VOLUME* and *WEIGHT*) as the database column names. Therefore, LINGO initializes the attributes *VOLUME* and *WEIGHT* from the columns also titled *VOLUME* and *WEIGHT* contained in the data table named *CAPACITY* found in the ODBC data source *TRUCKS*.

Example 3:

```
REQUIRED, DAYS = @ODBC();
```

In this example, we will assume a) we have titled the model *PRODUCTION*, b) *REQUIRED* is a derived set derived from the two primitive sets *JOB* and *WORKSTATION* (e.g., *REQUIRED(JOB, WORKSTATION)*), and c) *DAYS* is a set attribute defined on the set *REQUIRED*. All arguments to the *@ODBC* function have been omitted, so LINGO supplies *PRODUCTION* as the data source name; the set name *REQUIRED* as the data

table name; and the three data column names *JOB*, *WORKSTATION, and DAYS.* Had we wanted to be more specific, we could have explicitly included all the arguments to the *@ODBC* function with the equivalent statement:

```
REQUIRED, DAYS = @ODBC( 'PRODUCTION', 'JOB',
 'WORKSTATION', 'DAYS');
```

Importing Data with ODBC in a PERT Model

We will now modify the project scheduling model, *PERT* introduced in Chapter 2, *Using Sets*, to demonstrate the use of *@ODBC* to import the set names of the project's tasks from a Microsoft Access database. The modified model appears below, with changes listed in bold type:

```
SETS:
   TASKS: TIME, ES, LS, SLACK;
   PRED(TASKS, TASKS);
ENDSETS

DATA:
   TASKS = @ODBC('PERTODBC', 'TASKS', 'TASKS');
   PRED = @ODBC('PERTODBC', 'PRECEDENCE',
    'BEFORE', 'AFTER');
   TIME = @ODBC('PERTODBC');
ENDDATA

@FOR(TASKS(J)| J #GT# 1:
 ES(J) = @MAX(PRED(I, J): ES(I) + TIME(I))
);

@FOR(TASKS(I)| I #LT# LTASK:
 LS(I) = @MIN(PRED(I, J): LS(J) - TIME(I));
);

@FOR(TASKS(I): SLACK(I) = LS(I) - ES(I));

ES(1) = 0;
LTASK = @SIZE(TASKS);
LS(LTASK) = ES(LTASK);
```

Model: PERTODBC

With the statement:

```
TASKS = @ODBC('PERTODBC', 'TASKS', 'TASKS');
```

we now fetch the members of the *TASKS* set from our ODBC data source, as opposed to explicitly listing them in the model. Specifically, we get the members of the *TASKS* set from the data column, or field, *TASKS* contained in the table named *TASKS* from the ODBC data source *PERTODBC*. Here is the data table as it appears in Access:

TASKS : Table

TASKS
DESIGN
FORECAST
SURVEY
PRICE
SCHEDULE
COSTOUT
TRAIN

Record: 1

Access Database: PERTODBC.MDB

Next, we use the statement:

```
PRED = @ODBC('PERTODBC', 'PRECEDENCE',
    'BEFORE', 'AFTER');
```

to fetch the members of the *PRED* set from an ODBC data source, as opposed to explicitly listing them in the model. More specifically, we pull the members of the *PRED* set from the data columns *BEFORE* and *AFTER* contained in the table named *PRECEDENCE* from the ODBC data source *PERTODBC*. Here is the data table showing the precedence relations as it appears in Access:

PRECEDENCE : Table

BEFORE	AFTER
DESIGN	FORECAST
DESIGN	SURVEY
FORECAST	PRICE
FORECAST	SCHEDULE
SURVEY	PRICE
SCHEDULE	COSTOUT
PRICE	TRAIN
COSTOUT	TRAIN

Record: 8

Access Database: PERTODBC.MDB

Note that the *PRECEDENCE* set is a two-dimensional set. Thus, we must supply two database columns containing the set members.

In order to retrieve the values for the task times, we create the ODBC link:

```
TIME = @ODBC('PERTODBC');
```

Note that we only specified the ODBC data source name—the table and column names have been omitted. In which case, LINGO supplies default values for the table and column. The object being initialized, *TIME*, is a set attribute. Thus, LINGO supplies its parent set name, *TASKS*, as the default table name. For the default column name, LINGO supplies the set attribute's name, *TIME*. Had we wanted to be specific, however, we could have explicitly entered all arguments to *@ODBC* with:

```
TIME = @ODBC('PERTODBC', 'TASKS', 'TIME');
```

Exporting Data with @ODBC

As is the case with most interface functions, *@ODBC* can export data as well as import it. Specifically, you can use the *@ODBC* function in the data section of a model to export set members and attribute values to ODBC data sources. In order to export solutions with *@ODBC*, you place calls to *@ODBC* in the data section of your model. These *@ODBC* export instructions are executed each time your model is solved.

The first form of syntax for using *@ODBC* to export data is:

@ODBC(['*data_source*'[, '*table_name*'[, '*column_name_1*'[,…,
'*column_name_n*']]]]) = *object_list*;

Note: When *importing*, *@ODBC* appears on the *right* of the equality sign. When *exporting*, the *@ODBC* function appears on the *left* of the equals sign.

The *object_list* is a list, optionally separated by commas, containing model objects (i.e., attributes, sets, or variables) that are to be exported to the ODBC data source. *Object_list* may contain up to one set and/or multiple set attributes. All set attributes in *object_list* must be defined on the same set. If *object_list* contains a set, then all attributes in *object_list* must be defined on this set. The *data_source* argument is the name of the ODBC data source containing the data table that will receive the exported values. The *table_name* argument is the name of the data table within the data source that will receive the data. Finally, the *column_name* arguments are the names of the receiving columns, or fields, in the data table *table_name.* Set attributes and primitive sets require one receiving column name each. Derived sets require one receiving column name for each dimension of the set. Thus, a two-dimensional derived set would require two receiving columns in a data table.

If the *data_source* argument is omitted, the model's title is used in its place (see the discussion of the *TITLE* statement in Chapter 1, *Getting Started with LINGO*). If *table_name* is omitted, the name of any set in the *object_list* is used in its place. If there is no set in *object_list*, then the name of the set where the attributes in *object_list* are defined is used.

If the *column_name* arguments are omitted, LINGO will choose default names based on whether the corresponding object in *object_list* is either a set attribute, a primitive set, or a derived set. When the object to be initialized is a set attribute or a primitive set, LINGO will use the name of the object as the default column name. When the object is a derived set, LINGO will generate one default column name for each dimension of the derived set, with each name being the same as the parent set that the given

dimension is derived from. As an example, a two-dimensional set named *LINKS* derived from the *SOURCE* and *DESTINATION* primitive sets would default to being exported to the two columns titled *SOURCE* and *DESTINATION*.

Keep in mind that set members are exported as text, while set attributes are exported as double precision floating point values.

Some examples of using *@ODBC* to export data values to an ODBC data source are:

Example 1:

```
@ODBC( 'TRANSPORTATION',
 'LINKS', 'VOLUME') = VOLUME;
```

LINGO sends the values of the *VOLUME* attribute to the column also titled *VOLUME* in the data table *LINKS* in the ODBC data source *TRANSPORTATION*.

Example 2:

```
@ODBC() = NUMBER_WORKING;
```

All arguments to the *@ODBC* function have been omitted and will default to the model's title for the data source, the attributes parent set for the data table, and the attribute's name for the column name. So, assuming we have used the *TITLE* statement to name this model *SCHEDULING*, and the attribute *NUMBER_WORKING* is defined on the set *SCHEDULES*, then LINGO exports the attribute *NUMBER_WORKING* to the column also titled *NUMBER_WORKING* in the data table *SCHEDULES* in the ODBC data source *SCHEDULING*.

The first form of syntax will generally be sufficient for most database export operations. However, there may be times when you need to export only portions of the attributes, or you need to export quantities computed from the attribute values. Our second form of syntax uses the *@WRITEFOR* reporting function to handle these more general cases:

@ODBC('*data_source*', '*table_name*', '*column_name_1*'[,…, '*column_name_n*']) = @WRITEFOR(*setname* [(*set_index_list*) [| *conditional_qualifier*]] : *output_obj_1*[,…, *output_obj_n*]);

@WRITEFOR functions like any other set looping function in that, as a minimum, you will need to specify the set to loop over. Optionally, you may also specify an explicit set index list and a conditional qualifier. If a conditional qualifier is used, it is tested for each member of the looping set and output will not occur for any members that don't pass the test. It's this feature of being able to base output on the results of a condition that distinguish this second style of syntax.

The list of output objects, of course, specifies what it is you want to output. As with the first form of syntax, the output objects may be labels, set members and variable values. However, you have additional latitude in that the output objects may now consist of complex expressions of the variable values (e.g., you could compute the ratio of two variables). This is a useful feature when you need to report statistics and quantities derived from the variable values. By placing these calculations in the data section, as opposed to the model section, you avoid adding unnecessary complications to the constraints of the model.

In general, you can do everything in the second form of syntax that you can do in the first, and more. However, the first form has an advantage in that it can be very concise.

Some examples of using *@WRITEFOR* for ODBC exports follow:

Example 1:

```
@ODBC( 'TRANSPORTATION',
 'SOLUTION', 'FROM', 'TO', 'VOLUME') =
  @WRITEFOR( LINKS( I, J) | VOLUME( I, J) #GT# 0:
   WAREHOUSE( I), CUSTOMER( J), VOLUME( I, J));
```

In this example, we exploit the ability to specify a conditional expression to weed zero shipments out of the export. The nonzero values of the *VOLUME* attribute are sent to the *SOLUTION* table in the *TRANSPORTATION* data source. The shipping warehouse set name is placed in column *FROM*, the receiving customer set name goes to column *TO*, and the shipping volume for the arc in placed in the *VOLUME* column.

Example 2:

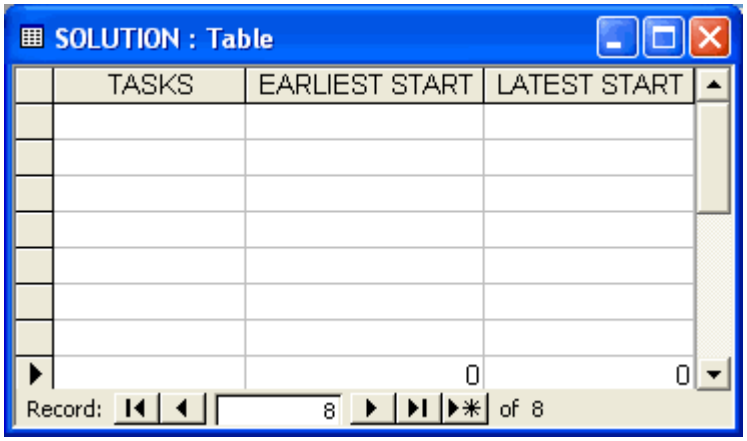
```
@ODBC( 'STAFFREP', 'STATS', 'RATIO') =
 @WRITEFOR( DEPARTMENTS( D): ON_DUTY( D) / NEEDS( D));
```

Here, we make use of *@WRITEFOR*'s ability to perform computations to compute a ratio of two variables. Specifically, the ratio of on-duty staff to staffing needs by department is placed into the column *RATIO* of table *STATS* in data source *STAFFREP*.

Note: When exporting to data tables, receiving columns that are longer than the number of exported values can be filled out by either erasing the contents of the extra cells or leaving the extra cells untouched. The default is to leave the extra cells untouched. If you would like to erase the contents of the extra cells, you'll need to enable the *Fill Out Ranges and Tables* option. If this option is enabled, extra text fields will be blanked out, while extra numeric fields will be zeroed out.

Exporting Data with ODBC in a PERT Model

Continuing from the PERT example used in the *Importing Data with ODBC in a PERT Model* section above, we can add modifications to export the solution values of the earliest and latest start times (*ES* and *LS*) back out to the *PERTODBC* data source. We will put these into a blank table titled *SOLUTION*. We will also export the members of the *TASKS* set in order to label our table. The data column, or field, that receives the *TASKS* members should be formatted as text, while the columns receiving the *ES* and *LS* attributes should be declared as numeric. Here is a look at the blank table we will be exporting to:



Access Database: PERTODBC.MDB

After modifying the model to export the data back to the *TASKS* table, we have (with the relevant changes in bold):

```
SETS:
   TASKS: TIME, ES, LS, SLACK;
   PRED(TASKS, TASKS);
ENDSETS

DATA:
   TASKS = @ODBC('PERTODBC', 'TASKS', 'TASKS');
   PRED = @ODBC('PERTODBC', 'PRECEDENCE', 'BEFORE', 'AFTER');
   TIME = @ODBC('PERTODBC');
   @ODBC('PERTODBC', 'SOLUTION', 'TASKS',
    'EARLIEST START', 'LATEST START') =
      TASKS, ES, LS;
ENDDATA

@FOR(TASKS(J)| J #GT# 1:
 ES(J) = @MAX(PRED(I, J): ES(I) + TIME(I))
);

@FOR(TASKS(I)| I #LT# LTASK:
 LS(I) = @MIN(PRED(I, J): LS(J) - TIME(I));
);

@FOR(TASKS(I): SLACK(I) = LS(I) - ES(I));

ES(1) = 0;
LTASK = @SIZE(TASKS);
LS(LTASK) = ES(LTASK);
```

Model: PERTODBC

With the data statement:

```
@ODBC('PERTODBC', 'SOLUTION', 'TASKS',
  'EARLIEST START', 'LATEST START') =
    TASKS, ES, LS;
```

we are sending the set *TASKS* to the text column *TASKS*, and the *ES* and *LS* attributes to the numeric columns *EARLIEST START* and *LATEST START*. The data table is called *SOLUTION*, while the ODBC data source name is *PERTODBC*.

Once the model has been solved, the updated data table will resemble:

SOLUTION : Table

TASKS	EARLIEST START	LATEST START
DESIGN	0	0
FORECAST	10	10
SURVEY	10	29
PRICE	24	32
SCHEDULE	24	24
COSTOUT	31	31
TRAIN	35	35

Record: 7 of 8

Access Database: PERTODBC.MDB

At the top of the solution report window, you will also notice an *export summary report*. There will be one report for each *@ODBC* statement in the model used for exporting data. This report lists details as to the operation of the *@ODBC export*. In the case of our PERT model, you should see the following report:

```
Export Summary Report
---------------------
Transfer Method:        ODBC BASED
ODBC Data Source:       PERTODBC
Data Table Name:        TASKS
Columns Specified:      3
   TASKS
   EARLIEST
   LATEST
LINGO Column Length:            7
Database Column Length:         7
```

The *Transfer Method* will always list "ODBC BASED" when doing ODBC exports. Next, the data source and table names are listed along with the number of columns specified and the column names. The *LINGO Column Length* field lists the number of elements in each attribute. The *Database Column Length* lists the length of the receiving columns in the database. In general, the LINGO Column Length will agree with the Database Column Length. If not, LINGO must either truncate its output or it will have insufficient data to fill out the columns in the database.

Export summary reports are not displayed when LINGO is in terse output mode. To place LINGO in terse output mode, click on the Terse Output checkbox on *the Interface tab* of the *Solver*|Options dialog box.

This version of the PERT model and its supporting database are contained in the *SAMPLES* directory. Feel free to run the model to experiment with it if you like—you will find it under the name PERTODBC. The supporting Access database file, *PERTODBC.MDB*, is also in the SAMPLES subdirectory and you will need to register it with the *ODBC* Administrator as described above in *ODBC Data Sources*.

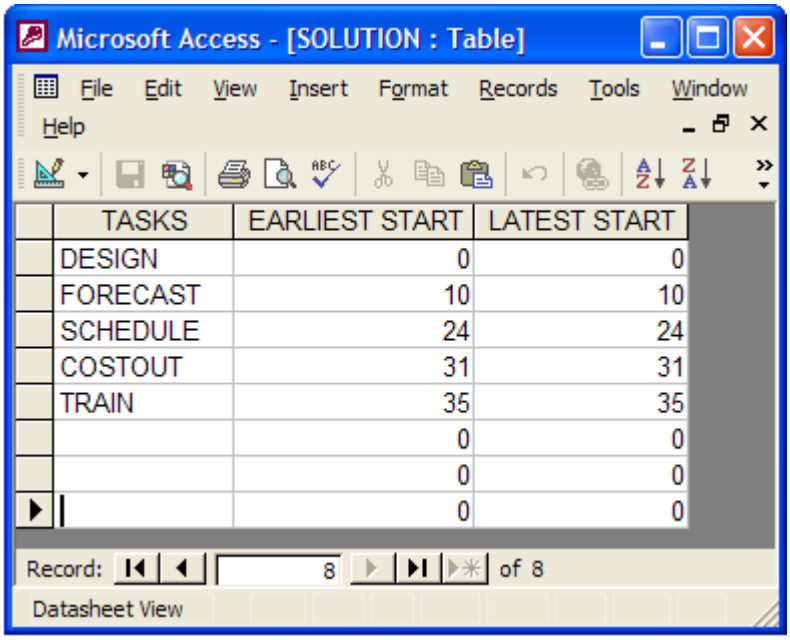
Note that we exported start and finish times for all the tasks in the project. If we were dealing with a large project there could be thousands of tasks to consider. With such an abundance of tasks, we might be interested in reporting only those tasks that lie on the critical path. We'll modify our *PERTODBC* example one last time to accomplish this using the *@WRITEFOR* reporting function.

For those unfamiliar with the concept of a critical path, it is the subset of tasks such that if any are delayed the entire project will be delayed. Generally, and somewhat counterintuitive to what one would expect, the set of tasks on the critical path will tend to be quite small compared to the total number of tasks in a large project. A task is considered to be on the critical path when its earliest start time is equal to its latest start time (i.e., there is no slack with respect to when the task must be started).

Below, we have modified *PERTODBC* to export only those tasks on the critical path.

```
MODEL:
SETS:
   TASKS: TIME, ES, LS, SLACK;
   PRED( TASKS, TASKS);
ENDSETS

DATA:
   TASKS = @ODBC( 'PERTODBC', 'TASKS', 'TASKS');
   PRED = @ODBC( 'PERTODBC', 'PRECEDENCE', 'BEFORE', 'AFTER');
   TIME = @ODBC( 'PERTODBC');
   @ODBC( 'PERTODBC', 'SOLUTION', 'TASKS',
    'EARLIEST START', 'LATEST START') =
     @WRITEFOR( TASKS( I) | ES( I) #EQ# LS( I):
      TASKS( I), ES( I), LS( I));
ENDDATA

@FOR( TASKS( J)| J #GT# 1:
 ES( J) = @MAX( PRED( I, J): ES( I) + TIME( I))
);

@FOR( TASKS( I)| I #LT# LTASK:
 LS( I) = @MIN( PRED( I, J): LS( J) - TIME( I));
);

@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));

ES( 1) = 0;
LTASK = @SIZE( TASKS);
LS( LTASK) = ES( LTASK);
END
```

We specify a conditional expression to test for the earliest start time equaling the latest start time, thereby restricting the list of exported tasks to those that lie on the critical path. Note that if you limit the number of output records there won't be enough records to completely fill our output table. We can have LINGO fill out any extra fields with blanks and zeroes by enabling the *Fill Out Ranges and Tables* option. Doing this, the solution table will resemble the following after solving the model:



TASKS	EARLIEST START	LATEST START
DESIGN	0	0
FORECAST	10	10
SCHEDULE	24	24
COSTOUT	31	31
TRAIN	35	35
	0	0
	0	0
	0	0

Note that that LINGO nulled out all the extra records.

11 Interfacing with Other Applications

Although LINGO has a convenient, interactive interface and a large library of callable functions that make it easy to set up and solve models, there may be times when you would like to bundle LINGO's functionality into your own applications, or call functions from within your LINGO models that were written in an external programming language. LINGO makes use of the *Dynamic Link Library* (DLL) standard under Windows to provide you with a “hook” to access LINGO's functionality from within your own custom applications. This gives you the ability to build application specific front-ends to LINGO that allow naïve users to input data and view solutions in a simple, familiar way without having to worry about the details of LINGO modeling. Your application handles the details of driving LINGO in the background, invisible to the user. LINGO also allows you to provide custom functions in the DLL format that you can call from within any model with the *@USER* function.

In the following section, *The LINGO Dynamic Link Library*, we document how to call the LINGO DLL in order to add LINGO's functionality to your own applications. Following this, in the section *User Defined Functions*, we will show you how to build a function in an external programming language and call it from a LINGO model with the *@USER* function.

The LINGO Dynamic Link Library

Windows versions of LINGO include a callable DLL. The ability to call a DLL is a standard feature of all Windows development environments (e.g., Visual Basic, Delphi, and Visual C++). The LINGO DLL is supplied in either 32-bit of 64-bit configurations and, thus, will run under all current releases of Windows (Win32 and Win64).

The interface to the LINGO DLL is relatively simple and gives you the ability to run a LINGO command script from within your application. Given that you can access all the major features of LINGO from a command script, the LINGO DLL interface is very powerful. You will, however, need to familiarize yourself with LINGO's command language in order to build useful command scripts. For more details on the commands available in the command language, see *Command-line Commands*. For an example of a script file, see *A Command Script Example.*

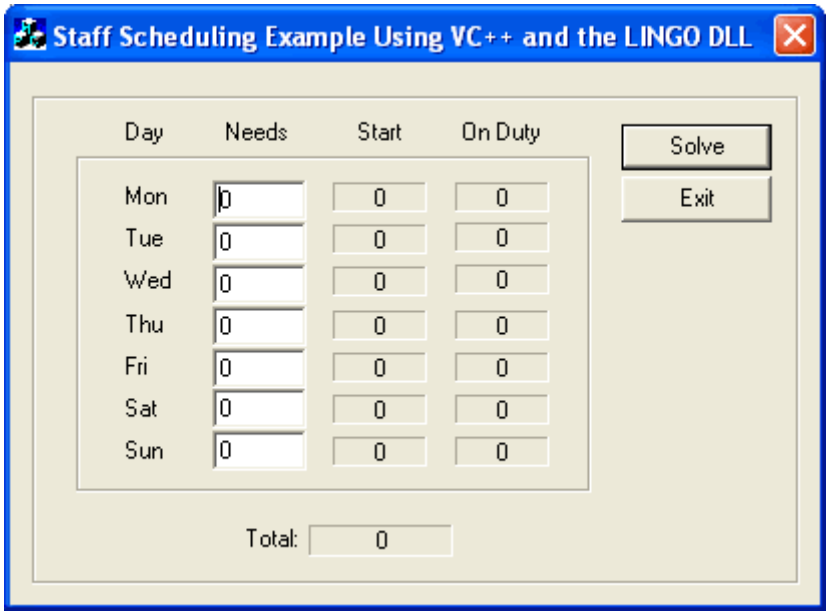
When LINGO is installed a number of examples on calling the DLL are installed, too. These examples may be found in the *Programming Samples* folder below the main LINGO folder. You will find examples for each of the following development environments:

- Visual C/C++
- Visual Basic
- Excel
- FORTRAN
- ASP .NET
- C# .NET
- VB .NET
- Delphi
- Java

In this chapter, we will walk through detailed examples on calling the LINGO DLL using both Visual C/C++ and Visual Basic. Users of other development environments will also be interested in these programming examples. Many of the ideas presented carry over to other development environments.

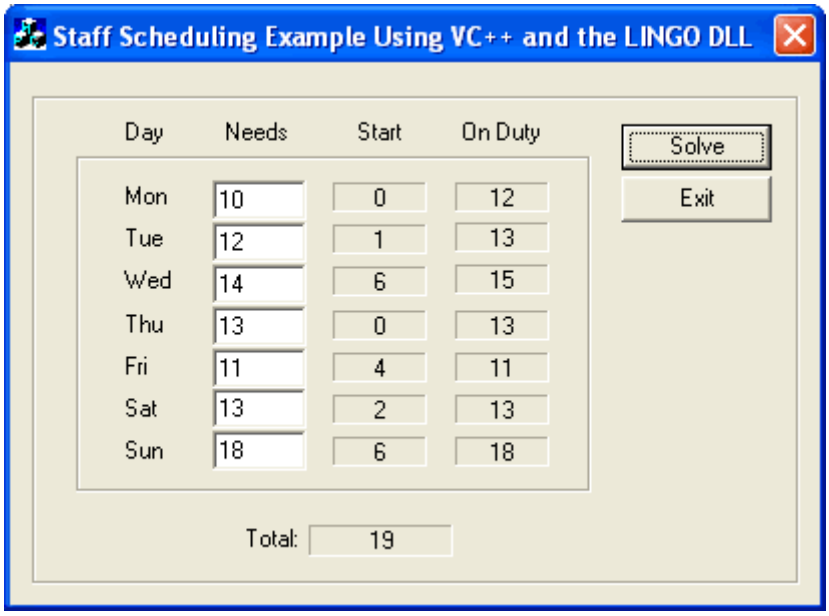
Staff-Scheduling Example Using the LINGO DLL

To illustrate interfacing with the LINGO DLL, we will again make use of the Pluto Dogs staff-scheduling example introduced on page 54. We will create examples using both the Visual C++ and Visual Basic programming languages. We will construct a dialog box that resembles the following:



The user enters the staffing requirements in the cells of the *Needs* column and presses the *Solve* button. The application then extracts the staffing requirements from the dialog box, passes them along with a model to the LINGO DLL for solution, and places the results back into the dialog box for viewing. Specifically, it shows the number of employees to start on each given day in the *Start* column, the number of staff on duty in the *On Duty* column, and the total number of staff required in the *Total* cell.

As an example, here is how the dialog box will appear after a sample run:



The Model

The model passed to LINGO to solve this problem is the familiar staffing model we've used before with some modifications, and appears below with important changes in bold:

```
MODEL:
 SETS:
    DAYS / MON TUE WED THU FRI SAT SUN/:
     NEEDS, START, ONDUTY;
 ENDSETS

 [OBJECTIVE] MIN = @SUM(DAYS(I): START(I));

 @FOR(DAYS(TODAY):

 ! Calculate number on duty;
    ONDUTY(TODAY) =
     @SUM(DAYS(D)| D #LE# 5:
      START(@WRAP(TODAY - D + 1,
      @SIZE(DAYS))));

 ! Enforce staffing requirement;
    ONDUTY(TODAY) >= NEEDS(TODAY);

    @GIN(START);
 );

 DATA:
    NEEDS = @POINTER(1);
    @POINTER(2) = START;
    @POINTER(3) = ONDUTY;
    @POINTER(4) = OBJECTIVE;
    @POINTER(5) = @STATUS();
 ENDDATA
END
```

Model: STAFFPTR

Since the LINGO script processor will read the model, it must begin with the *MODEL:* command and end with the *END* command. The script processor treats all text between the *MODEL:* and *END* keywords as model text, as opposed to script commands.

We have added the *ONDUTY* attribute to compute the number of employees on duty each day. We compute these figures for the purpose of passing them back to the calling application, which, in turn, posts them in the dialog box in the *On Duty* column.

We have named the objective row *OBJECTIVE* for the purpose of returning the objective's value to the calling application, so it may place the value in the *Total* cell.

The @POINTER Function

The *@POINTER* function in data, init and calc sections acts as direct memory link between the calling application and the LINGO solver. This ability to do direct memory transfers of data in and out of LINGO is very powerful. It allows for the fast transfer of data without the hassle of building and parsing disk based files.

When you call the LINGO DLL, you can pass it a list of memory pointers. *@POINTER(n)* refers to the *n-th* pointer in the passed pointer list. The *@POINTER* function only makes sense in a model when accessing LINGO through the DLL or OLE interfaces.

If the *@POINTER* function appears on the right-hand side of a data statement, as in the case of the following:

```
NEEDS = @POINTER( 1);
```

then LINGO will initialize the objects(s) on the left-hand side of the statement from the values in memory beginning at the location referenced by the pointer on the right-hand side of the statement. On the other hand, when the *@POINTER* function appears on the left-hand side of a data statement, LINGO will export the values of the objects(s) on the right-hand side of the data statement to the memory location referenced by the pointer on the left-hand side of the statement.

The *@POINTER* function reads and writes all numeric data (i.e., attribute values) using double precision floating point format. BASIC and C/C++ developers know this as the *double* data type, while FORTRAN developers refer to it as either *REAL*8* or *DOUBLE PRECISION*. Set members are passed as strings of ASCII text, with each member separated by a line feed character, with the list being terminated with an null, or ASCII 0.

When setting aside memory locations for *@POINTER*, you must be sure to allocate adequate space. LINGO assumes adequate space and/or supplied values are available at the passed memory locations. If you do not allocate sufficient space or values, you may experience memory protection faults or, even worse, erroneous results without warning.

So, reviewing the model's data section:

```
DATA:
    NEEDS = @POINTER( 1);
    @POINTER( 2) = START;
    @POINTER( 3) = ONDUTY;
    @POINTER( 4) = OBJECTIVE;
    @POINTER( 5) = @STATUS();
ENDDATA
```

we see the staffing requirements attribute, *NEEDS,* is read from the first memory location. The *START*, *ONDUTY*, and *OBJECTIVE* values are written to the second, third, and fourth memory locations, respectively. Finally, the value for the *@STATUS* function is written to the fifth memory location. See the following section for more details on the *@STATUS* function.

The @STATUS Function

The *@STATUS* function returns the status of the solution process using the following codes:

@STATUS Code	Interpretation
0	*Global Optimum* - The optimal solution has been found, subject to current tolerance settings.
1	*Infeasible* - No solution exists that satisfies all constraints.
2	*Unbounded* - The objective can be improved without bound.
3	*Undetermined* - The solution process failed.
4	*Feasible* - A feasible solution was found that may, or may not, be the optimal solution.
5	*Infeasible or Unbounded* - The preprocessor determined the model is either infeasible or unbounded. Turn off presolving and re-solve to determine which.
6	*Local Optimum* - Although a better solution may exist, a locally optimal solution has been found.
7	*Locally Infeasible* - Although feasible solutions may exist, LINGO was not able to find one.
8	*Cutoff* - The objective cutoff level was achieved.
9	*Numeric Error* - The solver stopped due to an undefined arithmetic operation in one of the constraints.

In general, if *@STATUS* does not return a code of 0, 4, 6, or 8, the solution is of little use and should not be trusted. In fact, if *@STATUS* does not return 0, 4, 6, or 8, in many cases LINGO will not even export data to the *@POINTER* memory locations.

The *@POINTER* function reads and writes all data using double precision floating point format. BASIC and C/C++ developers know this as the *double* data type, while FORTRAN developers refer to it as either *REAL*8* or *DOUBLE PRECISION*.

When setting aside memory locations for *@POINTER*, you must be sure to allocate adequate space. LINGO assumes adequate space and/or supplied values are available at the passed memory locations. If you do not allocate sufficient space or values, you may experience memory protection faults or, even worse, erroneous results without warning.

Functions Exported by the LINGO DLL

The LINGO DLL exports twelve functions.The exported functions are contained in the file *Lingo19\Lingd19.Dll* (or *Lingo64_19/Lingd64_19.dll* for 64-bit versions). The library file *Lingo19\Programming Samples\Lingd65.lib* may be used to import these functions into your own custom application.

Below is a list of the functions exported by the LINGO DLL along with a brief description of each routine's functionality. The definitions are written using C language conventions. Refer to the programming samples in the following sections for specific examples of the use of each of these routines using Visual C and Visual Basic.

void LSclearPointersLng(pLSenvLINGO pL)

This clears out the list of *@POINTER()* pointers to user memory transfer areas established through calls to *LSsetPointerLng()*.

Arguments:

pL	Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

int LScloseLogFileLng(pLSenvLINGO pL)

This closes LINGO's log file that was opened previously by a call to *LSopenLogFileLng()*.

Arguments:

pL	Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

pLSenvLINGO CALLTYPE LScreateEnvLng()

This creates a LINGO environment object. All other LINGO DLL routines require a valid pointer to a LINGO environment object. You should free this object at the end of your application by calling *LSdeleteEnvLng()*. *LScreateEnvLng()* will search the application's startup directory and the system path for a valid LINGO license file. If you would prefer to pass LINGO the license key directly, refer to the following function, *LScreateEnvLicenseLNG()*.

Return Value:

Returns 0 if an error occurred, otherwise, a pointer to a LINGO environment object is returned.

pLSenvLINGO CALLTYPE LScreateEnvLicenseLng(char pcLicenseKey, int* pnErr)*

This creates a LINGO environment object. All other LINGO DLL routines require a valid pointer to a LINGO environment object. You should free this object at the end of your application by calling *LSdeleteEnvLng()*. *LScreateEnvLicenseLng()* requires that you pass the license key directly. If you would prefer to have LINGO search the application's startup directory and the system path for a valid LINGO license file, refer to the previous function, *LScreateEnvLng()*.

Arguments:

pcLicenseKey — Pointer to a text string containing a LINGO license key. The license key may be found at the bottom of your LINGO license file, typically stored in the main LINGO directory under the file name LNDLNG??.LIC, where ?? is LINGO's version number. The license key is delimited in the license file by a greater-than (>) character.

pnError — Pointer to an integer that returns 0 if no problem or an error code listed in section LINGO DLL Error Codes.

Return Value:

Returns 0 if an error occurred, otherwise, a pointer to a LINGO environment object is returned.

int LSdeleteEnvLng(pLSenvLINGO pL)

This deletes a LINGO environment object previously created through a call to *LScreateEnvLng()*, which frees up the system memory allocated to the LINGO object.

Arguments:

pL — Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

int LSexecuteScriptLng(pLSenvLINGO pL, char pcScript)*

This routine is the main workhorse of the LINGO DLL that processes LINGO command scripts. The script may be contained entirely in memory, or it may contain one or more *TAKE* commands to load scripts from disk.

Arguments:

pL — Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

pcScript — Pointer to a character string containing a LINGO command script. Each line must be terminated with a linefeed character (ASCII 10), and the entire script must be terminated with a NULL (ASCII 0).

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

int LSgetCallbackInfoLng(pLSenvLINGO pL, int nObject, void pResult)*

You may establish a function in your application that the LINGO solver calls back to at regular intervals to keep you abreast of its progress. We refer to this type of routine as being a *callback function* and you may refer to section *Callback Functions* for more information. Your callback function may then call LINGO through *LSgetCallbackInfoLng()* to request specific information from the solver while it is processing a model.

Arguments:

pL Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

nObject Index of the information item you're seeking. Current possibilities are:

Index	Name	Type	Information Item
0	*LS_IINFO_VARIABLES_LNG*	*Int*	Total number of variables
1	*LS_IINFO_VARIABLES_INTEGER_LNG*	*Int*	Number of integer variables
2	*LS_IINFO_VARIABLES_NONLINEAR_LNG*	*Int*	Number of nonlinear variables
3	*LS_IINFO_CONSTRAINTS_LNG*	*Int*	Total number of constraints
4	*LS_IINFO_CONSTRAINTS_NONLINEAR_LNG*	*Int*	Number of nonlinear constraints
5	*LS_IINFO_NONZEROS_LNG*	*Int*	Total nonzero matrix elements
6	*LS_IINFO_NONZEROS_NONLINEAR_LNG*	*Int*	Number of nonlinear nonzero matrix elements
7	*LS_IINFO_ITERATIONS_LNG*	*Int*	Number of iterations
8	*LS_IINFO_BRANCHES_LNG*	*Int*	Number of branches (IPs only)
9	*LS_DINFO_SUMINF_LNG*	*Double*	Sum of infeasibilities
10	*LS_DINFO_OBJECTIVE_LNG*	*Double*	Objective value
11	*LS_DINFO_MIP_BOUND_LNG*	*Double*	Objective bound (IPs only)
12	*LS_DINFO_MIP_BEST_OBJECTIVE_LNG*	*Double*	Best objective value found so far (IPs only)

pResult Pointer to where you want LINGO to store the results of your query. LINGO will place the result at this address. You must

allocate four bytes of memory for *ints* and eight bytes of memory for *doubles* beforehand.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

int LSgetCallbackVarPrimalLng(pLSenvLINGO pL, char pcVarName, double* pdPrimals)*

You may establish a function in your application that the LINGO solver calls back to at regular intervals to keep you abreast of its progress. We refer to this type of routine as being a *callback function*, and you may refer to section *Callback Functions* for more information. Your callback function may then call LINGO through *LSgetCallbackVarPrimal()* to request the current values for attributes and/or scalar variables. This will allow you to keep your users posted on the values of the model's variables in the current best solution found so far. This routine will only return values successfully when solving integer models, or when solving models with the global solver.

Arguments:

pL — Pointer to a LINGO environment created by a previous call to LScreateEnvLng().

pcVarName — Pointer to a character string containing the name of the scalar variable or attribute. The name must be terminated with either a blank or a NULL (ASCII 0).

pdPrimals — Points to where LINGO should write the variable value(s). You must ensure that enough space is available for the full set of values to avoid memory protection faults.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed in section *LINGO DLL Error Codes* is returned.

*int LSopenLogFileLng(pLSenvLINGO pL, char *pcLogFile)*

This creates a file for LINGO to write a log to while processing your script. In general, you should always try to create a log file (at least in the early stages of your project) to assist with debugging. If an error is occurring and you are not exactly sure why, then it is always a good idea to refer to the log file for a clue.

Arguments:

pL — Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

pcLogFile — Pointer to a character string containing the pathname for your log file.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

int LSsetCallbackErrorLng(pLSenvLINGO pL, lngCBFuncError_t pcbf, void pUserData)*

Use this routine to specify a callback function that LINGO will call whenever an error is encountered. This allows your application to keep close watch on any unusual conditions that might arise while processing your script.

Arguments:

pL	Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.
pcbf	Pointer to your callback routine.
pUserData	This is a user specified pointer. You may use it to point to any data you might need to reference from your callback function. LINGO merely passes the value of this pointer through to your callback function. You may set this pointer to *NULL* if it is not required.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

The callback function you supply must use the standard call convention and must have the following interface:

int MyErrCallback(pLSenvLINGO pL, void pUserData, int nErrorCode, char* pcErrorText);*

Your computer will most likely crash if you don't follow this interface specification *exactly*. The LINGO error code is reported through the *nErrorCode* argument, along with the error text in *pcErrorText*. You should set aside at least 200 bytes for the error text. The list of LINGO error codes can be found in Appendix B, *Error Messages*.

int LSsetCallbackSolverLng(pLSenvLINGO pL, lngCBFuncError_t pcbf, void pUserData)*

Use this routine to specify a callback function that LINGO will call at frequent intervals when solving a model.

Arguments:

pL	Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.
pcbf	Pointer to your callback routine.
pUserData	This is a user specified pointer. You may use it to point to any data you might need to reference from your callback function. LINGO merely passes the value of this pointer through to your callback function. You may set this pointer to *NULL* if it is not required.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

The callback function you supply must use the standard call convention and must have the following interface:

int MySolverCallback(pLSenvLINGO pL, int nReserved, void pUserData);*

Your computer will most likely crash if you don't follow this interface specification *exactly*. The *nReserved* argument is reserved for future use and may be ignored.

int CALLTYPE LSsetPointerLng(pLSenvLINGO pL, double pdPointer, int* pnPointersNow)*

Call this routine one or more times to pass a list of memory pointers to LINGO. These pointers are used by LINGO's *@POINTER()* function for moving data into and solutions out of the solver. In other words, this allows you to have direct memory links with LINGO for fast and convenient data transfer.

Arguments:

pL	Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.
pdPointer	Pointer to a memory transfer location to be used by an instance of *@POINTER()*.
pnPointersNow	This is a pointer to an integer variable in which LINGO returns the current number of entries in the *@POINTER()* pointer list. Thus, on the first call to *LSsetPointersLng()*, this argument will return a value of 1. The pointer list may be cleared at any time by a call to *LSclearPointersLng()*.

Return Value:

Returns 0 if no error occurred. Otherwise, one of the nonzero error codes listed below in the section *LINGO DLL Error Codes* is returned.

LINGO DLL Error Codes

Most of the LINGO DLL functions return an error code. Below is a list of all possible codes:

Value	Name	Descriptions
0	LSERR_NO_ERROR_LNG	No error.
1	LSERR_OUT_OF_MEMORY_LNG	Out of dynamic system memory.
2	LSERR_UNABLE_TO_OPEN_LOG_FILE_LNG	Unable to open the log file.
3	LSERR_INVALID_NULL_POINTER_LNG	A NULL pointer was passed to a routine that was expecting a non-NULL pointer.
4	LSERR_INVALID_INPUT_LNG	An input argument contained invalid input.
5	LSERR_INFO_NOT_AVAILABLE_LNG	A request was made for information that is not currently available.
6	LSERR_UNABLE_TO_COMPLETE_TASK_LNG	Unable to successfully complete the specified task.
7	LSERR_INVALID_LICENSE_KEY_LNG	The license key passed to LScreateEnvLicenceLng() was invalid.
8	LSERR_INVALID_VARIABLE_NAME_LNG	A variable name passed to *LSgetCallbackVarPrimal()* was invalid.
1000	LSERR_JNI_CALLBACK_NOT_FOUND_LNG	A valid callback function was not found in the calling Java application.

Supporting DLLs

The main LINGO DLL requires many additional component DLLs. All the required DLLs are installed as part of the normal LINGO installation in the main LINGO folder. If you need a specific list of all the supporting DLLs required by the LINGO DLL, then you can open *Lingd14.Dll* in the *Dependency Walker* utility available at no charge from the following website: http://www.dependencywalker.com. The Dependency Walker program is a useful tool that lists all the required DLLs for an application, their current location on your system, and flags any DLLs that are not currently available.

Building the Application in C++

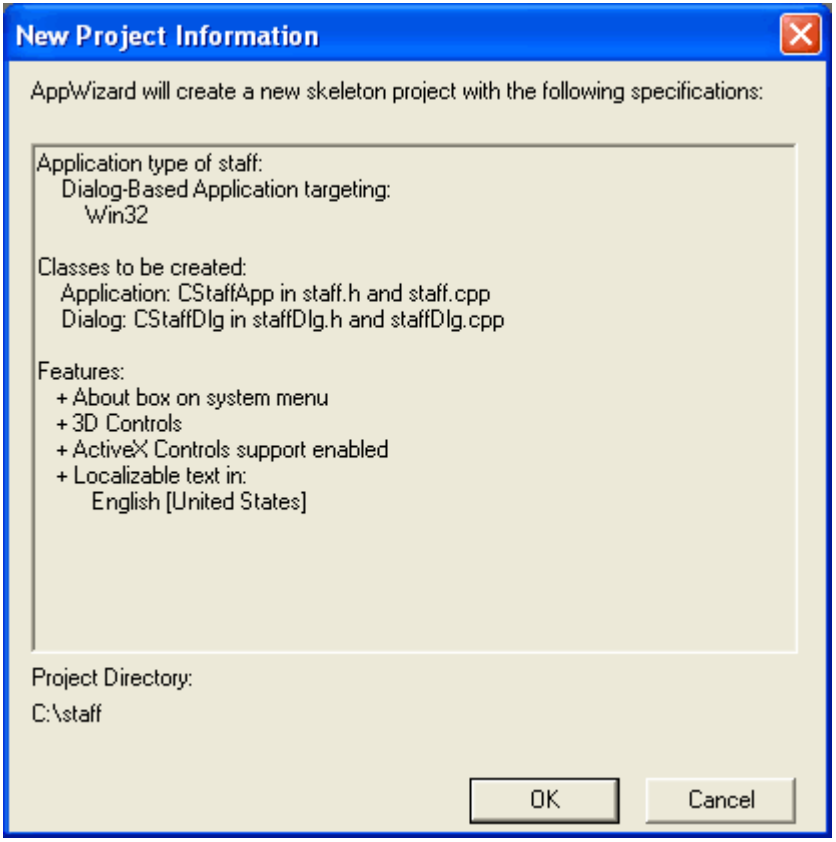
In this section, we will walk you through the steps involved in building the project to solve the staff scheduling model. If you would rather skip the details involved in constructing this project and would prefer to experiment with the completed application, it can be found under the path *\Lingo19\Programming Samples\VC++\STAFF1\STAFF.EXE.*

We will be working from a code base generated using Visual C++'s AppWizard facility. The generated code is for a *dialog based application*. In other words, the interface to the application will consist of a single dialog box. This dialog box will be modified, so it resembles the one illustrated below with fields for staffing requirements, daily employee starts, and so on.

The source code for this project may be found in the *Programming Samples\STAFF1* subdirectory or you can create your own project using Visual C/C++ 6.0 as follows:

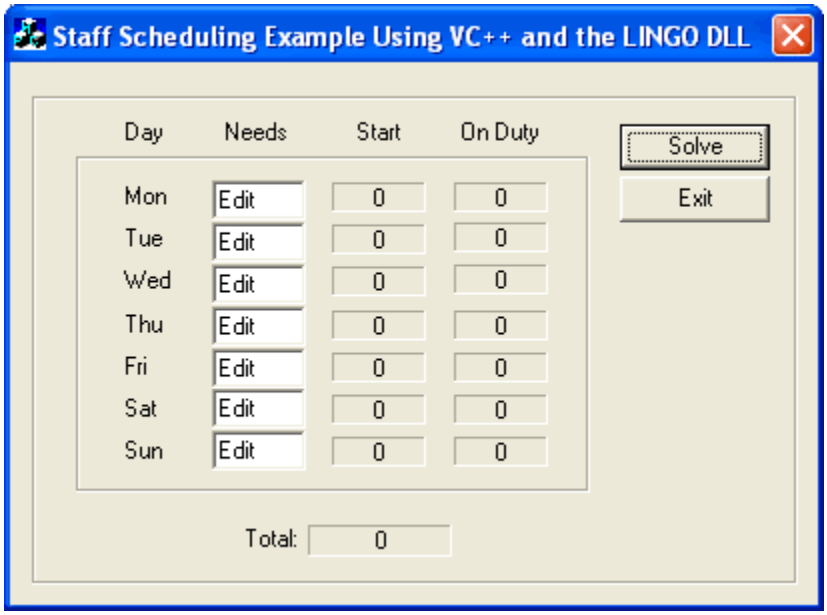
1. Start Visual C++ and issue the *File|New* command.
2. Select *Projects* tab, give the project a name, select *MFC AppWizard(Exe)* as the type of application, and then click the *OK* button.
3. Click the *dialog based* radio button and then the *Next* button.
4. Clear the *About box* button, check the *OLE Automation* button, check the *OLE Controls* button, and press the *Next* button. OLE support is required for some of the features in the LINGO DLL.
5. Click the *Next* button again.
6. Click the *Finish* button.

After completing these steps, the following summary of your project should be shown:



Click the *OK* button, and AppWizard will generate the skeleton code base.

Use the resource editor to modify the application's dialog box, so it resembles the following:



Next, you will need to use the ClassWizard in Visual C++ to associate member variables with each of the fields in the dialog box. From the View menu, select the *ClassWizard* command and then select the *Member Variables* tab.

At this point, the LINGO DLL import library must be added to the project in order to make the LINGO DLL available to the code. Do this by running the *Project|Add to Project|Files* command and select the file *\LINGO19\Programming Samples\LINGD19.LIB* (or *LINGD64_19.LIB* for 64-bit versions) for addition to the project.

After doing that, add the definitions of the LINGO DLL routines to the project. Simply include the *Lingd19.h* header file at the top of the dialog class header file as follows (code changes listed in bold):

```
// staffDlg.h : header file
//

#include "lingd19.h"

#if
!defined(AFX_STAFFDLG_H__74D746B7_CA4D_11D6_AC89_00010240D2AE__INCLUD
ED_)
#define
AFX_STAFFDLG_H__74D746B7_CA4D_11D6_AC89_00010240D2AE__INCLUDED_

#if _MSC_VER > 1000
#pragma once
#endif // _MSC_VER > 1000

/////////////////////////////////////////////////////////////////////
   ////////
// CStaffDlg dialog
                                .
                                .
                                .
```

All the groundwork has now been laid down and we're ready to begin writing the actual code to call LINGO to solve the staffing model. Go to the *Resource View* of the project, open the dialog box resource, and then double click on the *Solve* button. You will be prompted to create a handler function for the button, which should be named *OnSolve.* Now, edit the stub version of *OnSolve*, so it contains the following code:

```
void CStaffDlg::OnSolve()
{
  int nError, nPointersNow;
  CString csScript, cs;
  double dNeeds[7], dStart[7], dOnDuty[7], dStatus, dTotal;

  // Get user's staffing requirements from our dialog box
  UpdateData();

  // Load staffing requirements into the LINGO transfer array.
  // LINGO uses double precision for all values.
  dNeeds[ 0] = (double) m_nNeedsMon;
  dNeeds[ 1] = (double) m_nNeedsTue;
  dNeeds[ 2] = (double) m_nNeedsWed;
  dNeeds[ 3] = (double) m_nNeedsThu;
  dNeeds[ 4] = (double) m_nNeedsFri;
  dNeeds[ 5] = (double) m_nNeedsSat;
  dNeeds[ 6] = (double) m_nNeedsSun;

  // create the LINGO environment object
  pLSenvLINGO pLINGO;
  pLINGO = LScreateEnvLng();
  if ( !pLINGO)
  {
```

```
   AfxMessageBox("Unable to create LINGO Environment");
   return;
}

// Open LINGO's log file
nError = LSopenLogFileLng( pLINGO, "LINGO.log");
if ( nError) goto ErrorExit;

// Pass memory transfer pointers to LINGO
// @POINTER(1)
nError = LSsetPointerLng( pLINGO, dNeeds, &nPointersNow);
 if ( nError) goto ErrorExit;

 // @POINTER(2)
nError = LSsetPointerLng( pLINGO, dStart, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(3)
nError = LSsetPointerLng( pLINGO, dOnDuty, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(4)
nError = LSsetPointerLng( pLINGO, &dTotal, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(5)
nError = LSsetPointerLng( pLINGO, &dStatus, &nPointersNow);
if ( nError) goto ErrorExit;

// Here is the script we want LINGO to run
csScript = "SET ECHOIN 1\n";
csScript = csScript +
 "TAKE \\LINGO19\\SAMPLES\\STAFFPTR.LNG\n";
csScript = csScript +
 "GO\n";
csScript = csScript +
 "QUIT\n";

// Run the script
dStatus = -1.e0;
nError = LSexecuteScriptLng( pLINGO, (LPCTSTR) csScript);

// Close the log file
LScloseLogFileLng( pLINGO);

// Any problems?
if ( nError || dStatus)


   // Had a problem
   AfxMessageBox("Unable to solve!");

} else

   // Everything went ok ... load results into the dialog box
   m_csStartMon.Format( "%d", (int) dStart[0]);
   m_csStartTue.Format( "%d", (int) dStart[1]);
```

```
      m_csStartWed.Format( "%d", (int) dStart[2]);
      m_csStartThu.Format( "%d", (int) dStart[3]);
      m_csStartFri.Format( "%d", (int) dStart[4]);
      m_csStartSat.Format( "%d", (int) dStart[5]);
      m_csStartSun.Format( "%d", (int) dStart[6]);

      m_csOnDutyMon.Format( "%d", (int) dOnDuty[0]);
      m_csOnDutyTue.Format( "%d", (int) dOnDuty[1]);
      m_csOnDutyWed.Format( "%d", (int) dOnDuty[2]);
      m_csOnDutyThu.Format( "%d", (int) dOnDuty[3]);
      m_csOnDutyFri.Format( "%d", (int) dOnDuty[4]);
      m_csOnDutySat.Format( "%d", (int) dOnDuty[5]);
      m_csOnDutySun.Format( "%d", (int) dOnDuty[6]);

      m_csCost.Format( "%g", dTotal);

      UpdateData( FALSE);

   }

   goto Exit;

ErrorExit:
   cs.Format( "LINGO Errorcode: %d", nError);
   AfxMessageBox( cs);
   return;

Exit:
   LSdeleteEnvLng( pLINGO);
}
```

The first section of *OnSolve* is straightforward and deals with extracting the user's staffing requirements from the dialog box. Note that the data is stored in a double precision array rather than as integers. This is because these values will be passed to LINGO, which only passes values in double precision format.

Our first call to LINGO creates the LINGO environment object with the following code:

```
   // create the LINGO environment object
   pLSenvLINGO pLINGO;
   pLINGO = LScreateEnvLng();
   if ( !pLINGO)
   {
      AfxMessageBox("Unable to create LINGO Environment");
      return;
   }
```

The *pLSenvLINGO* data type is defined in the LINGO header file, *lingd19.h*.

Then, a log file for LINGO is established with the following call:

```
   // Open LINGO's log file
   nError = LSopenLogFileLng( pLINGO, "LINGO.log");
   if ( nError) goto ErrorExit;
```

As mentioned above, opening a log file for LINGO is good practice, at least when you're debugging the application. If something should go wrong, the log file will generally contain a helpful clue.

Our next step is to pass LINGO the physical addresses it will use to resolve the *@POINTER()* function references used in the data section of the model (refer to the *The Model* section for more details). This is done as follows:

```
// Pass memory transfer pointers to LINGO
// @POINTER(1)
nError = LSsetPointerLng( pLINGO, dNeeds, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(2)
nError = LSsetPointerLng( pLINGO, dStart, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(3)
nError = LSsetPointerLng( pLINGO, dOnDuty, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(4)
nError = LSsetPointerLng( pLINGO, &dTotal, &nPointersNow);
if ( nError) goto ErrorExit;

// @POINTER(5)
nError = LSsetPointerLng( pLINGO, &dStatus, &nPointersNow);
if ( nError) goto ErrorExit;
```

In summary, when LINGO is called to solve the model, the staffing needs are passed to LINGO in the *dNeeds* array via the *@POINTER(1)* reference. Solution information is passed from LINGO back to the application in the *dStart, dOnDuty, dTotal*, and *dStatus* structures via *@POINTER()* references 2 through 5, respectively. If any of this is unfamiliar, review *The @POINTER() Function* section above.

Next, the following code is used to build the command script:

```
 // Here is the script we want LINGO to run
csScript = "SET ECHOIN 1\n";
csScript = csScript +
 "TAKE \\LINGO19\\SAMPLES\\STAFFPTR.LNG\n";
csScript = csScript +
 "GO\n";
csScript = csScript +
 "QUIT\n";
```

The script consists of four commands, which are each terminated with a new line character (ASCII 10). The end of the script is terminated with a *NULL* character (ASCII 0). These commands and their functions are:

Command	Function
`SET ECHOIN 1`	Causes LINGO to echo input to the log file. This is a useful feature while building and debugging an application.
`TAKE`	Loads the model from a disk file. The *TAKE* command may be used to load model files, as in this example. It may also be used to run nested command scripts contained in files.
`GO`	Calls the solver to optimize the model.
`QUIT`	Closes down LINGO's script processor and returns control to the calling application.

At this point, the script is ready to be passed off to LINGO for processing with the following call:

```
// Run the script
  dStatus = -1.e0;
  nError = LSexecuteScriptLng( pLINGO, (LPCTSTR) csScript);
```

Note that *dStatus* is initialized to –1. LINGO returns the model status through memory transfer location number 5 (i.e., *@POINTER(5)*) to the *dStatus* variable. LINGO will only return a status value if it was able to solve the model. If an unexpected error were to occur, LINGO might not ever reach the solution phase. In that case, *dStatus* would never be set. Initializing *dStatus* to a negative value tests for this situation. Given that LINGO returns only *non-negative* status codes, a *negative* status code would indicate a problem. This method of error trapping is effective, but not particularly elegant. Another method that involves specifying an error callback routine is demonstrated below.

Now, LINGO's log file may be closed down by calling *LScloseLogFileLng()*:

```
// Close the log file
  LScloseLogFileLng( pLINGO);
```

Next, the following code tests to see if LINGO was able to find an optimal solution:

```
  // Any problems?
  if ( nError || dStatus)

     // Had a problem
     AfxMessageBox("Unable to solve!");

  } else
```

Note that the code returned in *nError* pertains only to the mechanical execution of the script processor. It has nothing to do with the status of the model's solution, which is returned in *dStatus* via the use of the *@POINTER()* and *@STATUS()* functions (see the *The Model* section). A model may actually be infeasible or unbounded, and the error code returned by *LSexecuteScript()* will give no indication. Thus, it is important to add a mechanism to return a solution's status to the calling application, as done here with the *@STATUS() -> @POINTER(5) -> dStatus* link. The end result in the sample code is that

"Unable to solve" is printed if either error condition occurs. A more user-friendly application would offer more specific information regarding the error condition.

As a final step, in order to avoid memory leaks in your application, remember to free up LINGO's environment when through:

```
Exit:
  LSdeleteEnvLng( pLINGO);
```

If everything has been entered correctly, you should now be able to build and run the project.

Visual Basic Staff Scheduling Using the LINGO DLL

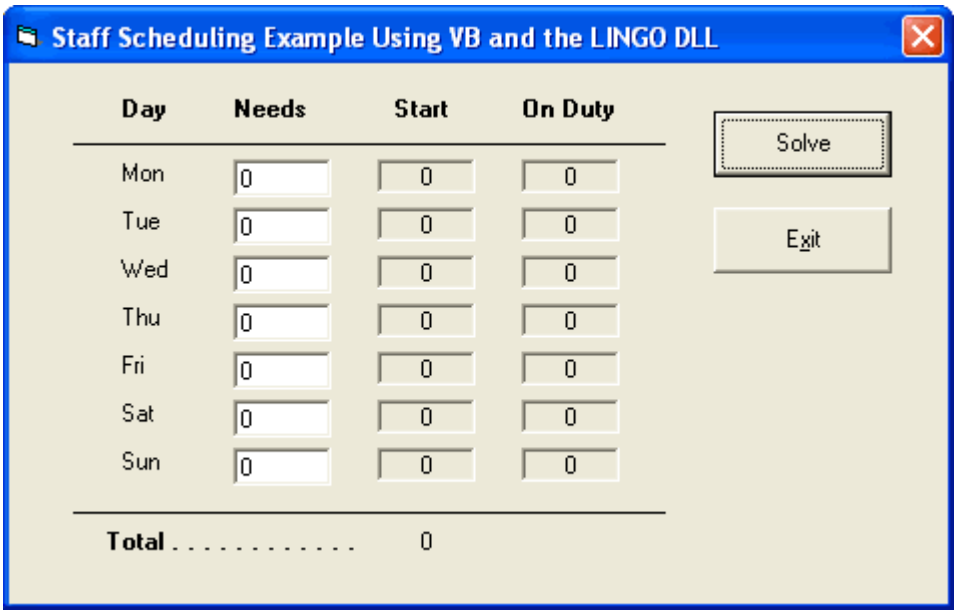
In the *Building the Application in Visual Basic* section, we will illustrate the use of Microsoft Visual Basic to build an application that interfaces with the LINGO DLL to solve the staff scheduling problem. This section assumes the reader is well versed in the use of Visual Basic.

Buiding the Application in Visual Basic

In this section, we will walk you through the steps involved in building the Visual Basic project. If you would rather skip the details involved in constructing this project and would prefer to experiment with the completed application, it can be found in the *\LINGO19\Programming Samples\VBasic\Staff1* folder.

You can build the project as follows:

1. Start Visual Basic and then issue the *File|NewProject* command.
2. Select *Standard Exe* and click the *OK* button.
3. Use the resource editor to format the project's form until it resembles the following:



Now, add handler code for the two buttons in the form. The *Exit* button is easy. All the *Exit* button does when the user clicks it is exit the application. So, double click on the *Exit* button and enter the following code:

```
Private Sub Exit_Click()
  End
End Sub
```

A little more work will be required to setup the *Solve* button. When the *Solve* button is pressed, the application will retrieve the staffing requirements from the form, pass them along with the model to the LINGO script processor (*LGVBSCRIPT*) to solve the model, retrieve the solution, and post the solution in the form.

First, we must declare the external LINGO function. Do this by adding the *\LINGO19\Programming Samples\LINGD19.BAS* module to the project using the *Project|Add Module* command in VB. This module contains the definitions of all the exported function in the LINGO DLL, and makes them available to our project.

Now, add the handler code for the *Solve* button. Go to the form, double click on the *Solve* button, and enter the following code:

```
Private Sub Solve_Click()

' Calls the LINGO DLL to solve the staffing
' model in STAFFPTR.LNG.  Staffing
' requirements are taken from the dialog
' box.

' Get the staffing needs from the dialog box
  Dim dNeeds(7) As Double
  For i = 1 To 7
     dNeeds(i) = Needs(i - 1).Text
  Next i

' Create the LINGO environment object
  Dim pLINGO As Long
  pLINGO = LScreateEnvLng()
  If pLINGO = 0 Then
    MsgBox ("Unable to create LINGO Environment.")
    GoTo FinalExit
  End If

' Open LINGO's log file
  Dim nError As Long
  nError = LSopenLogFileLng(pLINGO, "LINGO.log")
  If nError <> 0 Then GoTo ErrorExit

' Pass memory transfer pointers to LINGO
  Dim dStart(7) As Double, dOnDuty(7) As Double
  Dim dTotal As Double, dStatus As Double

' @POINTER(1)
  nError = LSsetPointerLng(pLINGO, dNeeds(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(2)
  nError = LSsetPointerLng(pLINGO, dStart(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(3)
  nError = LSsetPointerLng(pLINGO, dOnDuty(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(4)
  nError = LSsetPointerLng(pLINGO, dTotal, nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(5)
  nError = LSsetPointerLng(pLINGO, dStatus, nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' Build LINGO's command script (commands
' are terminated with an ASCII 10
  Dim cScript As String

' Causes LINGO to echo input
  cScript = "SET ECHOIN 1" & Chr(10)

' Read in the model file
```

```
    cScript = cScript & _
     "TAKE \LINGO\SAMPLES\STAFFPTR.LNG" & Chr(10)

  ' Solve the model
    cScript = cScript & "GO" & Chr(10)

  ' Quit LINGO DLL
    cScript = cScript & "QUIT" & Chr(10)

  ' Mark end of script with a null byte
    cScript = cScript & Chr(0)

  ' Run the script
    dStatus = -1#
    nError = LSexecuteScriptLng(pLINGO, cScript)

  ' Close the log file
    LScloseLogFileLng (pLINGO)

  ' Problems?
    If nError > 0 Or dStatus <> 0 Then
      MsgBox ("Unable to solve!")
      GoTo ErrorExit
    End If

  ' Place Start values in dialog box
    For i = 1 To 7
       Start(i - 1).Caption = dStart(i)
    Next i

  ' Place On Duty values in dialog box
    For i = 1 To 7
       OnDuty(i - 1).Caption = dOnDuty(i)
    Next i

  ' Put Total staffing in dialog box
    Total.Caption = dTotal

    LSdeleteEnvLng (pLINGO)
    GoTo FinalExit:

  ErrorExit:

    MsgBox ("LINGO Error Code: " & nError&)
    LSdeleteEnvLng (pLINGO)

  FinalExit:

  End Sub
```

The first section of the *Solve_Click* procedure is straightforward and deals with extracting the user's staffing requirements from the dialog box. Note that the data is stored in a double precision array, rather than as integers. This is because these values will be passed to LINGO, which only passes numeric values in double precision format.

The first call to LINGO creates the LINGO environment object with the following code:

```
' Create the LINGO environment object
  Dim pLINGO As Long
  pLINGO = LScreateEnvLng()
  If pLINGO = 0 Then
    MsgBox ("Unable to create LINGO Environment.")
    GoTo FinalExit
  End If
```

Next, a log file for LINGO is established with the call:

```
' Open LINGO's log file
  Dim nError As Long
  nError = LSopenLogFileLng(pLINGO, "LINGO.log")
  If nError <> 0 Then GoTo ErrorExit
```

As mentioned above, opening a log file for LINGO is good practice, at least when you're debugging the application. If something should go wrong, the log file will generally contain a helpful clue.

The next step is to pass LINGO the physical addresses it will use to resolve the *@POINTER()* function references used in the data section of the model (refer to the *The Model* section for more details). This is done as follows:

```
' Pass memory transfer pointers to LINGO
  Dim dStart(7) As Double, dOnDuty(7) As Double
  Dim dTotal As Double, dStatus As Double

' @POINTER(1)
  nError = LSsetPointerLng(pLINGO, dNeeds(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(2)
  nError = LSsetPointerLng(pLINGO, dStart(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(3)
  nError = LSsetPointerLng(pLINGO, dOnDuty(1), nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(4)
  nError = LSsetPointerLng(pLINGO, dTotal, nPointersNow)
  If nError <> 0 Then GoTo ErrorExit

' @POINTER(5)
  nError = LSsetPointerLng(pLINGO, dStatus, nPointersNow)
  If nError <> 0 Then GoTo ErrorExit
```

In summary, when LINGO is called to solve the model, the staffing needs are passed to LINGO in the *dNeeds* array via the *@POINTER(1)* reference. Solution information is passed from LINGO back to the application in the *dStart*, *dOnDuty*, *dTotal*, and *dStatus* structures via *@POINTER()* references 2 through 5, respectively. If any of this is unfamiliar, review *The @POINTER() Function* section.

Next, the following code is used to build the command script:

```
' Build LINGO's command script (commands
' are terminated with an ASCII 10
  Dim cScript As String

' Causes LINGO to echo input
  cScript = "SET ECHOIN 1" & Chr(10)

' Read in the model file
  cScript = cScript & _
   "TAKE \LINGO\SAMPLES\STAFFPTR.LNG" & Chr(10)

' Solve the model
  cScript  = cScript & "GO" & Chr(10)

' Quit LINGO DLL
  cScript = cScript & "QUIT" & Chr(10)

' Mark end of script with a null byte
  cScript = cScript & Chr(0)
```

The script consists of four commands, which are each terminated with a new line character (ASCII 10). The end of the script is terminated with a *NULL* character (ASCII 0). These commands and their functions are:

Command	Function
`SET ECHOIN 1`	Causes LINGO to echo input to the log file. This is a useful feature while building and debugging an application.
`TAKE`	Loads the model from a disk file. The *TAKE* command may be used to load model files, as in this example. It may also be used to run nested command scripts contained in files.
`GO`	Calls the solver to optimize the model.
`QUIT`	Closes down LINGO's script processor and returns control to the calling application.

At this point, the script is ready to be passed off to LINGO for processing with the following call:

```
' Run the script
  dStatus = -1#
  nError = LSexecuteScriptLng(pLINGO, cScript)
```

Note that *dStatus* is initialized to –1. LINGO returns the model status through memory transfer location number 5 (i.e., *@POINTER(5)*) to the *dStatus* variable. LINGO will only return a status value if it was able to solve the model. If an unexpected error were to occur, LINGO might not ever reach the solution phase. In that case, *dStatus* would never be set. Initializing *dStatus* to a negative value tests for this situation. Given that LINGO returns only *non-negative* status codes, a *negative* status code would indicate a problem. This method of error trapping is effective, but not particularly elegant. Another method that involves specifying an error callback routine is demonstrated below.

Now, LINGO's log file may be closed down by calling *LScloseLogFileLng()*:

```
' Close the log file
  LScloseLogFileLng (pLINGO)
```

Next, the following code tests to see if LINGO was able to find an optimal solution:

```
' Problems?
  If nError > 0 Or dStatus <> 0 Then
    MsgBox ("Unable to solve!")
    GoTo ErrorExit
  End If
```

Note that the code returned in *nError* pertains only to the mechanical execution of the script processor. It has nothing to do with the status of the model's solution, which is returned in *dStatus* via the use of the *@POINTER()* and *@STATUS()* functions (see *The Model* section). A model may actually be infeasible or unbounded, and the error code returned by *LSexecuteScriptLng()*will give no indication. Thus, it is important to add a mechanism to return a solution's status to the calling application, as done here with the *@STATUS() -> @POINTER(5) -> dStatus* link. The end result in the sample code is that "Unable to solve" is printed if either error condition occurs. A more user-friendly application would offer more specific information regarding the error condition.

As a final step, in order to avoid a memory leak in the application, remember to free up LINGO's environment when through:

```
  LSdeleteEnvLng (pLINGO)
  GoTo FinalExit:
```

If everything has been entered correctly, you should now be able to run the project.

Passing Set Members with @POINTER

In the previous examples, when using the *@POINTER* function, we only passed attribute values back and forth. You may also pass set members using *@POINTER*, with the primary difference being that the set members are passed as ASCII text, as opposed to double precision numeric values. In addition, each set member is separated by a line feed (ASCII 10), with the end of the list of set members denoted with a null (ASCII 0) character.

In order to illustrate passing set members via *@POINTER*, we will modify the knapsack problem discussed above in section *Binary Integer Example - The Knapsack Problem.* You will recall that the data in a knapsack problem consists of the list of potential items to include in the knapsack, their weight, their utility/value, and the knapsack's capacity. We will pass all this data to LINGO via the @POINTER function, including the set of potential items. After solving the model, we will create a new set, derived from the original set of items, which contains only those items that are included in the optimal solution. Finally, we will use the *@POINTER* function to pass the optimal set of items back to our calling program so that we can display them for the user.

We will develop this example using the C programming language, however, the concepts should carry over in a straightforward manner to all other development environments. The code for this example may be found in the *Programming Samples\VC++\Knapsack* folder off the main LINGO directory.

The Model

Here is a copy of the knapsack model our application will load into LINGO:

```
MODEL:

SETS:
   ITEMS: INCLUDE, WEIGHT, RATING;
ENDSETS

DATA:
   ITEMS  = @POINTER( 1);
   WEIGHT = @POINTER( 2);
   RATING = @POINTER( 3);
   KNAPSACK_CAPACITY = @POINTER( 4);
ENDDATA

SUBMODEL SACK:
   MAX = @SUM( ITEMS: RATING * INCLUDE);
   @SUM( ITEMS: WEIGHT * INCLUDE) <=
    KNAPSACK_CAPACITY;
   @FOR( ITEMS: @BIN( INCLUDE));
ENDSUBMODEL

CALC:
   !keep output to a minimum;
   @SET( 'TERSEO', 1);
   !solve the model;
   @SOLVE( SACK);
   !fix the INCLUDE attribute to it's optimal value;
   @FOR( ITEMS( I): INCLUDE( I) = INCLUDE( I));
ENDCALC

SETS:
   !construct a set of the optimal items;
   ITEMSUSED( ITEMS) | INCLUDE( &1) #GT# .5:;
ENDSETS

DATA:
   !send optimal items set back to caller;
   @POINTER( 5) = ITEMSUSED;
   !along with the solver status;
   @POINTER( 6) = @STATUS();
ENDDATA

END
```

Model: SACK

Note: Some of the model features in this example take advantage of the scripting capability in LINGO models. Scripting in models is discussed in more detail in Chapter 13, *Programming LINGO*.

In the model's sets section:

```
SETS:
   ITEMS: INCLUDE, WEIGHT, RATING;
ENDSETS
```

We define the *ITEMS* set, which will store the set of potential items for the knapsack. Each item has the following attributes:

- *INCLUDE* — a binary variable indicating whether or not the item is to be included in the optimal knapsack.
- *WEIGHT* — the item's weight
- *RATING* — the items utility, or value.

In the data section, we ask LINGO to import all the data for the model via the *@POINTER* function:

```
DATA:
   ITEMS  = @POINTER( 1);
   WEIGHT = @POINTER( 2);
   RATING = @POINTER( 3);
   KNAPSACK_CAPACITY = @POINTER( 4);
ENDDATA
```

Note that in addition to strictly numeric values, we are also importing the set *ITEMS*. This set will be passed from our calling application as an ASCII string.

Next, we have the actual knapsack model. We partition the model as a submodel within our main model (the concept of submodels is discussed further in Chapter 13):

```
SUBMODEL SACK:
   MAX = @SUM( ITEMS: RATING * INCLUDE);
   @SUM( ITEMS: WEIGHT * INCLUDE) <=
    KNAPSACK_CAPACITY;
   @FOR( ITEMS: @BIN( INCLUDE));
ENDSUBMODEL
```

This submodel contains three expressions. First, there's the objective that maximizes total utility of the selected items. Second, there is a constraint that forces total weight to not exceed capacity. Finally, via the *@BIN* function, we force the *INCLUDE* attribute members to be either 0 or 1, given that fractional solution do not make sense for this model.

The next section is a calc section where we perform three steps:

```
CALC:
   !keep output to a minimum;
   @SET( 'TERSEO', 1);
   !solve the model;
   @SOLVE( SACK);
   !fix the INCLUDE attribute to it's optimal value;
   @FOR( ITEMS( I): INCLUDE( I) = INCLUDE( I));
ENDCALC
```

The first step sets the *TERSEO* function to minimize output from LINGO. Next, we use the @SOLVE function to solve the knapsack submodel. Finally,. we loop over the *ITEMS* set, fixing the *INCLUDE* attribute members to their optimal values in the *SACK* submodel. The reason we need to fix their values is that we will need them in the following sets section where we use a set membership condition to generate the optimal set of items in the knapsack (set membership conditions will reject variables that aren't fixed in value).

Next, we derive the set *ITEMSUSED* from the original set of all potential items, however, we only include those items that have a nonzero value in the optimal solution:

```
SETS:
   !construct a set of the optimal items;
   ITEMSUSED( ITEMS) | INCLUDE( &1) #GT# .5:;
ENDSETS
```

As our last step, we construct a data section to send the set of optimal items back to the calling program via *@POINTER*:

```
DATA:
   !send optimal items set back to caller;
   @POINTER( 5) = ITEMSUSED;
   !along with the solver status;
   @POINTER( 6) = @STATUS();
ENDDATA
```

The Code

Here is a copy of the C code we will use to drive our application:

```
#include <stdlib.h>
#include <string.h>

#include "..\..\lingd19.h"

/*
  Solves a simple knapsack problem, passing
  all data to Lingo and retrieving the optimal
  set of knapsack items for display
*/
```

```
void main()
{
  // input data for model:

  // potential items in knapsack
  char pcItems[256] = "ANT_REPEL \n BEER \n BLANKET \n"
   "BRATWURST \n BROWNIES \n FRISBEE \n SALAD \n"
   "WATERMELON";

  // and their weights
  double pdItemWeight[8] = { 1, 3, 4, 3, 3, 1, 5,10};

  // and their rankings
  double pdItemRank[8]   = { 2, 9, 3, 8,10, 6, 4,10};

  // knapsack size
  double dSackSize = 15;

  // other declarations
  int i, nPointersNow, nError;
  double dStatus=-1.0;
  char pcScript[256];
  char pcItemsSolution[256];

  // create the LINGO environment object
  pLSenvLINGO pLINGO;
  pLINGO = LScreateEnvLng();
  if ( !pLINGO)
  {
     printf( "Can''t create LINGO environment!\n");
     goto FinalExit;
  }

  // Open LINGO's log file
  nError = LSopenLogFileLng( pLINGO, "LINGO.log");
  if ( nError) goto ErrorExit;

  // Pass memory transfer pointers to LINGO

  // @POINTER(1) - Items set
  nError = LSsetPointerLng( pLINGO, (void*) pcItems,
   &nPointersNow);
  if ( nError) goto ErrorExit;

  // @POINTER(2) - Item weights
  nError = LSsetPointerLng( pLINGO, (void*) pdItemWeight,
   &nPointersNow);
  if ( nError) goto ErrorExit;
```

```
  // @POINTER(3) - Item ranks
  nError = LSsetPointerLng( pLINGO, (void*) pdItemRank,
   &nPointersNow);
  if ( nError) goto ErrorExit;

  // @POINTER(4) - Sack size
  nError = LSsetPointerLng( pLINGO, (void*) &dSackSize,
   &nPointersNow);
  if ( nError) goto ErrorExit;

  // @POINTER(5) - Output region for optimal items set
  nError = LSsetPointerLng( pLINGO, (void*) pcItemsSolution,
   &nPointersNow);
  if ( nError) goto ErrorExit;

  // @POINTER(6) - Variable to receive solution status
  nError = LSsetPointerLng( pLINGO, &dStatus, &nPointersNow);
  if ( nError) goto ErrorExit;

  // Here is the script we want LINGO to run:
  //  Load the model, solve the model, exit.
  strcpy( pcScript, "TAKE SACK.LNG \n GO \n QUIT \n");

  // Run the script
  nError = LSexecuteScriptLng( pLINGO, pcScript);
  if ( nError) goto ErrorExit;

  // display solution status
  printf("\nSolution status (should be 0): %d\n", (int)
dStatus);

  // display items in optimal sack
  printf("\nItems in optimal sack:\n%s\n", pcItemsSolution);

  // Close the log file
  LScloseLogFileLng( pLINGO);

  // All done
  goto NormalExit;

ErrorExit:
  printf("LINGO Error Code: %d\n", nError);

NormalExit:
  LSdeleteEnvLng( pLINGO);
FinalExit: ;
}
```

SACK.C

There are a couple of interesting features to note in this code pertaining to the passing of set members. First off, there is the declaration of the original set members:

```
// potential items in knapsack
char pcItems[256] = "ANT_REPEL \n BEER \n BLANKET \n"
 "BRATWURST \n BROWNIES \n FRISBEE \n SALAD \n"
 "WATERMELON";
```

The set members are merely listed as one long string, separated by line feeds (\n). We also added blank spaces for readability, which LINGO strips out when it parses the names. Note, that since we are working in C, there is an implicit null byte at the end of this string due to the use of double quotes. This terminating null is important, because it lets LINGO know where the end of the list occurs.

We pass a pointer to the set to LINGO with the following call to LSsetPointerLng

```
// @POINTER(1) - Items set
nError = LSsetPointerLng( pLINGO, (void*) pcItems,
 &nPointersNow);
if ( nError) goto ErrorExit;
```

We pass a pointer to the set to LINGO with the following call to *LSsetPointerLng*. For receiving the optimal set of items back from LINGO we set aside the following text array: `char pcItemsSolution[256];`

We let LINGO know to store the solution set in this array with the following call to LSLsetPointerLng:

```
// @POINTER(5) - Output region for optimal items set
nError = LSsetPointerLng( pLINGO, (void*) pcItemsSolution,
 &nPointersNow);
```

Recall that this statement in the code pairs with the following statement in the model to establish the link for receiving the optimal set of items:

```
!send optimal items set back to caller;
@POINTER( 5) = ITEMSUSED;
```

If you have Visual C/C++ 6.0 installed on your machine, you should be able to build the application by going to the \LINGO19\Programming Samples\VC++\Knapsack folder and issuing the NMAKE command. Alternatively, you may simply go to the folder and run the sack.exe executable. After running the application, you should see the following:

```
Solution status (should be 0): 0

Items in optimal sack:
ANT_REPEL
BEER
BLANKET
BRATWURST
BROWNIES
FRISBEE
```

From the solution we see that all items except the salad an watermelon are included in the optimal solution.

Callback Functions

In many instances, solving a model can be a lengthy operation. Given this, it may be useful to provide some device to keep the user posted as to the progress of the optimization. The standard interactive version of LINGO displays a status window showing the iteration count, objective value, and various other model statistics each time the solver is invoked. In addition, the user has the ability to interrupt the solver by clicking on a button in the status window. You can provide similar functionality to users through the LINGO DLL by supplying LINGO with a *callback function*—so named because the code calls the LINGO solver, and the LINGO solver then *calls back* to the supplied callback routine.

In the next section, the calling sequences used in establishing a callback function are discussed. After that, there are sample callback routines to the Visual C++ and Visual Basic staff scheduling examples illustrated above.

Specifying a Callback Function

To specify a callback function, the LINGO exported routine *LSsetCallbackSolverLng()* needs to be called before calling LINGO's script processor. The callback function will be called frequently by the LINGO solver.

You will recall from above the calling sequence for *LSsetCallbackSolverLng*:

int LSsetCallbackSolverLng(pLSenvLINGO pL, lngCBFuncError_t pcbf, void* pUserData)

Arguments:

pL Pointer to a LINGO environment created by a previous call to *LScreateEnvLng()*.

pcbf Pointer to your callback routine.

pUserData This is a user specified pointer. You may use it to point to any data you might need to reference from your callback function. LINGO merely passes the value of this pointer through to your callback function. You may set this pointer to *NULL* if it is not required.

Return Value:

Returns 0 if no error occurred. Otherwise, it returns one of the nonzero error codes listed below in the section *LINGO DLL Error Codes*.

The callback function must use the standard call convention and must have the following interface:

int MySolverCallback(pLSenvLINGO pL, int nReserved, void pUserData);*

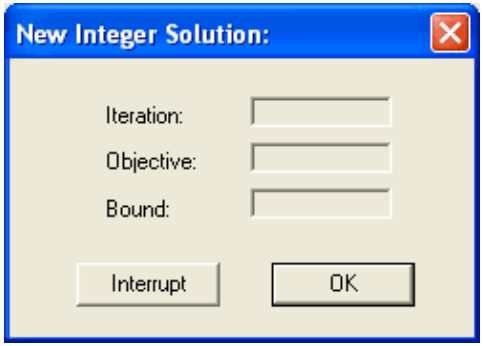
Your computer will most likely crash if this interface specification is not followed *exactly*. The *nReserved* argument is reserved for future use and may be ignored.

Once the callback function has control, information regarding the solver's status can be retrieved from LINGO using the *LSgetCallbackInfoLng()* function. See the section *Functions Exported by the LINGO DLL* above for more information on the *LSgetCallbackInfoLng()* interface.

A Visual C++ Callback Function

We will now extend the Visual C++ staffing example presented above by introducing a callback function. The callback function in this example will post a small dialog box each time the solver finds a better integer solution. The dialog box will display the solver's iteration count, the objective value for the new solution, the bound on the objective, and also include a button to allow the user to interrupt the solver if they desire. You can find the complete project for this sample in the directory *LINGO19\Programming Samples\VC++\STAFF2*.

The first step is to design the dialog box that we will post whenever a new integer solution is found. Here is a look at the dialog box for this example:



The box has three edit fields for the iterations, objective, and bound. There are two buttons—one to interrupt the solver and the other to clear the dialog box.

The next step is to use the ClassWizard to attach a handler class to the new dialog box. This class was named *CNewIPDlg*. When a class has been attached to the dialog box, then the ClassWizard must be used to assign member variables to handle the *Iterations*, *Objective*, and *Bound* edit fields. Once this is done, the header file for the dialog box will resemble:

```
// NewIPDlg.h : header file
//

/////////////////////////////////////////////////////////////////////////////
// CNewIPDlg dialog

class CNewIPDlg : public CDialog
{
// Construction
public:
    CNewIPDlg(CWnd* pParent = NULL);   // standard constructor

// Dialog Data
    //{{AFX_DATA(CNewIPDlg)
    enum { IDD = IDD_NEW_IP_SOLUTION };
    CString   m_csBound;
    CString   m_csIteration;
    CString   m_csObjective;
    //}}AFX_DATA

// Overrides
    // ClassWizard generated virtual function overrides
    //{{AFX_VIRTUAL(CNewIPDlg)
    protected:
    virtual void DoDataExchange(CDataExchange* pDX);
    //}}AFX_VIRTUAL

// Implementation
protected:

// Generated message map functions
    //{{AFX_MSG(CNewIPDlg)
// NOTE: the ClassWizard will add member functions here
    //}}AFX_MSG
    DECLARE_MESSAGE_MAP()
};
```

Callback Dialog Header File (NewIPDlg.h)

Here is the code to handle events from the new dialog box:

```
// NewIPDlg.h : header file
//

/////////////////////////////////////////////////////////////////////////////
// CNewIPDlg dialog

class CNewIPDlg : public CDialog
{
// Construction
public:
    CNewIPDlg(CWnd* pParent = NULL);   // standard constructor

// Dialog Data
    //{{AFX_DATA(CNewIPDlg)
    enum { IDD = IDD_NEW_IP_SOLUTION };
    CString   m_csBound;
    CString   m_csIteration;
    CString   m_csObjective;
    //}}AFX_DATA

// Overrides
    // ClassWizard generated virtual function overrides
    //{{AFX_VIRTUAL(CNewIPDlg) protected:
    virtual void DoDataExchange(CDataExchange* pDX);
    //}}AFX_VIRTUAL

// Implementation
protected:

    // Generated message map functions
    //{{AFX_MSG(CNewIPDlg)
        // NOTE: the ClassWizard will add member functions here
    //}}AFX_MSG
    DECLARE_MESSAGE_MAP()
};
```

Callback Dialog Handler (NewIPDlg.cpp)

The code in these two files was entirely generated by the ClassWizard, requiring no actual user input.

Next, the code for the callback function must be added. Here is a copy of the global routine that was added for this purpose:

```
int __stdcall MyCallback( void* pModel, int nReserved,
 void* pUserData)
{

// Callback function called by the LINGO solver
//
// return value: >= 0 if solver is to continue, else < 0 to interrupt

   // Get current best IP
   int nErr;
   double dBestIP;
   nErr = LSgetCallbackInfoLng( pModel,
    LS_DINFO_MIP_BEST_OBJECTIVE_LNG, &dBestIP);
   if ( nErr) return( 0);

   // Get best IP value published in dialog box
   double* pdBestIPShown = (double*) pUserData;

   // Is current better than incumbent?
   if ( dBestIP < *pdBestIPShown)
   {

      // Yes ... save its value
      *pdBestIPShown = dBestIP;

      // Get iteration count from LINGO

      int nIterations;
      LSgetCallbackInfoLng( pModel, LS_IINFO_ITERATIONS_LNG,
       &nIterations);

      // Get bound on solution
      double dBound;
      LSgetCallbackInfoLng( pModel, LS_DINFO_MIP_BOUND_LNG, &dBound);

      // Create a dialog to show the current solution value
      CNewIPDlg dlgNewIP;

      // Initialize the fields of the dialog
      dlgNewIP.m_csIteration.Format( "%d", nIterations);
      dlgNewIP.m_csObjective.Format( "%d", (int) dBestIP);
      dlgNewIP.m_csBound.Format( "%d", (int) dBound);

      // Post the dialog and check for a user interrupt
      if ( dlgNewIP.DoModal() == IDCANCEL) return( -1);
   }

   return( 0);

}
```

Callback Routine

Of course, this routine has the same interface that was described in the previous section:

int __stdcall MySolverCallback(pLSenvLINGO pL, int nReserved, void pUserData)*

The following code uses the *LSgetCallbackInfo* routine to get the value of the current best integer solution:

```
// Get current best IP
int nErr;
double dBestIP;
nErr = LSgetCallbackInfoLng( pModel,
 LS_DINFO_MIP_BEST_OBJECTIVE_LNG, &dBestIP);
if ( nErr) return( 0);
```

The constant, *LS_DINFO_MIP_BEST_OBJECTIVE*, is defined in the *LINGD15.H* header file.

We then get the value for the best integer solution that we have currently posted in the dialog box with the statement:

```
// Get best IP value published in dialog box
double* pdBestIPShown = (double*)pUserData;
```

Note that this statement references the user data pointer, *pUserData.* This pointer is passed to LINGO when the callback function is established, and it is useful as a means of accessing the data from within the callback function. In this particular case, a single variable is being pointing to. If needed, the pointer could reference a large data structure containing whatever data desired.

Next, test to see if the latest integer solution is better than the incumbent solution with the statement:

```
// Is current better than incumbent?
if ( dBestIP < *pdBestIPShown)
```

If the new solution is better, then we get additional information from LINGO on the iteration count and solution bound. Also, an instance of the callback dialog box to display the latest information in is created with the line:

```
// Create a dialog to show the current solution value
CNewIPDlg dlgNewIP;
```

The new data is then loaded into the fields of the dialog box:

```
// Initialize the fields of the dialog
dlgNewIP.m_csIteration.Format( "%d", nIterations);
dlgNewIP.m_csObjective.Format( "%d", (int) dBestIP);
dlgNewIP.m_csBound.Format( "%d", (int) dBound);
```

As the final step in this callback routine, we display the dialog box, and if the user hits the interrupt button we return a –1, indicating that the solver should stop and return with the best answer found so far:

```
// Post the dialog and check for a user interrupt
if ( dlgNewIP.DoModal() == IDCANCEL) return( -1);
```

At this point, there is one more piece of code to add. We must make a call to the *LSsetCallbackSolverLng()* routine to pass LINGO a pointer to our callback routine. A good place to do this is right after creating the LINGO environment in the *OnSolve()* handler code for our *Solve* button. The changes are listed below in bold type:

```
// create the LINGO environment object
pLSenvLINGO pLINGO;
pLINGO = LScreateEnvLng();
if ( !pLINGO)
{
   AfxMessageBox("Unable to create LINGO Environment");
   return;
}

// Pass LINGO a pointer to our callback function
nError = LSsetCallbackSolverLng( pLINGO, &MyCallback,
 &dBestIPShown);
if ( nError) goto ErrorExit;

// Open LINGO's log file
nError = LSopenLogFileLng( pLINGO, "LINGO.log");
if ( nError) goto ErrorExit;
```

A Visual Basic Callback Function

We will now extend the Visual Basic staffing example by introducing a callback function. The callback function in this example will post a small dialog box each time the solver finds a better integer solution. The dialog box will display the solver's iteration count, the objective value for the new solution, and the bound on the objective. You can find the complete project for this sample in the directory *\LINGO19\Programming Samples\VBasic\STAFF2*.

First, we need to construct a callback function and place it in a separate Module file (.bas file). Here are the contents of our callback function file *Module1.bas*:

```
Public Function MySolverCallback(ByVal pModel As Long, _
ByVal nReserved As Long, ByRef dBestIP As Double) As Long

' Callback function called by the LINGO DLL
' during model solution.
'
' Return value: >= 0 if solver is to continue,
'  else < 0 to interrupt the solver
  Dim nReturnVal As Long
  nReturnVal = 0
' Get current best IP
  Dim dObj As Double
  Dim nError As Long
  nError = LSgetCallbackInfoDoubleLng(pModel, _
   LS_DINFO_MIP_BEST_OBJECTIVE_LNG, dObj)
' Check for any error
  If (nError = LSERR_NO_ERROR_LNG) Then
' Is it better than the best one displayed so far?
     If (dObj < dBestIP) Then
' Yes ... display this solution
' Save the new best objective value
       dBestIP = dObj
' Get the iteration count from LINGO
        Dim nIterations As Long
        nResult = LSgetCallbackInfoLongLng(pModel, _
         LS_IINFO_ITERATIONS_LNG, nIterations)
' Get the objective bound from LINGO
        Dim dBound As Double
        nResult = LSgetCallbackInfoDoubleLng(pModel, _
         LS_DINFO_MIP_BOUND_LNG, dBound)
' Display the information in a dialog box
        Dim nButtonPressed
        Dim cMessage As String
        cMessage = "Objective:" + Str(dBestIP) _
         + Chr(10) + "Bound:" + Str(dBound) _
         + Chr(10) + "Iterations:" + Str(nIterations)
        nButtonPressed = MsgBox(cMessage, vbOKCancel)
        If (nButtonPressed = vbCancel) Then
           nReturnVal = -1
        End If
    End If
  End If
  MySolverCallback = nReturnVal

End Function
```

VB Callback Module (Module1.bas)

Note: A VB callback function must be placed in a Module file (.bas file) . The callback function will not work if it is placed in a Forms file (.frm file). Also, the callback function and the module file must have different names. If the module file has the same name as the callback function, then the VB *AddressOf* operator will not be able to return the address of the callback function.

You will recall from the section *Specifying a Callback Function* that the callback routine must use the following calling sequence:

int __stdcall MySolverCallback(pLSenvLINGO pL, int nReserved, void pUserData)*

An equivalent function definition using VB code is:

```
Public Function MySolverCallback(ByVal pModel As Long, _
ByVal nReserved As Long, ByRef dBestIP As Double) As Long
```

VB uses the standard call (*__stdcall*) convention by default, so we need not specify this explicitly.

We will make use of the user data pointer to pass the value of the best objective displayed so far in the *dBestIP* argument. This variable will hold the objective value of the best integer solution found so far. We compare each new objective value to the best one found so far. If the latest is an improvement over the incumbent, then we display a dialog box summarizing the new solution.

The following code uses the *LSgetCallbackInfo* routine to get the value of the current best integer solution:

```
' Get current best IP
  Dim dObj As Double
  Dim nError As Long
  nError = LSgetCallbackInfoDoubleLng(pModel, _
   LS_DINFO_MIP_BEST_OBJECTIVE_LNG, dObj)
```

In the VB header file for LINGO (*LINGD19.BAS*), we created two aliases for the *LSgetCallbackInfoLng()* function: *LSgetCallbackInfoDoubleLng()* and *LSgetCallbackInfoLongLng()*. These were for retrieving, respectively, *double* and *long* data from LINGO. This is required due to VB not supporting the void data type found in C. We use *LSgetCallbackInfoDoubleLng()* to retrieve the objective value given that it is a double precision quantity.

Next, we check for any errors in retrieving the objective value. If none occurred, we check to see if the latest objective is better than the incumbent:

```
' Check for any error
  If (nError = LSERR_NO_ERROR_LNG) Then

' Is it better than the best one displayed so far?
     If (dObj < dBestIP) Then
```

If the new objective is better than the incumbent, then we save the new objective value, and retrieve the iteration count and objective bound:

```
' Save the new best objective value
        dBestIP = dObj

' Get the iteration count from LINGO
        Dim nIterations As Long
        nResult = LSgetCallbackInfoLongLng(pModel, _
         LS_IINFO_ITERATIONS_LNG, nIterations)

' Get the objective bound from LINGO
        Dim dBound As Double
        nResult = LSgetCallbackInfoDoubleLng(pModel, _
         LS_DINFO_MIP_BOUND_LNG, dBound)
```

We post a summary of the new solution in a dialog box:

```
' Display the information in a dialog box
        Dim nButtonPressed
        Dim cMessage As String
        cMessage = "Objective:" + Str(dBestIP) _
         + Chr(10) + "Bound:" + Str(dBound) _
         + Chr(10) + "Iterations:" + Str(nIterations)
        nButtonPressed = MsgBox(cMessage, vbOKCancel)
```

If the user pressed the *Cancel* button, as opposed to the *OK* button, then we set the return value to –1 before returning, which will cause the LINGO solver to interrupt:

```
    If (nButtonPressed = vbCancel) Then
           nReturnVal = -1
        End If

     End If
  End If
  MySolverCallback = nReturnVal
End Function
```

At this point, there is one more piece of code to add. We must make a call to the *LssetCallbackSolverLng()* routine to pass LINGO a pointer to our callback routine. This is accomplished at the start of the *Solve* button handler routine with the call:

```
' Pass LINGO a pointer to the callback routine
  Dim nError As Long
  Dim dBestObj As Double
  dBestObj = 1E+30
  nError = LSsetCallbackSolverLng(pLINGO, _
    AddressOf MySolverCallback, dBestObj)
```

Note the use of the VB *AddressOf* operator in the call to *LSsetCallbackSolverLng()*. This operator may be used only in function calls to pass the address of routines contained in module files.

Note: The *AddressOf* operator was added to VB starting with release 5.0. Thus, earlier releases of VB won't be able to exploit the callback feature in LINGO. Also, Visual Basic for Applications (VBA), the VB macro capability supplied with Microsoft Office, *does not* support the *AddressOf* operator. Thus, VBA applications calling LINGO will also not be able to establish callback routines.

Summary

The LINGO DLL has a very simple structure. You need only acquaint yourself with a handful of functions in order to access the DLL and add the power of the LINGO solver to your applications.

We've given brief examples on calling the DLL from Visual Basic and Visual C++. Additional examples are provided in the *Programming Samples*. Application developers working in other environments should be able to access the LINGO DLL in a fashion similar to the examples given here.

Finally, keep in mind that any application you build that makes use of the LINGO DLL is protected under the LINGO License Agreement. Thus, you may not distribute such applications without explicit permission from LINDO Systems. If you would like to make arrangements for distributing your application, please feel free to contact LINDO Systems regarding available runtime licensing arrangements.

User Defined Functions

The *@USER* function allows the use of custom functions of your own design in LINGO. In Windows versions of LINGO, you provide a Dynamic Link Library (DLL) that contains your *@USER* function. Most programming languages that support Windows should allow you to build a DLL. For platforms other than Windows, you provide LINGO with a compiled C or FORTRAN subroutine containing your *@USER* function.

From the perspective of a LINGO modeler, an *@USER* function is a function that can take any number of arguments, but must take at least one. It returns a result calculated by the user-written routine.

From the perspective of the programmer writing the custom function, an *@USER* function takes only two input arguments and returns a single result. The two input arguments consist of:

1. an integer specifying the number of arguments encountered in the *@USER* reference in the LINGO model, and
2. a vector containing the values of the arguments in the order in which they were encountered in the *@USER* reference in double precision format (i.e., an 8 byte floating point format).

In other words, although to the LINGO modeler an *@USER* function can appear to take any number of arguments, to the programmer implementing the *@USER* function, only two input arguments are passed.

It is possible to use multiple functions with *@USER* by writing and compiling each function as a separate subroutine and taking an argument to *@USER* as the index number of the subroutine that you want to branch to.

Installing @USER Under Windows

When LINGO for Windows starts up, it searches for the DLL file called *MYUSER.DLL*. LINGO searches for this file in your *startup directory*. The startup directory is the directory where you installed the LINGO program. If LINGO finds *MYUSER.DLL*, it loads the DLL into memory and calls the exported *MYUSER* routine every time a model references an *@USER* function.

On platforms other than Windows, you must link a compiled FORTRAN or C subroutine with the LINGO libraries in order to provide a customized *@USER* function. Refer to the *README* file for your version of LINGO for technical information on how to link your custom routines with LINGO.

In the following section, we illustrate the details of building such a DLL using Microsoft Visual C++.

Visual C++ Example

In this section, we will use Microsoft Visual C/C++ to create a 32-bit DLL that contains an *@USER* function to perform the square root function. This is very easy to do if we make use of the AppWizard in Visual C++ to build the base code for the DLL. Or, you can find the code for this example in the *USER\VC++* subdirectory off of your main LINGO directory. To build the base code, start the Visual C++ Developer Studio and do the following:

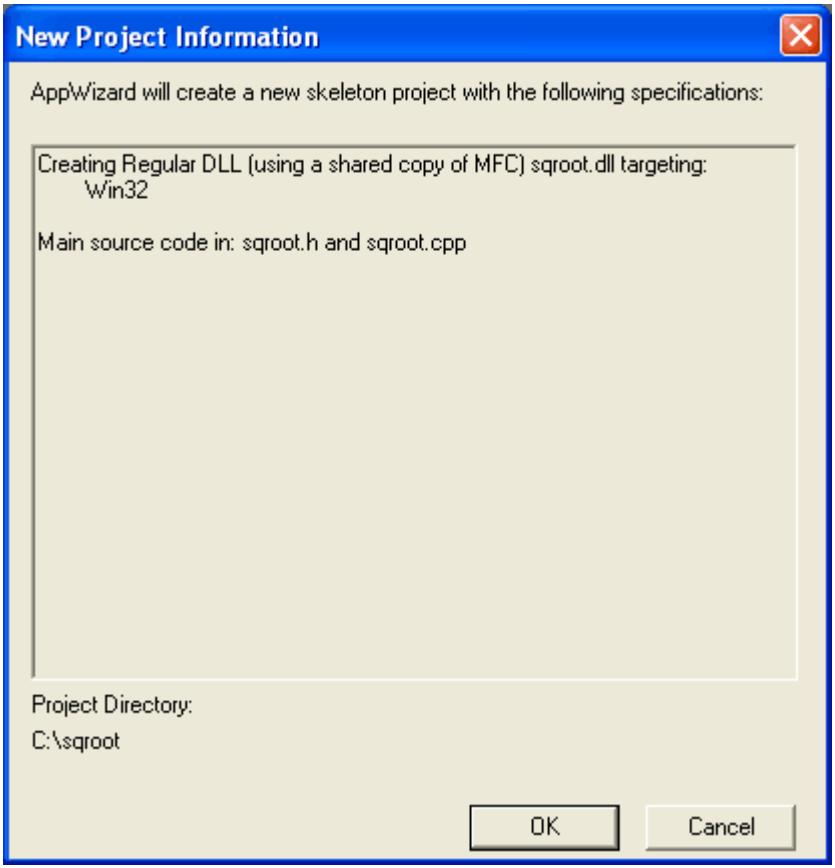
1. Issue the *File|New* command.

2. You should now see a *New* dialog box. Select the *Project Workspace* options and then click *OK*.

3. You will now see a *New Project Workspace* dialog box. Give the project the name *sqroot*. In the Type box, select the *MFC AppWizard* (dll) option. Click on the *Create* button.

4. A new *MFC AppWizard* dialog box should appear. Simply click on the *Finish* button.

5. You should now see a *New Project* Information box containing a summary of the options selected for your project that resembles:



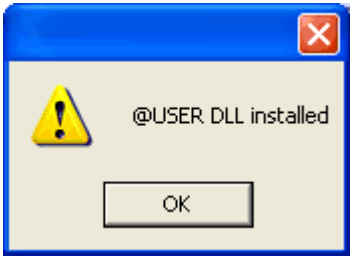
Click the *OK* button to finish creating the base code for our DLL.

Now, edit the *SQROOT.CPP* file and add the modifications listed below in bold:

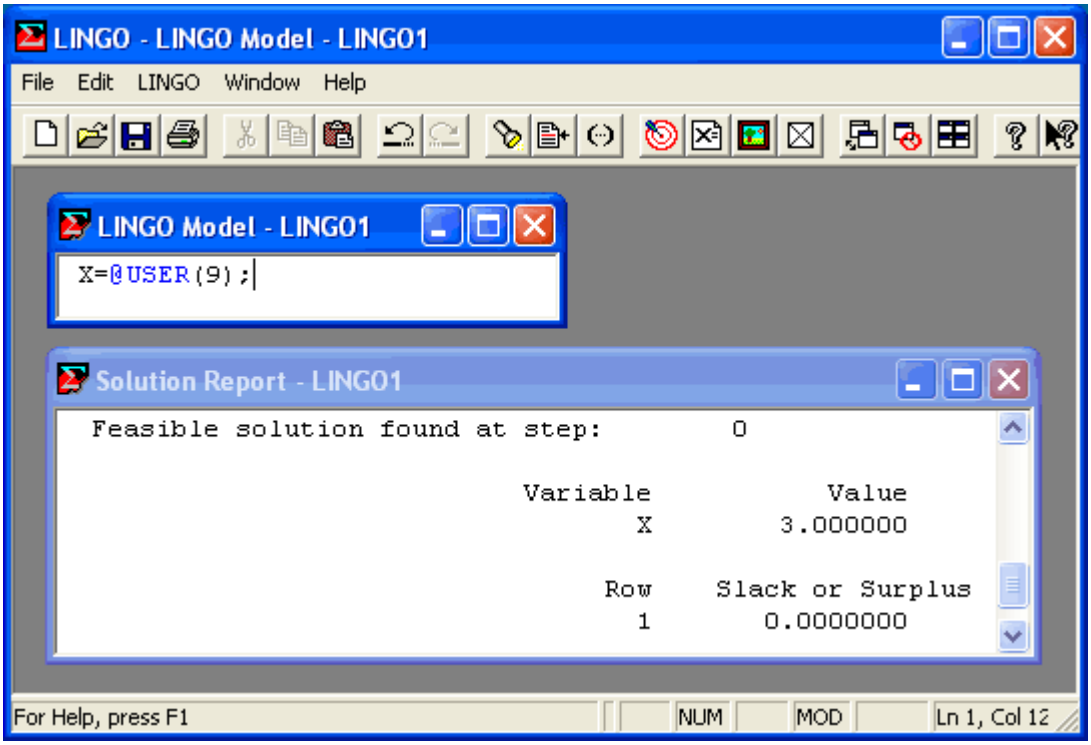
```
// sqroot.cpp : Defines the initialization
// routines for the DLL.
//
#include "stdafx.h"
#include "sqroot.h"
#ifdef _DEBUG
#define new DEBUG_NEW
#undef THIS_FILE
static char THIS_FILE[] = __FILE__;
#endif

/////////////////////////////////////////////////////////////////////////////
    // CSqrootApp
    BEGIN_MESSAGE_MAP(CSqrootApp, CWinApp)

                //{{AFX_MSG_MAP(CSqrootApp)
                // NOTE - the ClassWizard will add and
                // remove mapping macros here.
                // DO NOT EDIT what you see in these
                // blocks of generated code!
                //}}AFX_MSG_MAP
    END_MESSAGE_MAP()
CSqrootApp::CSqrootApp()
{
// The constructor
  // Remove next line for a "quiet" version
  // of MyUser.DLL
  AfxMessageBox("@USER DLL installed");
}
CSqrootApp theApp;
#include <math.h>
extern "C" __declspec( dllexport)
void MyUser( int* pnNumberOfArgs,
 double* pdArgs, double* dResult)

// This is an @USER routine callable by LINGO. In
// this particular case we simply take the
// square root of the first argument.
  *dResult = sqrt( *pdArgs);
}
```

File: SQROOT.CPP

You should now be able to build the DLL. When Visual C++ completes the build, copy the *SQROOT.DLL* file to LINGO's startup directory (the one where *LINGO19.EXE* or *LINGO64_19.EXE* is located), and rename *SQROOT.DLL* to be *MYUSER.DLL*. Now, start LINGO and you should see the following dialog box confirming the DLL was successfully loaded:



Input a small model to compute the square root of 9 and solve it to get the following results:



If you don't have a copy of Visual C++, you may experiment with this *@USER* routine by copying the DLL supplied with LINGO into your LINGO startup directory. You can find the *SQROOT.DLL* file in the *USER\VC++* subdirectory off the main LINGO directory.

12 Developing More Advanced Models

In this chapter, we will walk through the development of models in a handful of different application areas. The goal here is twofold. On the one hand, walking through a number of different models should help hone your skills at building LINGO models. Secondly, by applying LINGO to an array of different application areas, we hope to illustrate the potential benefits of mathematical modeling in almost any business situation. Below is a list of the application areas we will be touching upon in this chapter:

- Production Management
- Logistics
- Finance
- Queuing
- Marketing

The reader interested in additional examples of LINGO modeling should refer to *Optimization Modeling with LINGO*, by Linus Schrage, published by LINDO Systems. This book contains an exhaustive set of modeling examples from a wide array of application areas. In many cases, you should be able to find a model that will suit your needs with only modest modifications. Interested readers may also refer to Appendix A, *Additional Examples of LINGO Modeling*, to find additional samples of LINGO models.

Production Management Models

Blending Model — Model Name: Blend

Background

In blending problems, two or more raw materials are to be blended into one or more finished goods satisfying one or more quality requirements on the finished goods. Blending problems are of interest in a number of different fields. Some examples include determining an optimal blend for animal feed, finding the least costly mix of ores that produce an alloy with specified characteristics, or deciding on the most efficient combination of advertising media purchases to reach target audience percentages.

The Problem in Words

As a production manager at a fuel processing plant, you would like to maximize profit from blending the raw materials butane, catalytic reformate, and naphtha into the two finished products Regular and Premium. These final products must satisfy quality requirements on octane, vapor pressure, and volatility. Each raw material has a known limited availability and cost per unit. You know the minimum required for each finished good, the maximum you will be able to sell, and the profit contribution per unit.

The Model

```
MODEL:
 TITLE   BLEND;
 SETS:
   !Each raw material has an availability
     and cost/unit;
   RAWMAT/ BUTANE, CATREF, NAPHTHA/: AVAIL, COST;

   !Each finished good has a min required,
     max sellable, selling price,
     and batch size to be determined;
   FINGOOD/ REGULAR, PREMIUM/:
    MINREQ, MAXSELL, PRICE, BATCH;

   !Here is the set of quality measures;
   QUALMES/ OCTANE, VAPOR, VOLATILITY/;

   !For each combo of raw material and
     quality measure there is a quality
     level;
   RXQ(RAWMAT, QUALMES): QLEVEL;

   !For each combination of quality
     measure and finished good there are
     upper and lower limits on quality,
     and a slack on upper quality to be
     determined;
   QXF(QUALMES, FINGOOD):
    QUP, QLOW, QSLACK;

   !For each combination of raw material
```

```
    and finished good there is an amount
    of raw material used to be solved for;
  RXF(RAWMAT, FINGOOD): USED;
ENDSETS

DATA:
!Raw material availability;
 AVAIL = 1000, 4000, 5000;

!Raw material costs;
 COST = 7.3, 18.2, 12.5;

!Quality parameters of raw
 materials;
 QLEVEL = 120,   60, 105,
          100,  2.6,   3,
           74,  4.1,  12;

!Limits on finished goods;
 MINREQ  = 4000, 2000;
 MAXSELL = 8000, 6000;

!Finished goods prices;
 PRICE = 18.4,  22;

!Upper and lower limits on
 quality for each finished good;
 QUP = 110, 110,
        11,  11,
        25,  25;
 QLOW = 90,  95,
         8,   8,
        17,  17;
ENDDATA

!Subject to raw material availability;
 @FOR(RAWMAT(R):
  [RMLIM] @SUM(FINGOOD(F): USED(R, F))
   <= AVAIL(R);
 );

 @FOR(FINGOOD(F):

  !Batch size computation;
  [BATCOMP] BATCH(F) =
   @SUM(RAWMAT(R): USED(R, F));

  !Batch size limits;
  @BND(MINREQ, BATCH, MAXSELL);

  !Quality restrictions for each
   quality measure;
  @FOR(QUALMES(Q):

   [QRESUP] @SUM(RAWMAT(R):
     QLEVEL(R, Q) * USED(R, F))
      + QSLACK(Q, F) = QUP(Q, F) *
       BATCH(F);
```

```
      [QRESDN] QSLACK(Q, F) <=
       (QUP(Q, F) - QLOW(Q, F)) *
        BATCH(F);

    );
   );
   ! We want to maximize the profit contribution;
   [OBJECTIVE] MAX =
    @SUM(FINGOOD: PRICE * BATCH) -
     @SUM(RAWMAT(R): COST(R) *
      @SUM(FINGOOD(F): USED(R, F)));
END
```

Model: BLEND

The Sets

We have three primitive sets in this model — raw materials (*RAWMAT*), finished goods (*FINGOOD*), and quality measures (*QUALMES*).

From these three primitive sets, we create three derived sets.

The first derived set, *RXQ,* is a dense derived set and is a cross on the raw materials set and the quality measures set. We need this derived set in order to define an attribute to hold the quality values for the raw materials.

The second derived set, *QXF*, is a cross on the quality measures and the finished goods. We need this set because we will be concerned with the levels of the quality measures in the finished goods.

The final derived set, *RXF*, is a cross on the raw materials and finished goods. We need this set because we need to compute the amount of raw material used in each finished good.

The Variables

The primary variable that drives this model is the amount of each raw material used in each finished good (*USED*). We have also added two other variables for computation purposes. First, there is the *BATCH* variable, which is used to compute the batch size of each finished good. There is also the *QSLACK* variable, which computes the slack on the upper quality limit for each finished good and each quality measure.

The Objective

The objective in this model is to maximize the total profit contribution. This is computed using the following expression:

```
[OBJECTIVE] MAX =
 @SUM(FINGOOD: PRICE * BATCH) -
  @SUM(RAWMAT(R): COST(R) *
   @SUM(FINGOOD(F): USED(R, F)));
```

Total profit, of course, is total revenue minus total expenses. Total revenue is the sum over the finished goods of the price of each finished good multiplied by the batch size of each finished good, or, in LINGO syntax:

```
@SUM(FINGOOD: PRICE * BATCH)
```

Total expenses are computed by taking the sum over the raw materials of the price of the raw material multiplied by the amount of each raw material used. This is computed with the expression:

```
@SUM(RAWMAT(R): COST(R) *
     @SUM(FINGOOD(F): USED(R, F))
```

The Constraints

There are four sets of constraints in this model. Two of them are merely computational.
This first computational set of constraints computes the batch size of each finished good with the following expression:

```
!Batch size computation;
  [BATCOMP] BATCH(F) =
   @SUM(RAWMAT(R): USED(R, F));
```

In words, the batch size of finished good *F* is the sum of all the raw materials used in the production of the finished good.

The second set of computational constraints computes the slack on the upper quality limit for each finished good and each quality measure as follows:

```
[QRESUP] @SUM(RAWMAT(R):
 QLEVEL(R, Q) * USED(R, F))
  + QSLACK(Q, F) = QUP(Q, F) *
   BATCH(F);
```

In words, the actual quality level plus the slack on the upper quality level is equal to the upper quality level.

The first true constraint is on the finished products' batch sizes, which must fall within minimum and maximum levels. We use the *@BND* function to set this up as follows:

```
!Batch size limits;
@BND(MINREQ, BATCH, MAXSELL);
```

Note that we could have entered explicit constraints for the batch size limits, but the *@BND* function is more efficient at handling simple bounds on variables.

The final constraint forces the quality level of each finished good for each quality measure to fall within specifications. We do this with the following:

```
[QRESDN] QSLACK(Q, F) <=
 (QUP(Q, F) - QLOW(Q, F)) *
  BATCH(F);
```

In words, the slack must be less than the difference between the upper limit and the lower limit. If this were not true, quality would be beneath the lower limit. The fact that the slack variable cannot be negative guarantees we won't exceed the quality limit.

The Solution

Rounding all solution values to the nearest whole integer, total profit is $44,905 with batch sizes of 4,000 Regular and 4,095 Premium. The following matrix shows the recommended quantities of each raw material to use in the Regular and Premium blends:

Raw Material	Regular	Premium
Butane	534	466
Catalytic Reformate	1,516	2,484
Naphtha	1,950	1,146

Material Requirements Planning Model: MRP

Background

Material Requirements Planning, or MRP, is used to generate production schedules for the manufacture of complex products. MRP uses the demand schedule for a finished product, the lead times to produce the finished product, and all the various subcomponents that go into the finished product to work backwards and develop a detailed just-in-time production schedule that meets the demand schedule.

MRP's main focus is finding a feasible just-in-time production schedule to meet demand. MRP does not, however, attempt to optimize the production schedule to minimize total production costs. In many cases, the problems solved by MRP are so complex optimization would be prohibitive.

The Problem in Words

Suppose you are a manufacturer of human powered vehicles. You have a given schedule of demands for finished products over time. You need to know when and how many of each component, subcomponent, etc. is needed to meet demand.

Your final products are as follows:

1. Unicycles, made from a seat and a wheel,
2. Bicycles, made from a seat, two wheels, and a chain,
3. Tandems (bicycles built for two), made from two seats, two wheels, and two chains.

Each product is assembled from a set of components. Each component in turn may be assembled from other subcomponents. For simplicity and generality, we will refer to all products, components, and subcomponents as parts.

The subcomponents are as follows:

1. Seats,
2. Wheels, made from a hub and 36 spokes,
3. Chains, made from 84 links,
4. Hubs,
5. Spokes, and
6. Links.

It takes a certain amount of time, called the lead-time, to produce each batch of parts. The component parts for each part must be on hand when you begin production of a part.

The sales department has estimated demand for two months (eight weeks) in the future. According to their findings, there will be a demand for 10 unicycles in week eight, and 20 bicycles and 20 tandems in week nine.

The Model

```
MODEL:
! Data for this model is read from MRP.LDT;
SETS:
! The set of parts;
 PART: LT;
 ! LT(i) = Lead time to produce part i;

! The set of time periods;
 TIME;

! A relationship called USES between pairs of parts;
 USES( PART, PART): NEEDS;
 ! Parent part i needs NEEDS(i, j) units of
   child part j;

! For each part and time period we're interested in;
 PXT( PART, TIME): ED, TD;
 ! ED(i, j) = External demand for part i at time j;
 ! TD(i, j) = Total demand for part i at time j;
ENDSETS

DATA:

! Load the data from an external file;

 ! Parts list;
 PART = @FILE( 'MRP.LDT');

 ! Time periods;
 TIME = @FILE( 'MRP.LDT');

 ! Get the parent child relations and the
   number of parts required;
 USES, NEEDS = @FILE( 'MRP.LDT');

! Get the lead times from the file;
 LT  = @FILE( 'MRP.LDT');

! Get the external demands
  over time for each part;
 ED  = @FILE( 'MRP.LDT');

ENDDATA

! Set NP = no. of time periods in the problem;
 NP = @SIZE( TIME);

! For each part P and period T, the total demand =
  external demand + demand generated by parents
  one lead time in the future;
 @FOR( PXT( P, T) | T + LT( P) #LE# NP :
```

```
  TD( P, T) = ED( P, T + LT( P)) +
   @SUM( USES( P2, P): TD( P2, T + LT( P)) *
    NEEDS( P2, P));
 );

DATA:

! Display a table showing the production schedule;
 @TEXT() = ' The production schedule:';
 @TEXT() = @TABLE( TD);
ENDDATA
END
```

Model: MRP

The Data

An interesting feature of this model is all of the data is imported from an external text file using the *@FILE* function (for more information on *@FILE*, see Chapter 8, *Interfacing with External Files*). More specifically, *@FILE* reads all the data from a single file, *MRP.LDT*, that is pictured below:

```
! Parts list;
   U, ! Unicycles;
   B, ! Bicycles;
   T, ! Tandems;
   S, ! Seats;
   W, ! Wheels;
   C, ! Chains;
   H, ! Hubs;
   P, ! sPokes;
   L~ ! Links;

!  The set of periods;
   1..9 ~

! The parent-child use relationships
  (i.e., Unicycles  use Seats, etc.)
  The number of child parts required
  in each parent- child relationship,
  respectively (i.e, Unicycles use 1
  Seat, Wheels use 36 sPokes, etc.);
U S 1,  U W 1,  B S 1,  B W 2,  B C 1,
T S 2,  T W 2,  T C 2,  W H 1,  W P 36,
C L 84~

! The lead times for each part;
   1, 2, 1, 1, 3, 1, 1, 2, 2~

! The external demands or master schedule;
!             Time period;
!         1  2  3  4  5  6  7  8  9;
   ! U;   0, 0, 0, 0, 0, 0, 0,10, 0,
   ! B;   0, 0, 0, 0, 0, 0, 0, 0,20,
   ! T;   0, 0, 0, 0, 0, 0, 0, 0,20,
```

```
! S;   0, 0, 0, 0, 0, 0, 0, 0, 0,
! W;   0, 0, 0, 0, 0, 0, 0, 0, 0,
! C;   0, 0, 0, 0, 0, 0, 0, 0, 0,
! H;   0, 0, 0, 0, 0, 0, 0, 0, 0,
! P;   0, 0, 0, 0, 0, 0, 0, 0, 0,
! L;   0, 0, 0, 0, 0, 0, 0, 0, 0
```

File: MRP.LDT

You will recall the tilde (~) is LINGO's end-of-record mark. Whenever a tilde is encountered in a data file, LINGO stops reading input from the data file and begins reading where it left off in the model file.

The Sets

We have two primitive sets in this model—the component parts (*PARTS*) and the time periods (*TIME*). From these two primitive sets, we create two derived sets.

The first derived set, *USES,* is a sparse set derived from the cross of the *PARTS* set on itself. We use this set to construct a data table, or input-output matrix (*NEEDS*), containing the parts usage data, which tells us how many of each of the other parts are required to produce a given part. The set is sparse because not all parts are required in the production of certain other parts (e.g., chains are not required to produce a spoke).

The other derived set, *PXT,* is a dense set formed from the cross on the parts and time periods sets. We need this set because we will be concerned with the demand for each part in each period and the amount of each part to begin producing in each period.

The Variables

The only unknown in this model is the total demand (*TD*) attribute, where *TD(p, t)* is the total demand for product *p* in period *t*. Total demand stems from two sources—external demand (*ED*) from customers and internal demand for production requirements. We compute *TD* by incorporating the part's lead time. Thus, when *TD* is nonzero in a period, production must begin in that period.

The Formulas

The key computation in this model is:

```
! For each part P and period T, the total demand =
  external demand + demand generated by parents
  one lead time in the future;
 @FOR(PXT(P, T) | T + LT(P) #LE# NP:
  TD(P, T) = ED(P, T + LT(P)) +
   @SUM(USES(P2, P): TD(P2, T + LT(P)) *
    NEEDS(P2, P));
 );
```

For each part in each time period, the amount of that part we must begin producing in the period is the amount we will need one lead time away to satisfy 1) external demand, and 2) internal production requirements. The subexpression that gives external demand is simply:

```
ED(P, T + LT(P))
```

The remainder of the expression:

```
@SUM(USES(P2, P): TD(P2, T + LT(P)) *
    NEEDS(P2, P));
```

sums up the amount of the part needed for internal production of all other parts one lead time away. Note, we place a logical condition on the outer *@FOR* loop in this calculation (shown here in bold):

```
@FOR(PXT(P, T) | T + LT(P) #LE# NP:
```

Without this condition, the calculation would extend beyond the final period in the *TIME* set.

The Solution

Solving the model, we get the following nonzero values for *TD*:

```
Variable          Value
TD(U, 7)        10.00000
TD(B, 7)        20.00000
TD(T, 8)        20.00000
TD(S, 6)        30.00000
TD(S, 7)        40.00000
TD(W, 4)        50.00000
TD(W, 5)        40.00000
TD(C, 6)        20.00000
TD(C, 7)        40.00000
TD(H, 3)        50.00000
TD(H, 4)        40.00000
TD(P, 2)        1800.000
TD(P, 3)        1440.000
TD(L, 4)        1680.000
TD(L, 5)        3360.000
```

Solution: MRP

Putting this solution in tabular form, we get the following production schedule:

	2	3	4	5	6	7	8
Unicycles						10	
Bicycles						20	
Tandems							20
Seats					30	40	
Wheels			50	40			
Chains					20	40	
Hubs		50	40				
Spokes	1,800	1,440					
Links			1,680	3,360			

MRP Production Schedule

Note the use of the *@TABLE* output function in the data section at the end of the model that displays the *TD* attribute in table form similar to the table above:

```
The production schedule:

      1    2    3    4     5    6    7    8    9
 U    0    0    0    0     0    0   10    0    0
 B    0    0    0    0     0    0   20    0    0
 T    0    0    0    0     0    0    0   20    0
 S    0    0    0    0     0   30   40    0    0
 W    0    0    0   50    40    0    0    0    0
 C    0    0    0    0     0   20   40    0    0
 H    0    0   50   40     0    0    0    0    0
 P    0 1800 1440    0     0    0    0    0    0
 L    0    0    0 1680  3360    0    0    0    0
```

Assembly Line Balancing Model: ASLBAL

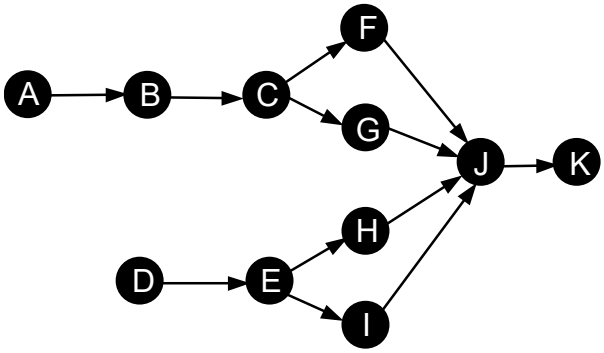
Background

In the assembly line balancing problem, tasks are assigned to workstations on an assembly line, so the line's cycle time is minimized. An assembly line consists of a series of workstations, which each perform one or more specialized tasks in the manufacture of a final product. The cycle time is the maximum time it takes any workstation to complete its assigned tasks. The goal in balancing an assembly line is to assign tasks to stations, so equal amounts of work are performed at each station. Improperly balanced assembly lines will experience bottlenecks—workstations with less work are forced to wait on preceding stations that have more work assigned.

The problem is complicated further by precedence relations amongst the tasks, where some tasks must be completed before others may begin (e.g., when building a computer, installing the disk drives must precede putting on the outer casing). The assignment of tasks to workstations must obey the precedence relations.

The Problem in Words

For our example, we have eleven tasks (A through K) to assign to four stations (1 through 4). The task precedence diagram looks like this:



The times to complete the various tasks are given in the table below:

Task:	A	B	C	D	E	F	G	H	I	J	K
Minutes:	45	11	9	50	15	12	12	12	12	8	9

We need to find an assignment of tasks to workstations that minimize the assembly line's cycle time.

The Model

```
MODEL:
 ! Assembly line balancing model;
   !This model involves assigning tasks to stations in an
     assembly line so bottlenecks are avoided. Ideally, each
     station would be assigned an equal amount of work.;

 SETS:
   ! The set of tasks to be assigned are A through
     K, and each task has a time to complete, T;
   TASK/ A B C D E F G H I J K/: T;

   ! Some predecessor,successor pairings must be
     observed(e.g. A must be done before B, B
     before C, etc.);
   PRED(TASK, TASK)/ A,B  B,C  C,F  C,G  F,J  G,J
     J,K  D,E  E,H  E,I  H,J  I,J /;

   ! There are 4 workstations;
   STATION/1..4/;

   TXS(TASK, STATION): X;
   ! X is the attribute from the derived set TXS
     that represents the assignment. X(I,K) = 1
     if task I is assigned to station K;
 ENDSETS

 DATA:
   ! Data taken from Chase and Aquilano, POM;
   ! There is an estimated time required for each
     task:
        A  B  C  D  E  F  G  H  I  J  K;
   T = 45 11  9 50 15 12 12 12 12  8  9;
 ENDDATA

  ! The model;
  ! *Warning* may be slow for more than 15 tasks;
  ! For each task, there must be one assigned station;
  @FOR(TASK(I): @SUM(STATION(K): X(I, K)) = 1);

  ! Precedence constraints;
  ! For each precedence pair, the predecessor task
    I cannot be assigned to a later station than
    its successor task J;
  @FOR(PRED(I, J):
   @SUM(STATION(K):
    K * X(J, K) - K * X(I, K)) >= 0);
```

```
  ! For each station, the total time for the assigned tasks must
    less than the maximum cycle time, CYCTIME;
  @FOR(STATION(K):
   @SUM(TXS(I, K): T(I) * X(I, K)) <= CYCTIME);

  ! Minimize the maximum cycle time;
  MIN = CYCTIME;
  ! The X(I,J) assignment variables are
    binary integers;
  @FOR(TXS: @BIN(X));

END
```

Model: ASLBAL

The Sets

We have two primitive sets in this model—the tasks (*TASK*) and the workstations (*STATION*). From these two primitive sets, we create two derived sets.

The first derived set, *PRED,* is a sparse derived set and is based on a cross of the *TASK* set on itself. The members of this set are the precedence relations amongst the tasks. For instance, the first member of this set is the pair (A,B), indicating task *A* must precede task *B*.

The other derived set, *TXS,* is a dense derived set formed by taking the cross of the task set on the workstation set. We need this set because we will be determining what tasks get assigned to what workstations.

The Variables

The decision variables in this model are the members of the *X* attribute that is defined in the *TXS* set. *X(t, s)* is a binary variable that is 1 if task *t* is assigned to station *s*, otherwise 0. The *X* attribute is forced to being binary in the expression:

```
! The X(I,J) assignment variables are
    binary integers;
  @FOR(TXS: @BIN(X));
```

We also introduce the scalar variable, *CYCTIME*, to represent the entire assembly line's cycle time, which is computed by taking the maximum cycle time over the workstations.

The Objective

The objective in this model is simply to minimize total cycle time for the line and is given as:

```
! Minimize the maximum cycle time;
MIN = CYCTIME;
```

The Constraints

We have the following three types of constraints in this model:

1. each task must be assigned to one station,
2. precedence relations must be observed amongst the tasks, and
3. the line cycle time variable, *CYCTIME*, must be greater-than-or-equal-to the actual cycle time.

The following expression sums up the assignment variable for each task, and sets the sum to equal 1:

```
! For each task, there must be one assigned station;
  @FOR(TASK(I): @SUM(STATION(K): X(I, K)) = 1);
```

This forces each task to be assigned to a single station.

We use the following expression to enforce the precedence relationships amongst the tasks:

```
! Precedence constraints;
  ! For each precedence pair, the predecessor task
    I cannot be assigned to a later station than its
    successor task J;
  @FOR(PRED(I, J):
   @SUM(STATION(K):
    K * X(J, K) - K * X(I, K)) >= 0);
```

Suppose task *I* is a predecessor to task *J*. If *I* were incorrectly assigned to a workstation later than *J,* the sum of the terms *K * X(I, K)* would exceed the sum of the terms *K * X(J, K)* and the constraint would be violated. Thus, this constraint effectively enforces the predecessor relations.

We restrain the cycle time using the following constraints:

```
! For each station, the total time for the
    assigned tasks must be less than the maximum
    cycle time, CYCTIME;
  @FOR(STATION(K):
   @SUM(TXS(I, K): T(I) * X(I, K)) <= CYCTIME);
```

The quantity:

```
@SUM(TXS(I, K): T(I) * X(I, K))
```

in this constraint computes the cycle time for station *K*. We use the *@FOR* statement to make the *CYCTIME* variable greater-than-or-equal-to the cycle times for all the workstations. If we couple this with the fact that we are minimizing *CYCTIME* in the objective, *CYCTIME* will be "squeezed" into exactly equaling the maximum of the cycle times for the workstations.

By "squeezing" *CYCTIME* to the correct value, we avoid using the *@MAX* function. Had the *@MAX* function been used, LINGO would have had to resort to its nonlinear solver to handle the piecewise linear *@MAX*. Avoiding nonlinear models whenever possible is a critical modeling practice.

The Solution

Solving the model, we get the following nonzero values for the assignment *X* variable:

```
Variable            Value
 X(A, 2)         1.000000
 X(B, 3)         1.000000
 X(C, 4)         1.000000
 X(D, 1)         1.000000
 X(E, 3)         1.000000
 X(F, 4)         1.000000
 X(G, 4)         1.000000
 X(H, 3)         1.000000
 X(I, 3)         1.000000
 X(J, 4)         1.000000
 X(K, 4)         1.000000
```

Solution: ASLBAL

Summarizing this solution, we have:

Workstation	Assigned Tasks	Cycle Time
1	D	50
2	A	45
3	B, E, H, I	50
4	C, F, G, J, K	50

The cycle time for the entire line is 50 minutes—the maximum of the cycle times across all the workstations. We have a well-balanced line in that only workstation 2 has slack time totaling 5 minutes.

Logistics Models

Capacitated Plant Location Model: CAPLOC

Background

The capacitated plant location model is a generalization of the transportation model we introduced in Chapter 1, *Getting Started with LINGO*. The capacitated plant location problem allows greater latitude of decision making in that the points of origin (plant locations) are variable. Manufacturers and wholesale businesses are likely to encounter problems of this sort in matching existing customer demand to product availability and minimizing transportation costs.

The Problem in Words

Your firm has a choice of three locations to operate a manufacturing facility in. Four customers exist with a known demand for your product. Each potential plant location has an associated monthly operating cost, and shipping routes to the demand cities have varying costs. In addition, each potential plant will have a shipping capacity that must not be exceeded. You need to determine what plant(s) to open and how much of a product to send from each open plant to each customer to minimize total shipping costs and fixed plant operating costs.

The Model

```
MODEL:
! Capacitated Plant Location Problem;
 SETS:
    PLANTS: FCOST, CAP, OPEN;
    CUSTOMERS: DEM;
    ARCS( PLANTS, CUSTOMERS)  : COST, VOL;
 ENDSETS

DATA:
  ! The plant, their fixed costs
    and capacity;
    PLANTS, FCOST, CAP =
      P1      91    39
      P2      70    35
      P3      24    31;
  ! Customers and their demands;
    CUSTOMERS, DEM =
      C1      15
      C2      17
      C3      22
      C4      12;
  ! The plant to cust cost/unit
    shipment matrix;
    COST =   6  2  6  7
             4  9  5  3
             8  8  1  5;
 ENDDATA
```

```
! The objective;
   [TTL_COST] MIN = @SUM( ARCS: COST * VOL) +
    @SUM( PLANTS: FCOST * OPEN);

! The demand constraints;
   @FOR( CUSTOMERS( J): [DEMAND]
    @SUM( PLANTS( I): VOL( I, J)) >= DEM( J)
   );

! The supply constraints;
   @FOR( PLANTS( I): [SUPPLY]
    @SUM( CUSTOMERS( J): VOL( I, J)) <=
     CAP( I) * OPEN( I)
   );

! Make OPEN binary(0/1);
   @FOR( PLANTS: @BIN( OPEN));
END
```

Model: CAPLOC

The Sets

We have two primitive sets in this model—the plants (*PLANTS*) and the customers (*CUSTOMERS*).

From these two primitive sets, we create a dense derived set, *ARCS*, which is the cross of the plants and customers sets. We use this set to represent the shipping arcs between the plants and customers.

The Variables

There are two sets of decision variables in this model. The *VOL* attribute, defined on the *ARCS* set, represents the shipment volume from the plants to the customers along each arc. The *OPEN* attribute, defined on the *PLANTS* set, is used to represent the plants that are open. Specifically, *OPEN(p)* is 1 if plant *p* is opened, else it is 0. The members of the *OPEN* attribute are set to being binary using the expression:

```
! Make OPEN binary(0/1);
    @FOR(PLANTS: @BIN(OPEN));
```

The Objective

The objective in this model is to minimize total costs, which is the sum of the shipping costs and fixed plant costs. This is computed using the following expression:

```
! The objective;
    [TTL_COST] MIN = @SUM(ARCS: COST * VOL) +
     @SUM(PLANTS: FCOST * OPEN);
```

The shipping cost component of the objective is computed with:

```
@SUM(ARCS: COST * VOL)
```

while the fixed plant costs component is given by:

```
@SUM(PLANTS: FCOST * OPEN)
```

The Constraints

There are two sets of constraints in the model:

1. each customer must be sent enough product to satisfy demand, and
2. each plant can't supply more than its capacity.

The following expression guarantees each customer receives the quantity of product demanded:

```
! The demand constraints;
   @FOR(CUSTOMERS(J): [DEMAND]
    @SUM(PLANTS(I): VOL(I, J)) >= DEM(J)
   );
```

For each customer, we sum the amount being shipped to that customer and set it to be greater-than-or-equal-to the customer's demand.

To limit shipments from a plant to the plant's capacity, we use:

```
! The supply constraints;
   @FOR(PLANTS(I): [SUPPLY]
    @SUM(CUSTOMERS(J): VOL(I, J)) <=
     CAP(I) * OPEN(I)
   );
```

For each plant, we sum up the amount being shipped from the plant and set this quantity to be less-than-or-equal-to the plant's capacity multiplied by the plant's 0/1 *OPEN* indicator. Note that, in order for the plant to be able to ship any quantity of product, the *OPEN* binary variable will be forced to 1 by these constraints.

The Solution

Solving the model, we get the following solution:

```
Global optimal solution found.
Objective value:                 327.0000
Extended solver steps:                  4
Total solver iterations:               25

               Variable           Value
               OPEN(P1)        1.000000
               OPEN(P3)        1.000000
            VOL(P1, C1)        15.00000
            VOL(P1, C2)        17.00000
            VOL(P1, C4)        3.000000
            VOL(P3, C3)        22.00000
            VOL(P3, C4)        9.000000
```

Total costs are minimized at 327 by opening plants 1 and 3. From plant 1, we ship 15, 17, and 3 units respectively to customers 1, 2, and 4. From plant 3, we ship 22 units to customer 3, and 9 units to customer 4.

Shortest Route Problem Model: DYNAMB

Background

In the shortest route problem, we want to find the shortest distance from point *A* to point *B* in a network.

We will use an approach called dynamic programming (DP) to solve this problem. Dynamic programming involves breaking a large, difficult problem up into a series of smaller, more manageable problems. By solving the series of smaller problems, we are able to construct the solution to the initial large problem. Typically, the most difficult aspect of DP is not the mathematics involved in solving the smaller subproblems, but coming up with a scheme, or recursion, for decomposing the problem.

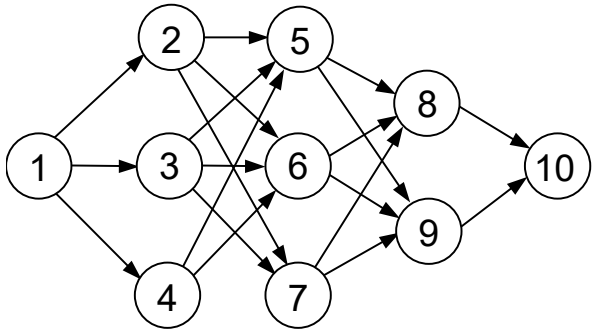
To find the distance of the shortest path through the network, we will use the following DP recursion:

$$F(i) = \min_{j} [\, D(i, j) + F(j)]$$

where *F(i)* is the minimal travel distance from point *i* to the final destination point, and *D(i, j)* is the distance from point *i* to point *j*. In words, the minimal distance from node *i* to the terminal node is the minimum over all points reachable along a single arc from *i* of the sum of the distance from *i* to the adjoining node plus the minimal distance from the adjoining node to the terminal node.

The Problem in Words

Suppose we have the following network of cities:



Links are assumed to be one-way. The distances between the cities are given to us. We want to determine the shortest distance between cities 1 and 10.

The Model

```
SETS:
 ! Dynamic programming illustration (see
   Anderson, Sweeney & Williams, An Intro to Mgt
   Science, 6th Ed.). We have a network of 10
   cities.  We want to find the length of the
   shortest route from city 1 to city 10.;
 ! Here is our primitive set of ten cities,
   where F(i) represents the shortest path
   distance from city i to the last city;
 CITIES /1..10/: F;
 ! The derived set ROADS lists the roads that
   exist between the cities (note: not all city
   pairs are directly linked by a road, and
   roads are assumed to be one way.);
  ROADS(CITIES, CITIES)/
   1,2  1,3  1,4
   2,5  2,6  2,7
   3,5  3,6  3,7
   4,5  4,6
   5,8  5,9
   6,8  6,9
   7,8  7,9
   8,10
   9,10/: D;
 ! D(i, j) is the distance from city i to j;
ENDSETS
DATA:
 ! Here are the distances that correspond to the
   above links;
  D =
     1    5    2
    13   12   11
     6   10    4
    12   14
     3    9
     6    5
     8   10
     5
     2;
ENDDATA
! If you are already in City 10, then the cost
  to travel to City 10 is 0;
 F(@SIZE(CITIES)) = 0;
! The following is the classic dynamic
  programming recursion. In words, the shortest
  distance from City i to City 10 is the minimum
  over all cities j reachable from i of the sum
  of the distance from i to j plus the minimal
  distance from j to City 10;
 @FOR(CITIES(i)| i #LT# @SIZE(CITIES):
  F(i) = @MIN(ROADS(i, j): D(i, j) + F(j))
```

```
);
```

Model: DYNAMB

The Sets

We have the single primitive set, *CITIES*, which corresponds to the cities in the network. From this primitive set, we form a single derived set, *ROADS*, to represent the links between the cities. We specify the members of this set. Thus, *ROADS* is a sparse derived set.

The Variables

The *F* attribute defined on the *CITIES* set is used to store the distance from each city to the destination city.

The Formulas

The recursion, discussed above, is entered in LINGO with the following statement:

```
@FOR(CITIES(i)| i #LT# @SIZE(CITIES):
  F(i) = @MIN(ROADS(i, j): D(i, j) + F(j))
);
```

The Solution

Solving the model, we get the following values for *F*:

Variable	Value
F(1)	19.00000
F(2)	19.00000
F(3)	14.00000
F(4)	20.00000
F(5)	8.000000
F(6)	7.000000
F(7)	12.00000
F(8)	5.000000
F(9)	2.000000
F(10)	0.000000

F(1), the shortest distance from city 1 to city 10, gives us the distance of the shortest path of 19. For the curious reader, this distance corresponds to the path $1 \rightarrow 3 \rightarrow 5 \rightarrow 8 \rightarrow 10$. Refer to model *DYNAMB2.LG4* to see how to extend this model to compute the actual path as well as its distance.

Financial Models

Markowitz Portfolio Selection Model: GENPRT

Background

In the March, 1952 issue of *Journal of Finance*, Harry M. Markowitz published an article titled *Portfolio Selection*. In the article, he demonstrates how to reduce the risk of asset portfolios by selecting assets whose values aren't highly correlated. At the same time, he laid down some basic principles for establishing an advantageous relationship between risk and return. This has come to be known as diversification of assets. In other words, don't put all your eggs in one basket.

A key to understanding the Markowitz model is to be comfortable with the statistic known as the *variance* of a portfolio. Mathematically, the variance of a portfolio is:

$$\Sigma_i \Sigma_j X_i X_j \sigma_{i,j}$$

where,

X_i	is the fraction of the portfolio invested in asset i,
$\sigma_{i,j}$	for $i \neq j$: the covariance of asset i with asset j, and
	for $i=j$: the variance of asset i.

Variance is a measure of the expected fluctuation in return—the higher the variance, the riskier the investment. The covariance is a measure of the correlation of return fluctuations of one stock with the fluctuations of another. High covariance indicates an increase in one stock's return is likely to correspond to an increase in the other. A covariance close to zero means the return rates are relatively independent. A negative covariance means that an increase in one stock's return is likely to correspond to a decrease in the other.

The Markowitz model seeks to minimize a portfolio's variance, while meeting a desired level of overall expected return.

The Problem in Words

You're considering investing in three stocks. From historical data, you have calculated an expected return, the variance of the return rate, and the covariance of the return between the different stocks. You want to reduce variability, or risk, by spreading your investment wisely amongst the three stocks. You have a target growth rate of 12%. As an additional safety feature, you decide to invest no more than 75% in any single asset. What percentages of your funds should you invest in the three stocks to achieve this target and minimize the risk of the portfolio?

The Model

```
! GENPRT: Generic Markowitz portfolio;
 SETS:
  ASSET/1..3/: RATE, UB, X;
  COVMAT(ASSET, ASSET): V;
 ENDSETS

 DATA:
! The data;
!  Expected growth rate of each asset;
  RATE = 1.3   1.2  1.08;
!  Upper bound on investment in each;
  UB   = .75   .75   .75;
!  Covariance matrix;
  V    =   3     1   -.5
           1     2   -.4
         -.5   -.4     1;
!  Desired growth rate of portfolio;
  GROWTH = 1.12;
ENDDATA

! The model;
! Min the variance;
 [VAR] MIN = @SUM(COVMAT(I, J):
              V(I, J) * X(I) * X(J));

! Must be fully invested;
 [FULL] @SUM(ASSET: X) = 1;

! Upper bounds on each;
 @FOR(ASSET: @BND(0, X, UB));

! Desired value or return after 1 period;
 [RET] @SUM(ASSET: RATE * X) >= GROWTH;
```

Model: GENPRT

The Sets

We define a single primitive set, *ASSETS*, corresponding to the three stocks in the model. From the *ASSETS* set, we derive the dense set named *COVMAT*, which is the cross of the *ASSETS* set on itself. We use the *COVMAT* set for defining the covariance matrix.

The Attributes

We define four attributes in this model.

The *RATE*, *UB* and *V* attributes are for storing data. *RATE* stores the expected return for each asset, *UB* stores the upper bound on the fraction of the asset allowed in the portfolio, and *V* stores the covariance matrix. (Note the covariance matrix is symmetric and larger portfolio models would benefit from storing just half of the matrix, rather than the entire matrix as we have done here for simplicity reasons.)

The final attribute, *X*, constitutes the decision variables for the model. Specifically, *X(i)* is the fraction of the portfolio devoted to asset *i*.

The Objective

The objective in this model is to minimize the portfolio's risk. As we mentioned above, we use the portfolio's variance as a measure of risk in the following expression:

```
! Min the variance;
 [VAR] MIN = @SUM(COVMAT(I, J):
              V(I, J) * X(I) * X(J));
```

The Constraints

There are three forms of constraints in the model:

1. we must be fully invested,
2. we can't invest too much in any one asset, and
3. expected return should meet, or exceed, our threshold level of 12%.

The following expression enforces the 100% investment requirement:

```
! Must be fully invested;
 [FULL] @SUM(ASSET: X) = 1;
```

In words, the sum of all the weights of all the assets in the portfolio must equal 1. Without this constraint, LINGO will tend to under invest in order to get a lower variance. You can confirm this by dropping the constraint and running the model.

To keep the solution from investing too much in any one asset, we use the *@BND* function:

```
! Upper bounds on each;
 @FOR(ASSET: @BND(0, X, UB));
```

As you recall, the *@BND* function is the most efficient method for placing simple bounds on variables.

We constrain the portfolio's expected return to be greater-than-or-equal-to our target with the expression:

```
! Desired value or return after 1 period;
 [RET] @SUM(ASSET: RATE * X) >= GROWTH;
```

The left-hand side of this expression is the expected rate of return, and is the sum of the fractions invested in each asset weighted by each asset's return.

The Solution

Solving the model, we get the following solution:

```
Local optimal solution found.
  Objective value:                0.4173749
  Total solver iterations:               13

                Variable           Value
                    X(1)       0.1548631
                    X(2)       0.2502361
                    X(3)       0.5949008

                      Row    Slack or Surplus
                      VAR       0.4173749
                     FULL       0.0000000
                      RET       0.2409821E-01
```

Solution: GENPRT

Total variance is minimized at .4174 when we put 15.5% in asset 1, 25% in asset 2, and 59.5% in asset 3.

Scenario Portfolio Selection Model: PRTSCEN

Background

Scenarios are outcomes of events with an influence on the return of a portfolio. Examples might include an increase in interest rates, war in the Middle East, etc. In the scenario-based approach to portfolio selection, the modeler comes up with a set of scenarios, each with a certain probability of occurring during the period of interest. Given this set of scenarios and their probabilities, the goal is to select a portfolio that minimizes risk, while meeting a target return level.

In the Markowitz portfolio model, presented above, we used a portfolio's variance as a measure of risk. As one might imagine, variance is not the only possible measure of risk. Variance is a measure of the fluctuation of return above and below its average. As a statistic, variance weights a scenario that returns 20% above average the same as a scenario that returns 20% below average. If you're like most investors, you're probably more worried about the risk that return will be *below* average. In our scenario-based model, we will expand our options by including two new measures of risk that focus on returns below the target level—*downside risk* and *semi-variance risk.*

Both semi-variance and downside risk only consider the option that returns will be below the target. Downside risk is a measure of the expected difference between the target and returns below the target, while semi-variance is a measure of the *squared* expected difference between the target and returns below the target. Therefore, semi-variance puts a relatively higher weight on larger shortfalls.

The Problem in Words

Again, you are considering investing in three stocks (ATT, GM, and USX). You have determined there are 12 equally likely scenarios in the forthcoming period. You have come up with predicted rates of return for the stocks under the twelve scenarios. You have a target growth rate of 15% for your portfolio. Your goal is to construct optimal portfolios that minimize expected risk while meeting the expected level of return using three different risk measures—variance, downside, and semi-variance.

The Model

```
! Scenario portfolio model;
SETS:
   SCENE/1..12/: PRB, R, DVU, DVL;
   STOCKS/ ATT,  GMT,  USX/:  X;
   SXI(SCENE, STOCKS): VE;
ENDSETS

DATA:
   TARGET = 1.15;
! Data based on original Markowitz example;
   VE =
    1.300    1.225    1.149
    1.103    1.290    1.260
    1.216    1.216    1.419
    0.954    0.728    0.922
    0.929    1.144    1.169
    1.056    1.107    0.965
    1.038    1.321    1.133
```

```
    1.089    1.305    1.732
    1.090    1.195    1.021
    1.083    1.390    1.131
    1.035    0.928    1.006
    1.176    1.715    1.908;
! All scenarios happen to be equally likely;
   PRB= .08333;
ENDDATA

! Compute expected value of ending position;
   AVG = @SUM(SCENE: PRB * R);

! Target ending value;
   AVG >= TARGET;
   @FOR(SCENE(S):

! Compute value under each scenario;
      R(S) = @SUM(STOCKS(J): VE(S, J) * X(J));

! Measure deviations from average;
      DVU(S) - DVL(S) = R(S) - AVG
   );

! Budget;
   @SUM(STOCKS: X) = 1;

! Our three measures of risk;
   [VARI] VAR = @SUM(SCENE: PRB * (DVU+DVL)^2);
   [SEMI] SEMIVAR = @SUM(SCENE: PRB * (DVL) ^2);
   [DOWN] DNRISK = @SUM(SCENE: PRB * DVL);

! Set objective to VAR, SEMIVAR, or DNRISK;
   [OBJ] MIN = VAR;
```

Model: PRTSCEN

The Sets

We define two primitive sets—*SCENE* and *STOCKS*. The *SCENE* set corresponds to the set of 12 scenarios, while *STOCKS* is the set of three candidate stocks. We form a single dense derived set, *STXSC*, which is the cross of these two sets. We need the *STXSC* set in order to establish the table of returns for each stock under each scenario.

The Attributes

We define four attributes on the scenarios set—*PRB*, *R*, *DVU*, and *DVL*. *PRB* stores the probabilities of the scenarios. *R* is the expected rate of return under each scenario for a given allocation amongst the stocks. *DVU* is the deviation above average return for each scenario for a given stock allocation. Finally, *DVL* is the deviation below average return for each scenario for a given stock allocation.

The *X* attribute is defined on the stocks set. *X* denotes the fraction of the portfolio allocated to each stock. The members of *X* must be determined and constitute the decision variables of the model.

Finally, on the *SXI* set, we define the *VE* attribute, which stores the table containing the returns of each stock under each scenario.

The Objective

Once again, the objective in this model is to minimize the portfolio's risk. The default version of the objective minimizes variance as follows:

```
! Set objective to VAR, SEMIVAR, or DNRISK;
   [OBJ] MIN = VAR;
```

To solve using the other two measures of risk, simply change the name of the variable from *VAR* to either *SEMIVAR* or *DNRISK*.

The Formulas

There are six categories of formulas in this model:

1. computing the expected rate of return (*AVG*),
2. constraining the expected rate of return to exceed our target,
3. computing the expected rate of return under each scenario (*R*),
4. computing deviations from average return for each scenario (*DVU* and *DVL*),
5. constraining the portfolio to be fully invested, and
6. computing the three measures of risk (*VAR*, *SEMIVAR*, and *DNRISK*).

The following expression computes the expected rate of return for the entire portfolio:

```
! Compute expected value of ending position;
   AVG = @SUM(SCENE: PRB * R);
```

We do this by taking the sum of the expected returns for each scenario (*R*) weighted by the probabilities of the scenarios (*PRB*).

We constrain expected return to be greater-than-or-equal-to our target using the constraint:

```
! Target ending value;
   AVG >= TARGET;
```

The expected return under scenario *S* is computed using:

```
R(S) = @SUM(STOCKS(J): VE(S, J) * X(J));
```

This is just the sum of the return of each stock under the scenario weighted by the fraction of the portfolio in each stock.

The deviations from average return for the scenarios are computed with:

```
DVU(S) - DVL(S) = R(S) - AVG
```

If the expected return from the scenario is greater than the average, *DVU* will be positive. If the expected return is less than average, *DVL* will be positive. Given the fact that *DVL* and *DVU* impact the objective, there will never be an optimal solution where both *DVL* and *DVU* are greater than 0. If both are greater than 0, then a better solution can always be obtained by driving one of the two to zero. By setting things up this way, we have partitioned the deviations into two parts. *DVU* represents deviations above the average, which only the variance measure is concerned with. On the other hand, *DVL* represents deviations below average, which both the downside risk and semi-variance measures are computed from.

The following constraint forces us to be fully invested and should be familiar from the discussion of the Markowitz model:

```
! Budget;
   @SUM(STOCKS: X) = 1;
```

We compute our three measures of risk using:

```
! Our three measures of risk;
   [VARI] VAR = @SUM(SCENE: PRB * (DVU + DVL)^2);
   [SEMI] SEMIVAR = @SUM(SCENE: PRB * (DVL) ^2);
   [DOWN] DNRISK = @SUM(SCENE: PRB * DVL);
```

These are simply the sum of a measure of deviation across the scenarios weighted by the probabilities of the scenarios. The variance measure takes the square of both the above and below average deviations. The semi-variance measure squares just the below average deviation. While, the downside risk measure uses the below average deviations without squaring.

The Solution

If we solve the model using the three risk measures in the objective, we get the following three solutions:

	Variance	Semi-variance	Downside Risk
ATT	.530	.575	.511
GMT	.357	.039	.489
USX	.113	.386	.000

The fraction of the portfolio devoted to *ATT* is fairly consistent. However, the fractions of *GMT* and *USX* can vary significantly when the risk measure changes.

Options Pricing Model: OPTION

Background

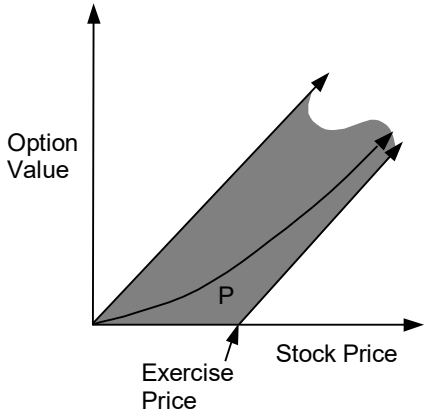
A *call option* is a financial instrument that gives the holder the right to buy one share of a stock at a given price (the exercise price) on or before some specified expiration date. A frequent question is, "How much should one be willing to pay for such an option?" We can easily come up with some broad bounds on what we would be willing to pay.

Suppose the stock price is $105 and we have a chance to buy an option with an exercise price of $100. We should certainly be willing to pay up to $5 for the option, because, as a minimum, we could buy the stock for $100 and immediately sell it for $105, realizing a profit of $5. Thus, when the stock price exceeds the exercise price, we should be willing to pay at least the difference in the two prices.

When the exercise price exceeds the stock price, buying at the exercise price and selling at the market price would not be an intelligent option. In this instance, we should be willing to "pay" at least $0.

In any case, the most we would ever be willing to pay for an option is the current price of the underlying stock. Suppose the option price was more than the stock price. Then, we could get the same expected return at a lower price by buying the stock.

The following graph illustrates these bounds on an option's value as a function of the price of the underlying stock:



Unfortunately, these bounds are not very tight. In reality, the option value function will resemble the curve *P* in the graph. The exact shape of this curve is influenced by three additional factors. These are, 1) the time to expiration, 2) the volatility, or variance, in the price movements of the stock, and 3) the interest rate. The underlying formula for curve *P* eluded researchers for many years until Fischer Black and Myron Scholes derived it in 1973. A Nobel Prize was subsequently awarded for their work in 1997.

The Problem in Words

You believe that the stock of National Semiconductor (symbol: NSM) is about to go up. To capitalize on this belief, you would like to purchase call options on NSM. Checking the quotes on NSM options with your online brokerage service, you find that NSM call options are trading at 6 5/8 per share. These call options have an exercise price of $40, and expire in 133 days from today. The current price of NSM shares is 40 3/4. Are the options trading at a fair price?

The Model

```
! Computing the value of an option using the Black
  & Scholes formula (see "The Pricing of Options
  and Corporate Liabilities", Journal of Political
  Economy, May-June, 1973);
SETS:
! We have 27 weeks of prices P(t), LOGP(t) is log
  of prices;
   WEEK/1..27/: P, LOGP;
ENDSETS

DATA:
! Weekly prices of National Semiconductor;
   P = 26.375, 27.125, 28.875, 29.625, 32.250,
       35.000, 36.000, 38.625, 38.250, 40.250,
       36.250, 41.500, 38.250, 41.125, 42.250,
       41.500, 39.250, 37.500, 37.750, 42.000,
       44.000, 49.750, 42.750, 42.000, 38.625,
       41.000, 40.750;

! The current share price;
   S = 40.75;

! Time until expiration of the option, expressed
  in years;
   T = .3644;

! The exercise price at expiration;
   K = 40;

! The yearly interest rate;
   I = .163;
ENDDATA

SETS:
! We will have one less week of differences;
   WEEK1(WEEK)| &1 #LT# @SIZE(WEEK): LDIF;
ENDSETS

! Take log of each week's price;
   @FOR(WEEK: LOGP = @LOG(P));

! and the differences in the logs;
   @FOR(WEEK1(J): LDIF(J) =
    LOGP(J + 1) - LOGP(J));

! Compute the mean of the differences;
   MEAN = @SUM(WEEK1: LDIF)/ @SIZE(WEEK1);
```

```
! and the variance;
   WVAR = @SUM(WEEK1: (LDIF - MEAN)^2)/
    (@SIZE(WEEK1) - 1);

! Get the yearly variance and standard deviation;
   YVAR = 52 * WVAR;
   YSD = YVAR^.5;

! The Black & Scholes option pricing formula;
   Z = ((I + YVAR/2) *
    T + @LOG(S/ K))/(YSD * T^.5);

! where VALUE is the expected value of the option;
   VALUE = S *@PSN(Z) - K *@EXP(- I * T) *
    @PSN(Z - YSD *T^.5);

! LDIF may take on negative values;
   @FOR(WEEK1: @FREE(LDIF));

! The price quoted in the Wall Street Journal for
  this option when there were 133 days left was $6.625;
```

Model: OPTION

The Sets and Attributes

In this model, there is a single primitive set, *WEEK*, which corresponds to the 27 weeks of price data. We define the two attributes *P* and *LOGP* on this set. *P* stores the raw price data on NSM, while *LOGP* stores the logarithm of the prices.

The Formulas

It is beyond the scope of this document to get into the theoretical details behind the Black & Scholes pricing model. The interested reader should refer to Black and Scholes (1973) for the details. However, we will briefly discuss some of the mechanics of the key formulas involved in the model.

In our model, we compute the option's value in two key steps. First, we compute *Z* as follows:

Z = ((*I* + *YVAR/2*) * *T* + @*LOG*(*S*/ *K*))/(*YSD* * *T*^.5);

where,

I	= the yearly interest rate,
YVAR	= variance of the stock's price,
T	= time until expiration in years,
S	= current share price,
K	= exercise price, and
YSD	= standard deviation on stock's price.

We then use the value of *Z* in the following formula to determine the expected value of the option:

VALUE = *S* *@*PSN*(*Z*)-*K* *@*EXP*(- *I* * *T*) * @*PSN*(*Z*-*YSD* **T*^.5);

where,

@*PSN(Z)* = returns the cumulative standard normal probability, and
@*EXP(X)* = returns e^x.

The Solution

Solving the model, LINGO comes up with a value of $6.58 for the call option—not too far from the quoted price of $6.63.

Bond Portfolio Optimization Model: PBOND

Background

In certain situations, a business or individual may be faced with financial obligations over a future number of periods. In order to defease (i.e., eliminate) this future debt, the debtor can determine a minimal cost mix of current assets (e.g., cash and bonds) that can be used to cover the future stream of payments. This problem is sometimes referred to as the *cash flow matching* problem or the *debt defeasance* problem.

The Problem in Words

You are the head of your state's lottery office. Lottery prizes are not paid out immediately, but are parceled out over a 15 year period. You know exactly how much your office needs to pay out in prizes over the next 15 years. You would like to set aside enough money from lottery receipts to invest in secure government bonds to meet this future stream of payments. All remaining lottery receipts will be turned over to the state's treasurer to help fund the education system. You would like to turn over as many of the receipts as possible to the treasurer, so your plan is to purchase a minimal cost mix of bonds that just meets your future cash needs.

Here is the amount of cash you will need (in millions) to make future prize payments:

Year	0	1	2	3	4	5	6	7	8	9	10	11	12
Needs	$10	$11	$12	$14	$15	$17	$19	$20	$22	$24	$26	$29	$31

There are two bonds currently being offered that you feel are of sufficient quality to guarantee the future stream of prize payments. These bonds are listed below:

Bond	Years to Maturity	Price ($M)	Coupon ($M)
A	6	.98	.06
B	13	.965	.065

If funds are not invested in bonds, they can be placed into short-term money market funds. You conservatively estimate that short-term rates will be about 4% over the 15 year time horizon.

How many of each bond should you buy, and how much additional cash should you allocate to money market funds to minimize your total outlay while still being able to meet all the future prize payments?

The Model

```
! Bond portfolio/cash matching problem. Given cash
  needs in a series of future periods, what
  collection of bonds should we buy to meet these
  needs?;
SETS:
   BOND/A B/ :
      MATAT,   ! Maturity period;
      PRICE,   ! Price;
      CAMNT,   ! Coupon;
      BUY;     ! Amount to buy;
   PERIOD/1..15/:
      NEED,    !Cash needed each period;
      SINVEST; !Short term investment each period;
ENDSETS

DATA:
   STRTE = .04;         !Short term interest rate;
   MATAT = 6, 13;       !Years to maturity;
   PRICE = .980, .965;  !Bond purchase prices;
   CAMNT = .060, .065;  !Bond coupon amounts;
   NEED = 10, 11, 12, 14, 15, 17, 19, 20, 22, 24,
          26, 29, 31, 33, 36; ! Cash needs;
ENDDATA
!Minimize total investment required to generate
  the stream of future cash needs;
   MIN = LUMP;

! First period is slightly special;
   LUMP = NEED(1) + SINVEST(1) +
    @SUM(BOND: PRICE * BUY);

! For subsequent periods;
   @FOR(PERIOD(I)| I #GT# 1:
    @SUM(BOND(J)| MATAT(J) #GE# I:
     CAMNT(J) * BUY(J)) +
    @SUM(BOND(J)| MATAT(J) #EQ# I:
     BUY(J)) +
    (1 + STRTE) * SINVEST(I - 1) =
    NEED(I) + SINVEST(I);
   );

! Can only buy integer bonds;
   @FOR(BOND(J): @GIN(BUY(J)));
```

Model: PBOND

The Sets

We define two primitive sets—*BOND* and *PERIOD*. The *BOND* set corresponds to the two bonds, while *PERIOD* is the set of 15 years in the time horizon.

The Attributes

We define four attributes on the *BONDS* set—*MATAT, PRICE, CAMNT,* and *BUY*. *MATAT* stores the bonds' maturities, *PRICE* the price, *CAMNT* the coupon amount, and *BUY* the number of each bond to buy.

The *NEED* and *SINVEST* attributes are defined on the *PERIOD* set. *NEED* stores the cash needs in each period and *SINVEST* stores the amount in short-term investments in each period.

The Objective

The objective of this model is to minimize the initial cash outlay. The initial cash outlay, or lump sum, is stored in the *LUMP* variable. Thus, our objective is written simply as:

```
! Minimize the total investment required to generate
  the stream of future cash needs;
   MIN = LUMP;
```

The Formulas

There are three categories of formulas in this model:

1. computing the initial lump sum payment (*LUMP*),
2. *sources=uses* constraints, which enforce the condition that all sources of cash in a period must equal uses for cash in a period, and
3. integer restrictions on the *BUY* variable limiting us to buying only whole numbers of bonds.

The following expression computes the lump outflow of cash in the initial period:

```
LUMP = NEED(1) + SINVEST(1) +
    @SUM(BOND: PRICE * BUY);
```

Cash is needed for three purposes in the initial period:

1. payment of lottery prizes (*NEED(1)*),
2. allocations to short-term money funds (*SINVEST(1)*), and
3. bond purchases (*@SUM*(*BOND*: *PRICE* * *BUY*)).

In the remaining 14 periods, sources of cash must equal uses of cash. We enforce this condition with:

```
! For subsequent periods;
   @FOR(PERIOD(I)| I #GT# 1:
    @SUM(BOND(J)| MATAT(J) #GE# I:
     CAMNT(J) * BUY(J)) +
    @SUM(BOND(J)| MATAT(J) #EQ# I:
     BUY(J)) +
    (1 + STRTE) * SINVEST(I - 1) =
    NEED(I) + SINVEST(I);
   );
```

The sources of cash in a period are threefold:

1. coupon receipts:

```
@SUM(BOND(J)| MATAT(J) #GE# I:
     CAMNT(J) * BUY(J))
```

2. maturing bonds:

```
@SUM(BOND(J)| MATAT(J) #EQ# I:
     BUY(J))
```

3. maturing short-term investments from the previous period:

```
(1 + STRTE) * SINVEST(I - 1)
```

These sources must equal the following two uses:

1. lottery prize payments:

```
NEED(I)
```

2. new short-term investments:

```
SINVEST(I)
```

Finally, to force bond purchases to be whole numbers, we add:

```
! Can only buy integer bonds;
   @FOR(BOND(J): @GIN(BUY(J)));
```

The Solution

If we solve the model, we get the following values:

```
Global optimal solution found.
 Objective value:                 195.7265
 Extended solver steps:                  4
 Total solver iterations:               27

               Variable           Value
                   LUMP        195.7265
                 BUY(A)        96.00000
                 BUY(B)        90.00000
             SINVEST(1)        4.796526
             SINVEST(2)        5.598387
             SINVEST(3)        5.432322
             SINVEST(4)        3.259615
             SINVEST(5)        0.000000
             SINVEST(6)        90.61000
             SINVEST(7)        81.08440
             SINVEST(8)        70.17778
             SINVEST(9)        56.83489
            SINVEST(10)        40.95828
            SINVEST(11)        22.44661
            SINVEST(12)       0.1944784
            SINVEST(13)        65.05226
            SINVEST(14)        34.65435
            SINVEST(15)       0.4052172E-01
```

Solution: PBOND

You have been able to cover a total future debt of $319 million with a lump payment of $195.73 million. To do this, you buy 96 *A* bonds, 90 *B* bonds, put $4.8 million into short-term investment funds, and hold $10 million in cash to pay prizes in the initial period.

Queuing Models

Erlang Queuing Models — Model: EZQUEUE

Background

The telephone, communications, and computer industries have long used queuing models to estimate the performance of a service system in the face of random demand. The two most frequently used models are the *Erlang loss* and *Erlang waiting* models. In both cases, customers arrive randomly at a number of identical servers. In the Erlang loss model, there is no queue, so any customer finding all servers busy is lost. In the Erlang waiting model, there is an infinite queue space, so any customer finding all servers busy waits until a server is free. In either case, the major measure of performance is the fraction of customers that find all servers busy.

To compute a system's performance, we must know the load placed on the system per unit of time and the number of servers. The load is a unitless measure of the amount of work arriving per unit of time. For example, if 20 customers arrive each hour and each requires ½ hour of work, then the arriving load is 10 (20 customers per hour multiplied by ½ hour per customer).

The most crucial probabilistic assumption in both cases is the number of arrivals per unit of time is Poisson distributed with a constant mean. The held case (with a queue) further requires that service times be exponentially distributed. If the arriving load is denoted *AL* and the number of servers by *NS*, then the expected fraction finding all servers busy is given in the loss case by *@PEL(AL, NS)* and in the held case by *@PEB(AL, NS)*.

The Problem in Words

You manage the customer service department of your business. Calls arrive at the rate of 25 customers per hour. Each call requires an average of 6 minutes to process. How many service representatives would be required for no more than 5% of customers to receive a busy signal?

The Model

```
! Arrival rate of customers/ hour;
   AR = 25;
! Service time per customer in minutes;
   STM = 6;
! Service time per customer in hours;
   STH = STM/ 60;
! Fraction customers finding all servers busy;
   FB = .05;
! The PEL function finds number of servers
  needed, NS;
   FB = @PEL(AR * STH, NS);
```

Model: EZQUEUE

The Solution

```
Feasible solution found at step:        0

             Variable           Value
                   AR        25.00000
                  STM        6.000000
                  STH       0.1000000
                   FB       0.5000000E-01
                   NS        5.475485
```

Because we cannot have fractional servers, we need at least six servers to meet our requirement that no more than 5% of customers find all servers busy.

Suppose you install a sufficient number of incoming lines, so customers finding all servers busy can wait. Further, you will still use six servers. You want to find the following:

- the fraction of customers finding all servers busy,
- the average waiting time for customers who wait,
- the average overall waiting time, and
- the average number waiting.

The following variation on the previous model computes these four statistics:

```
! Arrival rate of customers/ hour;
   AR = 25;
! Service time per customer in minutes;
   STM = 6;
! Service time per customer in hours;
   STH = STM/ 60;
! The number of servers;
   NS = 6;
! The PEL function finds number of servers
  needed, NS;
   FB = @PEB(AR * STH, NS);
! The conditional wait time for those who wait;
   WAITC = 1 / (NS / STH - AR);
! The unconditional wait time;
   WAITU = FB * WAITC;
! The average number waiting;
   NWAIT = AR * WAITU;
```

Note how we now use the *@PEB* function, rather than *@PEL*, to account for the presence of a queue to hold callers finding all lines busy. The solution to the modified model is:

Variable	Value
AR	25.00000
STM	6.000000
STH	0.1000000
NS	6.000000
FB	0.4744481E-01
WAITC	0.2857143E-01
WAITU	0.1355566E-02
NWAIT	0.3388915E-01

Remember the unit of time is an hour, so the expected waiting time for those who wait is .2857 * 60 = 1.7 minutes.

Machine Repairman Models Model: EZMREPAR

Background

Another queuing model illustrates service demand from a finite population of users. The underlying assumption is, if a significant fraction of these users are already waiting for service, then the arrival rate of further service demands decreases until more of the users are serviced and returned to the calling community. Models of this class are referred to as *Machine Repairman* models because the calling population can be viewed as a set of machines where individual machines occasionally break down and need repair.

The *@PFS* ((Poisson Finite Source) function computes the expected number of customers either in repair or waiting for repair, given the number of customers, number of repairmen, and the limiting load of the queue. The following model illustrates the use of the *@PFS* function to model a computer timesharing system. Each incoming port can be thought of as a customer. Usually, the number of incoming ports is limited. Thus, the finite source assumption is appropriate.

The Problem in Words

You manage the accounting department in your company. You are contemplating installing a new server to handle the growing computing needs of your department. However, you have your doubts as to whether or not the system proposed by the computing services department will be adequate to meet your current and future needs. Your department has 32 employees who will make continuous use of the server throughout the day. You do some research and determine the mean time between server requests is 40 seconds per user. On average, it will take the proposed server 2 seconds to process a user request. Will the proposed server provide the horsepower needed by your department?

The Model

```
! Model of a computer timesharing system;
! The mean think time for each user (more
  generally, Mean Time Between Failures in a
  repair system);
  MTBF = 40;
! The mean time to process each compute request
  (more generally, Mean Time To Repair in
  seconds);
  MTTR =  2;
! The number of users;
  NUSER = 32;
! The number of servers/repairmen;
  NREPR = 1;
! The mean number of users waiting or in service
  (more generally, the mean number of machines
  down);
  NDOWN =
   @PFS(MTTR * NUSER/ MTBF, NREPR, NUSER);
! The overall request for service rate (more
  generally, overall failure rate), FR, must
  satisfy;
  FR = (NUSER - NDOWN)/ MTBF;
! The mean time waiting for or in service (more
  generally, the mean time down), MTD, must
  satisfy;
  NDOWN = FR * MTD;
```

Model: EZMREPAR

The Solution

Variable	Value
MTBF	40.00000
MTTR	2.000000
NUSER	32.00000
NREPR	1.000000
NDOWN	12.06761
FR	0.4983098
MTD	24.21707

Solution: EZMREPAR

This would probably be considered a heavily loaded system—on average, about 12 users are waiting for a response from the server. Each request for processing requires an expected elapsed time of over 24 seconds, even though the average request is for only two seconds of processing. As a group, users request service at the rate of almost one request every 2 seconds.

Steady State Queuing Model Model: QUEUEM

Background

A useful approach for tackling general queuing models is to use the *Rate In = Rate Out Principle* (RIRO) to derive a set of steady state equations for a queuing system. RIRO assumes a system can reach a state of equilibrium. In equilibrium, the tendency to move out of a certain state must equal the tendency to move towards that state. Given the steady state equations derived from this assumption, we can solve for the probability that a system is in a given state at any particular moment.

The following example assumes a system with multiple servers and "customers" that arrive in batches.

The Problem in Words

You operate a motor rebuilding business that services auto repair shops throughout a several state area. Motors arrive on trucks in batches of up to four motors per truck. On average, trucks arrive 1.5 times a day. The probabilities that a truck contains 1, 2, 3 or 4 motors are, respectively, .1, .2, .3 and .4. You presently have five workstations that can each rebuild two motors per day.

You would like to introduce a priority service that, in exchange for a higher fee, guarantees your customers a one day turnaround. To do this effectively, at least one workstation must be free 90 percent of the time in order to begin immediate repair of any incoming priority job. Are your current facilities adequate for this level of service?

The Model

```
! Model of a queue with arrivals in batches.  In
  this particular example, arrivals may show up in
  batches of 1, 2, 3, or 4 units;

SETS:
! Look at enough states so P(i) for large i is
  effectively zero, where P(i) is the steady state
  probability of i-1 customers in the system;
   STATE/ 1..41/: P;

! Potential batch sizes are 1, 2, 3 or 4 ,
  customers and A(i) = the probability that an
  arriving batch contains i customers;
   BSIZE/ 1..4/: A;
ENDSETS

DATA:
   ! Batch size distribution;
   A    = .1, .2, .3, .4;
   ! Number of batches arriving per day;
   LMDA = 1.5;
   ! Number of servers;
   S    = 5;
   ! Number of customers a server can
     process per day;
   MU   = 2;
ENDDATA
```

```
! LAST = number of STATES;
   LAST = @SIZE(STATE);

! Balance equations for states where the number of
  customers in the system is less than or equal to
  the number of servers;
   @FOR(STATE(N)| N #LE# S:
    P(N) * ((N - 1)* MU + LMDA) =
     P(N + 1) * MU * N +
      LMDA * @SUM(BSIZE(I)| I #LT# N: A(I)
       * P(N - I))
   );

! Balance equations for states where number in
  system is greater than the number of servers,
  but less than the limit;
   @FOR(STATE(N)| N #GT# S #AND# N #LT# LAST:
    P(N) * (S * MU + LMDA) =
     P(N + 1) * MU * S +
      LMDA * @SUM(BSIZE(I)| I #LT# N: A(I) *
       P(N - I))
   );

! Probabilities must sum to 1;
   @SUM(STATE: P) = 1;
```

Model: QUEUEM

The Formulas

The model computes the probabilities *P(i)*, *i* = 1 to 41, where *P(i)* is the probability there are *i* - 1 motors in the system for repair. We have chosen to stop at 41 because the probability the system would have more than 40 machines waiting for repair is effectively 0.

In order to solve for the 41 unknown *P(i)*, we will need 41 equations. One of these equations comes from the fact that the probabilities must all sum to 1:

```
! Probabilities must sum to 1;
   @SUM(STATE: P) = 1;
```

The remaining 40 equations are derived from the steady state assumptions that the rate of movement into any state must equal the rate out. We partition these balance equations into two types. In the first case, there are the states where the number of motors in the system is less-than-or-equal-to the number of servers. The balance equations in this case are:

```
@FOR(STATE(N)| N #LE# S:
 P(N) * ((N - 1)* MU + LMDA) =
  P(N + 1) * MU * N +
   LMDA * @SUM(BSIZE(I)| I #LT# N: A(I)
    * P(N - I))
);
```

For each state where the number of motors in the system is less-than-or-equal-to the number of servers, we set the rate of flow out of the state:

```
P(N) * ((N - 1)* MU + LMDA)
```

equal to the rate of flow into the state:

```
P(N + 1) * MU * N +
 LMDA * @SUM(BSIZE(I)| I #LT# N: A(I)
  * P(N - I))
```

The rate of flow out is the rate of flow to states with more motors, *LMDA*, plus the rate of flow to lesser states, (*N* - 1) * *MU*, multiplied by the probability of being in the state, *P*(*N*). The rate of flow in is the rate of flow in from higher states, *P*(*N* + 1) * *MU* * *N*, plus the expected rate of flow in from lower states, *LMDA* * *@SUM*(*BSIZE*(*I*)| *I* #LT# *N*: *A*(*I*) * *P*(*N* - *I*)).

We generate equivalent balance equations for the higher states where the number of motors exceeds the number of servers with the following:

```
@FOR(STATE(N)| N #GT# S #AND# N #LT# LAST:
 P(N) * (S * MU + LMDA) =
  P(N + 1) * MU * S +
   LMDA * @SUM(BSIZE(I)| I #LT# N: A(I) *
    P(N - I))
);
```

The Solution

An excerpt from the full solution report showing the first 10 values of *P* is listed below:

Variable	Value
LMDA	1.500000
S	5.000000
MU	2.000000
LAST	41.00000
P(1)	0.2450015
P(2)	0.1837511
P(3)	0.1515947
P(4)	0.1221179
P(5)	0.9097116E-01
P(6)	0.5707410E-01
P(7)	0.4276028E-01
P(8)	0.3099809E-01
P(9)	0.2187340E-01
P(10)	0.1538003E-01

The probability at least one workstation is free is the probability that four or fewer motors are in the system, in other words, the sum of the first five *P(i)*. In this case, it works out that at least one workstation will be free only about 70% of the time. Experimenting with the model by increasing the number of servers reveals you will need at least seven workstations to provide the level of service required by the new priority plan.

Marketing Models

Markov Chain Model — Model: MARKOV

Background

A standard approach used in modeling random variables over time is the Markov chain approach. Refer to an operations research or probability text for complete details. The basic idea is to think of the system as being in one of a discrete number of states at each point in time. The behavior of the system is described by a transition probability matrix that gives the probability the system will move to a specified other state from some given state. Some example situations are:

System	States	Cause of Transition
Consumer brand switching	Brand of product most recently purchased by consumer	Consumer changes mind, advertising
Inventory System	Amount of inventory on hand	Orders for new material, demands

An interesting problem is to determine the long-term, steady state probabilities of the system. If we assume the system can reach equilibrium, then it must be true the probability of leaving a particular state must equal the probability of arriving in that state. You will recall this is the *Rate In = Rate Out Principle* (RIRO) we used above in building the multi-server queuing model, *QUEUEM*. If we let:

π_i = the steady state probability of being in state i, and
p_{ij} = the transition probability of moving from state i to j,

then, by our RIRO assumption, for each state i:

$$\sum_{j \neq i} p_j p_{ji} = \pi_i (1 - p_{jj})$$

Rewriting the above, we get:

$$\pi_i = \sum_j \pi_j p_{ji}$$

This gives us n equations to solve for the n unknown steady state probabilities. Unfortunately, it turns out this system is not of full rank. Thus, there is not a unique solution. To guarantee a valid set of probabilities, we must make use of one final condition—the sum of the probabilities must be 1.

The Problem in Words

Your company is about to introduce a new detergent and you're interested in whether it will clean up in the market. It will be competing against three other existing brands. As a result of a small test market and interviews with consumers familiar with all four detergents, we have derived the following purchase transition matrix:

		Next Purchase:			
		A	B	C	D
Previous Purchase:	A	.75	.1	.05	.1
	B	.4	.2	.1	.3
	C	.1	.2	.4	.3
	D	.2	.2	.3	.3

Our new detergent is brand *A* in this matrix. The interpretation of the matrix is, for example, if someone most recently purchased brand *A*, then with probability .75 his next purchase of this product will also be brand *A*. Similarly, someone who most recently purchased brand *B* will next purchase brand *D* with probability .3. An associate of yours who looked at this matrix said, "Aha, we should expect to get 75% of the market in the long run with brand *A*." Do you think your associate is correct?

The Model

```
! Markov chain model;
SETS:
 ! There are four states in our model and over
   time the model will arrive at a steady state
   equilibrium.
   SPROB(J) = steady state probability;
 STATE/ A B C D/: SPROB;

 ! For each state, there's a probability of moving
   to each other state. TPROB(I, J) = transition
   probability;
 SXS(STATE, STATE): TPROB;
ENDSETS

DATA:
 ! The transition probabilities. These are proba-
   bilities of moving from one state to the next
   in each time period. Our model has four states,
   for each time period there's a probability of
   moving to each of the four states. The sum of
   probabilities across each of the rows is 1,
   since the system either moves to a new state or
   remains in the current one.;
   TPROB = .75  .1  .05  .1
           .4   .2  .1   .3
           .1   .2  .4   .3
           .2   .2  .3   .3;
ENDDATA
```

```
!  Steady state equations;
!   Only need N equations, so drop last;
  @FOR(STATE(J)| J #LT# @SIZE(STATE):
   SPROB(J) = @SUM(SXS(I, J): SPROB(I) *
    TPROB(I, J))
  );

! The steady state probabilities must sum to 1;
  @SUM(STATE: SPROB) = 1;

! Check the input data, warn the user if the sum
  of probabilities in a row does not equal 1.;
  @FOR(STATE(I):
   @WARN('Probabilities in a row must sum to 1.',
    @ABS(1 - @SUM(SXS(I, K): TPROB(I, K)))
     #GT# .000001);
  );
```

Model: MARKOV

The Sets

The primitive *STATE* set represents the four states of purchasing detergents *A*, *B*, *C*, and *D*. We build one derived set, *SXS* that is the cross of the *STATE* set on itself. The *SXS* set is used to establish the state transition matrix.

The Attributes

We have two attributes. The first, *SPROB*, is defined on the *STATES* set and is used to store the steady state probabilities of the system. We will be solving for the values of the *SPROB* attribute. The second attribute, *TPROB*, is defined on the two-dimensional *SXS* set and is used to store the values of the state transition matrix.

The Formulas

First off, to ensure data integrity, we use the *@WARN* function to verify the probabilities in each row of the state transition matrix sum to 1 using:

```
! Check the input data, warn the user if the sum of
  probabilities in a row does not equal 1.;
  @FOR(STATE(I):
   @WARN('Probabilities in a row must sum to 1.',
    @ABS(1 - @SUM(SXS(I, K): TPROB(I, K)))
     #GT# .000001);
```

Due to the potential for roundoff error, we allow for a modest transgression by using a tolerance factor of .000001. If the probabilities in a row sum up to more than 1.000001 or less than .999999, the user will receive a warning.

Next, the steady state probabilities must be exhaustive. We guarantee this by setting their sum to 1 with:

```
! The steady state probabilities must sum to 1;
  @SUM(STATE: SPROB) = 1;
```

Finally, in addition to this last equation, we need an additional *n-1* equations to solve for the *n* steady state probabilities. We get these equations from the RIRO derivation above using:

```
!  Steady state equations;
!   Only need N equations, so drop last;
  @FOR(STATE(J)| J #LT# @SIZE(STATE):
   SPROB(J) = @SUM(SXS(I, J): SPROB(I) *
    TPROB(I, J))
  );
```

The Solution

The solution is:

Variable	Value
SPROB(A)	0.4750000
SPROB(B)	0.1525000
SPROB(C)	0.1675000
SPROB(D)	0.2050000

Solution: MARKOV

So, we see the long run share of our new brand *A* will only amount to about 47.5% of the market, which is considerably less than the conjectured share of 75%.

Conjoint Analysis Model: CONJNT

Background

When designing a product, it is useful to know how much customers value various attributes of that product. This allows us to design the product most preferred by consumers within a limited budget. For instance, if we determine consumers place a very high value on a long product warranty, then we might be more successful in offering a long warranty with fewer color options.

The basic idea behind *conjoint analysis* is, while it may be difficult to get consumers to accurately reveal their relative utilities for product *attributes*, it's easy to get them to state whether they prefer one product *configuration* to another. Given these rank preferences, you can use conjoint analysis to work backwards and determine the implied utility functions for the product attributes.

The Problem in Words

Your company is about to introduce a new vacuum cleaner. You have conducted customer surveys and you determined the relative preferences for the following warranty/price product configurations (9 being the most preferred):

	$129	$99	$79
2 Years	7	8	9
1 Year	3	4	6
None	1	2	5

In order to derive an optimal product configuration, you need to know the utility functions for warranty length and price implied by the preferences in the table above.

The Model

```
! Conjoint analysis model to decide how much
  weight to give to the two product attributes of
  warranty length and price;

SETS:
! The three possible warranty lengths;
     WARRANTY /LONG, MEDIUM, SHORT/ : WWT;
! where WWT(i) = utility assigned to warranty i;

! The three possible price levels (high,
  medium, low);
     PRICE /HIGH, MEDIUM, LOW/ : PWT;
! where PWT(j) = utility assigned to price j;

! We have a customer preference ranking for each
  combination;
     WP(WARRANTY, PRICE) : RANK;
ENDSETS

DATA:
! Here is the customer preference rankings running
  from a least preferred score of 1 to the most
  preferred of 9.  Note that long warranty and low
```

```
  price are most preferred with a score of 9,
  while short warranty and high price are least
  preferred with a score of 1;
     RANK = 7  8  9
            3  4  6
            1  2  5;
ENDDATA

SETS:
! The next set generates all unique pairs of
  product configurations such that the second
  configuration is preferred to the first;
     WPWP(WP, WP) | RANK(&1, &2) #LT#
      RANK(&3, &4): ERROR;
! The attribute ERROR computes the error of our
  estimated preference from the preferences given
  us by the customer;
ENDSETS

! For every pair of rankings, compute the amount
  by which our computed ranking violates the true
  ranking. Our computed ranking for the (i,j)
  combination is given by the sum WWT(i) + PWT(j).
  (NOTE: This makes the bold assumption that
  utilities are additive!);
     @FOR(WPWP(i, j, k, l): ERROR(i, j, k, l) >=
      1 + (WWT(i) + PWT(j)) - (WWT(k) + PWT(l))
     );

! The 1 is required on the right-hand-side of the
  above equation to force ERROR to be nonzero in
  the case where our weighting scheme incorrectly
  predicts that the combination (i,j) is equally
  preferred to the (k,l) combination.

  Since variables in LINGO have a default lower
  bound of 0, ERROR will be driven to zero when we
  correctly predict (k,l) is preferred to (i,j).

  Next, we minimize the sum of all errors in order
  to make our computed utilities as accurate as possible;
     MIN = @SUM(WPWP: ERROR);
```

Model: CONJNT

The Sets

We have two primitive sets in the model: *WARRANTY* is the set of warranty lengths and *PRICE* is the set of prices. We form the derived set *WP* by taking the cross of *WARRANTY* and *PRICE* in order to create an attribute to store the preferences for each (*WARRANTY, PRICE*) pair.

The interesting set in this model is the sparse derived set *WPWP*:

```
! The next set generates all unique pairs of product
  configurations such that the second configuration
  is preferred to the first;
     WPWP(WP, WP) | RANK(&1, &2) #LT#
      RANK(&3, &4): ERROR;
```

This set is derived from the cross of *WP* on itself. Each member of this set contains two product configurations. Furthermore, we use a membership condition to limit the set to combinations of product configurations where the first configuration is preferred to the second configuration. We need this set because our model will be testing its proposed utility functions against all unique pairings of configurations.

Note that this set grows on the order of n^2, where n is the number of product configurations. Thus, it will tend to get big for large numbers of product configurations.

The Attributes

The model defines four attributes: *WWT*, *PWT*, *RANK* and *ERROR*. *WWT* and *PWT* are used to store the utilities of the warranty and price configurations, respectively. The model will solve for values of *WWT* and *PWT* that minimize total prediction error. These values will then give us the implied utility functions for the two product attributes. *RANK* is used to store the customer preference rankings. Finally, *ERROR* stores the error in predicting the preference of one configuration over another given the implied utility values contained in *WWT* and *PWT*.

The Objective

The objective is quite simple—we want to minimize the total prediction error over all the unique product configurations:

```
MIN = @SUM(WPWP: ERROR);
```

The Constraints

The model has just one class of constraints that computes the error term in predicting the preferences over each product configuration:

```
@FOR(WPWP(i, j, k, l): ERROR(i, j, k, l) >=
      1 + (WWT(i) + PWT(j)) -
       (WWT(k) + PWT(l))
);
```

We need to add a 1 to the right-hand side of the constraint to account for the case where we predict that configuration *(i,j)* is equally preferred to *(k,l)*. Because of the way that we defined the *WPWP* set, *(i,j)* will always be preferred to *(k,l)*. Thus, it would be an error to predict they are equally preferred. Note, also, because the lower bound on *ERROR* is 0 and we are also minimizing *ERROR*, *ERROR(i,j,k,l)* will be driven to 0 when the model correctly predicts that *(i,j)* is preferred to *(k,l)*.

The Solution

Portions of the solution are reproduced below:

```
Global optimal solution found at step:        35
Objective value:                        0.000000

        Variable           Value
       WWT(LONG)        7.000000
     WWT(MEDIUM)        2.000000
      WWT(SHORT)        0.000000
       PWT(HIGH)        0.000000
     PWT(MEDIUM)        1.000000
        PWT(LOW)        4.000000
```

Solution: CONJNT

Short, medium and long warranties rate utilities of 0, 2, and 7, while high, medium and low prices rate utilities of 0, 1, and 4. Note that, given the objective value of zero, this utility weighting scheme exactly predicts preferences for each pair of product configurations.

13 Programming LINGO

Up to this point, we have primarily been concerned with self-contained models, which are solved once and don't receive input from, nor pass output to, other models. However, in many modeling situations, there may be requirements for one, or more, models to be run in a sequence. Furthermore, the outputs of one model may be required as inputs to a subsequent model. Such a series of models will typically be run in a loop until some termination criterion is met. LINGO provides a number of programming control structures to automate processing in situations where one has multiple, dependent models that one must run in a sequence.

In the section immediately following, we will provide a brief introduction to the programming features available in LINGO. We will then illustrate the use of these features in several models.

Programming Features

The programming capabilities in LINGO fall into seven categories:

- Model control
- Flow of control
- Model generation
- Output statements
- Setting parameters
- Stochastic Programming
- Utility functions

Model Control

In this category we have two types of statements. First are the *SUBMODEL* and *ENDSUBMODEL* statements used to identify submodels, which are separate models contained within larger models. We also have the *@SOLVE* statement for use in solving submodels.

SUBMODEL and ENDSUBMODEL:

These two statements are used to bracket a submodel within a larger, composite model. The *SUBMODEL* statement must be followed immediately by a unique name that follows the normal LINGO naming conventions. The name must then be followed by a colon. You must also place a *ENDSUBMODEL* statement immediately after the last statement of your submodel. Submodels may only exist in the model section and are not allowed in data, init and calc sections.

As an example, the following illustrates a submodel for solving a knapsack type model:

```
SUBMODEL Pattern_Gen:
   [R_OBJ] MAX = @SUM( FG(i): PRICE(i)* Y(i));
   [R_WIDTH] @SUM( FG(i): WIDTH(i)*Y(i)) <= RMWIDTH;
   @FOR( FG(i): @GIN(Y(i)));
ENDSUBMODEL
```

In this example, the submodel is titled *Pattern_Gen* and contains one objective, *R_OBJ*, one constraint, *R_WIDTH*, and one *@FOR* loop to force the *Y* variables to be general integers via the *@GIN* function.

@DEBUG([SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

Infeasible or unbounded submodels may be debugged in a calc section with the use of the *@DEBUG* statement. Refer to the *Solver|Debug* command for more information of model debugging.

If your model contains submodels, you can choose to debug a particular submodel by specifying its name as an argument to *@DEBUG*. If desired, you may also specify more than one submodel name, in which case, LINGO will simultaneously debug all the specified models as one combined model. If a submodel name is omitted, LINGO will solve all model statements occurring before the *@DEBUG* statement and not lying within a submodel section. It is the user's responsibility to make sure the submodels together make sense, e.g., at most one submodel in an *@DEBUG* invocation can have an objective function.

In the following example, we solve a small submodel and then invoke the debugger if the solution is found to be non-optimal:

```
MODEL:

SUBMODEL M:
   MIN = X + Y;
   X>4;
   Y<3;
   Y>X;
ENDSUBMODEL

CALC:
   @SOLVE( M);
   @IFC( @STATUS() #NE# 0: @DEBUG( M));
ENDCALC

END
```

Note: Submodels must be defined in the model prior to any references to them via the *@DEBUG* statement.

@SOLVE([SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

Submodels may be solved in a calc section with the use of the *@SOLVE* statement. If your model contains submodels, you can choose to solve a particular submodel by specifying its name as an argument to *@SOLVE*. If desired, you may also specify more than one submodel name, in which case, LINGO will simultaneously solve all the specified models as one combined model. If a submodel name is omitted, LINGO will solve all model statements occurring before the *@SOLVE* statement and not lying within a submodel section. It is the user's responsibility to make sure the submodels together make sense, e.g., at most one submodel in an *@SOLVE* can have an objective function.

As an example, to solve the submodel *Pattern_Gen* listed immediately above, you would add the following statement to a calc section:

```
@SOLVE( Pattern_Gen);
```

Note:	Submodels must be defined in the model prior to any references to them via the *@SOLVE* statement.

@NEXTKBEST()

@NEXTKBEST can be used in a calc section to find the "next best" solutions to binary integer programs. First you must call *@SOLVE* (see above) to perform an initial solve of the model, which yields a solution with the best objective value. At this point, you may iteratively call *@NEXTKBEST* to generate one or more additional solutions ranked by objective value. This gives you the ability to determine if there are solutions close or equal in value to the optimal solution that might be preferred due to some secondary criteria. An example would be in a staff scheduling model, where there might be two solutions of equal cost, however, one of the solutions requires fewer workers on Sundays.

The LINGO sample model set includes the model *LOOPKBEST*. This model generates the 5 best solutions to a knapsack problem.

The code of interest that generates these solutions is:

```
CALC:
  ! Reduce the amount of output;
  @SET( 'TERSEO', 2);

  NBEST = 0;
  MXBEST = 5;
  ! Solve original model;
  @SOLVE( KNAPSACK);

  ! Loop to display additional K-Best solutions;
  ! Stop when we either have a infeasible solution
    or we reach MXBEST solutions.;
  @WHILE( @STATUS() #EQ# 0 #AND# NBEST #LT# MXBEST:

     NBEST = NBEST + 1;

     ! Print summary of current solution;
     REPORT;

     !Solve for next best solution;
     @NEXTKBEST();

  );

ENDCALC
```

Model: CHARTSTAFF

As mentioned, *@SOLVE* is called first to get an optimal solution to the model. We then have a *@WHILE* loop that call *@NEXTKBEST* until we either have an infeasible solutions, or we've generated 5 total solutions. Note that the initial *@SOLVE* call counts as one solution. We also make use of a procedure, REPORT, to print a summary report of each solution. You may open the model file in LINGO if you care to view this reporting procedure.

Note: Calling *@NEXTKBEST* indefinitely can yield all the feasible solutions to a 0/1 program. However, for some models this may entail a very large number of solutions.

Note: If the model is not convex and you are not using the global solver, then the K-Best solutions may not be generated according to objective rank due to finding locally optimal solutions along the way.

Charting

LINGO has a number of features to generate charts of your data. Chart types currently included are Bar, Bubble, Contour, Curve, Gantt, Histogram, Line, Network (node based and arc based), Pie, Radar, Scatter, Stacked Bar, Surface and Tornado. A description of the functions for each of the various chart types follows.

@CHARTBAR('TITLE', 'X-AXIS LABEL', 'Y-AXIS LABEL', 'LEGEND1', ATTRIBUTE1[, ..., 'LEGENDN', ATTRIBUTEN]);

@CHARTBAR will generate a bar chart of one or more attributes. The bar chart will be displayed in a new window. *@CHARTBAR* argument descriptions follow: (

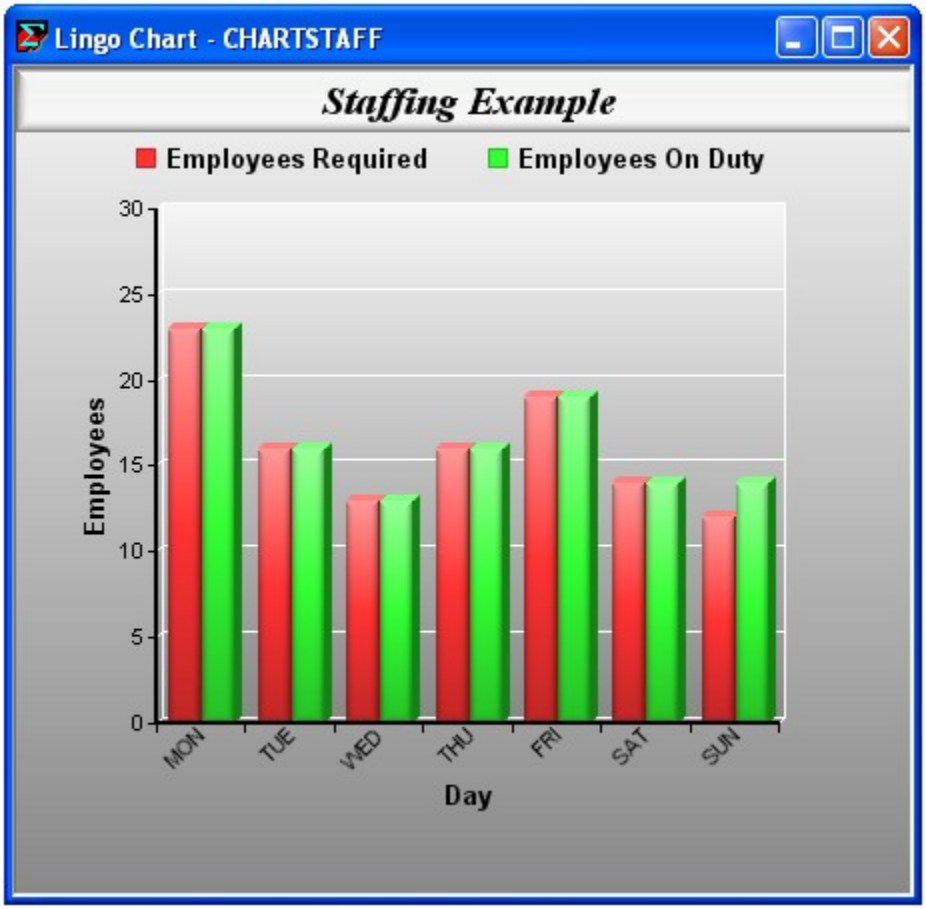
- TITLE - This is the title to display at the top of the chart. The title must be in quotes.
- X-AXIS LABEL - The x-axis label in quotes.
- Y-AXIS LABEL - The y-axis label in quotes.
- LEGENDi - The legend to be used to label attribute i in quotes.
- ATTRIBUTEi - This is the attribute to chart. At least one attribute argument must appear. If multiple attributes appear, the bars will be grouped together.

As an example, consider the following model, CHARTSTAFF, which is a modified version of the staff scheduling example presented in section Primitive Set Example in Chapter 2, Using Sets:

```
MODEL:

SETS:
  DAYS: REQUIRED, START, ONDUTY;
ENDSETS

DATA:
  DAYS =     MON TUE WED THU FRI SAT SUN;
  REQUIRED =  23  16  13  16  19  14  12;
ENDDATA

SUBMODEL MODSTAFF:
  MIN = @SUM( DAYS( I): START( I));

  @FOR( DAYS( J):
   ONDUTY( J) =
    @SUM( DAYS( I) | I #LE# 5:
     START( @WRAP( J - I + 1, 7)));

   ONDUTY( J) >= REQUIRED( J)

  );
ENDSUBMODEL

CALC:
  ! Solve the staffing model;
  @SOLVE( MODSTAFF);

  ! Bar chart of required vs. actual staffing;
  @CHARTBAR(
   'Staffing Example',   !Chart title;
   'Day',                !X-Axis label;
   'Employees',          !Y-Axis label;
   'Employees Required', !Legend 1;
   REQUIRED,             !Attribute 1;
   'Employees On Duty',  !Legend 2;
   ONDUTY                !Attribute 2;
  );
ENDCALC

END
```

Model: CHARTSTAFF

Here, we've added a reference to *@CHARTBAR* to generate a bar chart of staffing requirements vs. staff on duty for each day of the week. Here's is the bar chart that is generated:



Note that since we specified more than one attribute, the bars for the two attributes for each day of the week are grouped together. LINGO also automatically labeled each bar using the attributes' parent set DAYS. From the chart it's easy to see that staffing requirements are just met on days Monday through Saturday, while on Sunday we have slightly more staffing than is actually required.

All arguments to charting function are optional, with the exception of the attribute values. In many cases, if an argument is omitted, LINGO will provide a reasonable default. For example, if the title is omitted as we have done here:

```
! Bar chart of required vs. actual staffing;
@CHARTBAR(
 ,                      !Chart title will default to file name;
 'Day',                 !X-Axis label;
 'Employees',           !Y-Axis label;
 'Employees Required',  !Legend 1;
 REQUIRED,              !Attribute 1;
 'Employees On Duty',   !Legend 2;
 ONDUTY                 !Attribute 2;
);
```

then LINGO will use the model's file name instead. Note that you must still include the comma separator when omitting an argument.

The complete list of charting functions supported is given in the table below. You may also be interested in running the CHARTS.LG4 sample model, which generates examples of all the chart types.

Chart Type	Syntax	Notes
Bar	@CHARTBAR('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', Attribute1[, ..., 'Legendn', Attributen]);	Multiple attributes will result in bars being grouped. See below for stacked bar charts.
Bubble	@CHARTBUBBLE('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', X1, Y1, Size1,[, ..., 'Legendn', Xn, Yn, Sizen]);	Each series of bubbles is represented as a triplet of (*x*-location, *y*-location, bubble size).
Contour	@CHARTCONTOUR('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend', X, Y, Z);	You can thinks of a contour chart as a 3-dimensional surface chart compressed into two dimensions. Only one series is allowed and it is represented by a grid *(x,y,z)* coordinates.
Curve	@CHARTCURVE('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', X1, Y1[, ..., 'Legendn, Xn, Yn]);	Curves are good for representing two-dimensional *(x,y)* plots, e.g., *y=sin(x)* . Multiples curves may be drawn in a single chart, with each curve represented as a series of *(x,y)* coordinates. See the sample model *CHARTDISTRO* for an example of a curve chart.

Gantt	@CHARTGANTT('Title', 'X-Axis Label', 'Y-Axis', StartTimes, EndTimes);	Gantt charts are another form of a vertical bar chart. They are useful when planning and implementing projects consisting of many interdependent tasks. They do this by helping one to get an overview of the entire project and when all the various tasks of the project can start and how long they can run. The start and end times are defined on a 3-dimensional, derived set giving the assignment number, team and task of an assignment. More than one team may be working on one assignment, so the number of assignment may exceed the number of tasks. The start and end times also must be converted to LINGO's standard time with the *@YMD2STM* function. A example of a Gantt chart for project planning is show below.
Histogram	@CHARTHISTO('Title', 'X-Axis Label', 'Y-Axis', 'Legend', Bins, X);	If the bin count, *Bins*, is omitted, LINGO will automatically compute a number of bins that results in a reasonably distributed histogram. Only one histogram may be displayed per chart.
Line	@CHARTLINE('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', X1[, ..., 'Legendn', Xn]);	Multiple lines can be displayed on a single chart, with each being drawn in a different color.
Netarc	@CHARTNETARC('Title', 'X-Axis', 'Y-Axis', 'Legend1', X1, Y1, X2, Y2[, ..., 'Legendn', X1n, Y1n, X2n, Y2n]);	Netarc charts are network charts, where the network is represented by a list of 4-tuples, *(x1,y1,x2,y2)*, where *(x1,y1)-i* is one end point of arc i and *(x2,y2)-i* is the other end point of arc i. Multiple networks may be displayed on a single chart. An optional fifth argument may be

		added to a data series to specify whether an arc should display an arrowhead, or not. For example, our first data series on the left is: X1, Y1, X2, Y2. We could have added a fifth argument, say A1: X1, Y1, X2, Y2, A1, where A1 is a vector of length equal to the number of arcs. Each member of A1 should then be set to one of the following values: 0: Undirected arc with no arrow, 1: Arc points from first to second node, 2: Arc points from second to first node, or 3: Bidirectional arc with arrow at both ends.
Netnode	@CHARTNETNODE('Title', 'X-Axis Label', 'Y-Axis' Label, 'Legend1', X1, Y1, I1, J1[, ..., 'Legendn', Xn, Yn, In, Jn]);	Netnode charts are network charts, where the network is represented by a two lists of 2-tuples, *(x,y)* and (i,j), where the *(x,y)* are the coordinates of each node in the network and the *(i,j)* give the indices of two nodes from the *(x,y)* list that exist as an arc in the network. Multiple networks may be displayed on a single chart. An optional fifth argument may be added to a data series to specify whether an arc should display an arrowhead, or not. For example, our first data series on the left is: X1, Y1, I1, J1. We could have added a fifth argument, say A1: X1, Y1, I1, J1, A1, where A1 is a vector of length equal to the number of arcs. Each member of A1 should then be set to one of the following values: 0: Undirected arc with no arrow, 1: Arc points from first to

		second node, 2: Arc points from second to first node, or 3: Bidirectional arc with arrow at both ends.
Pie	@CHARTPIE('Title', 'Legend', X);	Only one attribute may be displayed in a pie chart.
Radar	@CHARTRADAR('Title', 'Legend1', X1[, ..., 'Legendn', Xn]);	Multiple attributes can be displayed, with each being in a different color.
Scatter	@CHARTSCATTER('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', X1, Y1[, ..., 'Legendn', Xn, Yn]);	Each scatter plot is represented by a list of *(x,y)* coordinates giving the location of each point in the scatter. Multiple scatter plots can be displayed in a single chart, with each being in a different color.
Spacetime	@CHARTSPACETIME('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', ARC1, TIME1, TIME2, [, ..., 'Legendn', ARCn, TIME1n, TIME2n]);	Spacetime charts are useful in visualizing the changes within a system over time, e.g., an airline's flights during some period of time. Time is typically measured on the x-axis and there are arcs representing movements or changes within the system over time. The *ARC* argument is a 3-dimensional set of arcs. The first dimension is the arc label/name, the second dimension store the origination point, while the final dimension store the arc destination. *TIME1* and *TIME2* store the start and end times for each arc. Multiple spacetime series may be plotted on teh same chart. The sample model *CHARTSPACETIME is* an example of a spacetime chart for an airline schedule. In this example, the cities in the schedule are listed on the y-axis, and each

		arc represents a scheduled flight.
Stacked Horizontal and Vertical Bar	@CHARTSHBAR('Title', 'X-Axis Label', 'Y-Axis Label', 'Legend1', DataTable); @CHARTSVBAR('Title', 'Y-Axis Label', 'X-Axis Label', 'Legend1', DataTable);	These bar charts will stack bar segments on top of one another, either horizontally or vertically. Data for the bars is specified in a 2-dimensional attribute. Each column represents one stacked bar. So, if your data table has three columns and five rows, then the chart will have three bars, each with 5 segments. This is similar to the way Excel handles stacked bars. The chart's legend, which gives the names of the bar segments, is automatically taken from the set name of the primitive set forming the first dimension of the data table's parent set. Note that when you display a horizontal chart, the x-axis will technically be on the y-axis. This is because the chart has been rotated 90 degrees, with the x-axis now running vertically. Please refer to the LINGO sample model, *CHARTSTACKEDBAR* for an example of stacked bar charts.
Surface	@CHARTSURFACE('Title', 'X-Axis Label', 'Y-Axis Label', 'Z-Axis Label', 'Legend', X, Y, Z);	Each surface plot is represented by a list of *(x,y,z)* coordinates, each containing the location of a points on the surface. A sufficient number of points should be included to avoid excessive interpolation between points. Only one surface may be drawn on a chart. See the sample model *CHARTSURF* below for an example of a surface chart.
Tornado	@CHARTTORNADO('Title', 'X-Axis Label', 'Y-Axis Label', Base,	A tornado chart is represented by a list of *(high, low)* pairs, giving the

	'Legend High', High, 'Legend Low', Low);	two extreme outcomes for a particular property (say profit) subject to changes in a particular parameter (say price). The *base* value is a single value for the property of interest given normal, or expected values, for each of the parameters. This is represented by one horizontal bar for each parameter, centered around the base value. The bars are then sorted by their lengths, giving the chart the appearance of a tornado. Tornado charts are useful for visualizing the sensitivity of a particular property to changes in a number of parameters. An example can be found in the LINGO sample models folder under the name *ASTROCOSTRNDO*.

Surface Chart Example

As another example, we will show how to create a surface chart by supplying a grid of (x,y,z) points lying on the surface: z = x * sin(y) + y * sin(x) using the following model, CHARTSURF:

```
MODEL:

! Creates a surface chart of z = x * sin( y) + y * sin( x);

SETS:
 !Set up a 21x21 grid of (X,Y,Z) points on the surface;
 SURF1 /1..21/;
 SURF2( SURF1, SURF1): X, Y, Z;
ENDSETS

CALC:
 !First point;
 XS = @FLOOR( - (@SIZE( SURF1) / 2) + .5);
 YS = XS;

 !Generate remaining points and compute z;
 @FOR( SURF2( I, J):
   X( I, J) = XS + I - 1;
   Y( I, J) = YS + J - 1;
   Z( I, J) = X( I, J) * @SIN( Y( I, J)) + Y( I, J ) * @SIN( X( I,
   J));
 );


 !Create the chart;
 @CHARTSURFACE(
   'z = x * sin( y) + y * sin( x)',  !Title;
   'X', 'Y', 'Z',                    !Axis labels;
   '(X,Y,Z)',                        !Legend;
   X, Y, Z                           !Points on the surface;
 );
ENDCALC

END
```

Model: CHARTSURF

First off, the CHARTSURF creates a 21x21 grid of points on the (X,Y) plane centered around (0,0) and computes Z for each of these points:

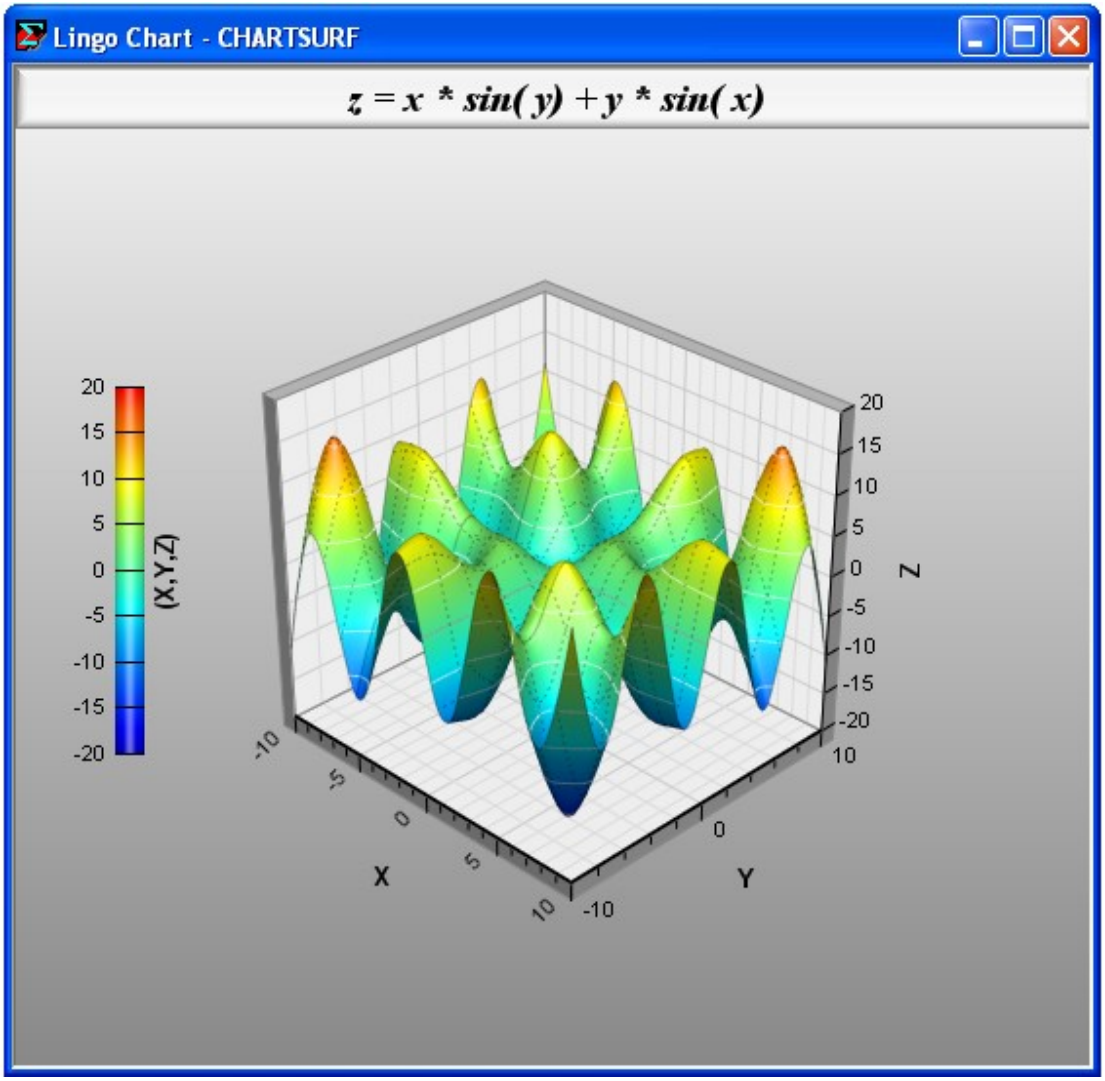
```
 !First point;
 XS = @FLOOR( - (@SIZE( SURF1) / 2) + .5);
 YS = XS;
```

```
!Generate remaining points and compute z;

@FOR( SURF2( I, J):
  X( I, J) = XS + I - 1;
  Y( I, J) = YS + J - 1;
  Z( I, J) = X( I, J) * @SIN( Y( I, J)) + Y( I, J ) * @SIN( X( I,
   J));
);
```

We then use the *@CHARTSURFACE* function to create the surface chart:

```
!Create the chart;
@CHARTSURFACE(
  'z = x * sin( y) + y * sin( x)',  !Title;
  'X', 'Y', 'Z',                    !Axis labels;
  '(X,Y,Z)',                        !Legend;
  X, Y, Z                           !Points on the surface;
);
```

The resulting chart will appear as follows:



Gantt Chart Example

In this next example, we will show how to create a Gantt chart showing a project that will bring a software package to market. This example can be found in your LINGO samples folder under the name CHARTGANT.

The sets section for this model follows:

```
SETS:
  TEAMS;
  TASKS;
  ASSIGNMENTS;
  TTA( ASSIGNMENTS, TEAMS, TASKS):
   START_YR, START_MO, START_DAY,
   END_YR, END_MO, END_DAY,
   START_STM, END_STM;
ENDSETS
```

Our primitive sets are TEAMS, TASKS and ASSIGNMENTS. There are three teams: Marketing, Planning and Development, that will be working on the nine tasks of the project. Some teams may be assigned to more than one task, as well as some tasks may be assigned to more than one team. Given this, the project may have more (team,task) assignment pairs than the number of tasks. In this project, there will be a total of 12 (team,task) assignments. Here are the team, task and assignment sets:

```
  TEAMS =
   Marketing
   Planning
   Development
  ;

  TASKS =
   Market_Research
   Define_Specifications
   Overall_Architecture
   Project_Planning
   Detail_Design
   Software_Development
   Test_Plan
   Testing_and_QA
   User_Documentation
  ;

  ASSIGNMENTS = 1..12;
```

The set TTA is a derived set, derived from the three primitive sets. It's used to store the 12 (team,task) assignments. We also define attributes on TTA to store starting and ending times for each (team,task) assignment. Here is the initialization of TTA and its attributes:

```
TTA, START_YR, START_MO, START_DAY, END_YR, END_MO, END_DAY =
  1 Marketing   Market_Research         2017  8 16    2017  8  30
  2 Marketing   Market_Research         2017 10  4    2017 10  18
  3 Marketing   Define_Specifications   2017  8 30    2017  9  13
  4 Planning    Overall_Architecture    2017  9 13    2017  9  27
  5 Planning    Project_Planning        2017  9 20    2017 10   4
  6 Development   Detail_Design         2017  9 27    2017 10  11
  7 Development   Software_Development  2017 10  4    2017 11   8
  8 Planning          Test_Plan         2017 10  4    2017 10  18
  9 Development   Test_Plan             2017 10 25    2017 11   8
 10 Development  Testing_and_QA         2017 11  1    2017 11  22
 11 Marketing       User_Documentation  2017 10 18    2017 11   1
 12 Development  User_Documentation     2017 11  8    2017 11  22
;
```

Here we see that the first (team,task) assignment has Marketing working on the Market Research task from 8/16/2017 to 8/30/2017.

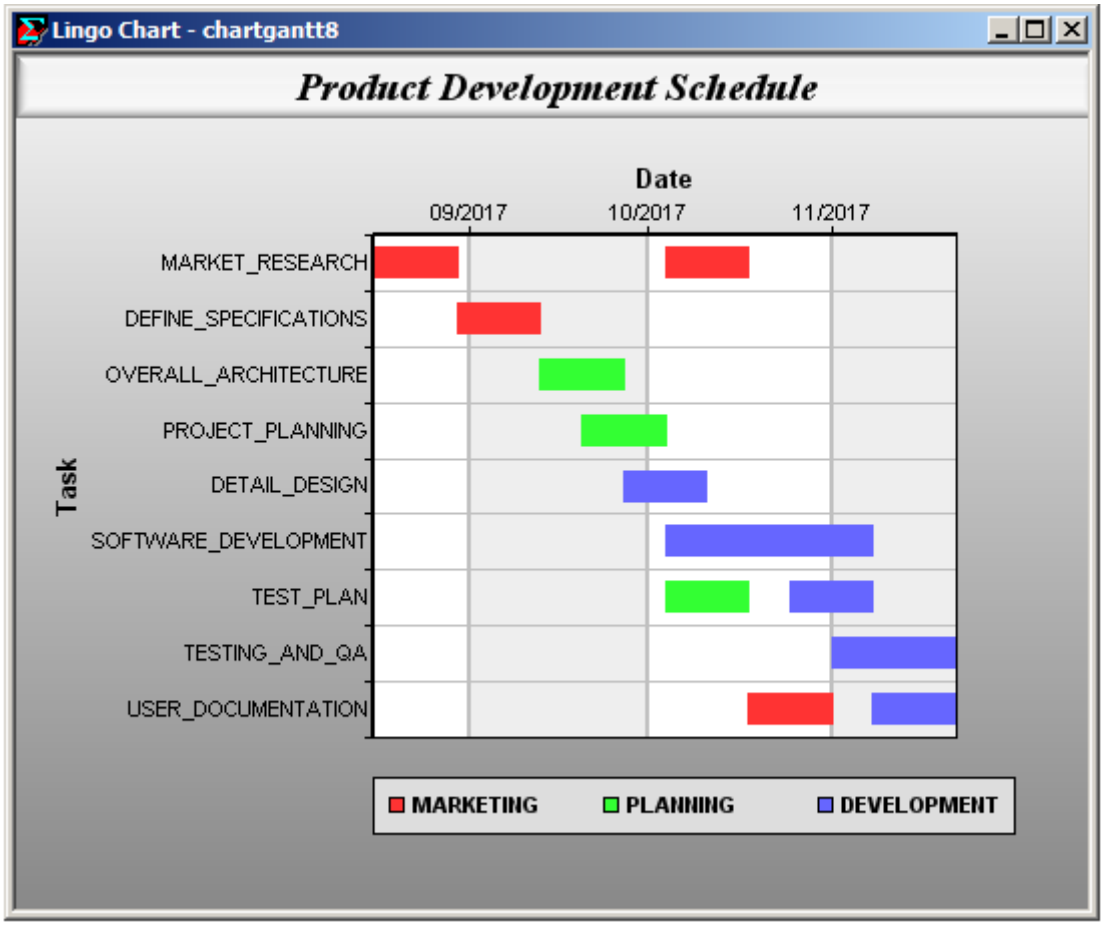
The Gantt chart routine requires that dates be passed in LINGO's standard time format, so we convert the dates above to standard time in the calc section using the *@YMD2STM* function:

```
@FOR( TTA( ASG, TM, TK):
 START_STM( ASG, TM, TK) =
  @YMD2STM( START_YR( ASG, TM, TK), START_MO( ASG, TM, TK),
   START_DAY( ASG, TM, TK) , 0, 0, 0);
 END_STM( ASG, TM, TK) =
  @YMD2STM( END_YR( ASG, TM, TK), END_MO( ASG, TM, TK),
   END_DAY( ASG, TM, TK) , 0, 0, 0);
);
```

At this point, everything is set to create the chart, which we do with the following statement:

```
@CHARTGANTT( 'Product Development Schedule', 'Date',
 'Step', START_STM, END_STM);
```

Note that we don't explicitly pass the TTA set. LINGO infers that TTA contains the assignment information given that it is the parent set of the start and end time attributes. LINGO also retrieves the task names from the TTA set. The chart appears as follows:

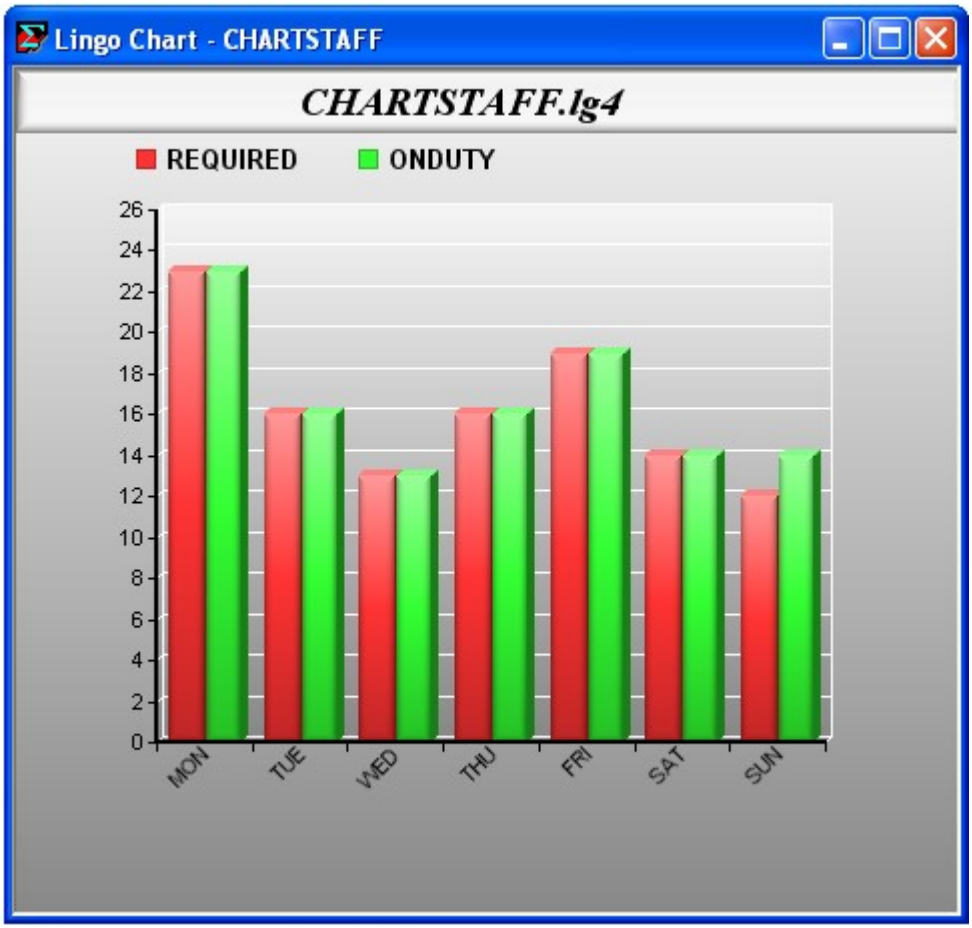


Simplified Chart Function Formats

Some of the chart functions above support a simplified format where all the arguments, with the exception of the data attributes, are omitted. For instance, in the bar chart example above, we could have simply used the following:

```
! Bar chart of required vs. actual staffing;
@CHARTBAR( REQUIRED, ONDUTY);
```

In which case, the resulting chart would be:



Note the following differences between the charts created using the simplified chart format and the example above where we specified all the arguments:

- The chart title is now the file name.
- The legends are simply the names of the attributes.
- The X and Y axis labels have been omitted.

The simplified syntax for the charting functions appears below:

Chart Type	Simplified Syntax
Bar	@CHARTBAR(Attribute1[, ..., Attributen]);
Bubble	@CHARTBUBBLE(X1, Y1, Size1,[, ..., Xn, Yn, Sizen]);
Contour	@CHARTCONTOUR(X, Y, Z);
Curve	@CHARTCURVE(X1, Y1[, ..., Xn, Yn]);
Histogram	@CHARTHISTO(X);
Line	@CHARTLINE(X1[, ..., Xn]);
Netarc	@CHARTNETARC(X1, Y1, X2, Y2[, ..., X1n, Y1n, X2n, Y2n]);
Netnode	@CHARTNETNODE(X1, Y1, I1, J1[, ..., Xn, Yn, In, Jn]);
Pie	@CHARTPIE(X);
Radar	@CHARTRADAR(X1[, ..., Xn]);
Scatter	@CHARTSCATTER(X1, Y1[, ..., Xn, Yn]);
Surface	@CHARTSURFACE(X, Y, Z);
Tornado	@CHARTTORNADO(Base, High, Low);

Using Procedures to Specify Chart Points

For curve, surface and contour charts, you may use a procedure to specify the function to be plotted. LINGO will automatically call this procedure to generate function points for the chart, thereby allowing you to skip the step of explicitly generating the points. The syntax for these procedure based charting functions is as follows:

Chart Type	Simplified Syntax
Contour	@CHARTPCONTOUR('Title', 'X-Axis Label', 'Y-Axis Label', 'Z-Axis Label', PROCEDURE, X, XL, XU, Y, YL, YU, 'Legend', Z);
Curve	@CHARTPCURVE('Title', 'X-Axis Label', 'Y-Axis Label', PROCEDURE, X, XL, XU, 'Legend1', Y1[, ...,'LegendN', YN]);
Surface	@CHARTPSURFACE('Title', 'X-Axis Label', 'Y-Axis Label', 'Z-Axis Label', PROCEDURE, X, XL, XU, Y, YL, YU, 'Legend', Z);

In the case of the two-dimensional curve chart, the PROCEDURE argument is the name of the procedure containing the formula for the curve, X is the independent variables, XL and XU its upper and lower bounds and Y1 is the dependent variable that gets assigned the function's value. The three-dimensional contour and surface charts have two independent variables (X and Y) and the dependent variable Z. As an example, the model below uses the *@CHARTPSURFACE* function to generate the same surface chart presented above:

```
MODEL:

! Creates surface chart of z = x * sin( y) + y * sin( x);
! Function points are provided by the WAVE procedure;

PROCEDURE WAVE:
  Z = X * @SIN( Y) + Y * @SIN( X);
ENDPROCEDURE

CALC:
  B = 10;
  @CHARTPSURFACE(
    'z = x * sin( y) + y * sin( x)', !Title;
    'X','Y','Z',  !Axis labels;
    WAVE,         !Procedure name;
    X, -B, B,     !Independent var 1 and its bounds;
    Y, -B, B,     !Independent var 2 and its bounds;
    'Z',          !Legend;
    Z             !Dependent var/function value;
  );
ENDCALC

END
```

Model: CHARTPSURF

Note: Charting capabilities are only available in the interactive versions of LINGO. If you are using the LINGO API and need a charting capability, then you should investigate third-party solutions or use charting tools available in your development environment.

Flow of Control

In a calc section, model statements are normally executed sequentially. Flow of control statements can be used to alter the execution order of statements. In effect, this gives you a programming, or scripting, capability within the LINGO environment.

@IFC and @ELSE

These statements provide you with conditional IF/THEN/ELSE branching capabilities. The syntax is as follows:

```
@IFC( <conditional-exp>:
  statement_1[; …; statement_n;]
[@ELSE
  statement_1[; …; statement_n;]]
);
```

with the *@ELSE* block of statements being optional in case a pure if statement is desired.

Note: Be aware of the use of the letter 'C' in *@IFC*. This is to distinguish the flow of control if statement (*@IFC*) from the arithmetic if (*@IF*).

To illustrate, the following sample code uses if/else blocks as part of a binary search for a key value in an array:

```
@IFC( KEY #EQ# X( INEW):
  LOC = INEW;
@ELSE
  @IFC( KEY #LT# X( INEW):
     IE = INEW;
  @ELSE
     IB = INEW;
  );
);
```

@FOR

You've encountered the *@FOR* set looping statement previously as a way to generate constraints in the model section. *@FOR* is also allowed in the calc section to perform looping. The main difference is that *@FOR* does not generate constraints when used in the calc section. Instead, it immediately executes any assignment statements contained within its scope. The following example shows a *@FOR* loop extracted from a portfolio model. The loop is used to solve the portfolio model for a number of different levels of desired return. For each level of return, the model minimizes variance, and the variance is stored for later use.

```
@FOR( POINTS( I):
  ! Compute new return level;
  RET_LIM = RET_MIN + (I-1)*INTERVAL;
  ! Re-solve the model;
  @SOLVE();
  ! Store the return value;
  YRET( I) = RET_LIM;
  ! Store the variance too;
  XSTD( I) = VARIANCE^0.5;

);
```

@WHILE

The *@WHILE* statement is used for looping over a group of statements until some termination criterion is met. The syntax is as follows:

```
@WHILE( <conditional-exp>: statement_1[; …; statement_n;]);
```

As long as the conditional expression is true, the *@WHILE* function will keep looping over its block of statements.

As an example, the following code uses an *@WHILE* loop to search for a key value in an array as part of a binary search procedure:

```
@WHILE( KEY #NE# X( LOC):
  INEW = @FLOOR( ( IE + IB) / 2);
  @IFC ( KEY #EQ# X( INEW):
       LOC = INEW;
  @ELSE
       IFC( KEY #LT# X( INEW):
         IE = INEW - 1;
       @ELSE
         IB = INEW + 1;
       );
  );
);
```

In this case, the loop executes until the current value selected from the array, *X(LOC)*, is equal to the key value, *KEY*.

@BREAK

The *@BREAK* statement is used to break out of the current loop. Execution resumes at the first statement immediately following the end of the current loop. The *@BREAK* statement is valid only within *@FOR* and *@WHILE* loops in calc sections and does not take any arguments. As an example, we extend the *@WHILE* loop in the binary search example above to include an *@BREAK* statement that will be executed when the key value can't be found:

```
@WHILE( KEY #NE# X( LOC):
 !exit loop if key can't be found;
 @IFC( IE - IB #LE# 1:
   @PAUSE( 'Unable to find key!!!');
   @BREAK;
 );
 INEW = @FLOOR( ( IE + IB) / 2);
 @IFC ( KEY #EQ# X( INEW):
   LOC = INEW;
 @ELSE
   @IFC( KEY #LT# X( INEW):
       IE = INEW - 1;
   @ELSE
       IB = INEW + 1;
   );
 );
);
```

@STOP(['MESSAGE'])

The *@STOP* statement terminates execution of the current model. The *@STOP* statement is valid only within calc sections and takes on optional text argument. When an *@STOP* is executed, LINGO will display error message 258:

```
[Error Code:  258]

Model execution halted.  STOP statement encountered.
```

If a text argument is included in the *@STOP*, then it will also be displayed as part of this error message.

As an example, we extend the *@WHILE* loop in the binary search example above to include an *@STOP* statement that will be executed when the key value can't be found:

```
@WHILE( KEY #NE# X( LOC):
  !exit if key can't be found;
 @IFC( IE -IB #LE# 1:
   @STOP( 'Unable to find key!!!');
 );
 INEW = @FLOOR( ( IE + IB) / 2);
 @IFC ( KEY #EQ# X( INEW):
   LOC = INEW;
 @ELSE
   @IFC( KEY #LT# X( INEW):
       IE = INEW;
   @ELSE
       IB = INEW;
   );
 );
);
```

Input Statements

There are three functions discussed in this section: @READLN (read line), @REAMRM (read to record marker) and @INPUT (user terminal input). These functions are user for programmatically reading text input data in calc sections. This contrasts with input functions in data sections. Data section input cannot be programmatically controlled and will be executed regardless whenever the model is solved. With @READLN, @REAMRM and @INPUT, you may dynamically test conditions to determine if and/or how the text reads are executed. You may also dynamically specify how much text to read based on things such as the dimensions of the current model.

@READLN(['FILE_NAME'][, LINES])

The *@READLN* function reads a specified number of lines (*LINES*) from a text file ('*FILE_NAME'*). The file name argument is optional, and if it is omitted the last file name used in either a *@READLN* or *@READRM* will be substituted. The number of lines argument, *LINES*, can be a constant or a scalar variable giving the number of lines to read. If *LINES* is omitted, then a single line will be read.

@READRM(['FILE_NAME'][, 'RECORD_MARKER'])

The *@READRM* function reads from a text file (*'FILE_NAME'*) until either it encounters any specified record marker string or the end of the file. The file name argument is optional, and if it is omitted the last file name used in either a *@READLN* or *@READRM* will be substituted. The record marker string may also be omitted, in which case, *@READRM* will read to the end of the file.

We will now illustrate the use of the two input functions above with the following transportation model, *TRAN*, found in the LINGO samples model set:

```
MODEL:
! A 3 Warehouse, 4 Customer
  Transportation Problem;
SETS:
  WAREHOUSE / WH1, WH2, WH3/   : CAPACITY;
  CUSTOMER   / C1, C2, C3, C4/ : DEMAND;
  ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

! The objective;
[OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
@FOR( CUSTOMER( J): [DEM]
 @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
  DEMAND( J));

! The supply constraints;
@FOR( WAREHOUSE( I): [SUP]
 @SUM( CUSTOMER( J): VOLUME( I, J)) <=
  CAPACITY( I));

! Here are the parameters;
DATA:
  CAPACITY =   30, 25, 21 ;
  DEMAND =   15, 17, 22, 12;
  COST =      6,  2,  6,  7,
              4,  9,  5,  3,
              8,  8,  1,  5;
ENDDATA
END
```

Model: TRAN

In this model, we see that all the data (the set names and the parameters) are explicitly contained in the model. This is really only practical for small models. For models with large amounts of data, we would typically want to keep the model separate from the data, given that the data tends to change frequently, while the model does not. Here is a version of the same model that removes the data, placing it in an external text file, and a calc section has been added to replace the original data section. The calc section uses *@READLN* and *@READRM* to read the data:

```
MODEL:
! A 3 Warehouse, 4 Customer
  Transportation Problem;
! Illustrates use of the file reading functions:
  @READLN and @READRM;
SETS:
  WAREHOUSE    : CAPACITY;
  CUSTOMER     : DEMAND;
  ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

CALC:
  !Read 1 line with number of warehouses
    and number of customers;
  NW, NC = @READLN( 'TRANREAD.LDT', 1);
  ! Read NW lines of warehouses;
  WAREHOUSE = @READLN( , NW);
  ! Read NC lines of customers;
  CUSTOMER = @READLN( , NC);
  ! Read cost data up to record
     marker: '~';
  COST = @READRM( , '~');
  ! Read capacities up to
     record marker;
  CAPACITY = @READRM( , '~');
  ! Read demand data up to end-of-file;
  DEMAND   = @READRM();
ENDCALC

! The objective;
[OBJ] MIN = @SUM( ROUTES: COST * VOLUME);

! The demand constraints;
@FOR( CUSTOMER( J): [DEM]
 @SUM( WAREHOUSE( I): VOLUME( I, J)) >=
  DEMAND( J));

! The supply constraints;
@FOR( WAREHOUSE( I): [SUP]
 @SUM( CUSTOMER( J): VOLUME( I, J)) <=
  CAPACITY( I));

END
```

Model: TRANREAD

The data file used for the model is:

```
3 4  !4 warehouses, 3 customers;
W1   !List of warehouse;
W2
W3
C1   !List of sustomers ;
C2
C3
C4
6  2  6  7
4  9  5  3
8  8  1  5 ~
30 25 21 ~
15 17 22 12
```

A brief description of the input statements used follows:

- First, we have an *@READLN* that reads one line containing the number of customers and warehouses. Note that we specify the input file name, given that this is the first call to one of the input functions.
- Second, now that we know the number of warehouses and customers, we use *@READLN* to read *NW* warehouses and another *@READLN* to read *NC* customers. Here, we do not have to specify the file name and are letting it default to the previous name.
- Finally, there are three calls to *@READRM* to read in the cost data, warehouse capacities and customer demands. The first two calls specify an end-of-record marker (here we use a tilde), but the final call has no record marker specified, meaning we simply read to the end of the file.

@INPUT('PROMPT')

The *@INPUT* function displays the prompt string *'PROMPT'* to the user and then reads their typed response from the keyboard. As an example, you will recall the staff scheduling model, *STAFFDEM*, discussed in the section *A Staff Scheduling Problem*:

```
MODEL:
SETS:
  DAYS: REQUIRED, START;
ENDSETS

DATA:
  DAYS =     MON TUE WED THU FRI SAT SUN;
  REQUIRED =  20  16  13  16  19  14  12;
ENDDATA

MIN = @SUM( DAYS( I): START( I));

@FOR( DAYS( J):
  @SUM( DAYS( I) | I #LE# 5:
     START( @WRAP( J - I + 1, 7)))
        >= REQUIRED( J)
);
END
```

Model: STAFFDEM

In this example, you will note the the daily staffing requirements are hard coded in the model with the statement:

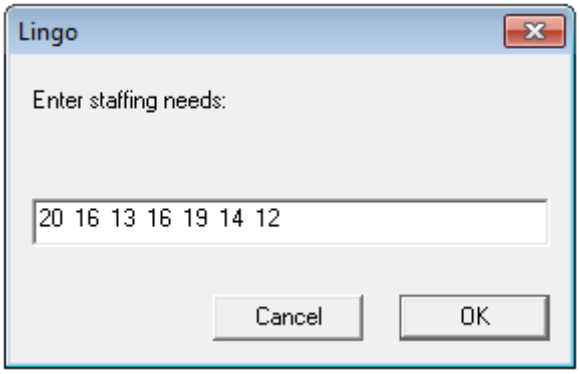
```
  REQUIRED =  20  16  13  16  19  14  12;
```

If this model were to be run frequently for different sets of staffing requirements, we might find it convenient to prompt the users for their current staffing needs. In this case, we could change the model as follows (modified sections in bold):

```
MODEL: SETS:
  DAYS: REQUIRED, START, ONDUTY;
ENDSETS

DATA:
  DAYS = MON TUE WED THU FRI SAT SUN;
ENDDATA

CALC:
  REQUIRED = @INPUT('Enter staffing needs:');
ENDCALC

MIN = @SUM( DAYS( I): START( I));


@FOR( DAYS( J):

 ONDUTY( J) =
   @SUM( DAYS( I) | I #LE# 5:
     START( @WRAP( J - I + 1, 7)));

 ONDUTY( J) >= REQUIRED( J)

);
END
```

The data is no longer hard coded, and we now prompt the user via *@INPUT* for their staffing requirements. Note that the *@INPUT* statement may only appear in programmatic (i.e., calc) sections of a model, so we have added a calc section to accommodate this.

When we run the model, we receive the following prompt for the staffing data:



To which, we've gone ahead and entered the original staffing requirements. Clicking the *OK* button will then result in the model continuing to solve, but using the runtime staffing requirements entered by the user.

Model Generation

The commands in this category are related to the model generator, i.e., the component in LINGO that translates your model's statements into a format compatible with the solver.

@GEN([SUBMODEL_NAME[, … SUBMODEL_NAME_N]]) (

The *@GEN* statement generates a model and displays the generated equations. *@GEN* converts the model into the appropriate format for the solver engine; however, it does not actually call the solver. You will primarily use *@GEN* for debugging your models.

@GEN produces a report showing all the equations in the expanded model. This report is identical to the report created by the *Solver|Generate* command. By default, the report will be sent to the terminal output window. You may use the *@DIVERT* statement to route the report to a file.

The *@GEN* statement accepts an optional argument of one or more submodel names. If a submodel is specified, LINGO will only generate the model contained within the specified submodel section. If multiple submodel names are specified, LINGO will combine them all into a single, larger model. If submodel names are omitted entirely, then LINGO will generate only those model statements occurring before the *@GEN* statement and not contained in any submodels.

As an example, below is a small staffing model that uses the *@GEN* statement in a calc section:

```
MODEL:
SETS:
 DAY / MON, TUE, WED, THU,
  FRI, SAT, SUN/ :
   NEED, START, COST;
ENDSETS

! Minimize total staffing costs;
[OBJ] MIN = @SUM( DAY( TODAY) :
 START( TODAY) * COST( TODAY));

! Subject to meeting daily needs;
   @FOR( DAY( TODAY): [CONS]
    @SUM( DAY( COUNT)| COUNT #LE# 5:
     START( @WRAP(TODAY - COUNT+1,
      @SIZE( DAY)))) >= NEED( TODAY));

DATA:
   NEED =  18 15 12 16 19 14 12;
COST = 400;
ENDDATA

CALC:
 @GEN();
ENDCALC

END
```

Running this sample model yields the following generated model report:

```
MODEL:
[OBJ] MIN= 400 * START_MON + 400 * START_TUE +
 400 * START_WED + 400 * START_THU + 400 * START_FRI +
 400 * START_SAT + 400 * START_SUN ;
[CONS_MON] START_MON + START_THU + START_FRI +
 START_SAT + START_SUN >=  18 ;
[CONS_TUE] START_MON + START_TUE + START_FRI +
 START_SAT + START_SUN >=  15 ;
[CONS_WED] START_MON + START_TUE + START_WED +
 START_SAT + START_SUN >=  12 ;
[CONS_THU] START_MON + START_TUE + START_WED +
 START_THU + START_SUN >=  16 ;
[CONS_FRI] START_MON + START_TUE + START_WED +
 START_THU + START_FRI >=  19 ;
[CONS_SAT] START_TUE + START_WED + START_THU +
 START_FRI + START_SAT >=  14 ;
[CONS_SUN] START_WED + START_THU + START_FRI +
 START_SAT + START_SUN >=  12 ;
END
```

@GENDUAL([SUBMODEL_NAME[, …, SUBMODEL_NAME_N]])

The *@GENDUAL* statement generates the *dual* formulation of a linear programming model and displays the generated equations. Every linear programming model has a corresponding, mirror-image formulation called the dual. If the original model has *M* constraints and *N* variables, then its dual will have *N* constraints and *M* variables.

Some interesting properties of the dual are that any feasible solution to the dual model provides a bound on the objective to the original, primal model, while the optimal solution to the dual has the same objective value as the optimal solution to the primal problem. It's also true that the dual of the dual model is, once again, the original primal model. You may wish to refer to any good linear programming text for a further discussion of duality theory.

For instance, replacing the *@GEN* statement with an *@GENDUAL* statement in the staffing model above, we'd have the following calc section:

```
CALC:
 @GENDUAL();
ENDCALC
```

Running this sample model yields the following generated model report showing the original model's dual formulation:

```
MODEL:
MAX = 18 * CONS_MON + 15 * CONS_TUE + 12 * CONS_WED + 16 *
CONS_THU + 19 * CONS_FRI + 14 * CONS_SAT + 12 * CONS_SUN;
[ START_MON]  CONS_MON +  CONS_TUE +  CONS_WED +  CONS_THU
+
CONS_FRI <= 400;
[ START_TUE]  CONS_TUE +  CONS_WED +  CONS_THU +  CONS_FRI
+
CONS_SAT <= 400;
[ START_WED]  CONS_WED +  CONS_THU +  CONS_FRI +  CONS_SAT
+
CONS_SUN <= 400;
[ START_THU]  CONS_MON +  CONS_THU +  CONS_FRI +  CONS_SAT
+
CONS_SUN <= 400;
[ START_FRI]  CONS_MON +  CONS_TUE +  CONS_FRI +  CONS_SAT
+
CONS_SUN <= 400;
[ START_SAT]  CONS_MON +  CONS_TUE +  CONS_WED +  CONS_SAT
+
CONS_SUN <= 400;
[ START_SUN]  CONS_MON +  CONS_TUE +  CONS_WED +  CONS_THU
+
CONS_SUN <= 400;
END
```

Note that the dual is the transpose of the original linear program, with variables becoming the new row names and row names becoming the new variable names.

@PIC([SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

The *@PIC* statement works much like *@GEN*, with the exception that it displays a matrix picture rather than the generated equations. For instance, replacing the *@GEN* statement with an *@PIC* statement in the staffing model above, we'd have the following calc section:

```
CALC:
 @PIC();
ENDCALC
```

Running the modified model will generate the following matrix picture:

```
            S S S S S S S
            T T T T T T T
            A A A A A A A
            R R R R R R R
            T T T T T T T
            ( ( ( ( ( ( (
            M T W T F S S
            O U E H R A U
            N E D U I T N
            ) ) ) ) ) ) )

      OBJ: C C C C C C C  MIN
CONS(MON): 1     1 1 1 1  > B
CONS(TUE): 1 1'  ' 1'1 1  > B
CONS(WED): 1 1 1 '   1 1  > B
CONS(THU): 1 1 1 1     1  > B
CONS(FRI): 1 1'1 1 1'  '  > B
CONS(SAT): ' 1 1 1 1 1 '  > B
CONS(SUN): '   1 1 1 1 1  > B
```

Refer to the *PICTURE* command for a discussion on how to interpret matrix picture reports.

@RELEASE(VARIABLE_NAME)

When a variable is assigned a value in a calc section, that variable is marked as being fixed at the value, i.e., subsequent optimization via *@SOLVE* will not affect the variable's value. The *@RELEASE* statement is used to release such a fixed variable so that it may once again be optimizable.

The following calc section, developed for the staff scheduling model presented above, will help to illustrate the use of *@RELEASE*:

```
CALC:
   @SOLVE();
   @FOR( DAY( D):
      ! Force starts to 0 for today;
      START( D) = 0;
      @SOLVE();
      OBJ0( D) = OBJ;
      ! Allow START( D) to be optimizable again;
      @RELEASE( START( D));
   );
   @FOR( DAY( D):
      @WRITE( DAY( D), 10*' ', OBJ0( D), @NEWLINE( 1));
   );
ENDCALC
```

This calc section loops through each day of the week and does the following:

1. Sets the number of employees starting on the day to zero: `START( D) = 0`
2. Solves for the low-cost staffing pattern given that the current day will have no starting employees: `@SOLVE()`
3. Stores the objective for a later report: `OBJ0( D) = OBJ`
4. Releases the number of employee starts for the current day so that it can be optimized in the next solve: `RELEASE( DAY( D))`

The last part of the calc section writes the following report summarizing the results:

```
MON          9800
TUE          9000
WED          8600
THU          9600
FRI          9200
SAT          8800
SUN          8600
```

@SMPI('FILE_NAME'[, SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

The *@SMPI* statement generates the model and then writes it to a specified file in MPI format. MPI is a special format developed by LINDO Systems for representing all classes of mathematical programs: linear, integer, and nonlinear. This format is not intended for permanent storage of your models. LINDO API users may be interested in this format for exporting models to the LINDO API.

The *@SMPI* statement also accepts an optional argument of one or more submodel names. If a single submodel is specified, LINGO will only generate and save the model contained within the specified submodel section. If multiple submodel names are specified, LINGO will group them all together as one larger, combined model before writing the MPI file. If no submodel name is specified, then only the model statements occurring before the *@GEN* statement and not contained in any submodels will be used in generating the MPI file.

As an example, we could write the staffing example listed above to an MPI file by modifying the calc section to be:

```
CALC:
 @SMPI( 'MYMPIFILE.MPI');
ENDCALC
```

@SMPS('FILE_NAME'[, SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

The *@SMPS* statement generates the model and then writes it to a specified file in MPS format file. The MPS file format is an industry standard format developed by IBM, and is useful for passing models from one solver or platform to another. LINDO API users may be interested in this format for exporting models to the LINDO API.
The *@SMPS* statement also accepts an optional argument of one or more submodel names. If a single submodel is specified, LINGO will only generate and save the model contained within the specified submodel section. If multiple submodel names are specified, LINGO will group them all together as one larger, combined model before writing the MPS file. If no submodel name is specified, then only

the model statements occurring before the *@GEN* statement and not contained in any submodels will be used in generating the MPS file.

As an example, we could write the staffing example listed above to an MPS file by modifying the calc section to be:

```
CALC:
 @SMPS( 'MYMPSFILE.MPS');
ENDCALC
```

Note:	A model must be linear or quadratic to successfully export it in MPS format.

Note:	When exporting an MPS file, LINGO truncates all variable names to 8 characters. For instance, the two distinct LINGO names SHIP(WH1, C1) and SHIP(WH1, C2) would both be truncated to the single 8 character name SHIPWH1C under MPS. Either choose names to avoid collisions of truncated names, or enable the option for converting names to RC format when doing MPS I/O. LINGO will display an error message if potential collisions exist.

Note:	The MPS file format is intended primarily for exporting models to other applications or platforms. The MPS format is purely scalar in nature—all set-based information is lost upon converting a LINGO model to MPS format. Thus, when saving copies of a model on your own machine, you should always use the File|Save command in order to preserve your model in its entirety.

@STATS([SUBMODEL_NAME[, ..., SUBMODEL_NAME_N]])

The *@STATS* statement works much like *@GEN*, with the exception that it displays a report containing summary statistics about the size of the generated model. Please refer to the *Solver|Model Statistics* command for more information on the fields of a STATS report.

Output Statements

The features in this section are for displaying output. In addition to the output functions discussed here, all the reporting functions discussed in Chapter 7, *Report Functions*, for data and init sections are also available for use in calc section scripts.

@SOLU([0|1[, MODEL_OBJECT[, 'REPORT_HEADER']]])

The *@SOLU* statement mimics the functionality of the *SOLU* command-line command. If all arguments are omitted, then *@SOLU* will display the default, LINGO solution report. All reports are sent to the screen unless an *@DIVERT* (see below) command is in effect, routing output to a text file.

If the first argument to *@SOLU* is 0, then only nonzero variables and binding rows will be displayed. If the first argument is 1, then all information will be displayed.

If you wish to narrow the scope of the solution report, then the optional *MODEL_OBJECT* argument can be either an attribute or row name. In which case, the report will be restricted to the specified object.

The optional third argument, '*REPORT_HEADER',* is used when you wish to place a header string on the report.

You can refer to the *LOOPOLE* model in the *Samples* folder for an example of a model that uses the *@SOLU* command

@WRITE('TEXT1'|VALUE1[, ..., 'TEXTN'|VALUEN])

The *@WRITE* statement is the primary tool you'll use for displaying output in calc sections. *@WRITE* can display both text and variable values. Text strings are delimited with single or double-quotes. All output will be directed to the screen unless *@DIVERT* (discussed below) is used to route output to a file.

@WRITE is useful for both debugging your calc section code and for generating custom reports. As an example, the following product mix model uses *@WRITE* statements in a calc section to construct a custom report:

```
MODEL:
   [PROFIT] MAX = 200 * WS + 300 * NC;
   [CHASSIS1] WS <= 60;
   [CHASSIS2] NC <= 40;
   [DRIVES] WS + 2 * NC <= 120;
CALC:
   @SOLVE();
   @WRITE( 'Total profit = ', PROFIT, @NEWLINE( 1));
   @WRITE( 'Production:', @NEWLINE( 1));
   @WRITE( '   WS = ', WS, @NEWLINE( 1),
      '   NC = ', NC, @NEWLINE( 1),
      'Total units = ', WS + NC,
      @NEWLINE( 2)
   );
   @WRITE( 'Dual analysis:', @NEWLINE( 1),
      '   Chassis1: ', @DUAL( CHASSIS1),
      @NEWLINE( 1),
      '   Chassis2: ', @DUAL( CHASSIS2),
      @NEWLINE( 1),
      '   Drives:   ', @DUAL( DRIVES),
      @NEWLINE( 1)
   );
ENDCALC
END
```

Running this model will yield the following report:

```
Total profit = 21000
Production:
   WS = 60
   NC = 30
Total units = 90

Dual analysis:
   Chassis1: 50
   Chassis2: 0
   Drives:   150
```

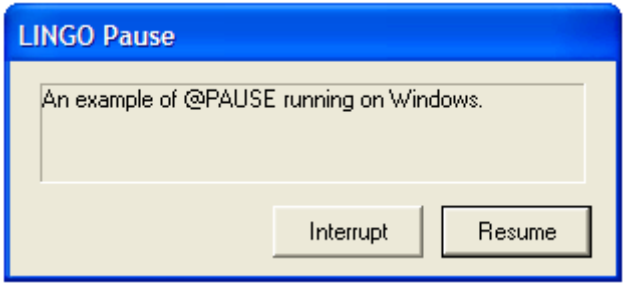
Note that in addition to the *@WRITE* statement, we also made use of both the *@NEWLINE* report function to produce line feeds and the *@DUAL* report function to return the dual values, or shadow prices, on the constraints. Additional information on these and other report functions may be found in section *Report Functions* of Chapter 7.

@PAUSE('TEXT1'|VALUE1[, ..., 'TEXTN'|VALUEN])

The *@PAUSE* statement has the same syntax as the *@WRITE* statement, however, *@PAUSE* causes LINGO to pause execution and wait for a user response. For example, under Windows, the following reference to *@PAUSE*:

```
CALC:
 @PAUSE( 'An example of @PAUSE running on Windows.');
ENDCALC
```

will cause the following dialog box to be displayed:



The user has the option of pressing either *the Resume* button to continue normal processing, or the *Interrupt* button to immediately terminate execution of the model. On platforms other than Windows, the output will be routed to the terminal and LINGO will wait for the user to press the *Enter* key.

@DIVERT(['FILE_NAME'])

By default, output generated by the *@WRITE* statement will be sent to the screen. However, you may wish to capture this output in a file. *@DIVERT* allows you to do this. As an example, we modified the product mix example from above to route its custom report to file *MYREPORT.TXT* as follows:

```
CALC:
   @SOLVE();
   @DIVERT( 'MYREPORT.TXT');
   @WRITE( 'Total profit = ', PROFIT, @NEWLINE( 1));
   @WRITE( 'Production:', @NEWLINE( 1));
   @WRITE( '   WS = ', WS, @NEWLINE( 1),
      '   NC = ', NC, @NEWLINE( 1),
      'Total units = ', WS + NC,
      @NEWLINE( 2)
   );
   @WRITE( 'Dual analysis:', @NEWLINE( 1),
      '   Chassis1: ', @DUAL( CHASSIS1),
      @NEWLINE( 1),
      '   Chassis2: ', @DUAL( CHASSIS2),
      @NEWLINE( 1),
      '   Drives:   ', @DUAL( DRIVES),
      @NEWLINE( 1)
   );
   @DIVERT();
ENDCALC
```

Note the two occurrences of *@DIVERT*. In the first instance, we specify the file we want to open, and subsequent *@WRITE* output is diverted to that file. The second reference to *@DIVERT* closes the file and reverts output back to the terminal device.

@DIVERT also accepts an optional second argument of the letter 'A'. If this argument is present, LINGO will append output to the end of the file if it already exists. If the argument is omitted, LINGO will overwrite the file if it already exists

Note: *@DIVERT* statements can be nested so that multiple levels of output files may be simultaneously in use.

Setting Parameters

LINGO has many optional settings that can be controlled with the *Solver|Options* command. At times, you may find it necessary to alter these parameters dynamically in your model's calc sections. For this reason, LINGO provides the *@SET* statement, which gives you access to the entire set of system parameters. There is also and additional function, *@APISET*, for setting more obscure parameters in the LINDO API (LINGO's solver library) that aren't available through the standard LINGO options set.

@SET('PARAM_NAME', PARAMETER_VALUE)

To change a parameter's setting, *@SET* requires that you pass it a parameter name as a text string, along with the parameter's new value. For example, to set the integer solver's relative optimality tolerance to .05 we'd use:

```
@SET( 'IPTOLR', .05);
```

A list of all the parameter names can be found in Chapter 6's discussion of the *SET* command-line command.

A parameter may be returned to its default value by omitting the *parameter_value* argument. For instance,

```
@SET('IPTOLR');
```

would return the *IPTOLR* parameter to its default value.

Use:

```
@SET( 'DEFAULT');
```

to return *all* parameters to their default settings:

Note:	Parameters whose values have been changed with *@SET* in a calc section will be restored to their original settings once the model run is completed.

@APISET(PARAM_INDEX, 'INT|DOUBLE', PARAMETER_VALUE)

LINGO uses the LINDO API solver library as its underlying solver engine. The LINDO API has a wealth of parameters that may be set by the user. Many of these parameters would be of use only in rare instances. Given this, LINGO does not provide direct access to all possible LINDO API parameters. However, if you need access to certain API parameters that aren't in the standard LINGO set, you may do so with the *@APISET* command.

To change a parameter's setting, *@APISET* requires that you pass it a parameter index, a text string of either 'INT' or "DOUBLE' indicating if the parameter is an integer or double precision quantity, along with the parameter's new value. You can refer to the LINDO API documentation (available on LINDO Systems' Web site) for a list of available parameters. A list of parameters and their indices is also in the lindo.h file included as part of your LINGO installation.

As an example, the LINDO API adds cuts in the branch-and-bound tree every 10 nodes. If you would like to add cuts more frequently, say every 5 nodes, then you could do so with the:

```
@APISET( 318, 'INT', 5);
```

You may also force all API parameters back to their default values with:

```
@APISET( 'DEFAULT');
```

Note: Parameters whose values have been changed with *@APISET* will be restored to their original settings once the model run is completed.

Procedures

PROCEDURE and ENDPROCEDURE:

LINGO allows for blocks of code known as procedures. These blocks can be called from anywhere in a calc section by simply referring to the procedure's name. Procedures can be useful for portions of calc sections that must be executed multiple times. The *PROCEDURE* and *ENDPROCEDURE* statements are used to bracket a procedure's code. The *PROCEDURE* statement must be followed immediately by a unique name that follows the normal naming conventions. The name must then be followed by a colon. You must also place an *ENDPROCEDURE* statement immediately after the last statement of your procedure. Procedures form their own model section that must lie outside data, init and calc sections. Unlike a calc section, procedures are not automatically executed when solving a model. Instead, you must reference the procedure's name in a calc section to execute its code.
As an example, the following procedure was extracted from the sample model, *PROCEDURE.LG4*:

```
PROCEDURE PRINT_REPORT:
  @WRITE( '     Start: ');
  @WRITEFOR( DAY: @FORMAT( START, '6.0f'));
  @WRITE( @FORMAT( @SUM( DAY: COST*START), '8.0f'));
  @WRITE( @NEWLINE( 1));
  @WRITE( '   On Duty: ');
  @WRITEFOR( DAY: @FORMAT( NEED + EXCESS, '6.0f'));
  @WRITE( @NEWLINE( 1));
  @WRITE( '  Required: ');
  @WRITEFOR( DAY: @FORMAT( NEED, '6.0f'));
  @WRITE( @NEWLINE( 1));
  @WRITE( '    Excess: ');
  @WRITEFOR( DAY: @FORMAT( EXCESS, '6.0f'));
  @WRITE( @NEWLINE( 2));
ENDPROCEDURE
```

This procedure prints a small report for a staffing model. The procedure is called several times from the model's main calc section to print solution reports for several permutations of the original model. Here is a code fragment from the models calc section that illustrates calling the procedure:

```
@WRITE( 'Solution 1 - min cost:', @NEWLINE( 1));
PRINT_REPORT;
```

The first line prints a header file for the report, while the second line generates the actual report by calling the *PRINT_REPORT* procedure above. A sample of one of the reports follows:

```
                MON   TUE   WED   THU   FRI   SAT   SUN TOTAL
Solution 1 - min cost:
     Start:      5     4     0     6     2     2     4  4600
   On Duty:     19    17    15    19    17    14    14
  Required:     19    17    15    19    17    14    12
    Excess:      0     0     0     0     0     0     2
```

You may wish to open *PROCECURE.LG4* in LINGO to experiment with the use of procedure calls.

You may also pass one, or more, scalar arguments to procedures. For example, the following illustrates passing a variable to a procedure that prints out the value of the variable and its square root:

```
MODEL:

SETS:
 S1: X;
ENDSETS

PROCEDURE PRINT_ROOT( Z):
  @WRITE( 'The square root of ', Z, ' is: ', @SQRT( Z));
  @WRITE( @NEWLINE( 1));
ENDPROCEDURE

DATA:
  X = 1 4 9;
ENDDATA

CALC:
    @FOR( S1( I):
    PRINT_ROOT( X( I));
 );
 @WRITE( @NEWLINE( 1));
ENDCALC


END
```

If you run this model, you should see the output:

```
The square root of 1 is: 1
The square root of 4 is: 2
The square root of 9 is: 3
```

R Interface

R is a popular statistical and graphics language (see: https://www.r-project.org). LINGO's R interface allows you to access the many features of R directly from within your LINGO models. The R interface is straightforward in that it consists of just two functions; these functions are *@R* for transferring values to and from R, and *@RCMD* for passing a command to R for execution. These two functions are available within calc sections of a LINGO model and are discussed below.

Note:	Currently, the @R and @RCMD functions are only available on Windows versions of LINGO.

In order to link to R from LINGO, you will need to do the following set up steps:

- Install a version of R that has the same bit level (32 or 64 bits) as the copy of LINGO you are using.
- The R bin folder must be on the system path. Typically, the R bin folder will have a name like C:\R\R-<version number>\bin\x64 when running a 64-bit version of R on Windows. R most likely installed this folder name in your system path during installation, but, if not, you will need to modify the path to include this folder.
- The R_HOME environment must point to the main R folder. For instance, on Windows, if you enter the SET command in a command window, you should see something like the following: R_HOME=C:/R/R-<version>, where <version> is the version number of R.
- Finally, you must install the RInside library in your copy of R. You may do this by running R and then entering the command: install.packages("RInside"). You can verify the installation of RInside by entering the R command: library("RInside").

@R(OBJECT_NAME);
This function may be used for transferring values to and from R. @R supports the transfer of sets, attributes and scalar variables to and from R. Some examples follows:

Example 1: `@R( 'r_x') = X;`
passes the LINGO attribute X to R, placing it in the R object called r_x.

Example 2: `MYSET = @R( 'r_set');`
loads the set MYSET LINGO from a R text object called r_set.

Note:	Currently, *@R* supports the transfer of one item at a time. To pass multiple items, use multiple *@R* statements.

@RCMD('COMMAND_STRING(S)');
This function may be used to pass one, or more, command strings to R. These commands need to be written in the syntax of R's command language. Some examples follow:

Example 1: `@RCMD( 'sink( /myfolder/rsink.txt')););`
opens an output sink for R in /myfolder/rsink.txt.

Example 2: `@RCMD( 'cat("x=\n"); print( x);'););`
writes out the string x= and then prints the object x.

Note: For two examples of interfacing with R, refer to the CUBS and MTCARS examples in you LINGO Samples folder. CUBS illustrates forecasting the Cubs win/loss percentage using the forecasting tools of R and MTCARS shows how to retrieve the MTCARS predefined data

Stochastic Programming

LINGO 12.0 introduced a stochastic programming solver (SP) for multistage stochastic programs with recourse. Details may be found in Chapter 14, *Stochastic Programming*. Some additional programming features were also added for use with SP models. These additional programming features are discussed below.

@SPLOADSCENE(SCENARIO_INDEX)

This function loads the solution for scenario index *SCENARIO_INDEX*. The solution for this scenario then becomes the current/active solution. As an example, the following would load the solution for scenario 1:

```
@SPLOADSCENE( 1);
```

Suppose you have an SP model with an objective row call PROFIT_ROW. The following code will compute the expected/average profit over each scenario:

```
I = 1;
EXP_PROFIT = 0;
@WHILE ( I #LE# @SPNUMSCENE()):
   @SPLOADSCENE( I);
   EXP_PROFIT = EXP_PROFIT + PROFIT_ROW;
   I = I + 1;
);
EXP_PROFIT = EXP_PROFIT / @SPNUMSCENE();
@WRITE( 'Expected Profit = ', EXP_PROFIT);
```

@SPMAX(VARIABLE|ROW)

Use *@SPMAX* to determine the maximum value across all scenarios for any specified variable or row name.

@SPMEAN(VARIABLE|ROW)

Use *@SPMEAN* to determine the mean across all scenarios for any specified variable or row name.

@SPMIN(VARIABLE|ROW)

Use *@SPMIN* to determine the minimum value across all scenarios for any specified variable or row name.

@SPNUMSCENE()

Use *@SPNUMSCENE* to retrieve the total number of scenarios in the model. You can refer to the code example immediately above for an example of using *@SPNUMSCENE()* to compute the expected/average profit over each scenario in an SP model.

@SPPRBSCENE(SCENARIO_INDEX)

Use *@SPPRBSCENE* to retrieve the probability of a specified scenario index, *SCENARIO_INDEX*. In the following example, we loop to display the probabilities for each scenario in the model:

```
CALC:
   ! Solve the SP;
   @SOLVE();

   ! Loop to display scenario probabilities;
   I = 1;
   @WHILE( I #LE# @SPNUMSCENE():
      @WRITE( ' Scenario: ', I, ' Probability: ',
       @SPPRBSCENE( I), @NEWLINE( 1));
      I = I + 1;
   );
ENDCALC
```

The output from the loop is:

```
 Scenario: 1 Probability: 0.1666666666666667
 Scenario: 2 Probability: 0.3333333333333333
 Scenario: 3 Probability: 0.5
```

@SPSTDDEV(VARIABLE|ROW)

Use *@SPSTDDEV* to determine the standard deviation of any variable or row across all scenarios.

Utility Functions

LINGO offers a pair of utility functions for sorting attributes and sets, *@RANK* and *@SORT*:

@RANK(SET|ATTRIBUTE(S))
@SORT(SET|ATTRIBUTE(S))
These functions are used to either rank or sort the values of an attribute or the members of a set. *@RANK* returns a ranking vector, listing each member of the set or attributes position/rank once sorted into ascending order, while *@SORT* returns the ordering of the set members or attributes when sorted. Both functions are only available for use in calc sections.

As an example, consider the following small model:

```
model:

sets:
 s1: x, rankx, sortx;
endsets

data:
 x = 3.2 5.7 -1.1;
enddata

calc:
 rankx = @rank( x);
 sortx = @sort( x);
endcalc

end
```

The *@RANK* function will place the following values into the RANKX attribute: 2, 3, 1, indicating that *X*(1) is the second from smallest value, *X*(2) is the largest, and *X*(3) is the smallest. *@SORT*, on the other hand, will return the indices of X in sorted order, i.e.: 3, 1, 2. You will note that these two functions are inverses of one another and that the following conditions hold:

RANK(SORT(i)) = i, and
SORT(RANK(i)) = i

In this next example, we generate a list of random numbers and then rank them with *@RANK*:

```
model:

! Generates 40,000 random numbers, ranks them
 with @RANK(), and then moves them into sorted
 order.;

sets:
 s1 /1..40000/: xrand, xrank, xsort;
endsets

calc:
  @set( 'terseo', 2);

  seed = .5;

  t0 = @time();

  @for( s1( i):
     xrand( i) = @rand( seed);
     seed = xrand( i);
  );

  t1 = @time();

  xrank = @rank( xrand);

  t2 = @time();

  @for( s1( i):
     xsort( xrank( i)) = xrand( i);
  );

  t3 = @time();

  @write(

     '  Time to generate: ', @format( t1 - t0, '8.2g'), @newline( 1),
     '  Time to rank:     ', @format( t2 - t1, '8.2g'), @newline( 1),
     '  Time to move:     ', @format( t3 - t2, '8.2g'), @newline( 2)
  );

endcalc

end
```

Model: SORTRAND

The following code generates the random numbers through the use of the *@RAND* function:

```
@for( s1( i):
   xrand( i) = @rand( seed);
   seed = xrand( i);
);
```

Then, we rank the random values via *@RANK* with:

```
xrank = @rank( xrand);
```

The random values are then moved into sorted order using an *@FOR* loop:

```
@for( s1( i):
   xsort( xrank( i)) = xrand( i);
);
```

Finally, we display the time these three operations used with the *@WRITE* statement:

```
@write(
   '  Time to generate: ', @format( t1 - t0, '8.2g'), @newline( 1),
   '  Time to rank:     ', @format( t2 - t1, '8.2g'), @newline( 1),
   '  Time to move:     ', @format( t3 - t2, '8.2g'), @newline( 2)
);
```

Running the model in LINGO will yield a report similar to the following:

```
Time to generate:     0.07
Time to rank:         0.01
Time to move:         0.05
```

Note: Multiple attributes can be passed to both *@RANK* and *@SORT* to perform multidimensional sorts, e.g.: *@SORT(X, Y, Z)*. Here, the sort would be performed first on *X*, with any ties in *X* being broken by *Y*, and so on. When specifying multiple attributes, all the attributes must be defined on the same set.

Note: Sets may also be sorted by passing the set name as the argument to both *@RANK* and *@SORT*, e.g.: *@SORT(MY_SET_NAME)*. Only one set name may be specified when sorting sets. Combining set names and attributes in the same sort is not supported. Sorting a set can be useful to present written reports in sorted order.

Note: To rank or sort an object in descending order, precede the object with a minus sign, e.g., *@RANK(-X)* would rank *X* in descending order.

Programming Example: Binary Search

In this section we will illustrate some of LINGO's programming features by developing a model to perform a *binary search*. Binary searches are an efficient way to look up a key in a sorted list. In the worst case, a binary search should perform no more than $log_2(n)$ comparisons to determine if a key is on a sorted list, where n is the number of items on the list.

The basic idea behind a binary search is that you bracket the key in the list. You then iterate by reducing the size of the bracket by a factor of 2 each pass. This process continues until you either find the key, or conclude that the key is not on the list when the bracket size becomes 0.

Note: A binary search is not typically something you would do in LINGO, and we use it here merely as an example of a simple algorithm for illustrative purposes.

The Model

The following model is an example of implementing a binary search in LINGO:

```
MODEL:
! Illustrates programming looping
  capabilities of LINGO by doing a
  binary search;
SETS:
  S1: X;
ENDSETS

DATA:
 ! The key for which we will search;
  KEY = 16;
 ! The list (must be in sorted
   increasing order);
  X = 2 7 8 11 16 20 22 32;
ENDDATA
! Do a binary search for key;
CALC:
 ! Set begin and end points of search;
  IB = 1;
  IE = @SIZE( S1);
 ! Loop to find key;
  @WHILE( IB #LE# IE:
   ! Cut bracket in half;
    LOC = @FLOOR((IB + IE)/2);
   @IFC( KEY #EQ# X(LOC):
     @BREAK; ! Do no more loops;
    @ELSE
      @IFC( KEY #LT# X( LOC):
        IE = LOC-1;
      @ELSE
        IB = LOC+1;
      );
    );
  );

@IFC( IB #LE# IE)
   ! Display key's location;
    @PAUSE( 'Key is at position: ', LOC);
  @ELSE
   ! Key not in list;
    @STOP( ' Key not on list!!!');
  @ENDIF
ENDCALC
END
```

Model: LOOPBINS

The Details

Our first step is to define a set, *S1*, and give it the attribute *X* using the following sets section:

```
SETS:
  S1: X;
ENDSETS
```

The *X* attribute will store the list that we will search for the key value. We populate *X* and input our key in the data section:

```
DATA:
 ! The key for which we will search;
  KEY = 16;
 ! The list (must be in sorted
   increasing order);
  X = 2 7 8 11 16 20 22 32;
ENDDATA
```

Note that for the purposes of this example, the list of values must be input in sorted order. An interesting exercise would be to extend this model so that it tests to determine if the data is sorted. Or, if you are really ambitious, extend the model so that it will sort a list with arbitrary ordering.

Next comes the calc section, which contains our code to do the binary search. In the first part of the calc section:

```
CALC:
   ! Set begin and end points of search;
   IB = 1;
   IE = @SIZE( S1);
```

we bracket the key by pointing to the beginning of the list, *IB*, and the end of the list, *IE*. Note that we make use of the *@SIZE* function to dynamically compute the size of the list.

Next, we have the *@WHILE* loop to search for the key value:

```
! Loop to find key;
 @WHILE( IB #LE# IE:
  ! Cut bracket in half;
   LOC = @FLOOR((IB + IE)/2);
   @IFC( KEY #EQ# X(LOC))
     @BREAK; ! Do no more loops;
 @ELSE
      @IFC( KEY #LT# X( LOC):
        IE = LOC-1;
      @ELSE
        IB = LOC+1;
      );
    );
);
```

The *@WHILE* loop tests if the candidate range has become empty at the start of each iteration:

```
@WHILE( IB #LE# IE:
```

If not, we continue by dissecting the bracket in half:

```
!cut bracket in half;
INEW = @FLOOR( ( IE + IB) / 2);
```

We then determine if the new bracket point resides above or below the key and set *IB* and *IE* accordingly:

```
@IFC( KEY #EQ# X(LOC):
  @BREAK; ! Do no more loops;
@ELSE
   @IFC( KEY #LT# X( LOC):
     IE = LOC-1;
   @ELSE
     IB = LOC+1;
   );
);
```

Finally, when we fall out of the *@WHILE* loop we display the result of our search:

```
@IFC( IB #LE# IE:
   ! Display key's location;
    @PAUSE( 'Key is at position: ', LOC);
  @ELSE
   ! Key not in list;
    @STOP( ' Key not on list!!!');
  );
```

If the eligible range is empty, then we did not find the key. Otherwise, the key was found and report its location on the list.

If you run this model, LINGO should successfully find the key and display:

Programming Example: Markowitz Efficient Frontier

In the March 1952 issue of *Journal of Finance*, Harry M. Markowitz published an article titled *Portfolio Selection*. In the article, he demonstrates how to reduce the risk of asset portfolios by selecting assets whose values aren't highly correlated. The concepts behind the Markowitz portfolio model were discussed in detail in the previous chapter in section *Markowitz Portfolio Selection Model*. The basic idea is that, given a desired level of expected return, one should select a basket of assets that minimizes risk (i.e., variance in return). Any other basket of assets would be inefficient in that it entails taking on extra risk without extra compensation.

The Markowitz model allows you to evaluate tradeoffs between risk and return. By running the model for a series of different levels of return, you can see how portfolio risk must increase as desired return increases. The return/risk tradeoff may be graphed, and this graph is known as the *efficient frontier*. If we place risk on the vertical axis and return on the horizontal, then portfolios to the left of the efficient frontier are inefficient. This is because portfolios exist with the same amount of return but less risk. Conversely, all (return,risk) pairs that lie to the right of the curve cannot be achieved given the current available asset pool. Finally, all portfolios with (return,risk) combinations that lie on the curve are efficient—no portfolio can be found with the same level of return and lower risk.

We will now demonstrate how you can use the programming capabilities in LINGO to generate points along the efficient frontier by repeatedly solving the Markowitz model for a series of portfolio return values.

The Model

Here's the model we will use to generate points along the efficient frontier:

```
MODEL:
! Solves the generic Markowitz portfolio
  model in a loop to generate the points
  on the efficient frontier;
SETS:
   ASSET: RATE, UB, X;
   COVMAT( ASSET, ASSET): V;
   POINTS: XRET, YVAR;
ENDSETS

DATA:
! Number of points on the
   efficient frontier graph;
  NPOINTS = 10;
  POINTS = 1..NPOINTS;
! The stocks;
  ASSET = GOOGLE, YAHOO, CISCO;
! Expected growth rate of each asset;
  RATE = 1.3   1.2  1.08;
! Upper bound on investment in each;
```

```
  UB   = .75   .75   .75;
! Covariance matrix;
  V    =   3     1   -.5
           1     2   -.4
         -.5   -.4     1;
ENDDATA

! Below are the three objectives we'll use;
SUBMODEL SUB_RET_MAX:
   [OBJ_RET_MAX] MAX = RETURN;
ENDSUBMODEL


SUBMODEL SUB_RET_MIN:
   [OBJ_RET_MIN] MIN = RETURN;
ENDSUBMODEL

SUBMODEL SUB_MIN_VAR:
   [OBJ_MIN_VAR] MIN =
    @SUM( COVMAT( I, J): V( I, J) * X( I) * X( J));
ENDSUBMODEL

!and the constraints;
SUBMODEL SUB_CONSTRAINTS:
   ! Compute return;
   RETURN = @SUM( ASSET: RATE * X);
   ! Must be fully invested;
   @SUM( ASSET: X) = 1;
   ! Upper bounds on each;
@FOR( ASSET: @BND( 0, X, UB));
   ! Must achieve target return;
   RETURN >= RET_LIM;
ENDSUBMODEL

CALC:
! Set some parameters;
   ! Reset all params;
   @SET( 'DEFAULT');
   ! Minimize output;
   @SET( 'TERSEO', 1);
   ! Suppress status window;
   @SET( 'STAWIN', 0);

! Capture unwanted output;
   @DIVERT( 'LINGO.LOG');

! Solve to get maximum return;
   RET_LIM = 0;
   @SOLVE( SUB_RET_MAX, SUB_CONSTRAINTS);

! Save maximum return;
   RET_MAX = OBJ_RET_MAX;

! Solve to get minimum return;
   @SOLVE( SUB_RET_MIN, SUB_CONSTRAINTS);
```

```
! Save minimum return;
   RET_MIN = OBJ_RET_MIN;

! Interval between return points;
   INTERVAL =
    ( RET_MAX - RET_MIN) / ( NPOINTS-1);

! Loop over range of possible returns
  minimizing variance;
   RET_LIM = RET_MIN;
   @FOR( POINTS( I):
      @SOLVE( SUB_MIN_VAR, SUB_CONSTRAINTS);
      XRET( I) = RET_LIM;
      YVAR( I) = OBJ_MIN_VAR;
      RET_LIM = RET_LIM + INTERVAL;
   );

! Close log file;
   @DIVERT();

! Display the results;
   @WRITE( '     Return     Variance', @NEWLINE( 1));
   @FOR( POINTS: @WRITE( @FORMAT( XRET, '#12.6G'),
    @FORMAT( YVAR, '#12.6G'), @NEWLINE( 1))
   );
ENDCALC
END
```

Model: LOOPPORT

The Details

First consider the sets section:

```
SETS:
   ASSET: RATE, UB, X;
   COVMAT( ASSET, ASSET): V;
   POINTS: XRET, YVAR;
ENDSETS
```

It defines three sets: *ASSET*, *COVMAT* and *POINTS*.

The *ASSET* set will contain the set of assets available for investment. Each asset has an expected rate of return (*RETURN*), an upper bound on the amount of the asset we'll allow in the portfolio (*UB*), and the fraction of the portfolio devoted to the asset (*X*). Note that the fraction of the portfolio devoted to each asset, *X*, constitutes our decision variables.

The *COVMAT* set is a cross of the *ASSET* set on itself. We create this set for the attribute *V*, which will store the covariance matrix for all the assets.

The *POINTS* set is used to represent the points on the efficient frontier that we will be generating. For each point, we will determine its x-coordinate, *XRET*, and its y-coordinate, *YVAR*. Note that the x-coordinate will represent risk, while the y-coordinate will represent return. What we intend to do is to solve a portfolio model once for each member of the *POINTS* set to get a new point on the efficient frontier. These points will be stored in *XRET* and *YVAR*. Once the loop is completed, the list of points in *XRET* and *YVAR* will give us a glimpse of what the efficient frontier looks like.

Next we have the data section:

```
DATA:
! Number of points on the
   efficient frontier graph;
  NPOINTS = 10;
  POINTS = 1..NPOINTS;
! The stocks;
  ASSET = GOOGLE, YAHOO, CISCO;
! Expected growth rate of each asset;
  RATE = 1.3   1.2  1.08;
! Upper bound on investment in each;
  UB   = .75   .75   .75;
! Covariance matrix;
  V    =   3     1   -.5
           1     2   -.4
         -.5   -.4     1;
ENDDATA
```

In this data section we are setting the number of points that we will generate along the efficient frontier (*NPOINTS*) to 10. Once we have the number of points established, we dynamically create the *POINTS* set. Next, we input the set of assets, their expected rate of return, their upper bounds, and their covariance matrix.

The next section of the model:

```
! Below are the three objectives we'll use;
SUBMODEL SUB_RET_MAX:
   [OBJ_RET_MAX] MAX = RETURN;
ENDSUBMODEL

SUBMODEL SUB_RET_MIN:
   [OBJ_RET_MIN] MIN = RETURN;
ENDSUBMODEL

SUBMODEL SUB_MIN_VAR:
   [OBJ_MIN_VAR] MIN =
    @SUM( COVMAT( I, J): V( I, J) * X( I) * X( J));
ENDSUBMODEL
```

makes use of the *SUBMODEL and ENDSUBMODEL* statements to establish three different objectives. The first two objectives respectively maximize and minimize portfolio return, which are used later on in the model to determine that maximum and minimum possible returns that can be generated with out basket of available stocks. The third objective, *SUB_MIN_VAR*, minimizes portfolio risk as measured by its variance.

Following the three submodels containing our various objectives, we have another submodel that contains our three constraints:

```
!and the constraints;
SUBMODEL SUB_CONSTRAINTS:
   ! Compute return;
   RETURN = @SUM( ASSET: RATE * X);
   ! Must be fully invested;
   @SUM( ASSET: X) = 1;
   ! Upper bounds on each;
   @FOR( ASSET: @BND( 0, X, UB));
   ! Must achieve target return;
   RETURN >= RET_LIM;
ENDSUBMODEL
```

The first constraint of our constraint section computes portfolio return. The second constraint says that we must invest 100 percent of our capital. The third constraint puts an upper bound on the percentage of the portfolio invested in each asset. Finally, the fourth constraint forces total portfolio return to achieve some desired level.

The next section, the calc section, is of primary interest. It contains the logic to solve the model multiple times to generate and store the points on the efficient frontier. First, we make use of *@SET* to set some parameter values:

```
! Set some parameters;
   ! Reset all params to their defaults; @SET( 'DEFAULT');
   ! Minimize output;
   @SET( 'TERSEO', 1);
   ! Suppress status window;
   @SET( 'STAWIN', 0);
```

The first call to *@SET* restores all parameters to their default values, the second call minimizes the level of LINGO's output to improve performance, while the third call suppresses the status window that would normally pop up when solving a model. We suppress the status window so it does not obscure the custom report we are creating at the end of the run.

Next, we use *@DIVERT* to route LINGO's output to a log file:

```
! Capture spurious output;
   @DIVERT( 'LINGO.LOG');
```

We do this to capture the output LINGO would normally generate while it's solving the various portfolio models. We capture this output so it doesn't distract from the custom report we will display at the end of the run.

Our next task is to find out the possible range of portfolio returns. We want this information so that we don't try to run our model for returns lying outside this range, which would result in an infeasible model. This requires two runs: one where we maximize return and one where we minimize return. Here is the code that performs the runs:

```
! Solve to get maximum return;
   RET_LIM = 0;
   @SOLVE( SUB_RET_MAX, SUB_CONSTRAINTS);

! Save maximum return;
   RET_MAX = OBJ_RET_MAX;

! Solve to get minimum return;
   @SOLVE( SUB_RET_MIN, SUB_CONSTRAINTS);

! Save minimum return;
   RET_MIN = OBJ_RET_MIN;
```

This is our first example of the the *@SOLVE* command. *@SOLVE* takes one or more submodel names as arguments. It then combines the submodels into a single model and solves them. In this case, we first solve submodel *SUB_RET_MAX* along with submodel *SUB_CONSTRAINTS* to get the maximum return possible subject to our constraint set. We then do a similar solve to find the minimal return subject to the constraints. Other items of interest are that we zero out *RET_LIM*, given that the constraint on return is temporarily not required, and we store the two extreme objective values in *RET_MAX* and *RET_MIN* for later use.

Our end goal is to solve the model for 10 values of portfolio return, with these values being equally spaced from *RET_MIN* to *RET_MAX*. The next step in the model computes the distance, or interval, between these points:

```
! Interval between return points;
INTERVAL =
    ( RET_MAX - RET_MIN) / ( NPOINTS-1);
```

Our next code segment is an *@FOR* loop that loops over each of the 10 return values and minimizes portfolio variance subject to attaining the desired level of return:

```
! Loop over range of possible returns
  minimizing variance;
   RET_LIM = RET_MIN;
   @FOR( POINTS( I):
      @SOLVE( SUB_MIN_VAR, SUB_CONSTRAINTS);
      XRET( I) = RET_LIM;
      YVAR( I) = OBJ_MIN_VAR;
      RET_LIM = RET_LIM + INTERVAL;
   );
```

We start by setting the desired level of return to *RET_MIN* and then increment this level of return each pass through the loop by the interval amount. Note that the solve command now uses the submodel containing the objective that minimizes return, *SUB_MIN_VAR*. Also, after each solve command, we store the coordinates of the point on the efficient frontier in *XRET* and *YRET*.

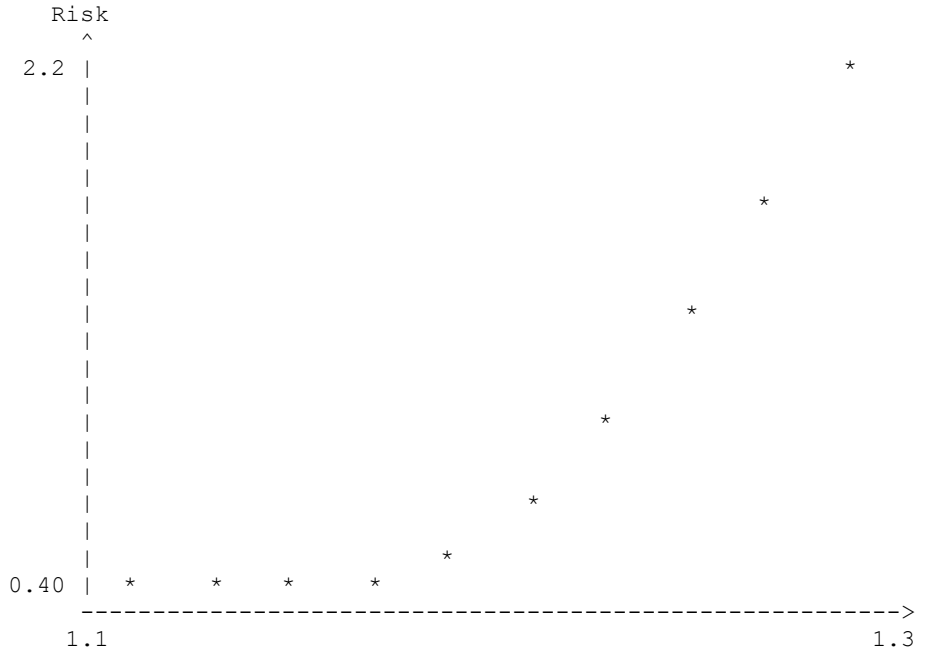
Our next section of code creates a custom report:

```
! Close log file;
   @DIVERT();

! Display the results;
   @WRITE( '     Return     Variance', @NEWLINE( 1));
   @FOR( POINTS: @WRITE( @FORMAT( XRET, '#12.6G'),
    @FORMAT( YVAR, '#12.6G'), @NEWLINE( 1))
   );
```

Once again, we make use of *@DIVERT*, but this time we do not pass an argument. This results in output once again going to the report window, which will allow us to view our report on screen. After restoring terminal output with *@DIVERT*, we use the *@WRITE* command inside an *@FOR* loop to write out the 10 points on the efficient frontier. If you run the model, you should see the following report:

```
Return     Variance
1.11000    0.417375
1.12833    0.417375
1.14667    0.418054
1.16500    0.462381
1.18333    0.575957
1.20167    0.758782
1.22000     1.01086
1.23833     1.33218
1.25667     1.72275
1.27500     2.18750
```

In summary, returns range from 11% to a high of 27.5%, with portfolio variance ranging from .417 to 2.18. One final note, if you load this model from your Lingo samples folder you will find additional Lingo code at the end devoted to graphing the frontier. We will not go into the details of that code at this point; however, the generated graph is pictured below:



Model: LOOPPORT – Graph of Efficient Frontier

Programming Example: Cutting Stock

An optimization problem in the paper, steel, and wood industries is the *cutting-stock* problem. The main feature of this problem is that finished goods of varying lengths are cut from larger raw material pieces of varying lengths. The goal is to cut the raw materials using an efficient set of patterns to minimize the total amount of raw materials required, while meeting demand for all finished products. Examples would include sheet metal and paper manufacturers that take larger rolls of product and cut them into smaller rolls for sale to customers.

As an example, suppose you are a steel fabricator that takes 45 foot steel beams and cuts them into 14, 12 and 7 foot beams for sale to construction firms. Cutting one 45 foot beam into one 14 foot piece, two 12 foot pieces and one 7 foot piece would be very efficient in that it would exactly use up the 45 foot raw material piece with no trim waste. On the other hand, a pattern of one 14 foot piece, one 12 foot piece and two seven foot pieces would not be as efficient due to a 5 foot piece of trim waste.

A brute force method for attacking the cutting-stock problem would be to generate all possible efficient patterns and run an optimization model to determine how may copies of each pattern to run to satisfy demand at minimal cost. The drawback here is that the number of patterns grows exponentially as the number of different finished good lengths increase. For all but the smallest problems, brute force pattern generation will yield large, intractable models.

Gilmore and Gomory published a paper in 1961 titled *A Linear Programming Approach to the Cutting-Stock Problem*. In this paper they outline a two-stage, iterative approach for solving cutting-stock problems that dramatically reduced the number of patterns one must generate to get good solutions. The basic idea involves solving a master problem containing a limited number of patterns. The dual prices on the finished goods are then passed to a small knapsack subproblem that selects a new cutting pattern that maximizes the sum of the dual values of all the finished goods contained in the pattern subject to not exceeding the length of the raw material. This pattern is then appended to the previous master problem, which is then re-solved. This iterative process continues until no further beneficial patterns can be generated by the knapsack subproblem. In which case, we have the optimal solution to the original, linear cutting-stock problem. The remarkable feature of this algorithm is that it typically takes relatively few passes to reach the optimal solution, thereby making it possible to solve very large cutting-stock models in an extremely reasonable amount of time. This approach of iteratively appending new columns to models is also referred to as *column generation*.

The Model

For our example, we will be cutting 45 foot wide rolls of paper into smaller rolls of widths: 34, 24, 15, 10 and 18. We use Lingo's programming capability to iteratively solve the master and subproblem until no further beneficial cutting patterns remain to be appended to the master problem.

```
MODEL:
! Uses Lingo's programming capability to do
  on-the-fly column generation for a
  cutting-stock problem;
SETS:
  PATTERN: COST, X;
  FG: WIDTH, DEM, PRICE, Y, YIELD;
  FXP( FG, PATTERN): NBR;
ENDSETS

DATA:
  PATTERN = 1..20; ! Allow up to 20 patterns;
  RMWIDTH = 45;    ! Raw material width;
  FG =  F34  F24  F15  F10 F18;!Finished goods...;
  WIDTH= 34   24   15   10  18;!their widths...;
  DEM = 350  100  800 1001 377;!and demands;
  BIGM = 999;
ENDDATA

SUBMODEL MASTER_PROB:
  [MSTROBJ] MIN= @SUM( PATTERN( J)| J #LE# NPATS:
   COST( J)*X( J));
  @FOR( FG( I):
   [R_DEM]
```

```
    @SUM( PATTERN( J)| J #LE# NPATS:
     NBR( I, J) * X( J)) >= DEM( I);
  );
ENDSUBMODEL

SUBMODEL INTEGER_REQ:
  @FOR( PATTERN: @GIN( X));
ENDSUBMODEL

SUBMODEL PATTERN_GEN:
  [SUBOBJ] MAX = @SUM( FG( I): PRICE( I)* Y( I));
  @SUM( FG( I): WIDTH( I)*Y( I)) <= RMWIDTH;
  @FOR( FG( I): @GIN(Y( I)));
ENDSUBMODEL

CALC:
  ! Send unwanted output to log file;
  @DIVERT( 'LINGO.LOG');

  ! Set parameters;
@SET( 'DEFAULT');
  @SET( 'TERSEO', 1);
  @SET( 'STAWIN', 0);

  ! Max number of patterns we'll allow;
  MXPATS = @SIZE( PATTERN);
  ! Make first pattern an expensive super pattern;
  COST( 1) = BIGM;
  @FOR( FG( I): NBR( I, 1) = 1);

  ! Loop as long as the reduced cost is
    attractive and there is space;
  NPATS = 1;
  RC = -BIGM;   ! Clearly attractive initially;
  @WHILE( RC #LT# 0 #AND# NPATS #LT# MXPATS:
    ! Solve for current best pattern runs;
    @SOLVE( MASTER_PROB);
    ! Copy dual prices to PATTERN_GEN submodel;
    @FOR( FG( I): PRICE( I) = -@DUAL( R_DEM( I)));
    ! Generate the current most attractive pattern;
    @SOLVE( PATTERN_GEN);
    ! Marginal value of current best pattern;
    RC = 1 - SUBOBJ;
    ! Add the pattern to the Master;
    NPATS = NPATS + 1;
    @FOR( FG( I): NBR( I, NPATS) = Y( I));
    COST( NPATS) = 1;
  );

  ! Finally solve Master as an IP;
  @SOLVE( MASTER_PROB, INTEGER_REQ);

  ! Redirect output back to terminal;
```

```
  @DIVERT();
ENDCALC
END
```

Model: LOOPCUT

The Details

First, we have the sets section:

```
SETS:
  PATTERN: COST, X;
  FG: WIDTH, DEM, PRICE, Y, YIELD;
  FXP( FG, PATTERN): NBR;
ENDSETS
```

The *PATTERN* set is used to represent the cutting patterns we will be generating. Each pattern has a cost, which, with one exception (discussed below), will be 1, i.e., each pattern uses 1 raw material piece. We also assigned an attribute called *X* to the patterns set. *X(p)* will be used to store the number of times each pattern *p* is to be cut and is one of the decision variables.

The *FG* set is used to represent the set of finished goods. As input, each finished good has a width and customer demand. The *PRICE* attribute will be used to store the dual prices on the finished goods. These prices will be updated each time we solve the master problem. *Y* will be an integer, decision variable that we will use in the knapsack subproblem to represent the number of pieces of each finished good to use in the next generated pattern. *YIELD* will be used to store the number of pieces of each finished good that gets produced.

The derived set, *FXP*, is derived on the finished goods and patterns sets. The attribute *NBR(i, j)* will store the number of finished goods *I* contained in pattern *j*.

Next, we have the data section

```
DATA:
  PATTERN = 1..20; ! Allow up to 20 patterns;
  RMWIDTH = 45;    ! Raw material width;
  FG =  F34  F24  F15  F10 F18;!Finished goods...;
  WIDTH= 34   24   15   10  18;!their widths...;
  DEM = 350  100  800 1001 377;!and demands;
  BIGM = 999;
ENDDATA
```

We initialize the pattern set to have 20 members. This will only allow for generation of up to 20 patterns; however, this should be more that adequate for this small example.

After inputting the raw material width of 45, we input the set of five finished goods and their widths. After that, we input the demands for the finished goods. Finally, we input a parameter called *BIGM*, the purpose of which is discussed below.

Next, we have our first submodel:

```
SUBMODEL MASTER_PROB:
  [MSTROBJ] MIN= @SUM( PATTERN( J)| J #LE# NPATS:
   COST( J)*X( J));
  @FOR( FG( I):
   [R_DEM]
    @SUM( PATTERN( J)| J #LE# NPATS:
     NBR( I, J) * X( J)) >= DEM( I);
  );
ENDSUBMODEL
```

This is the master problem we've been mentioning. The objective states that we wish to minimize the total cost of all the patterns used. The constraint says that we must meet, or exceed, demand for all the finished goods.

The next submodel:

```
SUBMODEL INTEGER_REQ:
  @FOR( PATTERN: @GIN( X));
ENDSUBMODEL
```

will be used in conjunction with the master problem to force the variables to take on integer values. Given that it's not possible to cut a fractional number of a particular pattern, for our final solution we need the *X* variables to be integer valued.

Our final submodel:

```
SUBMODEL PATTERN_GEN:
  [SUBOBJ] MAX = @SUM( FG( I): PRICE( I)* Y( I));
  @SUM( FG( I): WIDTH( I)*Y( I)) <= RMWIDTH;
  @FOR( FG( I): @GIN(Y( I)));
ENDSUBMODEL
```

is the pattern-generation subproblem we've mentioned. This is a knapsack problem that finds the best pattern that can be cut given the dual prices on the finished goods. Recall that we get the dual prices on the finished goods from the demand constraints in the master problem.

We then enter the calc section, which contains the programming logic to coordinate the iterative algorithm we've chosen. As with the previous Markowitz model, the start of the calc section is devoted to diverting output to a file and setting of parameters. You may refer to the previous Markowitz model for a discussion of why these steps are being performed.

One of the features of this model most likely to change in the future would be the maximum number of patterns to generate. It's probably not wise to assume that this number will always be fixed at 20 patterns. For this reason, we use the *@SIZE* function to get the current number of allowed patterns:

```
! Max number of patterns we'll allow;
MXPATS = @SIZE( PATTERN);
```

Next, we construct what we refer to as a "super pattern":

```
! Make first pattern an expensive super pattern;
COST( 1) = BIGM;
@FOR( FG( I): NBR( I, 1) = 1);
```

The supper pattern is a device to jumpstart our algorithm by guaranteeing that the model will be feasible from the start. We need the model to always be feasible in order to obtain a valid set of dual prices to feed into the pattern generation submodel. The super pattern is an artificial pattern that can create one piece of each finished good. In real life, such a pattern would not be possible because we can't physically fit one of each finished good on a single raw material piece. Given that the super pattern is not physically possible, we assign it a high cost, *BIGM*, so it will not be used in the final solution

Recall that *NBR(i, j)* represents the number of finished good *i* in pattern *j*. So, for our super pattern, we use a for loop over all the finished goods *i,* setting *NBR(i, 1)* to 1 for each of the finished goods. The second index of *NBR* is set at 1 given that the super pattern is the first pattern generated.

Next, the main loop that will alternate between solving the master and subproblem until an optimal solution is found:

```
! Loop as long as the reduced cost is
  attractive and there is space;
NPATS = 1;
RC = -BIGM;   ! Clearly attractive initially;
@WHILE( RC #LT# 0 #AND# NPATS #LT# MXPATS:
  ! Solve for current best pattern runs;
  @SOLVE( MASTER_PROB);
  ! Copy dual prices to PATTERN_GEN submodel;
  @FOR( FG( I): PRICE( I) = -@DUAL( R_DEM( I)));
  ! Generate the current most attractive pattern;
  @SOLVE( PATTERN_GEN);
  ! Marginal value of current best pattern;
  RC = 1 - SUBOBJ;
  ! Add the pattern to the Master;
  NPATS = NPATS + 1;
  @FOR( FG( I): NBR( I, NPATS) = Y( I));
  COST( NPATS) = 1;
);
```

First, we set the pattern count, *NPATS*, to 1 for the one pattern already generated, i.e., the super pattern. We will increment *NPATS* each time we generate a new pattern in our main loop.

Next, is the main *@WHILE* loop, which will execute until its condition evaluates false. We test two things in the condition.

First, we see if the marginal impact of the current pattern, *RC*, is negative and will, therefore, reduce the current cost. If the current pattern will reduce our cost, then this implies that we should continue looping and generating patterns, because subsequent patterns may also be helpful. Once the marginal benefit of the current best pattern goes to zero, generating subsequent patterns will be of no benefit; if the current best pattern can't help, subsequent patterns of less value will also be of no help. In which case, we should exit our loop.

The second condition on the loop tests to see if we've generated the maximum allowed number of patterns that we've allocated space for. If so, we must exit the loop.

Once in the loop, our first step is to invoke the *@SOLVE* command to solve the master problem. We then use an *@FOR* loop to pass the dual values on the finished goods to the pattern generation subproblem. We then solve the pattern generation subproblem and compute *RC,* the marginal rate of decrease in the main objective using the new pattern. Finally, at the end of the loop we append the new generated pattern to the master program for the next pass through the loop.

Once we can no longer find a cutting pattern that will further reduce our costs, we exit the loop and solve the master problem one last time:

```
! Finally solve Master as an IP;
@SOLVE( MASTER_PROB, INTEGER_REQ);
```

The one difference is that we now include the integer restrictions on the pattern usage array, *X.* As mentioned, cutting fractional quantities of a pattern is not physically possible, so we want our final solution to be integral.

The version of this model in the samples folder has an additional calc section to prepare a tabular report on the cutting patterns and their usage. We will not go into the details of this table-generating code in this discussion. However, if you run the model, you should receive the following report:

```
Total raws used:    985

Total feet yield:   43121
Total feet used:    44325

Percent waste:      2.7%

                    Pattern:
 FG  Demand Yield   1   2   3   4   5   6   7   8   9
=====================================================
F34    350   350    1   .   .   .   1   .   .   .   .
F24    100   100    1   .   .   .   .   1   .   .   1
F15    800   801    1   .   3   .   .   .   1   1   .
F10   1001  1002    1   4   .   .   1   2   1   3   .
F18    377   377    1   .   .   2   .   .   1   .   1
=====================================================
            Usage:  0   0 133   0 350   0 277 125 100
```

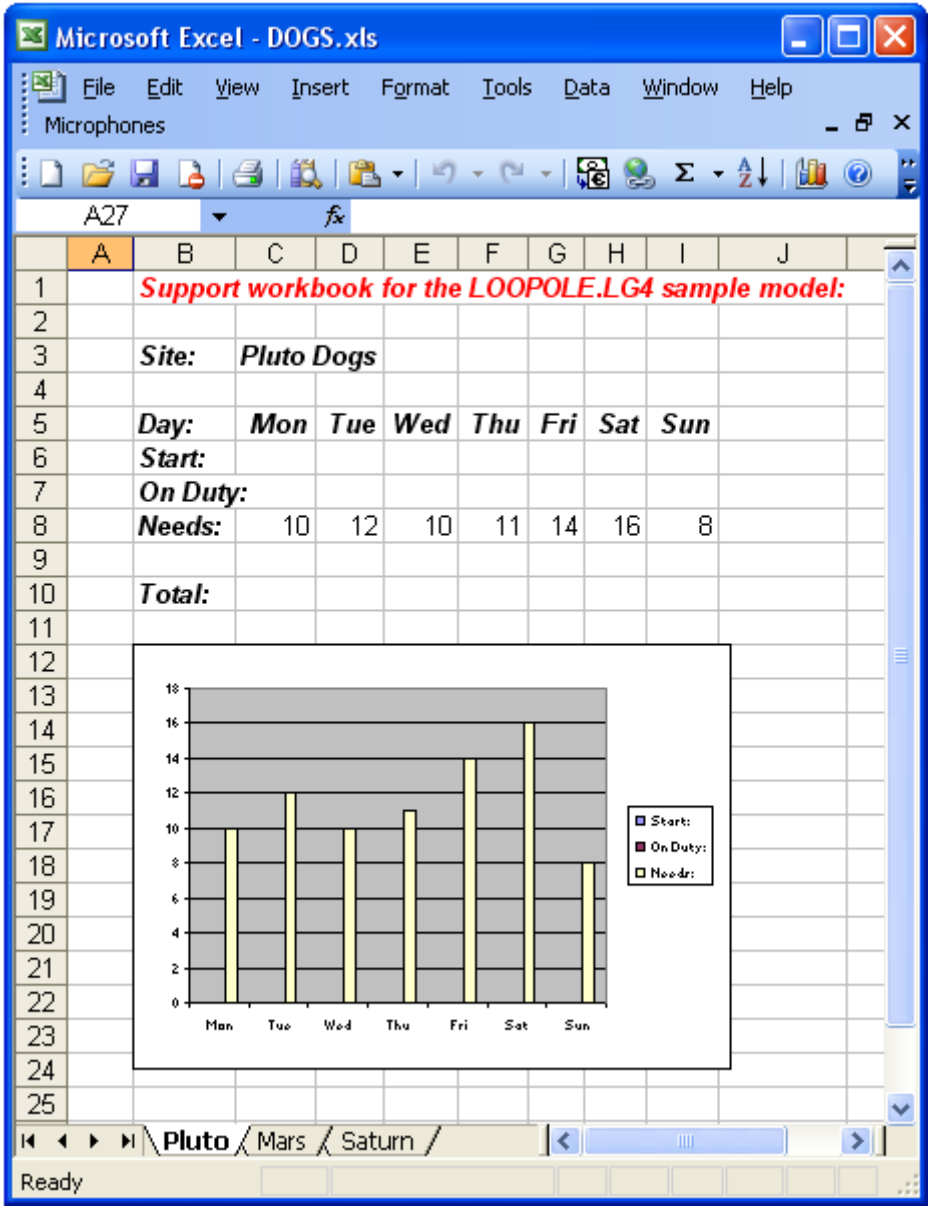
A total of 985 raw materials were used, with only 2.7% of the raw material input lost to trim waste. Note that only 9 patterns were needed to solve the model. Also note that Pattern 1, the super pattern, is not used in the final solution as per our design.

Programming Example: Accessing Excel

In this example, we will solve the familiar staff scheduling model once again. However, to make things interesting, we will solve several, independent models in a loop. Furthermore, the data for our models will be contained in a single Excel workbook, with each tab in the workbook storing data for an individual instance of the model. Solutions for each site will also be returned to their respective tabs in the Excel workbook.

Suppose we have three hot dog stands: Pluto Dogs, Saturn Dogs and Mars Dogs. Each site has daily staffing needs that vary throughout the week. We hire employees to work 5 continuous days in a row, followed by two days off. As an example, an employee starting on Tuesday would be on duty Tuesday through Saturday and off Sunday and Monday, while someone starting on Friday would work Friday through Tuesday and be off Wednesday and Thursday. We need to know how many employees to start on each day of the week at each site so as to minimize to total number of staff hired.

The data for this model may be found in the *DOGS.XLS* workbook contained in the *Samples* folder off the main LINGO folder. If you open this workbook, you'll see there are three tabs—*Pluto*, *Saturn* and *Mars*. Each tab contains the data for its particular site. The tab for the Pluto site appears below:



DOGS.XLS

Here we see that the Pluto site requires 10 people on Mondays, 12 on Tuesdays, and so on. The staffing needs are also represented in the bar graph on the tab. Similar tabs for the remaining two sites are also included in the workbook.

The Model

Here our model loops over the three sites/tabs, solving each individual staffing model:

```
MODEL:
! Uses a loop to solve three staff scheduling
  models, retrieving the data from and writing
  the solution to an Excel workbook;

SETS:
  SITES / PLUTO, MARS, SATURN/;
  DAYS / MON TUE WED THU FRI SAT SUN/:
   NEEDS, START, ONDUTY;
ENDSETS

SUBMODEL STAFF:
  [OBJROW] MIN = @SUM( DAYS: START);

  @FOR( DAYS( D):
     ONDUTY( D) = @SUM( DAYS( D2) | D2 #LE# 5:
       START( @WRAP( D - D2 + 1, @SIZE( DAYS))));
     ONDUTY( D) >= NEEDS( D);
  );

  @FOR( DAYS: @GIN( START));
ENDSUBMODEL

CALC:
  @SET( 'TERSEO', 2);

  @FOR( SITES( S):
     NEEDS = @OLE( '\LINGO\SAMPLES\DOGS.XLS',
      SITES( S)+'NEEDS');

     @SOLVE( STAFF);

     @SOLU( 0, ONDUTY, '  On Duty Report: ' + SITES( S));

     @OLE( , SITES( S)+'START', SITES( S)+'ONDUTY',
       SITES( S)+'TOTAL') = START, ONDUTY, OBJROW;
  );
ENDCALC
END
```

Model: LOOPOLE

This model may also be found in the Samples folder of the main LINGO folder.

Note: Versions of this example staffing model that interface with an Access database or that interface with text files are also available. Please see models *LOOPODBC* and *LOOPTEXT*.

The Details

In the model's sets section:

```
SETS:
  SITES / PLUTO, MARS, SATURN/;
  DAYS / MON TUE WED THU FRI SAT SUN/:
   NEEDS, START, ONDUTY;
ENDSETS
```

we declare two sets. The first set is our set of three sites followed by a set containing the days of the week. The days of the week set has three associated attributes: *NEEDS, START,* and *ONDUTY,* representing the staffing needs, the number of workers to start on each day of the week, and the total number of workers on duty each day.

Next, we declare a submodel that contains the staffing model that will be applied to each of the three sites:

```
SUBMODEL STAFF:
  [OBJROW] MIN = @SUM( DAYS: START);

  @FOR( DAYS( D):
     ONDUTY( D) = @SUM( DAYS( D2) | D2 #LE# 5:
       START( @WRAP( D - D2 + 1, @SIZE( DAYS))));
     ONDUTY( D) >= NEEDS( D);
  );

  @FOR( DAYS: @GIN( START));
ENDSUBMODEL
```

The objective function minimizes the total number of employees hired at the site. This is then followed by an *@FOR* loop over the days of the week that a) computes the number of employees on duty each day, and b) requires the number on duty to equal, or exceed, the number required on each day. We also require the number starting on each day to be an integer value using the *@GIN* statement.

The following calc section:

```
CALC:
  @SET( 'TERSEO', 2);

  @FOR( SITES( S):
     NEEDS = @OLE( '\LINGO\SAMPLES\DOGS.XLS',
      SITES( S)+'NEEDS');

     @SOLVE( STAFF);

     @SOLU( 0, ONDUTY, '  On Duty Report: ' + SITES( S));

     @OLE( , SITES( S)+'START', SITES( S)+'ONDUTY',
       SITES( S)+'TOTAL') = START, ONDUTY, OBJROW;
  );
ENDCALC
```

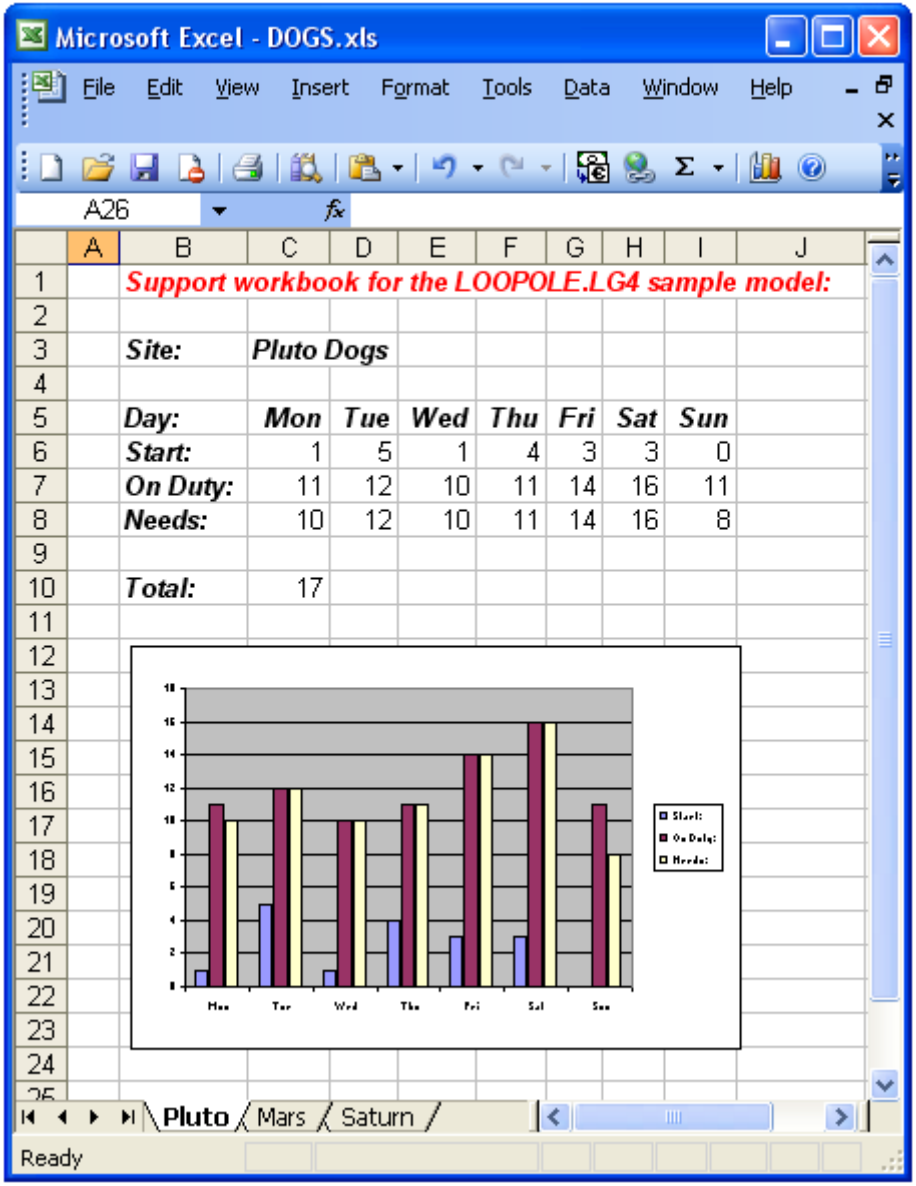
starts off by restricting the amount of LINGO's output by setting the *TERSEO* parameter to 2, which essentially eliminates all of LINGO"s output.

Next, we construct an *@FOR* loop that loops over the three sites, allowing us to solve the submodel *STAFF* for each site. The first statement in the loop is an *@OLE* statement that reads the staffing requirements from the appropriate tab of the workbook and places the requirements into the *NEEDS* attribute. If you look at the workbook closely, you'll see that there are three ranges named "SaturnNeeds", "PlutoNeeds" and "MarsNeeds" that contain the staffing requirements for each of the respective sites. We are building each of these range names via the `SITES(S)+'NEEDS'` argument in the *@OLE* function call.

Once the staffing requirements are loaded, we invoke the *@SOLVE* command to solve the staffing model for the current site. The *@SOLU* command prints out a small solution report to the screen showing the number of workers on duty each day for each of the sites.

At the bottom of the calc section, the last *@OLE* call sends the solution back to the workbook. Note that for each site there are three ranges titled *"<site_name>*START", "*<Isite_name>*ONDUTY" and "*<site_name>*TOTAL", where we return the daily number of workers to start, the number on duty each day, and the total number of employees required for the entire week. Note also that the workbook name was omitted in the call. When the workbook name is omitted, LINGO simply uses the current workbook in Excel. Given that we specified the workbook name in the original call to *@OLE*, it should now be the current workbook, thereby allowing us to omit its name in the final call.

If you solve the model, you'll see that the tab for each of the three sites contains the number of workers to start each day, the total number on duty each day, and the total number of employees required. For the Saturn site, the tab will resemble the following:



Summary

There are many instances where you will need to solve a model multiple times and/or use the results of one model as input into another. The programming capabilities in LINGO allow you to automate these classes of problems. The three examples presented in this chapter introduce the programming constructs available in LINGO.

In addition to the three examples presented in this chapter, several more examples of solving models in a loop may be found in the samples folder of your LINGO installation. These additional examples include:

> ***Loopdea.lg4*** – A Data Envelopment Analysis model with a loop over each of the decision-making units to compute their individual efficiency scores.
>
> ***Loopts.lg4*** – A traveling salesman model that loops to append subtour breaking constraints to the formulation, continuing until a complete tour of the cities is generated.
>
> ***Loopttt.lg4*** – A looping model that implements a game of ticktacktoe.

Finally, in addition to all the control structures and functions discussed in this chapter, all of the functions and operators listed in Chapter 7, LINGO's Operators and Functions, are also available for use in your calc section scripts, with the only exception being the variable domain functions. Variable domain functions are restricted for use in model section only.

14 Stochastic Programming

So far, we worked with deterministic mathematical programs where all model parameters (e.g. coefficients, bounds, etc.) are known constants. A stochastic program (SP) is a mathematical program (linear, nonlinear or mixed-integer) in which some of the model parameters are not known with certainty, and the uncertainty can be expressed with known probability distributions. Applications arise in a variety of industries:

- Financial portfolio planning over multiple periods for insurance and other financial companies, in face of uncertain prices, interest rates, and exchange rates
- Exploration planning for petroleum companies,
- Fuel purchasing when facing uncertain future fuel demand,
- Fleet assignment: vehicle type to route assignment in face of uncertain route demand,
- Electricity generator unit commitment in face of uncertain demand,
- Hydro management and flood control in face of uncertain rainfall,
- Optimal time to exercise for options in face of uncertain prices,
- Capacity and Production planning in face of uncertain future demands and prices,
- Foundry metal blending in face of uncertain input scrap qualities,
- Product planning in face of future technology uncertainty,
- Revenue management in the hospitality and transport industries.

Stochastic programs fall into two major categories: a) multistage stochastic programs with recourse, and b) chance-constrained programs.

Multistage Decision Making Under Uncertainty

In this chapter, the term *stochastic program* (SP) refers to a multistage stochastic model with recourse. The term *stage* is an important concept in this chapter. Usually it means the same as 'time period', however there are situations where a stage may consist of several time periods. The terms *random*, *uncertain* and *stochastic* are used interchangeably.

Multistage decision making under uncertainty involves making optimal decisions for a T-stage horizon before uncertain events (random parameters) are revealed while trying to protect against unfavorable outcomes that could be observed in the future.

Note: A stage boundary is either a) the beginning of the problem, b) the end of the problem, or c) a point just after a decision event but just before a random event. A stage is the sequence of random events followed by decision events between two successive stage boundaries. Thus, decisions made in stage k are based on all information revealed up to stage k, but no more.

In its most general form, a multistage decision process with T+1 stages follows an alternating sequence of random events and decisions. Slightly more explicitly:

0.1) in stage 0, we make a decision x_0, taking into account that…

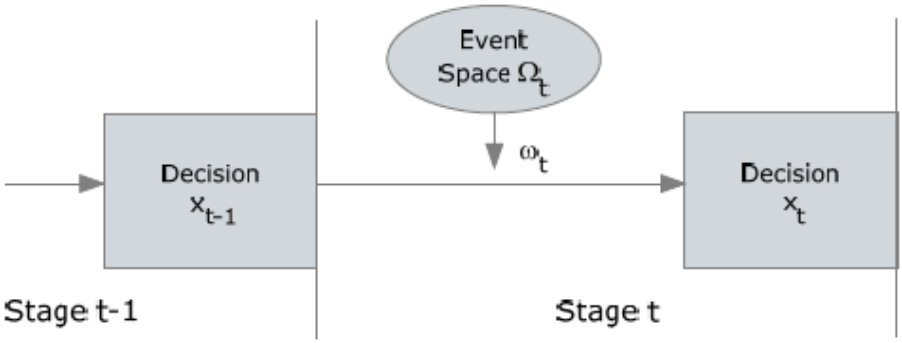
1.0) at the beginning of stage 1, "Nature" takes a set of random decisions ω_1, leading to realizations of all random events in stage 1, and…
1.1) at the end of stage 1, having seen nature's decision, as well as our previous decision, we make a recourse decision $x_1(\omega_1)$, taking into account that …

2.0) at the beginning of stage 2, "Nature" takes a set of random decisions ω_2, leading to realizations of all random events in stage-2, and…
2.1) at the end of stage 2, having seen nature's decision, as well as our previous decisions, we make a recourse decision $x_2(\omega_1, \omega_2)$, taking into account that …

.
.
.

T.0) At the beginning of stage T, "Nature" takes a random decision, ω_T, leading to realizations of all random events in stage T, and…
T.1) at the end of stage T, having seen all of nature's T previous decisions, as well as all our previous decisions, we make the final recourse decision $x_T(\omega_1,\ldots,\omega_T)$.

This relationship between the decision variables and realizations of random data can be illustrated as follows.



Each decision, represented with a rectangle, corresponds to an uninterrupted sequence of decisions until the next random event. And each random observation corresponds to an uninterrupted sequence of random events until the next decision point.

Recourse Models

The decision taken in stage 0 is called the *initial decision*, whereas decisions taken in succeeding stages are called *recourse decisions*. Recourse decisions are interpreted as corrective actions that are based on the actual values the random parameters realized so far, as well as the past decisions taken thus far. Recourse decisions provide latitude for obtaining improved overall solutions by realigning the initial decision with possible realizations of uncertainties in the best possible way.

Restricting ourselves to linear multistage stochastic programs for illustration, we have the following form for a multistage stochastic program with (T+1) stages.

$$\text{Minimize (or maximize):} \quad c_0x_0 + E_1[c_1x_1 + E_2[c_2x_2 \ldots + E_T[c_Tx_T] \ldots]]$$

such that:

$A_{00}x_0$				$\sim b_0$
$A(\omega_1)_{10}x_0$	$+ A(\omega_1)_{11}x_1$			$\sim b(\omega_1)_1$
$A(\omega_1,\omega_2)_{20}x_0$	$+ A(\omega_1,\omega_2)_{21}x_1$	$+ A(\omega_1,\omega_2)_{22}x_2$		$\sim b(\omega_1,\omega_2)_2$
…	…	…		…
$A(\omega_1,\ldots,\omega_T)_{T0}x_0$	$+ A(\omega_1,\ldots,\omega_T)_{T1}x_1 +$	…	$+ A(\omega_1,\ldots,\omega_T)_{TT}x_T$	$\sim b(\omega_1,\ldots,\omega_T)_T$
L_0	$\leq x_0$	$\leq U_0$		
$L(\omega_1)_1$	$\leq x_1$	$\leq U(\omega_1)_1$		
…	…	…		
$L(\omega_1,\ldots,\omega_T)_T$	$\leq x_T$	$\leq U(\omega_1,\ldots,\omega_T)_T$		

where,

$(\omega_1,\ldots,\omega_t)$ represents random outcomes from event space $(\Omega_1,\ldots,\Omega_t)$ up to stage-t,

$A(\omega_1,\ldots,\omega_t)_{tp}$ is the coefficient matrix generated by outcomes up to stage-t for all p=1…t, t=1…T,

$c(\omega_1,\ldots,\omega_t)_t$ is the objective coefficients generated by outcomes up to stage-t for all t=1…T,

$b(\omega_1,\ldots,\omega_t)_t$ is the right-hand-side values generated by outcomes up to stage-t for all t=1…T,

$L(\omega_1,\ldots,\omega_t)_t$ and $U(\omega_1,\ldots,\omega_t)_t$ are the lower and upper bounds generated by outcomes up to stage-t for all t=1…T,

'~' is one of the relational operators '≤' , '=', or '≥' ; and

x_0 and $x_t \equiv x(\omega_1,\ldots,\omega_t)_t$ are the decision variables (unknowns) for which optimal values are sought. The expression being optimized is called the cost due to initial-stage plus the expected cost of recourse.

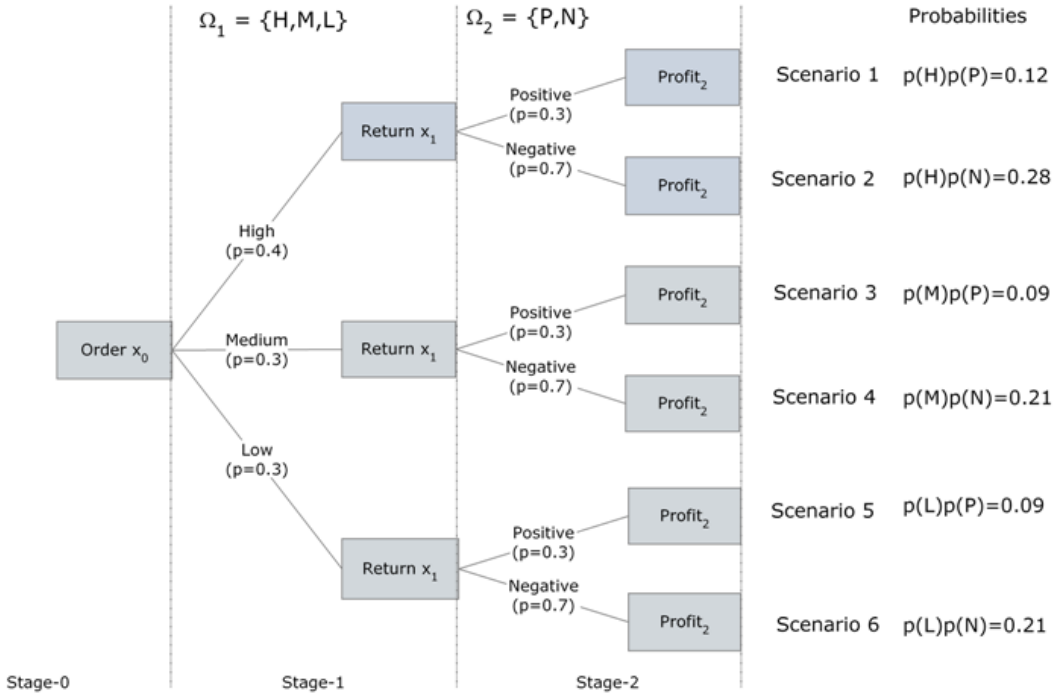
Note: LINGO can solve linear, nonlinear and integer multistage stochastic programming problems.

Scenario Tree

When the probability distributions for the random parameters (events) are discrete, there are only a finite number of outcomes in each stage. With each random parameter fixed to one of its possible outcomes, one can create a scenario representing one possible realization of the future. Enumeration of all possible combinations of outcomes allows us to represent all scenarios in a tree, with each scenario being a path from the root of the tree to one of its leaves. The nodes visited by each path correspond to values assumed by random parameters in the model.

We illustrate the construction of a scenario tree with a stochastic version of the well-known Newsvendor inventory problem. In this problem, we must decide how much to order initially and then later, how much of any unsold product to return before the end of the planning horizon. There is a shortage penalty when there are lost sales and a carrying cost for left over units. The decision process takes place under uncertain demand and uncertain price per returned item:

1. In stage 0, the order quantity has to be decided (under uncertain demand).
2. In stage 1, at the beginning, the demand is revealed. A recourse decision, at the end of stage 1, is the number of units to be returned to the publisher (for an uncertain refund price)
3. In stage 2 at the beginning, the refund price is announced by the publisher. The price per returned item can be either:
   - Positive (i.e. publisher accepts them at a high price which covers the cost of shipping and handling) or
   - Negative (i.e. publisher accepts them at a low price which doesn't cover the cost of shipping and handling).
4. The objective is to maximize the total expected profit at the end of planning horizon (stage 2).



In the scenario tree above, x_0 represents the initial decision, or order size to be determined before seeing any of the random outcomes. x_1 represents the quantity to return to the publisher of any portion of the unsold units. $Profit_2$ represents the total profit collected at the end of planning horizon. The notation Ω_1 represents the event space for the unknown demand, for which there are three different possible outcomes Ω_1 = {Low, Medium, and High} with probabilities {0.4, 0.3, 0.3}, respectively. Once we observe the demand $\omega_1 \in \Omega_1$, we make a recourse decision x_1 based upon which ω_1 nature chose and our previous decision x_0. The notation Ω_2 represents the event space for refund price per unsold newspapers if returned to the publisher in stage 2. This event has two different outcomes Ω_2 = {Positive, Negative} with probabilities {0.7, 0.3}. Once the refund price $\omega_2 \in \Omega_2$ in stage 2 is observed, the total profit would be computed by the model as the final decision $Profit_2$.

It should be clear from the scenario tree that,

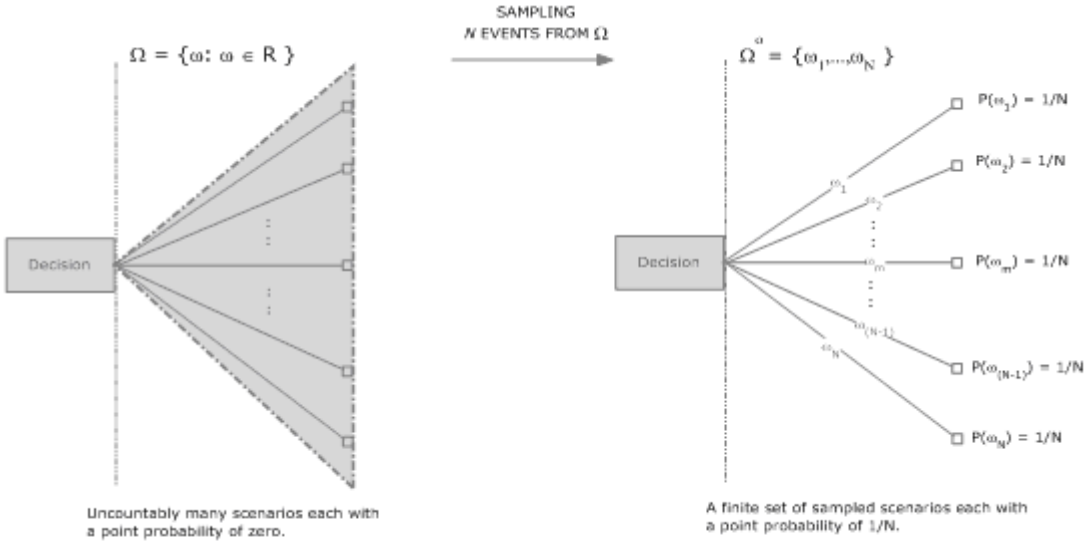
- There are as many distinct scenarios in the SP as there are leaf-nodes.
- Each root-leaf path defines a scenario, induced by a full observation of all random events.
- There is a one-to-one correspondence between the scenarios and the leaf-nodes in the tree.

- The unconditional probability of a node is computed by multiplying the conditional probabilities of the nodes positioned on the path, which starts from the root and terminates at that node.
- The unconditional probability of each leaf-node corresponds to the probability of the associated scenario.
- Each node in the tree corresponds to a vector of random parameter with a particular history up to that node in some scenario.

The branches out of each node enumerate all possible outcomes associated with random parameters associated with it in order to construct the history of random parameters that belong to the next stage.

Monte Carlo Sampling

In stochastic programming (SP), when one or more random variables have a continuous (or discrete but infinite) event space, it is obviously computationally impossible to explicitly account for the infinite number of possible outcomes. For such cases, Monte Carlo sampling (also called pre-sampling) can be used to approximate the problem using a finite scenario tree. As illustrated in the figure below, if the model has a single stochastic parameter with a continuous distribution such as the normal distribution, one can discretize the event space simply by generating N sample points and thereby construct a finite and tractable scenario tree. This is also the case for discrete distributions with infinite event spaces, such as the Poisson distribution.



Note: Since the point probability of each scenario in the original model is zero, it is customary to set the probabilities of sampled scenarios to 1/N.

Given the parametric distribution of each stochastic parameter, LINGO's sampling routines can be used to efficiently generate univariate samples from these distributions. When sampling, LINGO uses Latin-hyper-square techniques to reduce the sample variance. One may also specify a correlation matrix for the continuous random variable, which the sampling routines will use to induce correlation in the sample points. These sample points are then used to define discrete distributions, which approximate the original distributions. Repeating this for all continuous stochastic parameters, LINGO recasts the infinite event space model into an approximate model with a finite event space.

Setting up SP Models

There are four steps to setting up an SP model. Below is a list of these steps and the primary LINGO functions that you will call to perform each step. Each of these steps will be illustrated in detail in the example SP model later in this chapter.

Step	Task	Description	How
1	Defining core model	The core model is built just like any other deterministic LINGO model. The random variables are used directly in the core model's expressions as if they were determinate.	Entered like any other deterministic LINGO model.
2	Identifying the random variables	Each random variable must be identified along with the stage of the model where it becomes known.	*@SPSTGRNDV*
3	Identifying the initial decision and recourse variables	Each initial decision variable and each recourse variable must be identified along with the stage where it must be decided. The initial decision variables are assigned stage 0, while the subsequent recourse variables are assigned to stages 1,2,... .	*@SPSTGVAR*
4	Declaring distributions	The probability distributions of the random variables must be declared. The techniques for doing so will depend on whether the variable of interest has a distribution defined by either a) a discrete *outcome table*, or b) a *parametric distribution*. An outcome table is a finite set of all possible outcomes, their probabilities and values for the random variable(s). Outcome tables may be constructed using either scalar values or attribute vectors. A parametric distribution, on the other hand, is defined by a particular type of probability density function, e.g., the normal distribution. Parametric distributions may have a finite or infinite number of possible outcomes, depending on the distribution type.	Outcome Tables: Scalar-based: *@SPTABLESHAPE* *@SPTABLEOUTC* *@SPTABLEINST* *@SPTABLERNDV* Vector-based: *@SPDISTTABLE* Parametic Distributions: *@SPDIST<TYPE>* *@SPSAMPSIZE* *@SPCORR<METHOD>*

Language Features for SP Models

The primary features in the LINGO language used for building a stochastic programming model can be partitioned into the following three categories, corresponding to steps 2-4 of the *Setting up SP Models* section above:

1. *Identifying random variables*
2. *Identifying initial decision and recourse variables*
3. *Declaring distributions*

Each of the categories are discussed in the following five sections, which are primarily intended for reference use. The reader may wish to jump directly to the first SP modeling example, *Gas Buying Example,* in order to see LINGO's SP modeling features used in the context of an actual model.

There are additional SP language constructs available for use in calc sections for model's that exploit LINGO's programming capability. These features are discussed in Chapter 13, *Programming LINGO*.

Identifying Random Variables

The fundamental component of any SP model is its set of random variables. The random variables are the stochastic component of the model. Their values are not known with certainty beforehand, however, their uncertainty can be expressed with known probability distributions. LINGO requires each random variable to be explicitly identified. When we identify each random variable, we must also provide the stage in the scenario tree where each variable's value is revealed. The *@SPSTGRNDV* function is used for this purpose:

@SPSTGRNDV(STAGE, VARIABLE_NAME)
This function takes two arguments: the random variable's name and the integer valued stage at which the random variable's value is revealed. Note that this function merely flags a variable as being random, and does not communicate any information regarding the actual distribution of the variable. A distribution is associated with the random variable in the *Binding random variables to distribution instances*, discussed below. Some examples follow:

Example 1 : `@SPSTGRNDV( 1, DEMAND);`
Flags variable *DEMAND* as being a random variable in stage 1.

Example 2:
```
@FOR( TIME_ASSET( T, A) | T #GT# 1:
 @SPSTGRNDV( T - 1, RETURN( T, A)));
```
Flags *RETURN(T, A)* as being a random variable in stage *T*-1. Note that by using *@SPSTGRNDV* in an *@FOR* loop, we are able to identify multiple variables as being random.

Identifying Initial Decision and Recourse Variables

After identifying the random variables, all the remaining variables in the model are either *initial decision variables* or *recourse variables*. These are the variables that are in our control, as opposed to being stochastic and in the control of Mother Nature. The initial decision variables are those variables whose values must be determined at time 0, before nature has revealed values for any of the random variables. The recourse variables are the variables whose values are decided at a later period in response to outcomes for the random variables. Both initial decision and recourse variables are identified with the *@SPSTGVAR* function, with initial decision variables being distinguished by a stage of 0 and recourse variables by a stage >0. Row stages can be established using *@SPSTGROW*.

@SPSTGVAR(STAGE, VARIABLE_NAME)

This function takes two arguments: an integer value equal to the stage at which the variable's value must be decided and the variable's name. Some examples follow:

Example 1: `@SPSTGVAR( 0, PURCHASE);`
Flags variable *PURCHASE* as being an initial decision variable.

Example 2: `@FOR( TIME_ASSET( T, A): @SPSTGVAR( T - 1, INVEST( T, A)));`
Flags *INVEST(T, A)* as being recourse variable in stage *T*-1 (or an initial decision variable when *T-1=0)*. Note that by using *@SPSTGVAR* in an *@FOR* loop, we are able to identify multiple recourse variables.

Before an SP model can be passed to the SP solver engine, all initial and recourse variables must be assigned stage values. Having to explicitly assign stage values to all these variables can be a tedious and error-prone process. Fortunately, LINGO can deduce the stages for many of the variables based on the user inputting a minimal set of stages for the variables. We refer to this process a *stage deduction* and it is discussed more in section *The Solution* below of the *Gas Buying Example*. Guidelines for determining the variables that require explicit stage identification are listed below.

@SPSTGROW(STAGE, VARIABLE_NAME)

This function takes two arguments: an integer-valued stage at which the row's value becomes known and the row's name. This function should generally not be required, in that LINGO can automatically deduce all row stages. If you do use this function, you must name the rows whose stages you wish to set. The syntax of assigning names to rows may be found in the *Constraint Names* section of Chapter 1.

Example: `@SPSTGROW( 4, CAPACITY);`
Assigns row *CAPACITY* to stage 4.

Note: Guidelines for determining the variables and rows that must be explicitly staged are:

- All random variables must be assigned to a stage using *@SPSTGRNDV*.
- Any initial decision or recourse variable that belongs to stage *N* that does not depend either directly, or indirectly, on another variable (random or otherwise) declared to be in stage *N* must be explicitly assigned to stage *N* using *@SPSTGVAR*.
- If you are uncertain whether a particular variable must be explicitly assigned to a stage, or not, then it doesn't hurt to assign it anyway using *@SPSTGVAR*.
- In general, stage assignment for rows should be left to LINGO. Each row will be assigned to the stage equal to the maximum stage of the variables appearing in that row. However, if you wish to explicitly specify a row's stage, you may always do so using *@SPSTGROW*.

Declaring Distributions

After identifying the random variables, you will need to assign them to appropriate probability distributions. There are two classes of distributions: *parametric* distributions and *outcome table* distributions, with each requiring different declarations. Declaring parametric distributions involves declaring a particular type of probability density function (e.g., normal) and its parameters (e.g., mean and standard deviation). Outcome table distributions may be declared using scalar values or, assuming your model makes use of sets and attributes, they may also be declared very conveniently using attribute vectors and matrices. In either case, you will need to identify all possible outcomes and their respective probabilities.

Declaring Parametric Distributions

LINGO allows both independent and joint parametric probability distributions, as well as continuous and discrete distributions. The functions used to declare these distributions are of the form *@SPDIST<TYPE>,* where *<TYPE>* represents the type of distribution being declared. In addition, there are the *@SPSAMPSIZE* and *@SPCORR<METHOD>* functions for, respectively, setting sample sizes and inducing correlations, where *<METHOD>* refers to the particular method used to induce correlation.

@SPDIST<*TYPE*>(PARAM_1[, ... , PARAM_N], RANDOM_VAR)
This function is used to apply both continuous and discrete parametric probability distributions to random variables. At least one distribution parameter argument will always be required. Additional parameters may be required depending on the particular distribution. Parameter values are always numeric. The random variable must be declared beforehand via *@SPSTGRNDV*.

Below is a list of supported continuous distributions.

Distribution	Syntax	Parameters
Beta	@SPDISTBETA(A, B, RNDVAR)	*A* = alpha > 0 *B* = beta > 0
Cauchy	@SPDISTCACY(LOC, SCALE, RNDVAR)	*LOC* = location *SCALE* = scale > 0
Chi-Square	@SPDISTCHIS(DF, RNDVAR)	*DF* = degrees of freedom = a positive integer
Exponential	@SPDISTEXPO(LAMDA, RNDVAR)	*LAMDA* = rate parameter > 0
F	@SPDISTFDST(DF1, DF2, RNDVAR)	*DF1* = degrees of freedom 1 = a positive integer *DF2* = degrees of freedom 2 = a positive integer
Gamma	@SPDISTGAMM(SCALE, SHAPE, RNDVAR)	*SCALE* = scale parameter > 0 *SHAPE* = shape parameter > 0
Gumbel	@SPDISTGMBL(LOC, SCALE, RNDVAR)	*LOC* = location *SCALE* = scale > 0
Laplace	@SPDISTLAPL(LOC, SCALE, RNDVAR)	*LOC* = location *SCALE* = scale > 0
Logistic	@SPDISTLGST(LOC, SCALE, RNDVAR)	*LOC* = location *SCALE* = scale > 0
Lognormal	@SPDISTLOGN(MU, SIGMA, RNDVAR)	*MU* = mu parameter *SIGMA* = sigma parameter > 0
Normal	@SPDISTNORM(MU, SIGMA, RNDVAR)	*MU* = mean *SIGMA* = standard deviation > 0
Pareto	@SPDISTPRTO(SCALE, SHAPE, RNDVAR)	*SCALE* = scale parameter > 0 *SHAPE* = shape parameter > 0
Student's t	@SPDISTSTUT(DF, RNDVAR)	*DF* = degrees of freedom = a positive integer
Symetric Stable	@SPDISTSMST(A, RNDVAR)	*A* = alpha ∈ [.02,2.0]
Triangular	@SPDISTTRIA(L, U, M, RNDVAR)	*L* = lowest point *H* = high point *M* = middle point
Uniform	@SPDISTUNIF(L, U, RNDVAR)	*L* = lower point *U* = upper point
Weibull	@SPDISTWEIB(SCALE, SHAPE, RNDVAR)	*SCALE* = scale parameter > 0 *SHAPE* = shape parameter > 0

Example: `@SPDISTNORM( 82.3, 18.1, RND_TEMP);`

In this example, we apply a normal distribution to the random variable *RND_TEMP*, with a mean of 82.3 inches and a standard deviation of 18.1.

Here's the list of discrete distributions currently supported:

Distribution	Syntax	Parameters
Beta Binomial	@SPDISTBTBN(N, A, B, RNDVAR)	N = trials = a positive integer ≥ 0 A = alpha > 0 B = beta > 0
Binomial	@SPDISTBINO(N, P, RNDVAR)	N = trials = a positive integer ≥ 1 P = probability of success, $0 \le P \le 1$
Geometric	@SPDISTGEOM(P, RNDVAR)	P = probability of success, $0 < P \le 1$
Hypergeometric	@SPDISTHYPG(N, D, K, RNDVAR)	N = population = a positive integer ≥ 1 D = number defective $\in \{0,1,...,N\}$ K = sample size $\in \{1,2,...,N\}$
Logarithmic	@SPDISTLOGR(P, RNDVAR)	P = P-factor, $0 < P < 1$
Negative Binomial	@SPDISTNEGB(R, P, RNDVAR)	R = R-factor > *0* P = probability of success, $0 < P < 1$
Poisson	@SPDISTPOIS(LAMDA, RNDVAR)	*LAMDA* = arrival rate > 0

Example: `@SPDISTPOIS( 33, CUSTOMERS);`

In this example, we apply a Poisson distribution with an arrival rate of 33 to the random variable *CUSTOMERS*.

@SPSAMPSIZE(STAGE, SAMPLE_SIZE)

This function is used to set the sample size for probability distributions by stage. Most instances of the parametric distributions listed in this section have a large, if not an infinite, number of outcomes. Trying to incorporate all possible outcomes into the model would be impossible, therefore, LINGO must sample from the parametric distributions. If a specific sample size is not set using *@SPSAMPSIZE*, then LINGO defaults to a sample size equal to the *Default Sample Size/Stage* parameter, which is typically set to 2. However, in some models you may wish to have different sample sizes across stages, typically having larger samples in the more critical early stages, and less sampling in the latter stages. *@SPSAMPSIZE* can enforce this type of sampling strategy.

Example 1: `@SPSAMPSIZE( 1, 8);`

Here we set the sample size to 8 outcomes in stage 1.

Example 2:
```
@FOR( PERIODS( P) | P #GT# @INDEX( P0):
        @SPSAMPSIZE( P - 1, SAMP_SZ( P))
);
```

In this example, we set the sample size equal to *SAMP_SZ(P)* in periods 1 through *N*-1. Note that by adding the conditional expression *P #GT# @INDEX(P0)* to the *@FOR* loop, the loop starts with *P0*=2, and we avoid setting the sample size in stage 0. We do this because stage 0 never contains random variables. Random variables first occur in stage 1, with stage 0 being reserved for the initial decision variables. In this example, we have also not assigned a sample size to the final stage, because we either want the final stage to use the default sample size, or the final stage does not have random variables.

@SPCORR<METHOD>(RANDOM_VAR_1, RANDOM_VAR_2, RHO)

This function causes the sampling code to induce correlations between two random variables. The ordering of the two variables is not significant. The correlation coefficient, *RHO*, must lie in the interval [-1,1]. You must also choose between three methods available for inducing correlations: Pearson, Kendall or Spearman. Only one correlation method is allowed per model. Some examples follow:

Example 1: `@SPCORRPEARSON( X, Y, .9);`

Here we set the correlation between random variables X and Y to be .9 using the Pearson method.

Example 2:
```
@FOR( CORR_RNDM_PLANTS( P1, P2):
        @SPCORRKENDALL( CAPACITY( P1), CAPACITY( P2), RHO( P1,
P2));
      );
```

In this example, we have a derived, two-dimensional set. *CORR_RNDM_PLANTS*, that contains the pairs of electrical plants whose output capacities are correlated. We loop over these pairs, setting each pair's correlation to *RHO(P1, P2)* using the Kendall method.

Matrix Declarations of Outcome Table Distributions

LINGO allows probability density functions to be input as discrete outcome tables. You will need to input the values for the random variables under each outcome, as well as the probability of each outcome. Matrix-based declarations are considerably more concise than scalar-based declarations, in that one simple function call can declare the outcome table, it's probabilities, as well as assign multiple random variables to the distribution. The function used for vector-based outcome table declaration is *@SPDISTTABLE*:

@SPDISTTABLE(OUTCOME_TABLE, RANDOM_VARIABLE_ATTR[, PROBABILITY_ATTR])

This function identifies an outcome table, the random variables to be assigned to the outcome table, and, optionally, probabilities for each of the outcomes. The first argument, or outcome table, must be either a one or two-dimensional attribute containing the outcome values for the random variables. If it is one-dimensional, then the random variable argument must be a scalar. If the outcome table is two-dimensional, say *M* x *N,* then it must contain *M* outcomes, with each row being a single outcome for *N* random variables. Also, in the two-dimensional case the random variable argument must be a one-dimensional attribute of length *N.* If the probability argument is included, it must be a one-dimensional attribute of length equal to the number of outcomes. If the probability argument is omitted, then each outcome is assumed to be equally probable. Some examples follow:

Example 1: `@SPDISTTABLE( DIST_RETURNS, RETURNS);`

Declares an outcome table with the two-dimensional attribute *DIST_RETURNS* containing one outcome per row for the random variables in the one-dimensional *RETURNS* attribute. The number of columns in *DIST_RETURNS* must equal the number of elements in *RETURNS*. The optional probability argument was omitted in this example, meaning that all outcomes are equally likely.

Example 2: `@SPDISTTABLE( DIST_SALES, SALES, PROB);`

Declares an outcome table with the one-dimensional attribute *DIST_SALES* containing the possible outcomes for the scalar random variable *SALES*. The *PROB* attribute is a one-dimensional attribute of the same length as *DIST_SALES* that contains the probabilities of all the outcomes.

Sample SP models installed with LINGO that utilize *@SPDISTTABLE* include: *SPPUTOPT1*, *SPNEWSDISC* and *SPHYDRO*.

Scalar Declarations of Outcome Table Distributions

In the previous section, we discussed how to declare outcome tables using matrix-based declarations. In general, the matrix form of declaration is the simplest and easiest to use. However, there may be models that are not set-based that would be unable to use matrix declarations. Also, there may be instances when you only wish to declare portions of an attribute to be random. In these cases, you will need to use scalar-based outcome table declarations. The scalar-based functions used to declare outcome tables are: *@SPTABLESHAPE*, *@SPTABLEOUTC*., *@SPTABLEINST* and *@SPTABLERNDV*.

@SPTABLESHAPE('TABLE_NAME', NUMBER_OF_OUTCOMES, NUMBER_OF_RANDOM_VARS)

This function initializes an outcome table and requires three arguments: a unique name for the distribution table (entered as text), the number of possible outcomes, and the number of random variables in the distribution. Some examples follow:

Example 1: `@SPTABLESHAPE( 'WEATHER', 3, 1);`
Declares an outcome table called *WEATHER* with 3 outcomes and 1 independent variable.

Example 2: `@SPTABLESHAPE( 'RETURN', @SIZE( OUTCOMES), @SIZE( ASSETS));`
Declares an outcome table called *RETURN* with the number of outcomes equal to the size of set *OUTCOMES* and the number of jointly distributed variables equal to the size of set *ASSETS.*

@SPTABLEOUTC('TABLE_NAME', OUTC_PROB, OUTC_VAL1[,..., OUTC_VALN])

Once an outcome table has been declared with *@SPTABLESHAPE*, the next step is to load the table with its set of outcome values and their probabilities. This is accomplished through calls to *@SPTABLEOUTC*. *@SPTABLEOUTC* accepts a multiple number of arguments; however, there must always be at least two arguments, with argument 1 always being the distribution name and the remaining arguments being numeric values. The first numeric value passed will be taken as the probability of outcome 1. Assuming the distribution has *N* variables, the next *N* arguments will be taken as the values for variables 1 through *N* for outcome 1. This process repeats for each outcome until a total of $M * (N + 1)$ numeric values have been passed, where *M* is the number of outcomes. These numeric values may be passed in as many calls as is convenient, but, again, the first argument of each call must be the name of the relevant distribution. The following examples should help to illustrate:

Example 1: `@SPTABLEOUTC( 'WEATHER', .3, 56, .6, 77, .1, 92);`
In this example, we have a 3-outcome, single-variable, outcome table named 'WEATHER'. We make a single call to *@SPTABLEOUTC* to load all three outcomes. Outcome 1 has a probability of .3 and a value of 56, outcome 2 has a .6 probability and a value of 77, while outcome 3 has a .1 probability and a value of 92.

Example 2:
```
@FOR( OUTCOMES( O):
                @SPTABLEOUTC( 'D1', 1 / @SIZE( OUTCOMES));
                @FOR( ASSETS( A): @SPTABLEOUTC( 'D1', O_RETURN( O,
A)));
               );
```
Here we are making two calls to *@SPTABLEOUTC* for each member of the *OUTCOMES* set. The first call passes the probability of the outcome, while the second call passes the outcomes for each member of the *ASSETS* set. This example was taken from the *SPCOLLEGEDISC.LG4* example, which you may wish to review for additional clarification.

@SPTABLEINST('PARENT_TABLE_NAME', 'TABLE_INSTANCE_NAME')

In order to actually use an outcome table distribution in a model we must also declare an instance of the relevant distribution. This allows us to use a particular distribution a multiple number of times in a model, saving us from having to reenter the distribution's parameters each time. *@SPTABLEINST* is the function for declaring distribution instances. This function takes two arguments: the parent outcome table's name as established in the call to *@SPTABLESHAPE*, and a unique name to apply to the new instance of the distribution. A particular outcome table may be used many times in a single model, however, each particular instance requires a unique name. Some examples follow:

Example 1: `@SPTABLEINST( 'WEATHER', 'WEATHER_1');`
Declares an instance of the parent distribution 'WEATHER', giving it a name 'WEATHER_1'.

Example 2:
```
@FOR( TIME( T):
    @SPTABLEINST( 'D1', 'D1_' + TIME( T)));
```
Declares multiple instances of the parent distribution 'D1', assigning each one the name 'D1_'+*TIME(T)*. In this case, *TIME(T)* is a primitive set member whose name gets appended to the 'D1_' string to form the instance name. As an example, if set *TIME* has values *T1..T12*, then the instance name for the first distribution would be 'D1_T1'.

@SPTABLERNDV('DIST_INSTANCE_NAME', RANDOM_VAR_1[, ... , RANDOM_VAR_N])

The final step in setting up an outcome table distribution is binding the random variables to the distribution. This step associates specific random variables with the outcome table instance that was defined as described above. The *@SPTABLERNDV* function is used for this purpose. This function takes two, or more, arguments. The first argument is the distribution instance name, entered as text. The second argument is the name of the random variable to bind to the distribution instance. If the distribution has multiple variables, then you may include the additional variable names as arguments, or you may call *@SPTABLERNDV* multiple times, until all relevant random variable names have been assigned to the distribution instance. Some examples follow:

Example 1: `@SPTABLERNDV( 'WEATHER_1', RV_WEATHER_1);`
Binds random variable *RV_WEATHER_1* to the 'WEATHER_1' distribution instance.

Example 2:
```
@FOR( TIME( T): @FOR( ASSETS( A):
    @SPTABLERNDV( 'D1_' + TIME( T), RETURN( T, A))));
```
Assuming the *TIME* set consists of members *T1..T12*, this example assigns one random for each member of the *ASSETS* set to distribution instance 'D1_T*i*', for *i* =1,...,12. Assuming the *ASSETS* set has two members, then two random variables are assigned to each the 12 distribution instances.

Sample SP models installed with LINGO that utilize the above techniques include: *SPGAS1* and *SPCOLLEGEDISC*.

Gas Buying Example

Our first example of how to formulate and solve an SP is a gas buying example. This example was taken from the Committee on Stochastic Programming's (COSP) website: http://stoprog.org/index.html?spintroduction.html. The complete LINGO model can be found in the LINGO samples folder under the name *SPGAS1.LG4.*

A gas company must come up with a plan for its gas purchases under uncertain weather conditions. The demand for the current period (period 1) is known and is 100 units. The demand for the upcoming period is not known with certainty and will depend on how cold the weather is. There are three possible outcomes for the weather: normal, cold and very cold. Each of these outcomes are equally likely. The following table lists the costs and demands under the three outcomes:

Outcome	Probability	Gas Cost/Unit ($)	Units Demand
Normal	1/3	5.0	100
Cold	1/3	6.0	150
Very Cold	1/3	7.5	180

Gas for the current period is bought now and delivered directly to the customers at a cost of $5 per unit. Gas in the upcoming period can either be bought now and held in storage for period 2 use, or it can be purchased in period 2 at a price that will depend on the weather as per the table above. Storing gas bought in period 1 for use in period 2 costs the company $1 per unit. The question the company is faced with is: How much gas should be bought in periods 1 and 2 to meet total customer demand at minimal expected cost?

The Core Model

Step 1 - Defining the Core Model:

The core model is the same optimization model we would construct if all the random variables were known with certainty. There is nothing in the core model that addresses the stochastic nature of the model. The stochastic properties of the model will be introduced in a subsequent model section. For our current example, the core model is formulated as follows:

```
! Minimize Total Cost = Purchases + Holding;
[R_OBJ] MIN= PURCHASE_COST + HOLD_COST;

! Compute purchase cost;
[R_PC] PURCHASE_COST = 5 * PURCHASE_1 + COST_2 *
PURCHASE_2;

! Compute holding cost;
[R_HC] HOLD_COST = INVENTORY_1 + INVENTORY_2;

! Compute inventory levels;
[R_I1] INVENTORY_1 = PURCHASE_1 - 100;
[R_I2] INVENTORY_2 = INVENTORY_1 + PURCHASE_2 - DEMAND_2;
```

Core Model for Gas Buying Example

In the first expression:

```
! Minimize Total Cost = Purchases + Holding;
[R_OBJ] MIN= PURCHASE_COST + HOLD_COST;
```

we are saying that we want to minimize total cost, which consists of two components: the purchase cost of buying gas in periods 1 and 2, as well as holding costs of buying gas in a current period for use in a future period.

The following expression:

```
! Compute purchase cost;
[R_PC] PURCHASE_COST = 5 * PURCHASE_1 + COST_2 * PURCHASE_2;
```

calculates the total purchase cost, which is $5 per unit of gas purchased in period 1 plus the cost of gas in period 2 (*COST_2*) multiplied by the number of units of gas purchased in period 2. Note that *COST_2* is stochastic and will depend on how the weather plays out. We will convey the probabilistic nature of period 2 cost in the next section.

We next compute total holding costs, which, given the $1/unit holding cost, is simply the sum of units held over in periods 1 and 2:

```
! Compute holding cost;
[R_HC] HOLD_COST = INVENTORY_1 + INVENTORY_2;
```

Finally, we compute the inventory levels for the two periods:

```
! Compute inventory levels;
[R_I1] INVENTORY_1 = PURCHASE_1 - 100 ;
[R_I2] INVENTORY_2 = INVENTORY_1 + PURCHASE_2 - DEMAND_2;
```

Period 1's inventory is the amount purchased in period 1 (PURCHASE_1) less the known demand in period 1. While inventory in period 2 (INVENTORY_2) if equal to any inventory carried over from period 2 plus the purchase in period 2 (PURCHASE_2) minus the demand in period 2 (DEMAND_2). Once again, DEMAND_2 is not known with certainty. The uncertain nature of DEMAND_2 will be established in the next section of the model - the stochastic declarations section.

Stochastic Declarations

After the core model, the remainder of the model is the stochastic declarations section:

```
! *** STEP 2 *** - Define Random Variables;

!The random variables are period 2's demand and cost.;
@SPSTGRNDV( 1, COST_2);
@SPSTGRNDV( 1, DEMAND_2);

! *** STEP 3 *** - Define initial decision and recourse
   variables;

!The initial decision is how much to purchase in period 1;
@SPSTGVAR( 0, PURCHASE_1);
!Period 2 purchases are a recourse variable after
the weather reveals itself;
@SPSTGVAR( 1, PURCHASE_2);

! *** STEP 4 *** - Assign distributions to the random
   variables;

!Declare a discrete distribution called 'DST_DMD' with
three outcomes and two jointly distributed variables
(i.e., Demand and Cost);
@SPTABLESHAPE( 'DST_DMD', 3, 2);

!Load the three equally likely outcomes into 'DST_DMD';
!Dist Name  Probability Cost  Demand;
@SPTABLEOUTC( 'DST_DMD',       1/3,      5.0,      100);
@SPTABLEOUTC( 'DST_DMD',       1/3,      6.0,      150);
@SPTABLEOUTC( 'DST_DMD',       1/3,      7.5,      180);

!Declare a specific instance of the 'DST_DMD' distribution,
naming the instance 'DST_DMD_1';
@SPTABLEINST( 'DST_DMD', 'DST_DMD_1');

!Bind Period 2 Cost and Demand to the distribution instance;
@SPTABLERNDV( 'DST_DMD_1', COST_2, DEMAND_2);
```

Stochastic Declarations for Gas Buying Example

We use this section to identify all the random variables and their distributions. A detailed breakdown of the stochastic declarations follows.

Step 2 - Identifying the Random Variables:

The next step in building our sample SP model is to identify the random variables. The random variables are the variables that are stochastic by nature and whose values are not known before we must make our initial decisions. The *@SPSTGRNDV* function is used to identify random variables. *@SPSTGRNDV* accepts two arguments: the random variable's name and its stage.

In this example, there are two random variables, the second period cost and demand, and we identify them via *@SPSTGRNDV* in the following to statements:

```
!The random variables are period 2's demand and cost.;
@SPSTGRNDV( 1, COST_2);
@SPSTGRNDV( 1, DEMAND_2);
```

Note that we have indicated that the two variables are in stage 1. This may seem somewhat odd, in that they are relevant to *period 2*. However, you will recall that the first set of random variables to become known after our initial decision belong to stage 1. If this was a multiperiod model, period 3's random variables would belong to stage 2, and so on.

Step 3 - Identifying the Initial Decision and Recourse Variables:

The next step is to identify the the initial decision variables and the recourse variables. Unlike the random variables, which are under Mother Nature's control, the initial decision and recourse variables are under our control. The initial decision variables occur at the very outset, before any of the random variables become known, and are always assigned to stage 0. The recourse variables are the subsequent decisions we make after learning of the outcomes of the random variables. Recourse variables that are decided after the stage *N* random variables become known are assigned to stage *N* as well.

The *@SPSTGVAR* function is used to identify initial decision and recourse variables. *@SPSTGVAR* accepts two arguments: the variable's stage and its name.

In our example, there is one initial decision, which is *PURCHASE_1,* the amount of gas to purchase in period 1. The weather then reveals itself and our recourse variable is *PURCHASE_2,* the amount to purchase in period 2. We identify this information with the two statements:

```
!The initial decision is how much to purchase in period 1;
@SPSTGVAR( 0, PURCHASE_1);

!Period 2 purchases are a recourse variable after
the weather reveals itself;
@SPSTGVAR( 1, PURCHASE_2);
```

It turns out that before an SP model can be passed to the SP solver engine, all variables and rows must be assigned stage values. You will note that we have not done this. Specifically, we have not assigned stages to any of the rows nor to the variables: *PURCHASE_COST*, *HOLD_COST*, *PURCHASE_2*, *INVENTORY_1* and *INVENTORY_2*. The reason we have not made these stage assignments is that LINGO can deduce the stages of the remaining rows and variables from the minimal stage information we've already supplied. We will examine this stage deduction feature in more detail in the next section. In the meantime, the following guidelines should suffice:

Note: Guidelines for determining the variables and rows that must be explicitly staged are:

- All random variables must be assigned to a stage using *@SPSTGRNDV*.
- Any initial decision or recourse variable that belongs to stage *N* that does not depend either directly, or indirectly, on another variable (random or otherwise) declared to be in stage *N* must be explicitly assigned to stage *N* using *@SPSTGVAR*.
- If you are uncertain whether a particular variable must be explicitly assigned to a stage, or not, then it doesn't hurt to assign it anyway using *@SPSTGVAR*.
- In general, stage assignment for rows should be left to LINGO. Each row will be assigned to the stage equal to the maximum stage of the variables appearing in that row. However, if you wish to explicitly specify a row's stage, you may always do so using *@SPSTGROW*.

Step 4 - Declare Distributions

Next, we need to declare the joint probability distribution for the random variables *COST_2* and *DEMAMD_2*. In this case, we will be using an outcome table distribution, and in order to declare our distribution we will make use of the scalar-based functions: *@SPTABLESHAPE* and *@SPTABLEOUTC, @SPTABLEINST* and *@SPTABLERNDV*.

@SPTABLESHAPE initializes the distribution with a name, number of outcomes and number of jointly distributed variables, while *@SPTABLEOUTC* is called once for each outcome to load information relevant to each outcome:

```
!Declare a discrete distribution called 'DST_DMD' with
three outcomes and two jointly distributed variables
(i.e., Demand and Cost);
@SPTABLESHAPE( 'DST_DMD', 3, 2);

!Load the three equally likely outcomes into 'DST_DMD';
!            Dist Name  Probability     Cost     Demand;
@SPTABLEOUTC( 'DST_DMD',       1/3,      5.0,      100);
@SPTABLEOUTC( 'DST_DMD',       1/3,      6.0,      150);
@SPTABLEOUTC( 'DST_DMD',       1/3,      7.5,      180);
```

@SPTABLESHAPE accepts three arguments: a name for the distribution, the number of outcomes and the number of jointly distributed variables in the distribution. In this case, we've titled the distribution 'DST_DMD', and it has three outcomes along with two jointly distributed variables.

Now, to be able to actually apply the distribution to random variables we need to declare an instance of the distribution. By doing things this way, it's possible to reuse the same outcome table on more than one set of random variables. We declare an instance of a particular distribution using the *@SPTABLEINST* function, which accepts two arguments - the name of the parent distribution and a new name for the instance of the distribution. In the case of our example, we do the following:

```
!Declare a specific instance of the 'DST_DMD' distribution,
naming the instance 'DST_DMD_1';
@SPTABLEINST( 'DST_DMD', 'DST_DMD_1');
Or, in words, we create an instance of the DST_DMD distribution,
which we name DST_DMD_1.
```

Our last step is to associate, or bind, the random variables to the instance of the distribution. Specifically, we wish to bind the cost and demand random variables from period 2 to the *DST_DMD_1* distribution instance. We do this with the following expression:

```
!Bind Period 2 Cost and Demand to the distribution instance;
@SPTABLERNDV( 'DST_DMD_1', COST_2, DEMAND_2);
```

Our SP model has now been fully set up and we are now ready to have LINGO solve the model.

The Solution

SP models are solved just like all other models by using the *Solver|Solve* command. After solving our sample gas buying model, *SPGAS_SCALAR.LG4*, you will see a solution report that begins with the following section:

```
Global optimal solution found.
Objective value:                              1400.000
Infeasibilities:                              0.000000
Total solver iterations:                             2
```

The first line tells us that the solver found a globally optimal solution. This solution status information is critical. If the status is anything other than *globally optimal*, *locally optimal*, or *feasible* then the solution will not be valid.

The objective value is the expected objective value of the objective over all the scenarios. Thus, in this example, the gas companies expected cost over the three weather scenarios is $1400.

The infeasibilities figure measure the amount by which all constraints were violated in the deterministic equivalent (DE) model. The amount of infeasibilities should always be a very small number and should generally not exceed 1.e-7.

Total solver iterations lists the number of iterations, or passes, the solver required to solve the DE model.

The next section of the solution report lists a number of expected values:

```
Expected value of:
   Objective (EV):                                  1400.000
   Wait-and-see model's objective (WS):             1326.667
   Perfect information (EVPI = |EV - WS|):          73.33333
   Policy based on mean outcome (EM):               1479.444
   Modeling uncertainty (EVMU = |EM - EV|):         79.44444
```

These values are a guide as to how the stochastic nature of the model is impacting the objective value. The following is a brief description of these expected values:

Expected Value of Objective (EV) - is the expected value for the model's objective over all the scenarios, and is the same as the reported objective value for the model.

Expected Value of Wait-and-See Model's Objective (WS) - reports the expected value of the objective if we could wait and see the outcomes of all the random variables before making our decisions. Such a policy would allow us to always make the best decision regardless of the outcomes for the random variables, and, of course, is not possible in practice. For a minimization, it's true that WS <= EV, with the converse holding for a maximization. Technically speaking, WS is a relaxation of the true SP model, obtained by dropping the nonanticipativity constraints.

Expected Value of Perfect Information (EVPI) - is the absolute value of the difference between EV and WS. This corresponds to the expected improvement to the objective were we to obtain perfect information about the random outcomes. As such, this is a expected measure of how much we should be willing to pay to obtain perfect information regarding the outcomes of the random variables.

Expected Value of Policy Based On Mean Outcome (EM) - is the expected true objective value if we (mistakenly) assume that all random variables will always take on exactly their mean values. EM is computed using a two-step process. First, the values of all random variables are fixed at their means, and the resulting deterministic model is solved to yield the optimal values for the stage 0 decision variables. Next, a) the stage 0 variables are fixed at their optimal values from the previous step, b) the random variables are freed up, c) the nonanticipativity constraints are dropped, and d) this wait-and-see model is solved. EM is the objective value from this WS model.

Expected Value of Modeling Uncertainty (EVMU) - is the absolute value of the difference EV - EM. It is a measure of what we can expect to gain by taking into account uncertainty in our modeling analysis, as opposed to mistakenly assuming that random variables always take on their mean outcomes.

Note: The above approach for computing EM and EVMU makes unambiguous sense only for models with a stage 0 and a stage 1. If there are later random variables in stages 2, 3, etc., then there are complications. For example, for decisions in later stages, we have seen the outcomes from the random variables in earlier stages, so considering these random variables to take on their mean value does not make sense. For models with additional stages beyond 0 and 1, EVMU will merely be an approximation of the true expected value of modeling uncertainty.

Note: Computing these expected value statistics can be very time consuming for large models. If speed is an issue, you may wish to disable this feature on the *Solver|Options|SP Solver* tab.

The next component of the solution lists various statistics regarding the class and size of the model:

```
Model Class:                                        LP

Total scenarios/leaf nodes:                          3
Total random variables:                              2
Total stages:                                        1

                                   Core          Deteq
Total variables:                      6             18
Nonlinear variables:                  0              0
Integer variables:                    0              0

Total constraints:                    5             17
Nonlinear constraints:                0              0

Total nonzeros:                      12             30
Nonlinear nonzeros:                   0              0
```

Model class lists the DE's model class. In the case of our gas buying sample we have an LP, or linear program. Possible model classes are detailed in the *Solver Status Window* section of Chapter 1.

Total scenarios/leaf nodes lists the total number of scenarios in the model. In this example, there are three scenarios: Normal, Cold and Very Cold.

Note: The total number of scenarios will *grow exponentially in the number of stages*. For example, suppose we have just one random variable with two possible outcomes in each of N stages, then the number of scenarios will equal 2^N. Thus, it doesn't take many stages before an SP can become quite large.

Total random variables lists the number of random variables in the SP. In this example, we have just two: *COST_2* and *PURCHASE_2.*

The *Total stages* quantity is the model's stage count. In this case there is only one round of random variable realizations and recourse decisions, giving a stage count of 1.

Next, LINGO displays statistics regarding the model's size. Statistics for both the core model and DE are given. The more relevant statistics are the ones for the DE, which is the actual model that must be solved. Definitions for these statistics may be found in the *Solver Status Window* section of Chapter 1.

The next section of the solution reports lists the stage-0 solution:

```
Stage 0 Solution
----------------
                Variable           Value
              PURCHASE_1        200.0000
             INVENTORY_1        100.0000

                     Row    Slack or Surplus
                    R_I1        0.000000
```

The stage-0 solution lists the values for all variables and rows that are part of the initial decision. These values are of pressing importance, in that they must be implemented currently. For this reason, they are displayed in their own separate section near the top of the report.

In the case of our gas company, the optimal initial decision to minimize expected cost is to purchase 200 units of gas in period 1, storing 100 units in inventory. If period 2 is normal the company can fulfill demand entirely from inventory, otherwise it must make up the difference through additional purchases in period 2.

Next, we have the *Staging Report*:

```
Staging Report
--------------

      Random Variable        Stage  Distribution
               COST_2            1  DST_DMD(DST_DMD_1,1)
             DEMAND_2            1  DST_DMD(DST_DMD_1,2)

             Variable        Stage
        PURCHASE_COST           1*
            HOLD_COST           1*
           PURCHASE_1           0
           PURCHASE_2           1
          INVENTORY_1           0*
          INVENTORY_2           1*

                 Row        Stage
               R_OBJ           1*
                R_PC           1*
                R_HC           1*
                R_I1           0*
                R_I2           1*

              (*) Stage was inferred
```

The staging report lists all variables and rows along with their stages. When an SP model is solved, all variables and rows need to be assigned stage values before the model is passed to the SP solver engine. We could have done this explicitly through the use of the *@SPSTGVAR*, *@SPSTGROW* and *@SPSTGRNDV* functions; however, this can be a tedious process prone to errors - particularly for large models. On the one hand, all random variables must be flagged using *@SPSTGRNDV*, but only a minimal set of the recourse variables and the initial decision variables need to have their stages explicitly assigned. The reason we only need to specify a minimal set is that LINGO can deduce the stages of the remaining variables by tracing their dependency chains. We refer to this process as stage deduction, and any variable or row whose stage has been deduced will be flagged with an asterisk in the staging report. As an example, from the staging report we see that variable INVENTORY_1 was deduced to be in stage 0. LINGO determined this from the following row:

```
[R_I1] INVENTORY_1 = PURCHASE_1 - 100;
```

Given that we declared *PURCHASE_1* to be in stage 0, we can also compute *INVENTORY_1*'s value in stage 0. In which case, *INVENTORY_1* can be deduced to be in stage 0. In turn, we can then use this staging information for *INVENTORY_1* in the following expression:

```
[R_I2] INVENTORY_2 = INVENTORY_1 + PURCHASE_2 - DEMAND_2;
```

and deduce that *INVENTORY_2* is in stage 1, by the fact that the maximum of the stages of all the right-hand side variables is 1. LINGO continues in this fashion, iteratively tracing the dependencies in the model and assigning stages to each remaining row and variable. Explicit stage assignments made by the user are never overridden.

Guidelines for determining the variables and rows that require explicit staging were presented in section, *Identifying Initial Decision and Recourse Variables*.

The next report is the *Random Variable Distribution Report*, which lists information on the distribution assigned to each of the random variables, as well as their observed sample means and standard deviations:

```
Random Variable Distribution Report
-----------------------------------

                            Sample        Sample
     Random Variable          Mean        StdDev
Distribution
               COST_2      6.166667      1.027402
DST_DMD,DST_DMD_1,1
             DEMAND_2      143.3333      32.99832
DST_DMD,DST_DMD_1,2
```

From this, we can see that the random variable COST_2 is assigned to the first column of the DST_DMD_1 instance of the tabular distribution DST_DMD. Furthermore, COST_2 had a sample mean of 6.166667 and a sample standard deviation of 32.99832.

The remainder of the solution report contains sub-reports for each of the scenarios. Information regarding the each scenario's probability, objective value and variable values are displayed. The values for the random variables for the scenario are contained in a separate section of the report. Here we reproduce the report for scenario 1:

```
Scenario: 1   Probability: 0.3333333   Objective: 1100.000
------------------------------------------------------------

        Random Variable           Value
                 COST_2        5.000000
                DEMAND_2        100.0000

                Variable           Value
           PURCHASE_COST        1000.000
               HOLD_COST        100.0000
              PURCHASE_1        200.0000
              PURCHASE_2        0.000000
             INVENTORY_1        100.0000
             INVENTORY_2        0.000000

                     Row           Value
                    R_PC        0.000000
                    R_HC        0.000000
                    R_I1        0.000000
                    R_I2        0.000000
```

Note: Moderate to large SP models can have a huge number of scenarios. In which case, the standard solution report, with its detailed reporting of each scenario, can become unwieldy. In which case, you may wish to reduce the amount of output by placing LINGO into *Terse* output mode. You may then request more specific solution information using the *Solver|Solution* command or add output statements to your model (see *SPCOLLEGE2.LG4* for an example of adding output statements to an SP model).

If you examine the reports for each of the three scenarios in this example, you will find that the costs are as follows: $1,100 for Normal, $1,400 for Cold and $1,700 for Very Cold. This results in an expected cost of $1,400 = (1/3) * $1100 + (1/3) * $1400 + (1/3) * $1700. The company buys 200 units of gas in period 1, applying 100 units immediately to period 1 demand, while storing 100 units for period 2. If period 2"s weather is normal, then period 2 demand is satisfied entirely from inventory. If the period 2 turns out to be cold, an additional 50 units must be purchased in period 2. If the very cold scenario occurs, then an additional 80 units must be purchased.

Stock Option Example

Here we will walk through a stochastic programming example of a six-period *American put-option* model. The holder of an American put option has the right to sell a specified (underlying) stock at any time (the feature of American options) between now and a specified expiration date at a specified strike price. The holder makes a profit in the period of exercise if the strike price exceeds the market price of the stock at the time of sale, otherwise the profit is 0. The objective will be to maximize the expected present value of the option in period 0, which also corresponds to the maximum amount we should be willing to pay for the option.

In this example, the underlying stock's initial price is $100, with the option's strike price equal to $99. This initially places the option "out of the money" and make its value $0. However, should the stock's price decline over subsequent periods to the point where it becomes less than the strike price, then the option will have a positive value.

We will also use a risk free interest rate of 3% for discounting revenue back to the initial period.

The price behavior of the underlying stock is the stochastic element in this example. In any given period, we have determined that there are four equally likely outcomes for the stock's price:

Return (%)	Probability
+9.0	1/4
+3.0	1/4
-1.0	1/4
-8.0	1/4

Using the following reasoning, a casual observer might analyze the above data and declare the option as being worthless:

> *The stock is presently out of the money by $1. Before it will move into the money, the stock must decline by at least $1. However, this seems highly unlikely given that the expected return in any period is positive +0.75% = .25 * (9.0+3.0-1.0-8.0)%. On average, the stock will just go up in value, and the option will never go into the money. Given this, we should not be willing to pay anything for this option.*

We will see below if our casual observer in on the right track or not.

The complete LINGO model can be found in the LINGO samples folder under the name *SPPUTOPT1.LG4.*

The Core Model

Step 1 - Defining the Core Model:

First off, we have the model's sets section. Unlike the previous gas-buying model, which was entirely scalar, we will make use of LINGO's set-based modeling capabilities in this example. Any set-based model will have at least one sets section, and for the core model for this example we have the following:

```
SETS:
   PERIODS: !six periods (1 initial and 5 recourse);
    RETURN,        !return of underlying stock, a r.v.;
    PRICE,         !current price of underlying stock;
    SELL,          ! = 1 if option exercised in period;
    WLTH,          !wealth in period from exercise of
option;
    SAMP_SZ;       !sample size;
ENDSETS
```

Sets Section

As mentioned, this is a six-period (one initial and five recourse) model. Across the six periods, we have the following attributes:

- *RETURN* - the stock's return (fractional price change), a random variable,
- *PRICE* - the current price of the stock (a function of the *RETURN*),
- *SELL* - a recourse variable indicating the fraction of the option position sold,
- *WLTH* - wealth in the current period consisting of the wealth from the previous period multiplied by the risk free rate plus any proceeds from exercising all, or part, of the option, and
- *SAMP_SZ* - the sample sizes used for sampling the distribution for *RETURN*.

Next we have the data section containing the parameters we mentioned above, consisting of the stock's initial price, the option's strike price and the risk free rate for discounting:

```
DATA:
   PERIODS = P0..P5;      !names for the periods;
   INIT_PRICE = 100;      !initial underlying stock price;
   STRIKE_PRICE = 99;     !option strike price;
   RISK_FREE_RATE = .03; !risk free rate for discounting;
ENDDATA
```

Data Section

Next, we have the core model. You will recall that the core model is the model we would construct if the random variables were know with certainty.

```
! Core Model ------------------------------------------+;

NP = @SIZE( PERIODS); ! Number of periods;

! Maximize the present value of the option in period 0;
[R_OBJ] MAX = WLTH( NP)/( 1 + RISK_FREE_RATE) ^ ( NP - 1);

! Compute stock's price in each period;
[R_PRICE_INIT] PRICE( 1) = INIT_PRICE;
@FOR( PERIODS( P) | P #GT# 1:
 [R_PRICE] PRICE( P) = PRICE( P - 1) * ( 1 + RETURN( P));
  );

! Calculate our compounded wealth in each period;
[R_PV_1] WLTH( 1) =
 SELL( 1) *
  ( STRIKE_PRICE - PRICE( 1));

@FOR( PERIODS( P) | P #GT# 1:
 [R_PV] WLTH( P) = SELL( P) * ( STRIKE_PRICE - PRICE( P)) +
  WLTH( P - 1) * ( 1 + RISK_FREE_RATE);
   );

! Can't sell more than we have;
[R_SELL_EQ_1] @SUM( PERIODS( P): SELL( P)) <= 1;

! Random variables not allowed in initial period;
[R_RETURN_P0] RETURN( 1) = 0;
```

Core Model for Stock Option Example

As mentioned, our objective is to maximize the present value of the option in the initial period, which will tell us the maximum we should be willing to pay for the option. To compute the present value in period 0, we discount wealth in the final period back to the initial period using the risk-free rate:

```
! Maximize the present value of the option in period 0;
[R_OBJ] MAX = WLTH( NP)/( 1 + RISK_FREE_RATE) ^ ( NP - 1);
```

where *NP* is the number of periods, calculated using the *@SIZE* function:

```
NP = @SIZE( PERIODS); ! Number of periods;
```

Technically, *NP* is actually equal to the number of periods plus 1, due to *P0* being part of the *PERIODS* set. So, when we discount wealth, we discount by *NP* - 1 periods as opposed to *NP* periods.

In the next section of the core model we compute wealth in each period:

```
! Calculate our compounded wealth in each period;
[R_PV_1] WLTH( 1) =
 SELL( 1) *
  ( STRIKE_PRICE - PRICE( 1));

@FOR( PERIODS( P) | P #GT# 1:
 [R_PV] WLTH( P) = SELL( P) * ( STRIKE_PRICE - PRICE( P)) +
  WLTH( P - 1) * ( 1 + RISK_FREE_RATE);
   );
```

Wealth can potentially come from two sources: sale of the option and/or carryover wealth from the previous period. In the first period, carryover wealth is not possible, so period 1 is handled as a special case with sale of the option being the only source of wealth. Subsequent periods' wealth are then computed in an *@FOR* loop and include both sales and carryover sources of wealth. Revenue from sales is computed as: $SELL_p$ * (*STRIKE_PRICE* - $PRICE_p$). Note, that the *SELL* variable may be fractional if we choose to sell less than the full position on a period. Carryover wealth is computed as: $WLTH_{p-1}$ * (1 + *RISK_FREE_RATE*), where we multiply by the risk free rate to account for interest payments on wealth.

Clearly, we can't sell more of full option position, so the sum of the *SELL* attribute across all periods cannot exceed 1:

```
! Can't sell more than we have;
[R_SELL_EQ_1] @SUM( PERIODS( P): SELL( P)) <= 1;
```

Finally, there can be no return in period 0, so we set *RETURN* to 0 in the initial period:

```
! Random variables not allowed in initial period;
[R_RETURN_P0] RETURN( @INDEX( P0)) = 0;
```

In the next section, we will cover the final part of the model listing the stochastic properties.

Stochastic Declarations

The remainder of the model after the core model is the stochastic declarations section:

```
! SP Related Declarations ----------------------------+;

! Returns are the random variables;
@FOR( PERIODS( P) | P #GT# @INDEX( P0):
 @SPSTGRNDV( P - 1, RETURN( P));
  );

! Sell(P0) is the init decision,
  Sell(P1..P5) are the recourse variables;
@FOR( PERIODS( P):
 @SPSTGVAR( P - 1, SELL( P));
  );

SETS:

! Declare the outcome table distribution for returns;
   OUTCOMES: O_RETURN; !An outcome set with a single;
                       ! attribute of stock's return;
ENDSETS

DATA:
! Probability distribution and sampling data;
   SAMP_SZ = 0 4 4 4 4 4;        !sample size/period;
   O_RETURN = .09 .03 -.01 -.08; !4 outcomes for return;
ENDDATA

! Declare outcome table distribution for return;
@FOR( PERIODS( P) | P #GT# 1:
 @SPDISTTABLE( O_RETURN, RETURN( P));
  );

! Set the sample sizes for the stages;
@FOR( PERIODS( P) | P #GT# 1:
 @SPSAMPSIZE( P - 1, SAMP_SZ( P));
  );

END
```

Stochastic Declarations for Stock Option Example

Step 2 - Identifying the Random Variables:

As mentioned, the random variables in this example are the stock price returns, *RETURN*, for periods 1 through 5. We indicate this with the expression:

```
! Returns are the random variables;
@FOR( PERIODS( P) | P #GT# @INDEX( P0):
 @SPSTGRNDV( P - 1, RETURN( P));
  );
```

Note that return in the initial period, *P0*, is not valid in this model and we zeroed it out above.

Step 3 - Identifying the Initial Decision and Recourse Variables:

The initial decision is deciding whether or not to sell some, or all, of the option position in the initial period. The recourse decisions are whether or not to sell some of our position in periods 1 through 5. We set this up with the following:

```
! Sell(P0) is the initial decision,
  Sell(P1..P5) are the recourse variables;
@FOR( PERIODS( P):
 @SPSTGVAR( P - 1, SELL( P));
  );
```

Step 4 - Declare Distributions

We have a single discrete distribution for the returns on the underlying stock. As with the previous gas-buying example, this distribution is represented by an outcome table. However, unlike the previous example, we will make use of the matrix-based *@SPDISTTABLE* function for declaring our outcome table and its random variables. The code in the model devoted to declaring this distribution is:

```
SETS:
! Declare the outcome table distribution for returns;
   OUTCOMES: O_RETURN; !An outcome set with a single;
                       ! attribute of stock's return;
ENDSETS

DATA:
! Probability distribution and sampling data;
   SAMP_SZ = 0 4 4 4 4 4;        !sample size/period;
   O_RETURN = .09 .03 -.01 -.08; !4 outcomes for return;
ENDDATA

! Declare outcome table distribution for return;
@FOR( PERIODS( P) | P #GT# 1:
 @SPDISTTABLE( O_RETURN, RETURN( P));
  );

! Set the sample sizes for the stages;
@FOR( PERIODS( P) | P #GT# 1:
 @SPSAMPSIZE( P - 1, SAMP_SZ( P));
  );
END
```

First, we define a set *OUTCOMES* with an attribute *O_RETURN*. *O_RETURN* will be used to hold the four possible outcomes from the stock's return in a period. Next, we call *@SPDISTTABLE* to declare the outcome table for each *RETURN* random variable in periods 1 through 5.

We then set the samples size for each of the periods to 4 in the data section:

```
SAMP_SZ = 0 4 4 4 4 4;        !sample size/period;
```

and then pass them to LINGO's SP solver in the *@FOR* loop:

```
! Set the sample sizes for the stages;
@FOR( PERIODS( P) | P #GT# @INDEX( P0):
   @SPSAMPSIZE( P - 1, SAMP_SZ( P));
);
```

Note that a sample size is not relevant for the initial period, *P0*, given that no random variables are permitted in the initial period. So, we have explicitly avoided passing a sample size for period *P0.*

In the previous gas-buying example, we had only one stage and three outcomes, resulting in only three possible scenarios. This current model has a much larger set of scenarios. In this example, there are 5 stages with 4 possible outcomes for return in each stage. This results in 4^5=1024 scenarios. In order to keep the number of scenarios down to a more manageable size in larger models, you may need to reduce the sample sizes in each stage. As an example you might want to try running this same model with the following sample sizes:

```
SAMP_SZ = 0 4 4 2 2 2;        !sample size/period;
```

This cuts the number of scenarios in the underlying SP model to 4*4*2*2*2=128 scenarios. This should help to dramatically cut the runtime, but at the expense of potentially less accurate results due to sampling error.

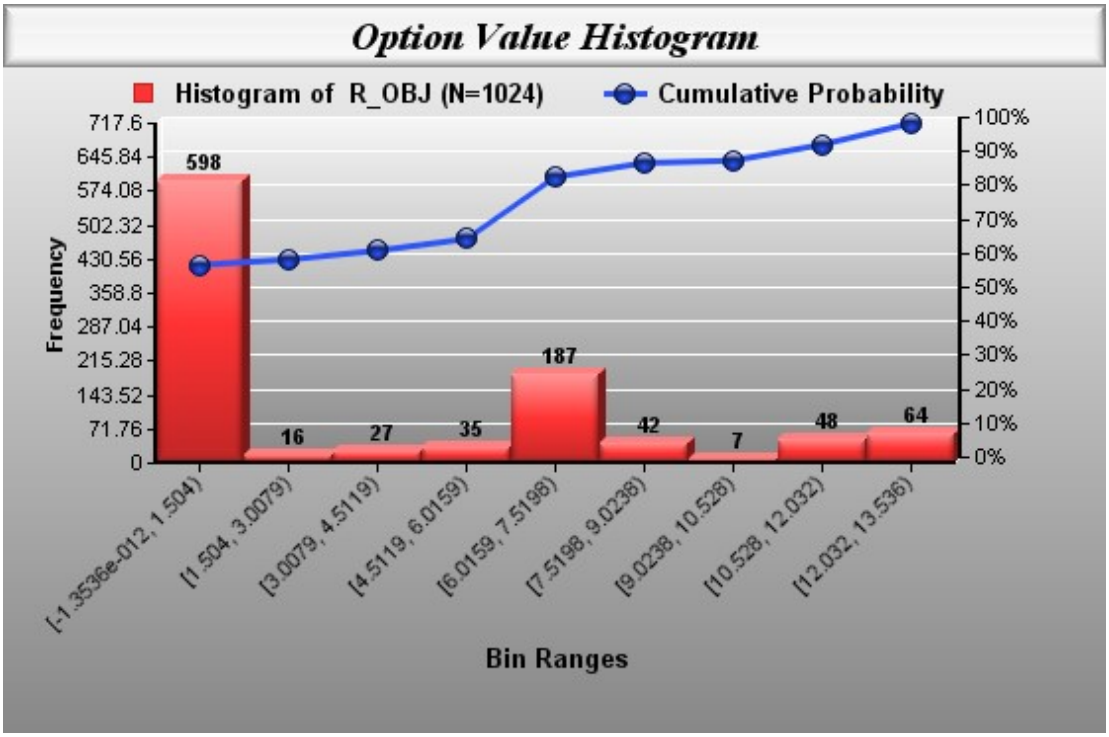
The Solution

After solving the model, you'll find that the expected value of the option is $3.37:

```
Local optimal solution found.
Objective value:                              3.451804
Infeasibilities:                              0.000000
Total solver iterations:                         19608
```

Thus, it turns out that our casual observation of the option being worthless was incorrect. Even though the option is initially out of the money, and, on average, is expected to go even further out of the money as time passes, the option still has a positive expected value of $3.45. This is due to the volatility in the stock price, which can temporarily send the stock price down, even though the long run expectation is for the price to climb.

Based on our sample size of 4 in each of teh 5 stages, there was a total of 1024 scenarios. In fully 598 of these scenarios, the option's value did prove to be worthless. However, in the remaining 426 scenarios the option had a positive value of as high as $13.54. Here's a histogram of the full range of outcomes:



The key initial decision was whether or not to sell the option. From the stage 0 (initial decision) solution we see that the optimal decision is to not sell:

```
Stage 0 Solution
----------------
          Variable           Value
         SELL( P0)        0.000000
```

Of course, we didn't really need LINGO to tell us not to sell in the first period — we could have deduced this ourselves. The worst outcome we could experience is to gain $0. By selling in the initial period, we lock in the $0 return, never allowing for the possibility of the stock to go down in price and the option to come into the money. We will rely on LINGO's solution, however, to tell us the best time to sell in subsequent periods. Here is a fragment of the solution report for scenario 65, which is one of the scenarios with the maximum option value of $13.54:

```
Scenario: 65   Probability: 0.9765625E-03   Objective: 13.53568
-------------------------------------------------------------

   Random Variable           Value
       RETURN( P1)      -0.8000000E-01
       RETURN( P2)      -0.8000000E-01
       RETURN( P3)      -0.1000000E-01
       RETURN( P4)      -0.1000000E-01
       RETURN( P5)       0.3000000E-01

          Variable           Value
                NP        6.000000
       RETURN( P0)        0.000000
        PRICE( P0)        100.0000
         SELL( P0)        0.000000
         WLTH( P0)        0.000000
        PRICE( P1)        92.00000
         SELL( P1)        0.000000
         WLTH( P1)        0.000000
        PRICE( P2)        84.64000
         SELL( P2)        1.000000
         WLTH( P2)        14.36000
        PRICE( P3)        83.79360
         SELL( P3)        0.000000
         WLTH( P3)        14.79080
        PRICE( P4)        82.95566
         SELL( P4)        0.000000
         WLTH( P4)        15.23452
        PRICE( P5)        85.44433
         SELL( P5)        0.000000
         WLTH( P5)        15.69156
```

Note that the stock goes down in the first four periods, but that we sold in period 2 for a gain of $14.36, which discounted back to the initial period is equal to a gain of $13.54. Had we been omniscient, we could have held on to the option for two more periods for additional gains. But, of course, we can't see the future, and we must base our decisions on maximizing the option's expected value over all scenarios.

Investing Under Uncertainty Example

In the previous two examples, the random variable distributions were expressed as discrete outcome tables. In this example, we will look at a model with distributions that are not discreet. In particular, we will be using a normal distribution for our random variables. Given the infinite number of outcomes in a normal distribution, we will also need to use *@SPSAMPSIZE* to declare the samples sizes to be used by the SP solver. In addition, we will use *@SPCORRPEARSON* to input a correlation coefficient for some of the random variables.

In this example, we are investing to fund a college education (Birge and Louveaux, 1997). We may invest in either stocks or bonds, both of which are assumed to have returns that are normally distributed as per the following table:

Asset	Mean Return	Standard Deviation
Bonds	12%	1%
Stocks	16%	10%

It also turns out that the returns on stocks and bonds are correlated, with a correlation coefficient of .5.

Our initial wealth is $55,000, while our target at the end of four periods is $80,000. Our goal is to maximize wealth at the end of the three periods, however, we will be very disappointed if we don't meet our goal. For this reason, we penalize ourselves by a factor of four for each dollar that we are short of our goal. We also have the option of readjusting our portfolio at the start of each period.

The complete LINGO model can be found in the LINGO samples folder under the name *SPCOLLEGENORM.LG4.*

The Core Model

The core model for our investing example follows:

```
! Core Model ------------------------------------------+;

SETS:
   TIME;
   ASSETS: MU, SIGMA;
   AXT( ASSETS, TIME): INVEST, RETURN;
   AXA( ASSETS, ASSETS) | &2 #GT# &1: RHO;
ENDSETS

DATA:
   INITIAL = 55;
   GOAL = 80;
   PENALTY = 4;
   TIME = T0..T3;
   ASSETS,   MU,  SIGMA =
    BONDS   1.12   .01
    STOCKS  1.16   .10;
   RHO = .5;
ENDDATA

MIN = PENALTY * UNDER - OVER;

@FOR( TIME( T) | T #GT# 1:
  @SUM( ASSETS( A): RETURN( A, T) * INVEST( A, T - 1)) =
   @SUM( ASSETS( A): INVEST( A, T))
);

FINAL = @SUM( ASSETS( A): INVEST( A, @SIZE( TIME)));
FINAL = GOAL + OVER - UNDER;

@SUM( ASSETS( A): INVEST( A, @INDEX( TIME, T0))) = INITIAL;

@FOR( ASSETS( A):
   RETURN( A, @INDEX( TIME, T0)) = 0;
);
```

Core Model for College Investing Example

In the sets section, we have two primitive sets: *TIME* and *ASSETS*. The *TIME* set will be used to represent our time periods and will have four members: *T0* will represent the initial decision period, while *T1*, *T2* and *T3* will represent the three time periods. The set *ASSETS* will contain two members: *BONDS* and *STOCKS*.

We also define two derived sets in the sets section: *AXT* and *AXA*. The *AXT* set is a dense, 2-dimensional set consisting of all (*ASSETS*,*TIME*) pairs. We define the two attributes *INVEST* and *RETURN* on *AXT*. The $INVEST_{a,t}$ attribute is our decision variable of how much to invest in asset *a* in period *t,* while $RETURN_{a,t}$ is a random variable representing the return of asset *a* in period *t*.

Next, in the data section we input the following pieces of data:

- INITIAL - the initial wealth,
- GOAL - the final goal for wealth
- PENALTY - the penalty factor to apply to goal shortfalls
- TIME - the four members of the TIME set
- ASSETS,MU,SIGMA - the normal distribution data for returns on bonds and stocks, and
- RHO - the correlation coefficient between bond and stock returns

Getting into the model section, the first expression:

```
MIN = PENALTY * UNDER - OVER;
```

is the objective function, which can be viewed as a penalty function that we will be minimizing. For each dollar under our goal (*UNDER*) we get penalized 4 units, while for each dollar over our goal (*OVER*) we reduce the penalty by 1 unit. This will force the solver to more heavily weight solutions that meet our goal, perhaps at the expense of maximizing total wealth.

In the next expression:

```
@FOR( TIME( T) | T #GT# 1:
@SUM( ASSETS( A): RETURN( A, T) * INVEST( A, T - 1)) =
  @SUM( ASSETS( A): INVEST( A, T))
);
```

we are setting total investments in period *t* equal to total investments plus returns in period *t*-1. Note that by formulating the investment flows in this manner, we are free to reallocate investments each period.

The following two expressions are used to calculate the amount that we are either over (OVER) or under (UNDER) goal:

```
FINAL = @SUM( ASSETS( A): INVEST( A, @SIZE( TIME)));
FINAL = GOAL + OVER - UNDER;
```

Next, we add a constraint to limit the initial investments to our initial wealth level:

```
@SUM( ASSETS( A): INVEST( A, @INDEX( TIME, T0))) = INITIAL;
```

Without this expression, the model would allow infinite investment, leading to an unbounded solution.

Finally, there are no investment returns in the initial decision period, so we zero out them out:

```
@FOR( ASSETS( A):
  RETURN( A, @INDEX( TIME, T0)) = 0;
);
```

Stochastic Declarations

After the core model, we enter the stochastic declarations identifying the stochastic features of the model:

```
! SP Related Declarations ----------------------------+;

! The initial decision and recourse variables;
@FOR( AXT( A, T):
   @SPSTGVAR( T - 1, INVEST( A, T));
);

@FOR( AXT( A, T) | T #GT# 1:
! Return is a random variable...;
   @SPSTGRNDV( T - 1, RETURN( A, T));
! ...and it's normally distributed;
   @SPDISTNORM( MU( A), SIGMA( A), RETURN( A, T))
);

@FOR( TIME( T)| T #GT# @INDEX( TIME, T0):
   !Set a sample size for each period;
   @SPSAMPSIZE( T-1, 8);

   !Load the stock and bond correlation for the period;
   @FOR( AXA( I, J):
      @SPCORRPEARSON( RETURN( I, T), RETURN( J, T), RHO( I,
J))
   );
);
```

Stochastic Declarations for College Investing Example

The initial decision variables in the model are how much to allocate to bonds and how much to allocate to stocks in the initial time period, or variables *INVEST*(*BONDS*, *T0*) and *INVEST*(*STOCKS*, *T0*). The recourse variables are how much to allocate to both assets in subsequent periods as the returns for each period are observed, or *INVEST*(*BONDS*, *T1*), *INVEST*(*STOCKS*, *T1*), ..., *INVEST*(*STOCKS*, *T3*). We identify these variables and assign them to their respective stages as follows:

```
! The initial decision recourse variables;
@FOR( AXT( A, T):
  @SPSTGVAR( T - 1, INVEST( A, T));
);
```

In the following loop:

```
@FOR( AXT( A, T) | T #GT# 1:
! Return is a random variable...;
  @SPSTGRNDV( T - 1, RETURN( A, T));
! ...and it's normally distributed;
  @SPDISTNORM( MU( A), SIGMA( A), RETURN( A, T))
);
```

we declare the asset return variables as random variables and declare that they have a normal distribution using the *@SPDISTNORM* function. Note that @SPDISTNORM requires the parameters of the distribution, which, in the case of a normal distribution, are the mean and standard deviation. Finally, we set the sample size to 8 samples in each stage/period and load the correlation coefficient between the two assets in each period in the loop:

```
@FOR( TIME( T)| T #GT# @INDEX( TIME, T0):
  !Set a sample size for each period;
  @SPSAMPSIZE( T-1, 8);

  !Load the stock and bond correlation for the period;
  @FOR( AXA( I, J):
     @SPCORRPEARSON( RETURN( I, T), RETURN( J, T), RHO( I, J))
  );
);
```

In this particular instance, we are using the Pearson method for inducing correlations in the sample values. Other alternatives are the Kendall and Spearman methods.

The Solution

After solving the model, you'll find that the expected value of the penalty function is approximately 2.78:

```
Global optimal solution found.
Objective value:                              2.775631
Infeasibilities:                              0.000000
Total solver iterations:                          1092
```

You will recall that the penalty function used in the model's objective is a weighted combination of 4 times the dollars under target minus the number of dollars over target. So, does the positive expected objective value mean that we miss our target on average? Perhaps not, given that we weight the under target dollars 4 times more than the over target dollars. To get a more meaningful result, a calc section was added to the model:

```
CALC:
  @SET( 'TERSEO', 1);
  @SOLVE();
  I = 1;
  NOVER = 0;
  NUNDER = 0;
  @WRITE( '                    Surplus     ');
  @FOR( TIME( T) | T #GT# @INDEX( T0): @WRITE( '    ', TIME( T)));
  @WRITE( @NEWLINE( 1));
  @WRITE( '   Scenario        Return      Prob');
  @FOR( TIME( T) | T #GT# @INDEX( T0): @WRITE( '  Bond   Stock'));
  @WRITE( @NEWLINE( 1));
  X_SURPLUS = 0;
  @WHILE( I #LE# @SPNUMSCENE():
     @SPLOADSCENE( I);
    @WRITE( @FORMAT( I, '10.0f'), @FORMAT( OVER - UNDER, '15.3f'),
        @FORMAT( @SPPRBSCENE( I), '10.3f')
     );
     @FOR( TIME( T) | T #GT# @INDEX( T0): @FOR( ASSETS( A):
        @WRITE( '   ', @FORMAT( 100*(RETURN( A, T) - 1), '4.1f'),
        '%')));
     @WRITE( @NEWLINE( 1));
     X_SURPLUS = X_SURPLUS + @SPPRBSCENE( I) * ( OVER - UNDER);
     I = I + 1;
     @IFC( UNDER #LE# 1.E-8
        NOVER = NOVER + 1;
     @ELSE
        NUNDER = NUNDER + 1;
     );
   );
   @WRITE( @NEWLINE( 1));
   @WRITE( '  Expected surplus: ', @FORMAT( X_SURPLUS, '15.3f'));
   @WRITE( @NEWLINE( 1));
   @WRITE( '  Scenarios over goal: ', @FORMAT( NOVER, '12g'));
   @WRITE( @NEWLINE( 1));
   @WRITE( ' Scenarios under goal: ', @FORMAT( NUNDER, '11g'));
   @WRITE( @NEWLINE( 1));
   @WRITE( ' Success ratio: ',
            @FORMAT( NOVER/(NOVER+NUNDER), '18.3f'));
ENDCALC
```

This calc section uses the scripting capabilities in LINGO to generate a custom report that displays each of the 512 scenarios in the model, their return values and their expected surpluses of funds over the goal. (More details on LINGO scripting capabilities may be found in Chapter 13, *Programming LINGO*.) The report also lists summary information on the expected surplus dollars, along with the number of scenarios over target and the number below target. Portions of this report follow:

	Surplus		T1		T2		T3	
Scenario	Return	Prob	Bond	Stock	Bond	Stock	Bond	Stock
1	0.528	0.002	11.0%	35.4%	13.3%	7.7%	9.5%	0.3%
2	14.604	0.002	11.0%	35.4%	13.3%	7.7%	11.9%	17.9%
3	12.203	0.002	11.0%	35.4%	13.3%	7.7%	11.0%	14.9%
4	27.423	0.002	11.0%	35.4%	13.3%	7.7%	11.6%	33.9%
5	18.706	0.002	11.0%	35.4%	13.3%	7.7%	12.9%	23.0%
				<...>				
500	9.946	0.002	12.7%	23.3%	11.4%	0.5%	11.6%	33.9%
501	3.249	0.002	12.7%	23.3%	11.4%	0.5%	12.9%	23.0%
502	-7.809	0.002	12.7%	23.3%	11.4%	0.5%	12.3%	5.3%
503	-4.413	0.002	12.7%	23.3%	11.4%	0.5%	12.3%	10.8%
504	1.961	0.002	12.7%	23.3%	11.4%	0.5%	13.2%	20.9%
505	6.928	0.002	12.7%	23.3%	12.8%	27.7%	9.5%	0.3%
506	22.123	0.002	12.7%	23.3%	12.8%	27.7%	11.9%	17.9%
507	19.531	0.002	12.7%	23.3%	12.8%	27.7%	11.0%	14.9%
508	35.960	0.002	12.7%	23.3%	12.8%	27.7%	11.6%	33.9%
509	26.551	0.002	12.7%	23.3%	12.8%	27.7%	12.9%	23.0%
510	11.241	0.002	12.7%	23.3%	12.8%	27.7%	12.3%	5.3%
511	15.953	0.002	12.7%	23.3%	12.8%	27.7%	12.3%	10.8%
512	24.741	0.002	12.7%	23.3%	12.8%	27.7%	13.2%	20.9%

Expected surplus:	4.413
Scenarios over goal:	344
Scenarios under goal:	168
Success ratio:	0.672

Based on this report, we see that the expected surplus is $4,413, meaning that on average we can expect to meet our goal by that amount. Furthermore, 344 of the 512 scenarios ended over target for a success ratio of 67.2%.

Chance-Constrained Programs (CCPs)

The second major class of models in stochastic programming is chance-constrained programs (CCP). A CCP model is a) similar to general stochastic programs in that the model contains random quantities with known distributions, but b) simpler in that the model has just a single decision stage and a single random outcome stage.

The goal in CCP is to make an optimal decision prior to realization of random data, while controlling the chance that constraints are violated. Consider an LP with random matrix Ξ and random right-hand-side ω:

$$\begin{aligned} &Min\ c\ x \\ &\Xi x_i \geq \omega_i \quad i=1...m \end{aligned}$$

If we required all m realizations of $\Xi\ x \geq \omega$ to be satisfied, then we would get a very conservative/expensive solution x, or no feasible solution at all. The distinctive feature of CCP is that we require that $\Xi\ x \geq \omega$ be satisfied with some prespecified probability, $0 < p < 1$, as opposed to it being satisfied for all possible realizations of (Ξ,ω).

An example of a CCP model would be a blending model where the quality level of the raw materials is not known with certainty, but have known probability distributions. Given this variability in raw material quality, it may not always be practical to satisfy quality requirements in the final blend 100 percent of the time. Instead, we seek to find a blend of the raw materials that will satisfy quality constraints to a specified precentage, say 90 percent, of the time. We present this example later in the chapter in section *A CCP Fuel Blending Model*.

Setting up CCP Models

Setting up a CCP model requires four steps. Three of the steps are identical to those used in setting up an SP. The one step found in SPs that's not found in CCPs is identifying initial decision and recourse variables, and this is because in a CCP all decision variables are considered to be in stage 0, i.e., they are all initial decision variables. This contrasts with an SP, which may have any number of recourse stages in addition to the initial decision stage. The fourth (and new) step in setting up CCPs is identifying the chance-constraint sets. These four steps are summarized in the table below and are illustrated in detail in the example CCP model that follows.

Step	Task	Description	How
1	Defining core model	The core model is built just like any other deterministic LINGO model. The random variables are used directly in the core model's expressions as if they were determinate.	Entered like any other deterministic LINGO model.
2	Identifying the random variables	Each random variable must be identified along with the stage of the model where it becomes known. In a CCP, random variables must always belong to stage 1.	*@SPSTGRNDV*
3	Declaring distributions	The probability distributions of the random variables must be declared. The techniques for doing so will depend on whether the variable of interest has a distribution defined by either a) a discrete *outcome table*, or b) a *parametric distribution*. An outcome table is a finite set of all possible outcomes, their probabilities and values for the random variable(s). Outcome tables may be constructed using either scalar values or attribute vectors. A parametric distribution, on the other hand, is defined by a particular type of probability density function, e.g., the normal distribution. Parametric distributions may have a finite or infinite number of possible outcomes, depending on the distribution type.	Outcome Tables: Scalar-based: *@SPTABLESHAPE* *@SPTABLEOUTC* *@SPTABLEINST* *@SPTABLERNDV* Vector-based: *@SPDISTTABLE*
4	Identify chance-constraint sets.	One or more sets of constraints are classified as being chance-constrained. An individual constraint may appear in no more than one chance-constraint set.	*@SPCHANCE*

Language Features for CCP Models

The primary features in the LINGO language used for building a stochastic programming model can be partitioned into the following three categories, corresponding to steps 2-4 of the *Setting Up CCP Models* section above:

1. *Identifying random variables*
2. *Declaring distributions*
3. *Identifying chance-constraint sets*

Each of the categories are discussed in the following three sections, which are primarily intended for reference use. The reader may wish to jump directly to the CCP modeling example, *A CCP Fuel Blending Example*, in order to see LINGO's CCP modeling features used in the context of an actual model.

There are additional CCP language constructs available for use in calc sections for model's that exploit LINGO's programming capability. These features are discussed in Chapter 13, *Programming LINGO.*

Identifying Random Variables

A fundamental component of any CCP model is its set of random variables. The random variables are the stochastic component of the model. Their values are not known with certainty beforehand, however, their uncertainty can be expressed with known probability distributions. LINGO requires each random variable to be explicitly identified. When we identify each random variable, we must also provide the stage in the scenario tree where each variable's value is revealed. Currently, unlike SPs, CCPs are limited to single stages, given this, the stage value must always be 1. The *@SPSTGRNDV* function is used for this for identifying the random variables in a CCP model:

@SPSTGRNDV(STAGE, VARIABLE_NAME)

This function takes two arguments: the random variable's name and the integer valued stage at which the random variable's value is revealed. Note that this function merely flags a variable as being random, and does not communicate any information regarding the actual distribution of the variable. A distribution is associated with the random variable in the *Binding random variables to distribution instances*, discussed below. Some examples follow:

Example 1: `@SPSTGRNDV( 1, DEMAND);`

Flags variable *DEMAND* as being a random variable in stage 1.

Example 2:

```
@FOR( CUSTOMERS( I):
    @SPSTGRNDV( 1, DEMAND( I))));
```

Flags *DEMAND(I)* as being a random variable in stage 1. Note that by using *@SPSTGRNDV* in an *@FOR* loop, we are able to identify multiple variables as being random.

Declaring Distributions

In a CCP, the distributions for the random variables are declared exactly as they are in SPs. You can refer to the section *Declaring Distributions* for SPs above to see how this is done.

Identifying Chance-Constraint Sets

A CCP model will have one or more sets of chance constraints. Each set is assigned a) a name, b) a direction and c) a probability. You make these declarations using the *@SPCHANCE* function using the syntax:

```
@SPCHANCE( 'Set_Name', '>='|'<=', Probability)
```

The set name must be unique, enclosed in quotes, and must follow normal *LINGO naming conventions*. The direction must be either '>=' or '<=', while the probability is a real number in the range of [0,1]. The direction argument indicates whether the probability that the set is satisfied is either greater-than-or-equal-to or less-than-or-equal-to the specified probability. An example follows:

Example 1: `@SPCHANCE( 'CCP_DEMAND', '>=', .95);`

Here we declare a CCP constraint set with name *CCP_DEMAND*, a direction of greater-than-or-equal-to, and a set probability of .95. This says that the constraints assigned to this set must all be satisfied in at least 95% of the scenarios. If just one constraint is unsatisfied in a particular scenario, then the whole set is considered unsatisfied for that scenario.

Once you've declared a CCP set, there is still an additional step of assigning actual constraints to the set. This is again done with the *@SPCHANCE* function, however, the syntax is slightly different:

```
@SPCHANCE( 'Set_Name',  Constraint_Ref)
```

The same set name is used as was used in the set declaration, however, you now must enter a constraint reference, consisting of the name of a constraint in the core model. Here's an example:

Example 2:

```
@FOR( CUST( I):
 @SPCHANCE( 'CCP_DEMAND', R_DEMAND( I)));
```

Here we use an *@FOR* loop to assign multiple constraints with name *R_DEMAND* to the CCP constraint set *CCP_DEMAND.*

When assigning constraints to a CCP constraint set there are a few rules to remember:

- A constraint may be assigned to no more than one CCP set.
- Each constraint belonging to a CCP set must reference at least one random variable.
- The objective function may not be assigned to a CCP set.

In the next section, we will walk through building a complete CCP model for fuel blending.

A CCP Fuel Blending Example – The Model

As production manager at a fuel processing plant, you would like to maximize profit from blending the raw materials butane, catalytic reformate, and naphtha into the two finished products Regular and Premium. These final products must satisfy certain ranges of quality requirements on octane, vapor pressure, and volatility. You also know the minimum required for each finished good, the maximum you will be able to sell, and the profit contribution per unit.

Each raw material has a limited availability and cost per unit which are known, however, the quality level of the three raw materials is not known with certainty. We do know that the quality levels are normally distributed with known means and standard deviations. We'd like to chose blending ratios so that the finished products don't fall below their lower limits on requirements at least 90 percent of the time and don't exceed their upper limits at least 70 percent of the time.

We will use a CCP to solve this problem, with the quality constraints forming the chance-constrained sets. The model we will use is available in the LINGO *Samples* folder under the name *BLENDCCP.LG4*.

Our model's sets section is:

```
SETS:
   !Each raw material has an availability
     and cost/unit;
   RAWMAT: AVAIL, COST;

   !Each finished good has a min required,
     max sellable, selling price,
     and batch size to be determined;
   FINGOOD: MINREQ, MAXSELL, PRICE, BATCH;

   !The quality measures set;
   QUALMES;

   !For each combo of raw material and
     quality measure there is an average
     quality level, its standard
     deviation, and the actual quality
     level received;
   RXQ( RAWMAT, QUALMES): QLEVMU, QLEVSD, QLEV;

   !For each combination of quality
     measure and finished good there are
     upper and lower limits on quality;
   QXF( QUALMES, FINGOOD): QUP, QLOW;

   !For each combination of raw material
     and finished good there is an amount
     of raw material used to be solved for;
   RXF( RAWMAT, FINGOOD): USED;
 ENDSETS
```

A feature to note in the sets section is the attribute *QLEV*, which will store the actual quality levels of the raw materials. The actual quality levels are not known with certainty, but they are normally distributed with means *QLEVMU* and standard deviations *QLEVSD*. Also, at the bottom of the sets section, we've also declared the attribute *USED*, which constitutes our decision variables and will tell us the amount of each raw material to use in each finished product

Next is the model's data section:

```
DATA:
!Raw materials;
 RAWMAT = BUTANE, CATREF, NAPTHA;

!Raw material availability;
 AVAIL =    1000,   4000,   5000;

!Raw material costs;
 COST =      7.3,   18.2,   12.5;

!Finished goods;
 FINGOOD = REGU, PREM;

!Limits on finished goods;
 MINREQ  =    4000,   2000;
 MAXSELL =    8000,   6000;

!Finished goods prices;
 PRICE =      18.4,     22;

!Quality measures (octane, vapor, volatility);
 QUALMES =    OCT,  VAP,  VOL;

!Average quality parameters of
 raw materials;
 QLEVMU =     120,   60,  105,
              100,  2.6,    3,
               74,  4.1,   12;

 QLEVSD =       5,    3,    6,
                2,  .11,  .13,
                3,   .2,   .4;

!Upper and lower limits on
 quality for each finished good;
 QUP = 100, 120,
        11,  11,
        25,  25;
 QLOW = 90,  95,
         8,   8,
        17,  17;
ENDDATA
```

At the bottom of the data section, we initialize the averages and standard deviations (*QLEVMU* and *QLEVSD*) for the raw material quality levels. As an example, the average vapor level for catalytic reformate is 60 with a standard deviation of 3. We also input the upper and lower quality limits for each quality and each finished product (*QUP* and *QLOW*). As an example here, the lower and upper limits on octane for Premium are 95 and 120. As mentioned, the quality constraints on the finished products will be chance constrained, so we will be able to violate these limits at times.

Next comes the core model:

```
!Core model ++++++++++++++++++++++++++++++++++++;

!We want to maximize the profit contribution;
[R_OBJ] MAX =
 @SUM( FINGOOD( F): PRICE( F) * BATCH( F)) -
  @SUM( RAWMAT( R): COST( R) *
   @SUM( FINGOOD( F): USED( R, F)));

!Subject to raw material availability;
@FOR( RAWMAT( R):
 [R_RMLIM] @SUM( FINGOOD( F): USED( R, F))
  <= AVAIL( R);
);

@FOR( FINGOOD( F):
 !Batch size limits;
 @BND( MINREQ( F), BATCH( F), MAXSELL( F));

 !Batch size computation;
 [R_BATCOMP] BATCH( F) =
  @SUM( RAWMAT( R): USED( R, F));
);

@FOR( QXF( Q, F):
 !Quality restrictions for each
  quality measure;
 [R_QUP] @SUM( RAWMAT( R):
  QLEV( R, Q) * USED( R, F)) <=
   QUP( Q, F) * BATCH( F);
 [R_QDN] @SUM( RAWMAT( R):
  QLEV( R, Q) * USED( R, F)) >=
   QLOW( Q, F) * BATCH( F);
);
```

Once again, the core model contains the deterministic version of the model without the introduction of any stochastic elements. Basically, we want to maximize profit, while not using more raw materials than are available, not exceeding batch size limits and strictly meeting the quality restrictions on the finished goods. As we transition the model to a CCP, we will relax this final condition on the finished goods' quality levels.

Finally, at the end of our model are the declarations that convert the core model into a CCP:

```
!CCP declarations ++++++++++++++++++++++++++++++;

 DATA:
  PROB_UP = .7;
  PROB_DN = .9;
  NSAMP   = 20;
 ENDDATA

 !Declare QLEV(Q,F) as normally distributed;
 @FOR( RXQ( R, Q):
  @SPSTGRNDV( 1, QLEV( R, Q));
  @SPDISTNORM( QLEVMU( R, Q), QLEVSD( R, Q),
   QLEV( R, Q));
 );

 !Establish two empty CCP row sets with
  a minimal satisfaction probability level;
 @SPCHANCE( 'CCP_QUAL_UP', '>=', PROB_UP);
 @SPCHANCE( 'CCP_QUAL_DN', '>=', PROB_DN);

 !Add all the quality constraints to the CCP set;
 @FOR( RXF( R, F):
  @SPCHANCE( 'CCP_QUAL_UP', R_QUP( R, F));
  @SPCHANCE( 'CCP_QUAL_DN', R_QDN( R, F));
 );

 !Set sample size;
 @SPSAMPSIZE( 1, NSAMP);
```

In the following section:

```
!Declare QLEV(Q,F) as normally distributed;
@FOR( RXQ( R, Q):
 @SPSTGRNDV( 1, QLEV( R, Q));
 @SPDISTNORM( QLEVMU( R, Q), QLEVSD( R, Q),
  QLEV( R, Q));
);
```

we declare the raw material quality levels as random variables in stage 1 via the *@SPSTGRNDV* function. We then declare them as being normally distributed using *@SPDISTNORM*, along with passing their means and standard deviations.

Next, we need to declare two CCP sets: one for the upper limits on quality, and one for the lower limits on quality. As mentioned above, we use the *@SPCHANCE* function to do this as follows:

```
!Establish two empty CCP row sets with
 a minimal satisfaction probability level;
@SPCHANCE( 'CCP_QUAL_UP', '>=', PROB_UP);
@SPCHANCE( 'CCP_QUAL_DN', '>=', PROB_DN);
```

Here, we assign the names *CCP_QUAL_UP* and *CCP_QUAL_DN* for, respectively, the upper and lower limit constraint sets. The direction for both sets is greater-than-or-equal-to ('>='), given that we want the probability that the sets are satisfied to equal, or exceed, a specified probability. The desired probability hurdles, *PROB_UP* and *PROB_DN*, were initialized in an earlier data section to be .7 and .9.

Once we've declared our CCP sets we need to assign constraints to them. We do this again with *@SPCHANCE*:

```
!Add all the quality constraints to the CCP set;
@FOR( RXF( R, F):
 @SPCHANCE( 'CCP_QUAL_UP', R_QUP( R, F));
 @SPCHANCE( 'CCP_QUAL_DN', R_QDN( R, F));
);
```

Here, we loop over every (raw material, finished good) pair, loading constraint references (i.e., constraint names) into their respective sets. You will note that the constraint names were assigned to the quality constraints earlier in the core model:

```
@FOR( QXF( Q, F):
 !Quality restrictions for each
  quality measure;
 [R_QUP] @SUM( RAWMAT( R):
  QLEV( R, Q) * USED( R, F)) <=
   QUP( Q, F) * BATCH( F);
 [R_QDN] @SUM( RAWMAT( R):
  QLEV( R, Q) * USED( R, F)) >=
   QLOW( Q, F) * BATCH( F);
);
```

You may refer to section *Constraint Names* for more information on how to establish constraint names.

Lastly, we set the sample size for the model to be 20 with the statement:

```
!Set sample size;
@SPSAMPSIZE( 1, NSAMP);
```

This is the number of times each random variable will be sampled, and, in the case of CCPs will also be the total number of scenarios. Note that currently for CCPs there is only one stage, so the sample size is always set for stage 1. Note that *NSAMP* was initialized earlier in the model to be 20.

In the next section, we will discuss the solution to our fuel blending example.

A CCP Fuel Blending Example – The Solution

Solving the CCP model from the previous section, we find the following summary at the top of the solution report:

```
Global optimal solution found.
Objective value:                              33396.85
Infeasibilities:                              0.000000
Extended solver steps:                             621
Total solver iterations:                         28081

Stochastic Model Class:             CHANCE-CONSTRAINED
Deteq Model Class:                                MILP

Total scenarios/leaf nodes:                         20
Total random variables:                              9

                                  Core          Deteq
Total variables:                     8             48
Nonlinear variables:                 0              0
Integer variables:                   0             40

Total constraints:                  18            248
Nonlinear constraints:               0              0

Total nonzeros:                     34           1262
Nonlinear nonzeros:                  0              0
```

The solver found a globally optimal solution of approximately $33,397 in 621 branches and 28,801 iterations. LINGO correctly identifies the model as `CHANCE-CONSTRAINED`. One interesting result is that the deterministic equivalent (deteq) generated by LINGO for solving the stochastic model was a mixed integer linear program (MILP). This is of note because the core model is a pure linear program with no integer variables. You can verify this further down on the line listing the integer variable counts of 0 in the core model and 40 in the deterministic equivalent. Binary variables were added to the deteq to work as indicator variables indicating whether or not a particular CCP set is satisfied in a particular scenario. Here, we have 2 sets and 20 scenarios resulting in 2 x 20 = 40 binary variables.

The remaining section of the summary report lists the sizes of the various components of the model. The main thing to note here is the size of the deteq relative to the core model, which turns out to be considerably larger. It is the dimensions of the deteq that are most relevant with respect to runtimes and any size limitations (e.g., variable limits) of your installation. Note that a deteq will grow as you increase the sample size; on the one hand, a large samples size is nice for reducing sampling error, but, on the other hand, you will experience longer runtimes when increasing sample size. Those interested in viewing the actual deteq can run the *Solver|Generate|Explicit Deteq* command.

Next up we have the *CCP Sets Report:*

```
  CCP Sets Report
  ---------------

                 Unsatisfied        Actual            Target
Set Name/Members  Scenarios   Probability   Dir Probability

       CCP_QUAL_UP        6     0.7000000    >=   0.7000000
 R_QUP( OCT, REGU)
 R_QUP( OCT, PREM)
 R_QUP( VAP, REGU)
 R_QUP( VAP, PREM)
 R_QUP( VOL, REGU)
 R_QUP( VOL, PREM)

       CCP_QUAL_DN        2    0.9000000     >=   0.9000000
 R_QDN( OCT, REGU)
 R_QDN( OCT, PREM)
 R_QDN( VAP, REGU)
 R_QDN( VAP, PREM)
 R_QDN( VOL, REGU)
 R_QDN( VOL, PREM)
```

The CCP Sets Report list the following information:

1. *Set Name/Members* - Each CCP set is listed by name immediately followed by the names of its member constraints.
2. *Unsatisfied Scenarios* - This reports on the number of scenarios where the CCP set was violated.
3. *Actual Probability* - This is the observed probability that a constraint set was satisfied in a particular scenario, and is computed for CCP set *i* as *(TotalScenarios - UnsatisfiedScenariosForSeti) / TotalScenarios*.
4. *Dir* - This is the direction of the CCP set, indicating whether the observed probability should be greater-than-or-equal-to or less-than-or-equal to the target probability.
5. *Target* - This column lists the target probability for the CCP set.

For this particular example, we see that the upper limits on quality were unsatisfied in a total of 6 out of 20 scenarios for an actual probability of .7, exactly matching the target probability. The lower limits set was violated only twice for an actual probability of .9, again exactly matching the target.

After the CCP Sets Report LINGO generates the *Stage 0 Solution:*

```
Stage 0 Solution
----------------

           Variable           Value
USED( BUTANE, REGU)        536.1166
USED( BUTANE, PREM)        401.7255
USED( CATREF, REGU)        2053.043
USED( CATREF, PREM)        1946.957
USED( NAPTHA, REGU)        1410.840
USED( NAPTHA, PREM)        569.2198

                Row    Slack or Surplus
   R_RMLIM( BUTANE)        62.15789
   R_RMLIM( CATREF)        0.000000
   R_RMLIM( NAPTHA)        3019.940
```

The Stage 0 Solution lists the values for all the non-random variables and constraints. These are all part of stage 0 and will not vary from one scenario the next, hence, their values are reported only once in the Stage 0 Solution. Here we've reproduced a a fragment of the report showing the items of most interest: the amount of each raw material used in each finished product (*USED*) and the slack values on the raw material supply constraints (*R_RMLIM*).

After the Stage 0 Solution LINGO generates the *Random Variable Distribution Report:*

```
Random Variable Distribution Report
-----------------------------------

                        Sample      Sample
  Random Variable         Mean      StdDev   Distribution
QLEV( BUTANE, OCT)     119.7697    4.981368   NORMAL,120,5
QLEV( BUTANE, VAP)     59.94561    2.969037   NORMAL,60,3
QLEV( BUTANE, VOL)     104.9087    6.133397   NORMAL,105,6
QLEV( CATREF, OCT)     99.94025    2.124726   NORMAL,100,2
QLEV( CATREF, VAP)     2.600531   0.1093538   NORMAL,2.6,0.11
QLEV( CATREF, VOL)     2.999666   0.1257882   NORMAL,3,0.13
QLEV( NAPTHA, OCT)     73.99625    3.077703   NORMAL,74,3
QLEV( NAPTHA, VAP)     4.102662   0.1975493   NORMAL,4.1,0.2
QLEV( NAPTHA, VOL)     12.00060   0.3970604   NORMAL,12,0.4
```

This report lists each of the random variables in the model, their sample mean and standard deviation, and their distribution type.

Finally, at the end of the report LINGO generates a individual reports for each of the scenarios. For brevity, we reproduce the report just for the 12th scenario:

```
Scenario: 12   Probability: 0.5000000E-01
-----------------------------------------

   Random Variable           Value
QLEV( BUTANE, OCT)        126.4475
QLEV( BUTANE, VAP)        65.45897
QLEV( BUTANE, VOL)        109.6616
QLEV( CATREF, OCT)        100.5477
QLEV( CATREF, VAP)        2.504830
QLEV( CATREF, VOL)        3.098372
QLEV( NAPTHA, OCT)        73.84414
QLEV( NAPTHA, VAP)        4.081760
QLEV( NAPTHA, VOL)        11.41373

           CCP Set          Status
       CCP_QUAL_UP     UNSATISFIED
       CCP_QUAL_DN       SATISFIED

               Row           Value
 R_QUP( OCT, REGU)        21598.34
 R_QDN( OCT, REGU)        18401.66
 R_QUP( OCT, PREM)        61555.49
 R_QDN( OCT, PREM)        11392.05
 R_QUP( VAP, REGU)       -1994.877
 R_QDN( VAP, REGU)        13994.88
 R_QUP( VAP, PREM)       -1399.827
 R_QDN( VAP, PREM)        10153.53
 R_QUP( VOL, REGU)        18744.58
 R_QDN( VOL, REGU)        13255.42
 R_QUP( VOL, PREM)        16364.39
 R_QDN( VOL, PREM)        6978.823
```

The scenario reports list the values for the random variables for the scenarios as well as the slack values of the CCP constraints. An interesting feature of the scenario reports for a CCP is the middle section, where each CCP set is listed along with whether or not the set was satisfied in the scenario. In this case, the upper limit quality constraints are listed as being unsatisfied, which is verified by the negative slacks on the row *R_QUP(VAP, REGU)* and *R_QUP(VAP_PREM)*. This then makes scenario 12 one of the 6 scenarios where the upper limit quality constraints were allowed to be violated.

15 On Mathematical Modeling

When developing a model in LINGO, it helps to understand how the model is processed internally by the LINGO solver. The relationships in your model influence the computation time, the solution methods used by LINGO, and the type of answer returned. Here we'll explain some of the different types of relationships in a LINGO model and how each type affects the solution search. An understanding of these topics is not required to use LINGO, but it can help you use the software more effectively.

Solvers Used Internally by LINGO

LINGO has four solvers it uses to solve different types of models. These solvers are:

- a direct solver,
- a linear solver,
- a nonlinear solver, and
- a branch-and-bound manager.

The LINGO solvers, unlike solvers sold with other modeling languages, are all part of the same program. In other words, they are linked directly to the modeling language. This allows LINGO to pass data to its solvers directly through memory, rather than through intermediate files. Direct links to LINGO's solvers also minimize compatibility problems between the modeling language component and the solver components.

When you solve a model, the direct solver first computes the values for as many variables as possible. If the direct solver finds an equality constraint with only one unknown variable, it determines a value for the variable that satisfies the constraint. The direct solver stops when it runs out of unknown variables or there are no longer any equality constraints with a single remaining unknown variable.

Once the direct solver is finished, if all variables have been computed, LINGO displays the solution report. If unknown variables remain, LINGO determines what solvers to use on a model by examining its structure and mathematical content. For a continuous linear model, LINGO calls the linear solver. If the model contains one or more nonlinear constraints, LINGO calls the nonlinear solver. When the model contains any integer restrictions, the branch-and-bound manager is invoked to enforce them. The branch-and-bound manager will, in turn, call either the linear or nonlinear solver depending upon the nature of the model.

The linear solver in LINGO uses the revised simplex method with product form inverse. A barrier solver may also be obtained, as an option, for solving linear models. LINGO's nonlinear solver employs both successive linear programming (SLP) and generalized reduced gradient (GRG) algorithms. Integer models are solved using the branch-and-bound method. On linear integer models, LINGO does considerable preprocessing, adding constraint "cuts" to restrict the noninteger feasible region. These cuts will greatly improve solution times for most integer programming models.

Type of Constraints

Through the use of the direct solver, LINGO substitutes out all the fixed variables and constraints from the model. The remaining reduced set of constraints and variables are then classified as being either linear or nonlinear. LINGO's solver status window, which by default opens every time you solve a model, gives a count of the linear and nonlinear variables and constraints in a model. If any nonlinear variables or constraints are found in the model, the entire model is considered nonlinear and the relatively slower nonlinear solver must be invoked in place of the linear solver.

Linear Constraints

If all the terms of a constraint are of the first order, the constraint is said to be linear. This means the constraint doesn't contain a variable squared, cubed, or raised to any power other than one, a term divided by a variable, or variables multiplied by each other. Also, proportionality must exist. In other words, for every unit increase or decrease in a variable, the value of the constraint increases or decreases by a fixed amount.

Linear formulas are "straight line" relationships. The basic form of a linear formula is:

$$Y = m\,X + b$$

where m and b are constants.

For example, suppose you're buying tomatoes for $1.50 per pound. The expression or function used to calculate the cost (C) in terms of the amount of tomatoes purchased (T) is:

$$C = 1.5 * T.$$

As you might expect, a graph of this function for cost is a straight line:

Linear expressions can have multiple variables. For example, if you added potatoes (P) at \$0.75 per pound and apples (A) at \$1.25 per pound, your cost function would become:

$$C = 1.5 * T + 0.75 * P + 1.25 * A$$

This new cost expression is also linear. You could think of it as the sum of three simpler linear expressions.

Because linear models can be solved much faster and with more accuracy than nonlinear models, it's preferable to formulate your models using linear expressions whenever possible. When the *Solver|Solve* command is issued, LINGO analyzes the relationships in the model. If all the expressions are linear, LINGO will recognize and take advantage of this fact.

Relative to other types of models, problems expressed using exclusively linear relationships can be solved quickly. If allowed to run to completion, LINGO will return the answer that yields the highest value for a maximization objective, or the lowest value for a minimization objective.

One way to learn whether or not all expressions in your model are linear is to note the classification statistics displayed during the solution process in the solver status window. The *Nonlinear* categories of the *Variables* and *Constraints* boxes display the number of nonlinear relationships in the model. If zeros appear in both these categories, the model is linear.

If LINGO displays a number greater than zero for the nonlinear relationships, you may want to investigate whether the constraints and variables in your model could be reformulated in a linear manner. For example, consider the following constraint:

$$X / Y = 10;$$

As written, this constraint is nonlinear because we are dividing by Y. By simply multiplying both sides of the equation through by Y, we can convert it to the equivalent linear constraint:

$$X = 10 * Y;$$

Nonlinear Constraints

By definition, all constraints that are not linear are nonlinear. Nonlinear expressions include relationships with variables that are squared, cubed, taken to powers other than one, or multiplied or divided by each other.

Models with nonlinear expressions are much more difficult to solve than linear models. Unlike linear models, nonlinear models may prevent LINGO from finding a solution, though one exists. Or LINGO may find a solution to a nonlinear model that appears to be the "best", even though a better one may exist. These results are obviously undesirable. For more on what you can do to help minimize the occurrence of these undesirable results, see the *Guidelines for Nonlinear Modeling* section below.

Local Optima vs. Global Optima

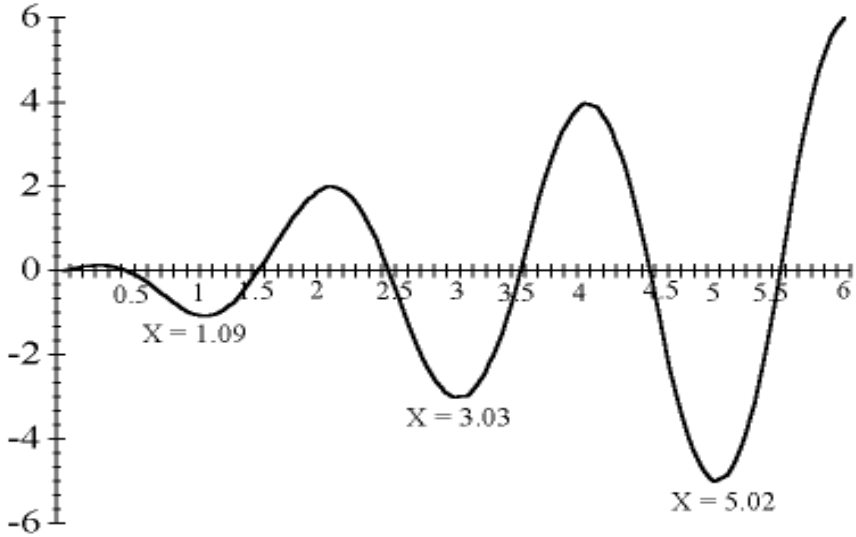
When LINGO finds a solution to a linear optimization model, it is the definitive best solution—we say it is the global optimum. Multiple optima may exist. However, a globally optimal solution is a feasible solution with an objective value that is as good as or better than all other feasible solutions to the model. The ability to obtain a globally optimal solution is attributable to certain properties of linear models.

This is not the case for nonlinear optimization. Nonlinear optimization models may have several solutions that are *locally optimal*. All gradient based nonlinear solvers converge to a locally optimal point (i.e., a solution for which no better feasible solutions can be found in the immediate neighborhood of the given solution). Additional local optimal points may exist some distance away from the current solution. These additional locally optimal points may have objective values substantially better than the solver's current local optimum. ***Thus, when a nonlinear model is solved, we say the solution is merely a local optimum. The user should be aware that other local optimums may, or may not, exist with better objective values.*** Conditions may exist where you may be assured that a local optimum is in fact a global optimum. See the *Convexity* section below for more information.

Consider the following small nonlinear model involving the highly nonlinear cosine function:

$$MIN = X * @COS(3.1416 * X);$$
$$X < 6;$$

The following graph shows a plot of the objective function for values of X between 0 and 6. If you're searching for a minimum, there are local optimal points at X values of 0, 1.09, 3.03, and 5.02 in the "valleys." The global optimum for this problem is at $X = 5.02$, because it is the lowest feasible valley.



Graph of X * @COS(3.1416 * X)

Imagine the graph as a series of hills. You're searching in the dark for the minimum or lowest elevation. If you are at X = 2.5, every step towards 2 takes you uphill and every step towards 3 takes you downhill. Therefore, you move towards 3 in your search for the lowest point. You'll continue to move in that direction as long as it leads to lower ground.

When you reach *X*=3.03, you'll notice a small flat area (slope is equal to zero). Continuing begins to lead uphill and retreating leads up the hill you just came down. You're in a valley, the lowest point in the immediate neighborhood—a local optimum. However, is it the lowest possible point? In the dark, you are unable to answer this question with certainty.

LINGO lets you enter initial values for variables (i.e., the point from which LINGO begins its search) using an *INIT* section. A different local optimum may be reached when a model is solved with different initial values for *X.* In this example, one might imagine that starting at a value of *X*=6 would lead to the global optimum at *X*=5.02. Unfortunately, this is not guaranteed, because LINGO approximates the true underlying nonlinear functions using linear and/or quadratic functions. In the early stages of solving the model, these approximations can be somewhat rough in nature. Therefore, the solver can be sent off to distant points, effectively passing over nearby local optimums to the true, underlying model.

When "good" initial values for the variables are known, you should input these values in an *INIT* section. Additionally, you may want to use the *@BND* function to bound the variables within a reasonable neighborhood of the starting point. When you have no idea what the optimal solution to your model is, you may find that observing the results of solving the model several times with different initial values can help you find the best solution.

LINGO has several optional strategies to help overcome the problem of stalling at local optimal points. The global solver employs branch-and-bound methods to break a model down into many convex sub-regions. LINGO also has a multistart feature that restarts the nonlinear solver from a number of intelligently generated points. This allows the solver to find a number of locally optimal points and

report the best one found. Finally, LINGO can automatically linearize a number of nonlinear relationships through the addition of constraints and integer variables so that the transformed linear model is mathematically equivalent to the original nonlinear model. Keep in mind, however, that each of these strategies will require additional computation time. Thus, whenever possible, you are always better off formulating models to be convex so that they contain a single extreme point.

Convexity

The characteristic of an expression called *convexity* along with a closely related feature called *concavity* are the most useful features of a model for guaranteeing that a local optimum is actually a global optimum. Let's consider the convexity property of a minimization problem. The mathematical definition of a *convex model* is as follows:

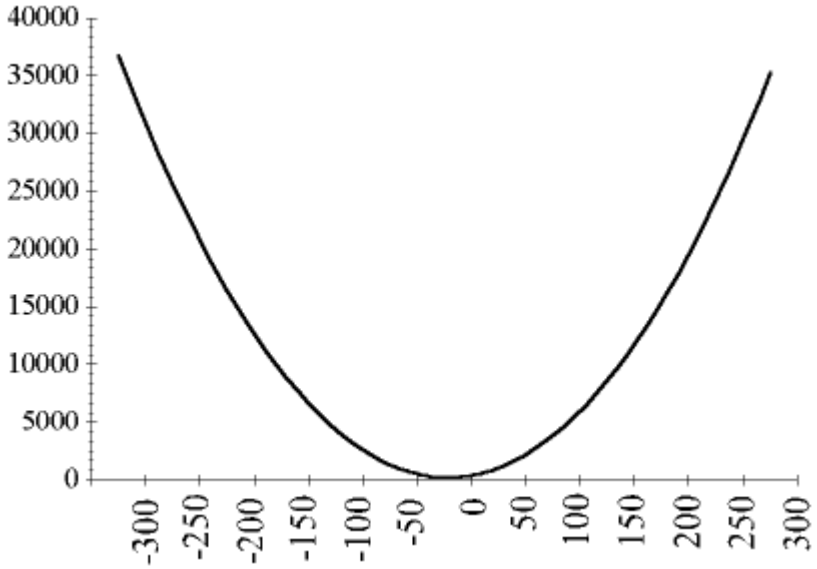
$$f(y) \leq a\, f(x) + (1-a)\, f(z), \text{ where } y=a*x + (1-a)*z$$

In words, a function is convex if, for any two points on the function, a straight line connecting the two points lies entirely on or above the function.

Determining the convexity of a multiple variable problem is no easy task. Mathematicians call a function convex if the matrix of the second derivatives is positive definite or has positive Eigen values. However, if you can identify your LINGO model as being convex for a minimization problem, you can ensure that any solution you reach is a global optimum (including nonlinear models).

Strictly Convex

A strictly convex function with no constraints has a single global minimum. Therefore, minimizing a strictly convex function will yield the unique global optimal solution regardless of the initial value of the variables. The graph below shows a convex function of a single variable:

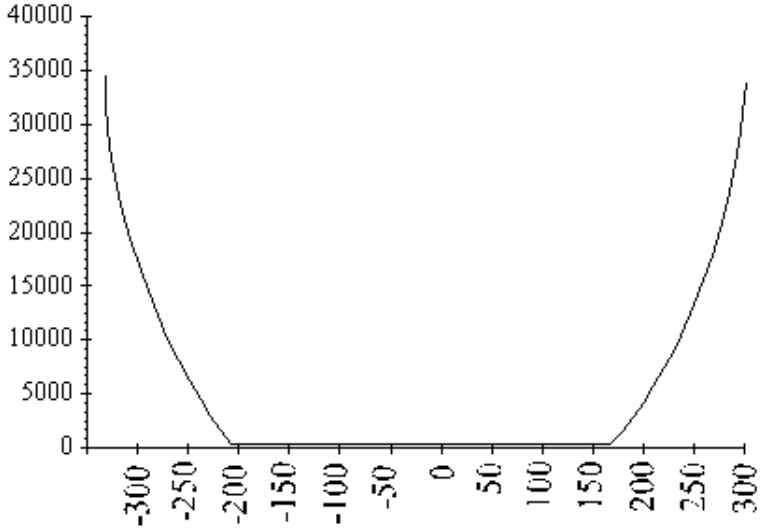


A Strictly Convex Function: .4*(*x*-3)2+.5

In this strictly convex function, the unique global minimum can be defined at the point on the function where the variable *x* is equal to 3. Changing the value of *x* to be more or less than 3 will increase the result of the function.

Loosely Convex

A loosely convex function with no constraints has multiple local minima, which are also global minima. Therefore, minimizing a loosely convex function may yield different solutions for different initial values of the variables. However, if you know the function is loosely convex, you will know the solution is a global minimum. For example, note the flat section (i.e., slope is zero) from approximately –200 to 150 in the following function that makes it loosely convex:



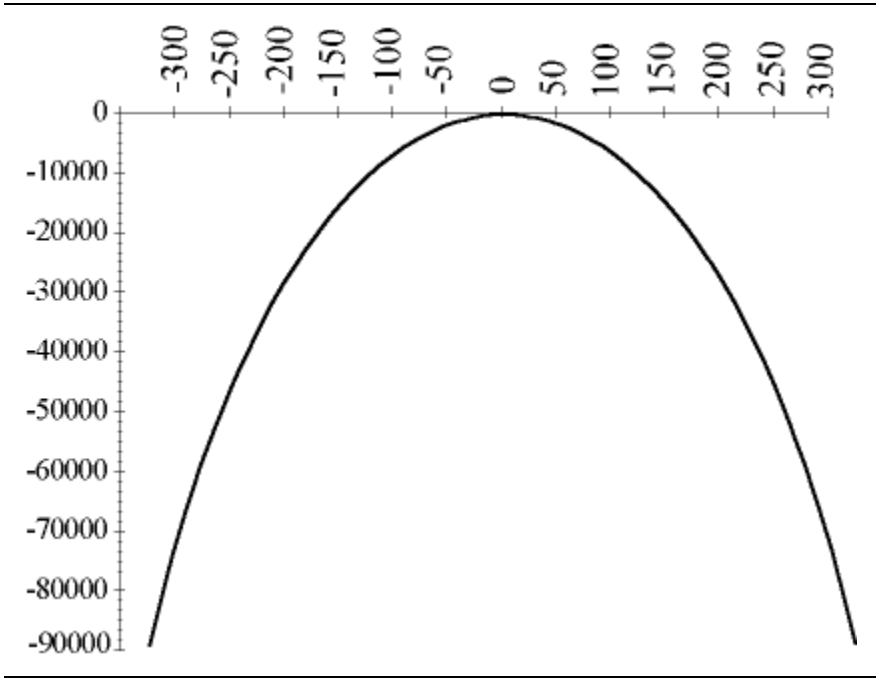
A Loosely Convex Function

In this loosely convex function, the global minimum of 3 can be found between about –200 through just over 150 on the x-axis. Although variable values may vary, the result of the function will be the same for all local minima.

Concavity

While convexity applies to minimization problems, concavity ensures the corresponding attribute of global optimality in maximization problems. *Concavity* can be defined as the negative of convexity (see above). In other words, a function is concave if, for any two points on the function, a straight line connecting the two points lies entirely on or below the function.

The following function is strictly concave:



A Strictly Concave Function: Graph of -(x^2)

In this strictly concave function, the unique global maximum can be defined at the point on the function where the variable x is equal to zero. Changing the value of the variable to be any more or less than 0 will decrease the result of the function. A loosely concave function might look similar to the negative of the loosely convex function shown in the previous section.

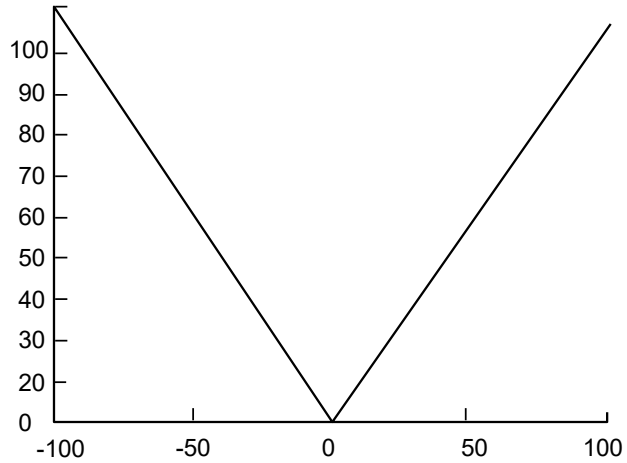
The only functions that are both convex and concave are straight lines (i.e., hyperplanes). Therefore LINGO classifies the solutions to all linear optimization problems as globally optimal. Due to the difficulty in determining convexity and concavity, LINGO classifies all nonlinear optimization models as locally optimal. However, you can ensure you have a global optimum if you can determine that your nonlinear optimization model is convex or concave, or you can reformulate it in such a way to make it convex or concave.

On the other hand, if you can determine your nonlinear optimization model is definitely not convex or concave, you know that it is a mixed function and multiple optima exist. It might be a good idea to try solving with different starting values for the variables. See the section above *Local Optima vs. Global Optima* for an example of a mixed function and more information on the strategies LINGO offers to help overcome the problems with solving mixed functions.

Smooth vs. Nonsmooth Functions

Smooth functions have a unique defined first derivative (slope or gradient) at every point. Graphically, a smooth function of a single variable can be plotted as a single continuous line with no abrupt bends or breaks. All the examples you've seen so far in this chapter have been smooth.

Nonsmooth functions include nondifferentiable and discontinuous functions. Functions with first derivatives with undefined regions are called nondifferentiable. Graphs of nondifferentiable functions may have abrupt bends. The absolute value of a variable, *@ABS(X)*, is an example of a nondifferentiable expression, as illustrated in the following graph:



Graph of *ABS*(*X*)

Here, there is an abrupt bend in the graph at zero. This can dramatically increase solution times as well as affect the accuracy of the solution. Additional nonsmooth functions are *@MAX*, *@MIN*, *@SMAX*, *@SMIN*, and any of the probability functions that use linear interpolation to return results for nonintegral arguments.

Perhaps even more confounding than functions with sharp bends are discontinuous functions that have actual breaks in their graphs. Discontinuous functions in LINGO include *@SIGN* and *@FLOOR*.

In simplified terms, LINGO searches along the path of a function to find a maximum or minimum point representing an optimal solution. As LINGO is searching along a function, it uses the function's derivatives to help guide the search. When derivatives vanish, as they do at sharp corners, LINGO is "blinded" and is unable to "see" around the sharp corner. Thus, dealing with functions containing breaks and sharp bends is considerably more difficult than dealing with smooth, continuous functions. Where possible, nonsmooth functions should be replaced with linear constraints or some combination of linear constraints and integer variables. As an example, refer to the assembly line balancing example (*ASLBAL.LNG*) on page 807 to see how we constructed a model to avoid the use of the *@MAX* function in order to maintain linearity.

Guidelines for Nonlinear Modeling

As shown in the previous sections, nonlinear models can be extremely complex to solve. Spending a little extra time to make sure the model is formulated in a way that is most efficient to solve can pay off in terms of solution speed and reliability. This section gives some general guidelines to consider when building and solving nonlinear models.

Supplying Bounds for Variables

Intelligent use of upper and lower bounds on variables can help make LINGO's solution search as efficient as possible. Supplying good bounds can keep LINGO from wasting time searching regions unlikely to yield good solutions. For example, suppose you know that, even though the feasible range for a particular variable is between 0 and 100, it is highly improbable the optimal value is outside the range of 50 to 75. In this case, using the *@BND* function to specify a lower bound of 50 and an upper bound of 75 could significantly reduce solution times.

Bounding can also help keep the solution search clear of mathematically troublesome areas like undefined regions. For example, if you have a constraint with the term $1/X$, it may be helpful to add a lower bound on X, so it does not get too close to zero.

Supplying Initial Values for Variables

The initial values you provide for the variables in a model can affect the "path" LINGO takes to the solution. Starting with values close to the optimal solution can noticeably reduce the solution time. In many situations, you may not know "good" initial values. However, when you do know reasonable ones, it may be to your benefit to use them in the *INIT* section.

Consider changing your supplied initial values and re-solving the model if you suspect: 1) there is an answer better than the answer returned by LINGO, or 2) a feasible solution exists even though LINGO returns the message "No feasible solution found".

Scale the Model to a Reasonable Range of Units

Try to model your problem such that the units involved are of similar orders of magnitude. If the largest number in the model is greater than 1000 times the smallest number in the model, LINGO may encounter problems when solving the model. This may also affect the accuracy of the solution by introducing rounding problems.

For example, consider a financial problem with equations expressing an interest rate of 8.5% (.085) and budget constraints of $12,850,000. The difference in magnitude between these numbers is on the order of 10^9 (1/100th compared to 10,000,000). A difference of 10^4 or less between the largest and smallest units would be preferable. In this case, the budget could be expressed in units of millions of dollars. That is, $12.85 would be used to represent $12,850,000. This lowers the difference in magnitude of the units of these numbers to 10^4.

Simplify Relationships

When practical, use linear rather than nonlinear relationships. Some nonlinear expressions can be reformulated in a linear manner. A simple example is a constraint on the ratio of two variables. Using the example from earlier in the chapter, consider the following constraint:

$$X / Y < 10;$$

This constraint is nonlinear because we are dividing by Y. To linearize the constraint, you can multiply both sides by Y. The equivalent, linear constraint becomes:

$$X < 10 * Y;$$

Avoid nonsmooth relationships when possible. Models with nonsmooth constraints are generally much more difficult to solve. When possible, approximate the nonsmooth relationship with smooth expressions and, perhaps, integer variables.

Reduce Integer Restrictions

Minimizing the use of integer restrictions can drastically reduce the solution time. In instances involving larger variable values, you may find solving the model without integer restrictions and then rounding yields acceptable answers in a fraction of the time required by the integer model. Be forewarned, however, that rounding a solution will not necessarily yield a feasible or optimal solution.

Appendix A: Additional Examples of LINGO Modeling

In this Appendix, we list, in alphabetical order by model name, many of the models contained in the *SAMPLES* directory off your main LINGO directory. Most of the models contain a brief background discussion. Many have appeared in more detailed descriptions earlier in this manual and are reproduced here for reference.

Assembly Line Balancing — Model: ASLBAL

The following model illustrates how to balance an assembly line in order to minimize its total cycle time. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
 ! Assembly line balancing model;
   ! This model involves assigning tasks to
     stations in an assembly line so bottlenecks
     are avoided. Ideally, each station would be
     assigned an equal amount of work.;

 SETS:
   ! The set of tasks to be assigned are A through
     K, and each task has a time to complete, T;
   TASK/ A B C D E F G H I J K/: T;

   ! Some predecessor,successor pairings must be
     observed(e.g. A must be done before B, B
     before C, etc.);
   PRED(TASK, TASK)/ A,B  B,C  C,F  C,G  F,J  G,J
     J,K  D,E  E,H  E,I  H,J  I,J /;

   ! There are 4 workstations;
   STATION/1..4/;

   TXS(TASK, STATION): X;
   ! X is the attribute from the derived set TXS
       that represents the assignment. X(I,K) = 1
     if task I is assigned to station K;
 ENDSETS
```

```
DATA:
  ! Data taken from Chase and Aquilano, POM;
  ! Each task has an estimated time required:
       A  B  C  D  E  F  G  H  I  J  K;
  T = 45 11  9 50 15 12 12 12 12  8  9;
ENDDATA

 ! The model;
 ! *Warning* may be slow for more than 15 tasks;

 ! For each task, there must be one assigned
   station;
 @FOR(TASK(I): @SUM(STATION(K): X(I, K)) = 1);

 ! Precedence constraints;
 ! For each precedence pair, the predecessor task
   I cannot be assigned to a later station than
   its successor task J;
 @FOR(PRED(I, J):
  @SUM(STATION(K):
   K * X(J, K) - K * X(I, K)) >= 0);

 ! For each station, the total time for the
   assigned tasks must be less than the maximum
   cycle time, CYCTIME;
 @FOR(STATION(K):
  @SUM(TXS(I, K): T(I) * X(I, K)) <= CYCTIME);

 ! Minimize the maximum cycle time;
 MIN = CYCTIME;

 ! The X(I,J) assignment variables are binary integers;
 @FOR(TXS: @BIN(X));

END
```

Model: ASLBAL

Bayes Rule; Conditional Probabilities Model: Bayes

```
MODEL:
SETS:  ! Computing probabilities using Bayes rule;
ACTUAL/1..3/:MPA;!Marginal probability of actual;
FCAST/1..3/:MPF;!Marginal probability of forecast;
FXA(FCAST, ACTUAL):  CAGF, !Conditional prob of actual given
   forecast;
CFGA, !Conditional prob of forecast given actual;
JP;  ! Joint probability of both;
ENDSETS

DATA:
!Conditional probability of forecast, given actual;
  CFGA = .80  .15  .20
         .10  .70  .20
         .10  .15  .60;
 ! Marginal probabilities of actual;
  MPA  = .5 .3 .2;
ENDDATA

 ! The calculations;
 ! Marginal probabilities are the sum of
   joint probabilities;
  @FOR(ACTUAL(J):
   MPA(J) = @SUM(FCAST(I): JP(I, J))
  );
  @FOR(FCAST(I):
   MPF(I) = @SUM(ACTUAL(J): JP(I, J))
  );

! Bayes rule relating joint to conditional
  probabilities;
 @FOR(FXA(I, J):
  JP(I, J) = MPF(I) * CAGF(I, J);
  JP(I, J) = MPA(J) * CFGA(I, J)
 );

END
```

Model: BAYES

Blending of Ingredients I — Model: BLEND

In blending problems, two or more raw materials are to be blended into one or more finished goods, satisfying one or more quality requirements on the finished goods. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
 TITLE   BLEND;
 SETS:
   !Each raw material has an availability
     and cost/unit;
   RAWMAT/ BUTANE, CATREF, NAPHTHA/: AVAIL, COST;

   !Each finished good has a min required,
     max sellable, selling price,
     and batch size to be determined;
   FINGOOD/ REGULAR, PREMIUM/:
    MINREQ, MAXSELL, PRICE, BATCH;

   !Here is the set of quality measures;
   QUALMES/ OCTANE, VAPOR, VOLATILITY/;

   !For each combo of raw material and
     quality measure there is a quality
     level;
   RXQ(RAWMAT, QUALMES): QLEVEL;

   !For each combination of quality
     measure and finished good there are
     upper and lower limits on quality,
     and a slack on upper quality to be
     determined;
   QXF(QUALMES, FINGOOD):
    QUP, QLOW, QSLACK;

   !For each combination of raw material
     and finished good there is an amount
     of raw material used to be solved for;
   RXF(RAWMAT, FINGOOD): USED;
 ENDSETS

 DATA:
 !Raw material availability;
  AVAIL = 1000, 4000, 5000;

 !Raw material costs;
  COST = 7.3, 18.2, 12.5;

 !Quality parameters of raw
  materials;
  QLEVEL = 120,   60, 105,
           100,  2.6,   3,
            74,  4.1,  12;

 !Limits on finished goods;
  MINREQ  = 4000, 2000;
```

```
 MAXSELL = 8000, 6000;

!Finished goods prices;
 PRICE = 18.4,  22;


!Upper and lower limits on
 quality for each finished good;
 QUP = 110, 110,
        11,  11,
        25,  25;
 QLOW = 90,  95,
         8,   8,
        17,  17;
ENDDATA

!Subject to raw material availability;
 @FOR(RAWMAT(R):
  [RMLIM] @SUM(FINGOOD(F): USED(R, F))
   <= AVAIL(R);
 );

 @FOR(FINGOOD(F):

  !Batch size computation;
  [BATCOMP] BATCH(F) =
   @SUM(RAWMAT(R): USED(R, F));

  !Batch size limits;
  @BND(MINREQ, BATCH, MAXSELL);

  !Quality restrictions for each
   quality measure;
  @FOR(QUALMES(Q):

   [QRESUP] @SUM(RAWMAT(R):
     QLEVEL(R, Q) * USED(R, F))
      + QSLACK(Q, F) = QUP(Q, F) *
       BATCH(F);

   [QRESDN] QSLACK(Q, F) <=
    (QUP(Q, F) - QLOW(Q, F)) *
     BATCH(F);

  );
 );

! We want to maximize the profit contribution;

 [OBJECTIVE] MAX =
  @SUM(FINGOOD: PRICE * BATCH) -
   @SUM(RAWMAT(R): COST(R) *
    @SUM(FINGOOD(F): USED(R, F)));

END
```

Model: BLEND

Plant Location Model: CAPLOC

In this example, we build a model to help decide what plants to open and how much product to ship from each plant to each customer. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Capacitated Plant Location Problem;
 SETS:
    PLANTS: FCOST, CAP, OPEN;
    CUSTOMERS: DEM;
    ARCS( PLANTS, CUSTOMERS)  : COST, VOL;
 ENDSETS

 DATA:
  ! The plant, their fixed costs
    and capacity;
    PLANTS, FCOST, CAP =
      P1      91    39
      P2      70    35
      P3      24    31;
  ! Customers and their demands;
    CUSTOMERS, DEM =
      C1      15
      C2      17
      C3      22
      C4      12;
  ! The plant to cust cost/unit
    shipment matrix;
    COST =   6  2  6  7
             4  9  5  3
             8  8  1  5;
 ENDDATA

 ! The objective;
    [TTL_COST] MIN = @SUM( ARCS: COST * VOL) +
     @SUM( PLANTS: FCOST * OPEN);

 ! The demand constraints;
    @FOR( CUSTOMERS( J): [DEMAND]
     @SUM( PLANTS( I): VOL( I, J)) >= DEM( J)
    );

 ! The supply constraints;
    @FOR( PLANTS( I): [SUPPLY]
     @SUM( CUSTOMERS( J): VOL( I, J)) <=
      CAP( I) * OPEN( I)
    );

! Make OPEN binary(0/1);
    @FOR( PLANTS: @BIN( OPEN));
END
```

Model: CAPLOC

Blending of Ingredients II Model: CHESS

In blending problems, two or more raw materials are to be blended into one or more finished goods, satisfying one or more quality requirements on the finished goods. In this example, we blend mixed nuts into four different brands with a goal of maximizing revenue. A detailed discussion of this model may be found in Chapter 2, *Using Sets.*

```
MODEL:
SETS:
   NUTS / PEANUTS, CASHEWS/: SUPPLY;
   BRANDS / PAWN, KNIGHT, BISHOP, KING/:
    PRICE, PRODUCE;
   NCROSSB(NUTS, BRANDS): FORMULA;
ENDSETS

DATA:
   SUPPLY = 750 250;
   PRICE = 2 3 4 5;
   FORMULA = 15 10  6  2
              1  6 10 14;
ENDDATA

MAX = @SUM(BRANDS(I):
 PRICE(I) * PRODUCE(I));

@FOR(NUTS(I):
 @SUM(BRANDS(J):
  FORMULA(I, J) * PRODUCE(J) / 16) <=
   SUPPLY(I)
);
END
```

Model: CHESS

Chemical Equilibrium Model: CHMBL1

In a chemical equilibrium problem of a closed system (such as a sealed container), there are two or more opposing processes, such as evaporation and condensation. The question is: at a given temperature and pressure, what portion of the material will be found in each possible state—water and vapor, for instance—at equilibrium. The typical equilibrium conditions are that the ratios of fractions must equal known temperature and/or pressure-dependent constants. Note that, in many cases, the numbers you will be dealing with in chemical equilibrium models will be infinitesimal. Such tiny numbers will make it next to impossible to solve your model. Transforming values by taking their logarithms is a useful strategy that is used here. This tends to result in a transformed model with more manageable data.

```
MODEL:
!Chemical equilibrium problem of Peters, Hayes &
!Hieftje. Calculate concentrations of various
!components of phosphoric acid(H3PO4) with pH of 8
!and total phosphate concentration of .10. The
!equilibrium equations in obvious form look like:
!
!    H2P * H/ H3P = .0075;
!
!However, for scale reasons it is better to take !logs thus;
   LH2P + LH - LH3P = @LOG(.0075);

! Ditto for other equilibrium equations;
   LHP + LH - LH2P = @LOG(6.2 * 10^-8);
   LH + LP - LHP = @LOG(4.8 * 10^-13);
   LH = @LOG(10 ^-8);

! Convert back to original variables;
   H   = @EXP(LH);
   P   = @EXP(LP);
   HP  = @EXP(LHP);
   H2P = @EXP(LH2P);
   H3P = @EXP(LH3P);
   H3P + H2P + HP + P = .1;

! Must unconstrain log variables;
 @FREE(LH2P); @FREE(LH); @FREE(LH3P);
 @FREE(LHP); @FREE(LP);

! Solution should be: LH2P= -4.2767, LH= -18.4207,
  LH3P= -17.8045, LHP= -2.4522, LP= -12.3965;
END
```

Model: CHMBL1

Conjoint Analysis Model: CONJNT

When designing a product, it's useful to know how much customers value various attributes of that product. This allows us to design the product most preferred by consumers within a limited budget. For instance, if we determine consumers place a very high value on a long product warranty, then we might be more successful in offering a long warranty with fewer color options.

The basic idea behind *conjoint analysis* is, while it may be difficult to get consumers to accurately reveal their relative utilities for product *attributes*, it's easy to get them to state whether they prefer one product *configuration* to another. Given these rank preferences, we can use conjoint analysis to work backwards and determine the implied utility functions for the product attributes. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Conjoint analysis model to decide how much
  weight to give to the two product attributes of
  warranty length and price;

SETS:
! The three possible warranty lengths;
     WARRANTY /LONG, MEDIUM, SHORT/ : WWT;
! where WWT(i) = utility assigned to warranty i;

!The three possible price levels(high,medium,low);
     PRICE /HIGH, MEDIUM, LOW/ : PWT;
! where PWT(j) = utility assigned to price j;

! We have a customer preference ranking for each
  combination;
     WP(WARRANTY, PRICE) : RANK;
ENDSETS

DATA:
! Here is the customer preference rankings running
  from a least preferred score of 1 to the most
  preferred of 9.  Note that long warranty and low
  price are most preferred with a score of 9,
  while short warranty and high price are least
  preferred with a score of 1;
     RANK = 7  8  9
            3  4  6
            1  2  5;
ENDDATA

SETS:
! The next set generates all unique pairs of
  product configurations such that the second
  configuration is preferred to the first;
     WPWP(WP, WP) | RANK(&1, &2) #LT#
      RANK(&3, &4): ERROR;
```

```
! The attribute ERROR computes the error of our
  estimated preference from the preferences given
  us by the customer;
ENDSETS

! For every pair of rankings, compute the amount
  by which our computed ranking violates the true
  ranking. Our computed ranking for the (i,j)
  combination is given by the sum WWT(i) + PWT(j).
  (NOTE: This makes the bold assumption that
  utilities are additive!);
     @FOR(WPWP(i, j, k, l): ERROR(i, j, k, l) >=
      1 + (WWT(i) + PWT(j)) -
       (WWT(k) + PWT(l))
     );

! The 1 is required on the right-hand-side of the
  above equation to force ERROR to be nonzero in
  the case where our weighting scheme incorrectly
  predicts that the combination (i,j) is equally
  preferred to the (k,l) combination.

  Since variables in LINGO have a default lower
  bound of 0, ERROR will be driven to zero when we
  correctly predict that (k,l) is preferred to
  (i,j).

  Next, we minimize the sum of all errors in order
  to make our computed utilities as accurate as
  possible;
     MIN = @SUM(WPWP: ERROR);
END
```

Model: CONJNT

Data Envelopment Analysis Model: DEAMOD

Data Envelopment Analysis (DEA) was developed to help compare the relative performance of decision-making units. DEA generates an efficiency score between 0 and 1 for each unit, indicating how effectively they are managing their resources. A compelling feature of DEA is it develops a unique rating system for each unit designed to make them look their best. This should facilitate acceptance of DEA within an organization. For more information on DEA, see Schrage (2006).

```
MODEL:
! Data Envelope Analysis of Decision Maker Efficiency;
 SETS:
  DMU/BL HW NT OP YK EL/: ! Six schools;
     SCORE; ! Each decision making unit has a;
            ! score to be computed;
  FACTOR/COST RICH WRIT SCIN/;
! There is a set of factors, input & output;
   DXF(DMU, FACTOR):  F; ! F(I, J) = Jth factor
                          of DMU I;
 ENDSETS
DATA:
! Inputs are spending/pupil, % not low income;
! Outputs are Writing score and Science score;
  NINPUTS = 2;  ! The first NINPUTS factors are inputs;
!      The inputs,    the outputs;
    F  =  8939  64.3     25.2   223
          8625  99       28.2   287
         10813  99.6     29.4   317
         10638  96       26.4   291
          6240  96.2     27.2   295
          4719  79.9     25.5   222;
 ENDDATA
 SETS:
   ! Weights used to compute DMU I's score;
   DXFXD(DMU,FACTOR) : W;
 ENDSETS
! Try to make everyone's score as high as possible;
   MAX = @SUM(DMU: SCORE);
! The LP for each DMU to get its score;
   @FOR(DMU(I):
    SCORE(I) = @SUM(FACTOR(J)|J #GT# NINPUTS:
     F(I, J)* W(I, J));
! Sum of inputs(denominator) = 1;
    @SUM(FACTOR(J)| J #LE# NINPUTS:
     F(I, J)* W(I, J)) = 1;
! Using DMU I's weights, no DMU can score better than 1;
    @FOR(DMU(K):
     @SUM(FACTOR(J)| J #GT# NINPUTS:
      F(K, J) * W(I, J))
       <= @SUM(FACTOR(J)| J #LE# NINPUTS:
        F(K, J) * W(I, J))
    )
   );
```

```
! The weights must be greater than zero;
   @FOR(DXFXD(I, J): @BND(.00001, X, 100000));
END
```

Model: DEAMOD

Generating Random Numbers Model: DEMRND

In cases where you are modeling under uncertainty, it is useful to have a random number generator to help simulate a situation. LINGO supports the *@RAND* function, which can be used to generate sequences of pseudo random numbers with uniform distribution between 0 and 1. By using the same seed value, you can recreate a series of numbers. In this example, we generate 15 random numbers, and then use them with the *@PSN* and *@PTD* functions to generate observations from both the unit Normal and T distributions.

```
MODEL:
! Generate a series of Normal and T distributed
  random variables ;
 SETS:
  SERIES/1..15/: U, ZNORM, ZT;
 ENDSETS

! First uniform is arbitrary;
  U(1) = @RAND(.1234);

! Generate the rest recursively;
  @FOR(SERIES(I)| I #GT# 1:
   U(I) = @RAND(U(I - 1))
  );

! Generate some...;
  @FOR(SERIES(I):
!   Normal deviates...;
   @PSN(ZNORM(I)) = U(I);
!   and t deviates(2 degrees of freedom);
   @PTD(2, ZT(I)) = U(I);
 !   ZNORM and ZT may take on negative values;
   @FREE(ZNORM(I)); @FREE(ZT(I));
  );
END
```

Model: DEMRND

Scenario-based Portfolio Model Model: DNRISK

In this model, we are attempting to come up with an optimal portfolio that meets a certain level of return while minimizing downside risk. Downside risk is a measure of the risk of falling below our target return. An additional feature of this model is it is scenario-based. More specifically, we have seven scenarios that will each occur with a given probability. The model incorporates this distribution of predicted outcomes in deriving the optimal portfolio.

```
MODEL:     ! (dnrisk.lng);
! Downside risk portfolio model;
 SETS:
    ASSET: INVEST; ! Amount to invest in each asset;
    SCENARIO:
        TRETRN,  ! Return under this scenario;
        DRISK;   ! Downside risk under this scenario;
    TABLE( SCENARIO, ASSET):
        ARETRN;  ! Return in scenario I of asset J;
 ENDSETS

 DATA:
! Number of scenarios;
    SCENARIO = 1..7;

! The available assets in which to invest;
    ASSET = ATT  GMC  USX;
    ARETRN =
    -.071  .144  .169
     .056  .107 -.035
     .038  .321  .133
     .089  .305  .732
     .090  .195  .021
     .083  .390  .131
     .035 -.072  .006;

! Desired return;
    DRETURN = .13;

! Threshold, below which we are unhappy;
    THRESH = .11;

! Power to use for risk(1 or 2);
!  When NPOW = 1, it is a linear program;
!  When NPOW = 2 and threshold = desired return;
!   it is the semi-variance;
    NPOW = 2;
 ENDDATA

    NSCEN = @SIZE( SCENARIO);

! Minimize average downside risk;
    MIN =  @SUM( SCENARIO: DRISK ^ NPOW)/ NSCEN;

! Compute return for each scenario;
    @FOR( SCENARIO( I):
     TRETRN( I) = @SUM( ASSET( J):
```

```
      ARETRN( I, J) * INVEST( J));

! .. and how much we fall short of threshold ;
     DRISK( I) >= THRESH - TRETRN( I);

! Return in a period could be negative;
     @FREE( TRETRN( I));
        );

! Our budget constraint(divided by a billion);
   [BUDGET] @SUM( ASSET: INVEST ) = 1;

! Our desired return;
   [PRICER] @SUM( SCENARIO( I): TRETRN( I))/ NSCEN >= DRETURN;
END
```

Model: DNRISK

Staff Scheduling Model: STAFFDEM

This *covering* model assigns start days to employees to cover staffing needs, minimizing the total number of staff, subject to the requirements that each staff member must have two consecutive days off each week. A detailed discussion of this model may be found in Chapter 2, *Using Sets*.

```
MODEL:
SETS:
   DAYS / MON TUE WED THU FRI SAT SUN/:
    REQUIRED, START;
ENDSETS

DATA:
   REQUIRED = 20 16 13 16 19 14 12;
ENDDATA

MIN = @SUM(DAYS(I): START(I));

@FOR(DAYS(J):
   @SUM(DAYS(I) | I #LE# 5:
      START(@WRAP(J - I + 1, 7)))
         >= REQUIRED(J)
);
END
```

Model: STAFFDEM

Dynamic Programming Model: DYNAMB

Dynamic programming (DP) is a creative approach to problem solving that involves breaking a large, difficult problem into a series of smaller, easy to solve problems. By solving this series of smaller problems, we are able to assemble the optimal solution to the initial large problem. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
SETS:
 ! Dynamic programming illustration (see Anderson, Sweeney &
   Williams, An Intro to Mgt Science, 6th Ed.). We have a network of
   10 cities. We want to find the length of the shortest route from
   city 1 to city 10.;
 ! Here is our primitive set of ten cities, where F(i) represents the
   shortest path distance from city i to the last city;
  CITIES /1..10/: F;
 ! The derived set ROADS lists the roads that exist between the
   cities (note: not all city pairs are directly linked by a road,
   and roads are assumed to be one way.);
  ROADS(CITIES, CITIES)/
   1,2  1,3  1,4
   2,5  2,6  2,7
   3,5  3,6  3,7
   4,5  4,6
   5,8  5,9
   6,8  6,9
   7,8  7,9
   8,10
   9,10/: D;
 ! D(i, j) is the distance from city i to j;
ENDSETS
DATA:
 ! The distances corresponding to the links;
  D =
     1    5    2
    13   12   11
     6   10    4
    12   14
     3    9
     6    5
     8   10
     5
     2;
ENDDATA
!  If you are already in City 10, then the cost to travel to City 10
   is 0;
 F(@SIZE(CITIES)) = 0;
! The following is the classic dynamic programming recursion. In
   words, the shortest distance from City i to City 10 is the minimum
   over all cities j reachable from i of the sum of the distance from
   i to j plus the minimal distance from j to City 10;
 @FOR(CITIES(i)| i #LT# @SIZE(CITIES):
```

```
    F(i) = @MIN(ROADS(i, j): D(i, j) + F(j))
  );
END
```

Capacitated EOQ — Model: EOQCAP

In this model, we have three products with known demand, production rates, setup costs, and holding costs. The question is how much of each product should we produce to minimize total setup and holding costs without exceeding the capacity of our production facility.

```
MODEL:
! Production capacity constrained EOQ;
! Demand rates for three products;
   d1 = 400; d2 = 300; d3 = 300;

! Production rates;
   p1 = 1300; p2 = 1100; p3 = 900;

! Setup costs for producing individual products;
   sc1 = 10000; sc2 = 12000; sc3 = 13000;

! Per unit holding costs;
   hc1 = 1; hc2 = 1.1; hc3 = 1.4;

!  The model;
!  Single machine capacity constraint; [CAP]
    d1 / p1 + d2 / p2 + d3 / p3
      + 1.5 * (d1 / q1 + d2 / q2 + d3 / q3) <= 1;

!  Minimize setup + holding costs;
    min = setup + holding;

!  Total setup costs;
    setup = (sc1 * d1 / q1) + (sc2 * d2 / q2)
                            + (sc3 * d3 / q3);

!  Total holding costs;
    holding = (hc1 * q1 * (1 - d1 / p1)
              + hc2 * q2 * (1 - d2 / p2)
              + hc3 * q3 * (1 - d3 / p3)) / 2;
!;

@BND(.01, Q1, 99999);
@BND(.01, Q2, 99999);
@BND(.01, Q3, 99999);
@FREE(SETUP);
@FREE(HOLDING);
END
```

Model: EOQCAP

Machine Repair Problem Model: EZMREPAR

This model analyzes a queuing system with a fixed population of members that periodically require servicing. Although we model a computer timesharing system in this example, this model is typically referred to as the *machine repair problem*, because one can think of it as applying to a group of machines periodically requiring repair by a number of service personnel. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
  ! Model of a computer timesharing system;
  ! The mean think time for each user (more
    generally, Mean Time Between Failures in a
    repair system);
    MTBF = 40;

    ! The mean time to process each compute request
    (more generally, Mean Time To Repair in
    seconds);
    MTTR =  2;

    ! The number of users;
    NUSER = 32;

  ! The number of servers/repairmen;
    NREPR = 1;

  ! The mean number of users waiting or in service
    (more generally, the mean number of machines
    down);
    NDOWN =
     @PFS(MTTR * NUSER/ MTBF, NREPR, NUSER);

  ! The overall request for service rate (more
    generally, overall failure rate), FR, must
    satisfy;
    FR = (NUSER - NDOWN)/ MTBF;

  ! The mean time waiting for or in service (more
    generally, the mean time down), MTD, must
    satisfy;
    NDOWN = FR * MTD;
END
```

Model: EZMREPAR

Newsboy Problem Model: EZNEWS

A common inventory management problem occurs when the product in question has limited shelf life (e.g., newspapers, produce, and computer hardware). There is a cost of over ordering, because the product will shortly become obsolete and worthless. There is also an opportunity cost of under ordering associated with forgone sales. Under such a situation, the question of how much of a product to order to maximize expected profit is classically referred to as the *newsboy problem*. In this example, we assume demand has a Poisson distribution. However, this is not mandatory. Refer to any operations research textbook for a derivation of the formulas involved.

```
MODEL:
 DATA:
 ! The average demand;
    MU = 144;

  ! Opportunity cost of each unit of lost demand;
    P = 11;

  ! Cost/unit of excess inventory;
    H = 5;

 ENDDATA

  ! Calculate the order-up-to point, S, using the
   newsboy problem equation;
  @PPS(MU, S) = P / (P + H);

  ! PS is the expected profit of being at S;
  PS=P * MU - H * (S -MU) - (P + H) * @PPL(MU, S);

END
```

Model: EZNEWS

Simple Queuing Example — Model: EZQUEUE

Given a queue with a certain arriving load and number of servers, the *@PEL* function determines the fraction of customers lost due to all servers being busy. In this example, we use *@PEL* to solve for the number of servers that limit customer loss to 5%.

```
MODEL:
! Arrival rate of customers/ hour;
   AR = 25;
! Service time per customer in minutes;
   STM = 6;
! Service time per customer in hours;
   STH = STM/ 60;
! Fraction customers finding all servers busy;
   FB = .05;
!The PEL function finds number of servers needed, NS;
   FB = @PEL(AR * STH, NS);
END
```

Model: EZQUEUE

General Equilibrium of an Economy Model: GENEQ1

```
MODEL:
! General Equilibrium Model of an economy;
! Data based on Kehoe, Math Prog, Study 23(1985);
! Find clearing prices for commodities/goods and
  equilibrium production levels for processes in
  an economy;

SETS:
 GOOD/1..4/: PRICE, H;
 SECTOR/1..4/;
 GXS(GOOD, SECTOR): ALPHA, W;
 PROCESS/1..2/: LEVEL;
 GXP(GOOD, PROCESS): MAKE;
ENDSETS

DATA:
! Demand curve parameter for each good and SECTOR;
 ALPHA =
   .5200  .8600  .5000  .0600
   .4000  .1     .2     .25
   .04    .02    .2975  .0025
   .04    .02    .0025  .6875;

! Initial wealth of Good I by Market J;
  W =
   50     0      0      0
    0    50      0      0
    0     0    400      0
    0     0      0    400;

! Amount produced of good I by process J;
  MAKE =
      6   -1
     -1    3
     -4   -1
     -1   -1;

! Weights for price normalization constraint;
  H = .25 .25 .25 .25;
ENDDATA

!————————————————————————————;
! Model based on Stone, Tech. Rep. Stanford OR(1988);
! Minimize the artificial variable;
MIN = V;

! Supply is >= demand;
@FOR(GOOD(G):
 @SUM(SECTOR(M): W(G, M))
  + @SUM(PROCESS(P): MAKE(G, P) * LEVEL(P))
   - @SUM(SECTOR(S):
    ALPHA(G, S) * @SUM(GOOD(I): PRICE(I) *
     W(I, S))/ PRICE(G)) + H(G) * V >= 0;
```

```
);

! Each process at best breaks even;
@FOR(PROCESS(P):
 @SUM(GOOD(G): - MAKE(G, P) * PRICE(G)) >= 0;
);

! Prices scale to 1;
@SUM(GOOD(G): - H(G) * PRICE(G)) = -1;
END
```

Model: GENEQ1

Markowitz Portfolio Example — Model: GENPRT

In the March, 1952 issue of *Journal of Finance*, Harry M. Markowitz published an article titled *Portfolio Selection*. In the article, he demonstrates how to reduce the risk of asset portfolios by selecting assets whose values aren't highly correlated. The following model implements these ideas in constructing a simple portfolio with three assets. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! GENPRT: Generic Markowitz portfolio;
 SETS:
  ASSET/1..3/: RATE, UB, X;
  COVMAT(ASSET, ASSET): V;
 ENDSETS

 DATA:
! The data;
!  Expected growth rate of each asset;
  RATE = 1.3   1.2  1.08;

!  Upper bound on investment in each;
  UB   = .75   .75   .75;

!  Covariance matrix;
  V    =   3     1   -.5
           1     2   -.4
         -.5   -.4     1;

!  Desired growth rate of portfolio;
  GROWTH = 1.12;
ENDDATA

! The model;
! Min the variance;
 [VAR] MIN = @SUM(COVMAT(I, J):
              V(I, J) * X(I) * X(J));

! Must be fully invested;
 [FULL] @SUM(ASSET: X) = 1;

! Upper bounds on each;
 @FOR(ASSET: @BND(0, X, UB));

! Desired value or return after 1 period;
 [RET] @SUM(ASSET: RATE * X) >= GROWTH;
END
```

Model: GENPRT

Job Shop Scheduling Model: JOBSLT

In this model, there are six jobs that can be done on one machine. The machine can only work on one job at a time. Each job has a due date. If we can't complete a job by its due date, we do not take the job. Our objective is to maximize the total value of the jobs selected.

Although we have based this model on a job shop scenario, the basic principles should be applicable in any area where time is the limiting factor in deciding what projects to undertake.

```
MODEL:
 ! One machine job selection;
 SETS:

 !There are six jobs each of which has a Due Date,
   Processing Time, Value, and a flag variable Y
   indicating if the job has been selected.;
  JOB/1..6/: ! Each job has a...;
   DD,  ! Due date;
   PT,  ! Processing time;
   VAL, ! Value if job is selected;
   Y;   ! = 1 if job is selected, else 0;
 ENDSETS

  DATA:
  VAL = 9  2  4  2  4  6;
  DD =  9  3  6  5  7  2;
  PT =  5  2  4  3  1  2;
  ENDDATA

  ! Maximize the total value of the jobs taken;
  MAX = TVAL;
  TVAL = @SUM(JOB: VAL * Y);

    ! For the jobs we do, we do in due date order;
  @FOR(JOB(J):

    ! Only jobs with earlier due dates can
      precede job J, and jobs must be completed
      by their due dates;
    @SUM(JOB(I)| DD(I) #LT# DD(J) #OR#
     (DD(I) #EQ# DD(J) #AND# I #LE# J):
      PT(I) * Y(I)) <= DD(J);

   ! Make the Y's binary;
    @BIN(Y);
  );
END
```

Model: JOBSLT

Knapsack Model — Model: KNAPSACK

In the knapsack model, one wants to select items to place into a knapsack to maximize a measure of utility without exceeding the capacity of the knapsack. This model can be generalized to many other areas such as truck loading, bin packing, choosing science experiments for the Space Shuttle, and so on. An in-depth description of this model can be found in Chapter 3, *Using Variable Domain Functions.*

```
SETS:
   ITEMS / ANT_REPEL, BEER, BLANKET,
    BRATWURST, BROWNIES, FRISBEE, SALAD,
     WATERMELON/:
      INCLUDE, WEIGHT, RATING;
ENDSETS
DATA:
   WEIGHT RATING =
     1      2
     3      9
     4      3
     3      8
     3     10
     1      6
     5      4
    10     10;
   KNAPSACK_CAPACITY = 15;
ENDDATA
MAX = @SUM(ITEMS: RATING * INCLUDE);
@SUM(ITEMS: WEIGHT * INCLUDE) <=
 KNAPSACK_CAPACITY;
@FOR(ITEMS: @BIN(INCLUDE));
```

Model: KNAPSACK

Learning Curve Model: LEARNC

The cost, labor, and/or time it takes to perform a task will often decrease the more times it is performed. A manufacturer may need to estimate the cost to produce 1,000 units of a product after producing only 100. The average unit cost of the first 100 is likely to be considerably higher than the average unit cost of the last 100. Learning curve theory assumes each time the quantity produced doubles, the cost per unit decreases at a constant rate.

In our example, we wish to estimate the cost (in hours) to produce paper based on the cumulative number of tons produced so far. The data is fitted to a curve of the form:

$$COST(i) = a * VOLUME(i)^{b}$$

where *COST* is the dependent variable and *VOLUME* is the independent variable. By taking logarithms, we can linearize the model:

$$\ln[\ COST(i)] = \ln(a) + b * \ln[\ VOLUME(i)]$$

We can then use the theory of linear regression to find estimates of ln(a) and b that minimize the sum of the squared prediction errors. Note that since the regression involves only a single independent variable, the formulas for computing the parameters are straightforward. Refer to any theoretical statistics text for a derivation of these formulas.

```
MODEL:
! Learning curve model;
  ! Assuming that each time the number produced
    doubles, the cost per unit decreases by a
    constant rate, predict COST per unit with
    the equation:
    COST(i) = A * VOLUME(i) ^ B;
 SETS:
  ! The OBS set contains the data for COST
    and VOLUME;
  OBS/1..4/:
     COST,   ! The dependent variable;
     VOLUME; ! The independent variable;
  ! The OUT set contains the outputs of the model.
    Note: R will contain the output results.;
  OUT/ A, B, RATE, RSQRU, RSQRA/: R;
 ENDSETS
  ! Data on hours per ton, cumulative tons for a
    papermill based on Balof, J. Ind. Eng.,
    Jan. 1966;
 DATA:
  COST = .1666,  .1428,  .1250,  .1111;
  VOLUME = 8,     60,     100     190;
 ENDDATA

! The model;
 SETS:
  ! The derived set OBSN contains the set of
    logarithms of our dependent and independent
    variables as well the mean shifted values;
```

```
 OBSN(OBS): LX, LY, XS, YS;
ENDSETS

NK = @SIZE(OBS);
 ! Take the logs;
  @FOR(OBSN(I):
    LX(I) = @LOG(VOLUME(I));
    LY(I) = @LOG(COST(I)); );
 ! Compute means;
XBAR = @SUM(OBSN: LX)/ NK;
YBAR = @SUM(OBSN: LY)/ NK;

 ! Shift the observations by their means;
@FOR(OBSN:
  XS = LX - XBAR;
  YS = LY - YBAR);

 ! Compute various sums of squares;
XYBAR = @SUM(OBSN: XS * YS);
XXBAR = @SUM(OBSN: XS * XS);
YYBAR = @SUM(OBSN: YS * YS);

 ! Finally, the regression equation;
SLOPE = XYBAR/ XXBAR;
CONS = YBAR - SLOPE * XBAR;
RESID = @SUM(OBSN: (YS - SLOPE * XS)^2);
 ! The unadjusted/adjusted fraction of variance
   explained;
[X1]R(@INDEX(RSQRU)) = 1 - RESID/ YYBAR;
[X2]R(@INDEX(RSQRA)) =  1 -  (RESID/ YYBAR) *
 (NK - 1)/(NK - 2);
[X3]R(@INDEX(A)) = @EXP(CONS);
[X4]R(@INDEX(B)) = - SLOPE;
[X5]R(@INDEX(RATE)) = 2 ^ SLOPE;
! Some variables must be unconstrained in sign;
@FOR(OBSN: @FREE(LY); @FREE(XS); @FREE(YS));
@FREE(YBAR); @FREE(XBAR); @FREE(SLOPE);
@FREE(XYBAR); @FREE(CONS);
END
```

Model: LEARNC

Markov Chain Model — Model: MARKOV

A standard approach used in modeling random variables over time is the Markov chain approach. The basic idea is to think of the system as being in one of a discrete number of states at each point in time. The behavior of the system is described by a transition probability matrix, which gives the probability the system will move to a specified other state from some given state. Some example situations are:

System	States	Cause of Transition
Consumer brand switching	Brand of product most recently purchased by consumer	Consumer changes mind, advertising
Inventory System	Amount of inventory on hand	Orders for new material, demands

In the following model, we use Markov chain analysis to determine the long-term, steady state probabilities of the system. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
 ! Markov chain model;
SETS:
 ! There are four states in our model and over
   time the model will arrive at a steady state
   equilibrium.
   SPROB(J) = steady state probability;
 STATE/ A B C D/: SPROB;
 ! For each state, there's a probability of moving
   to each other state. TPROB(I, J) = transition
   probability;
 SXS(STATE, STATE): TPROB;
ENDSETS
DATA:
 ! The transition probabilities. These are proba-
   bilities of moving from one state to the next
   in each time period. Our model has four states,
   for each time period there's a probability of
   moving to each of the four states. The sum of
   probabilities across each of the rows is 1,
   since the system either moves to a new state or
   remains in the current one.;
   TPROB = .75  .1  .05  .1
           .4   .2  .1   .3
           .1   .2  .4   .3
           .2   .2  .3   .3;
ENDDATA
! The model;
!  Steady state equations;
!   Only need N equations, so drop last;
  @FOR(STATE(J)| J #LT# @SIZE(STATE):
   SPROB(J) = @SUM(SXS(I, J): SPROB(I) *
    TPROB(I, J))
  );
```

```
! The steady state probabilities must sum to 1;
  @SUM(STATE: SPROB) = 1;

! Check the input data, warn the user if the sum
  of probabilities in a row does not equal 1.;
  @FOR(STATE(I):
   @WARN('Probabilities in a row must sum to 1.',
    @ABS(1 - @SUM(SXS(I, K): TPROB(I, K)))
     #GT# .000001);
  );

END
```

Model: MARKOV

Matching Model — Model: MATCHD

Pair-matching problems require a number of objects be grouped into pairs subject to some criteria. The objective may be to minimize cost, to group like objects, etc. As an example, the following matching model pairs workers into offices to minimize total incompatibilities between paired individuals. A detailed discussion of this model may be found in Chapter 2, *Using Sets*.

```
MODEL:
SETS:
   ANALYSTS / 1..8/;
   PAIRS(ANALYSTS, ANALYSTS) | &2 #GT# &1:
    RATING, MATCH;
ENDSETS

DATA:
   RATING =
      9  3  4  2  1  5  6
         1  7  3  5  2  1
            4  4  2  9  2
               1  5  5  2
                  8  7  6
                     2  3
                        4;
ENDDATA

MIN = @SUM(PAIRS(I, J):
 RATING(I, J) * MATCH(I, J));

@FOR(ANALYSTS(I):
 @SUM(PAIRS(J, K) | J #EQ# I #OR# K #EQ# I:
  MATCH(J, K)) = 1
);

@FOR(PAIRS(I, J): @BIN(MATCH(I, J)));
END
```

Model: MATCHD

Computing Demand Backlog Model: METRIC

The *METRIC* model estimates the average number of backorders in a two level inventory system composed of a single depot or distribution center and any number of outlets served by the depot. Inventory policy at each location is described by a single number *S*. Whenever the sum of the amount on hand and on order drops below *S*, an order is placed to bring inventory back up to *S*. This is an evaluation model, rather than an optimization model. The user is prompted for the *S* values (*SDEPOT* and *SOUTLET*).

```
MODEL:
! The two level METRIC inventory model. ;
 SETS:
  OUTLET/1..2/: ! Each outlet has a...;
   ROUTLET,  ! Resupply time from depot to outlet;
   DEM,      ! Demand rate at outlet;
   SOUTLET,  ! Stock level to use at outlet;
   ERT,      ! Effective resupply time to outlet;
   AL;       ! Average level of backlogged demand;
 ENDSETS

DATA:
  ROUTLET =  2   2;
  DEM =     .1  .1;
  RDEPOT = 14;

!  Get the stock levels from the user;
  SDEPOT = ?;
  SOUTLET = ?, ?;
 ENDDATA

! Compute total demand;
   DEM0 = @SUM(OUTLET: DEM);

! Effective expected wait at depot;
   EWT0 = @PPL(DEM0 * RDEPOT, SDEPOT)/ DEM0;
  @FOR(OUTLET(I):

! Estimate the resupply time including depot delay;
    ERT(I) = ROUTLET(I) + EWT0;

! Expected demand on backorder;
    AL(I) = @PPL(DEM(I) * ERT(I), SOUTLET(I));
  );

! Total expected demand on backorder;
  TBACK = @SUM(OUTLET: AL);
END
```

Model: METRIC

The Mexican Steel Problem Model: MEXICO

The following model is a production-planning model adapted from the GAMS documentation.

```
MODEL:
  ! The Mexican Steel problem;
 SETS:
   ! Steel plants;
  PLANT /AHMSA FUNDIDA SICARTSA HYLSA HYLSAP/:
   RD2, MUE;

   ! Markets;
  MARKET / MEXICO MONTE GUADA /: DD, RD3, MUV;

   ! Final products;
  CF /STEEL/: PV, PE, EB;

   ! Intermediate products;
  CI / SPONGE PIGIRON/;

   ! Raw materials;
  CR /PELLETS COKE NATGAS ELECTRIC SCRAP/ : PD;

   ! Processes;
  PR /PIGIRON1 SPONGE1 STEELOH STEELEL STEELBOF/;

   ! Productive units;
  UNIT / BLASTFUR OPENHEAR BOF DIRECT ELECARC/;

   ! ;
  CRXP(CR, PR) : A1; CIXP(CI, PR) : A2;
  CFXP(CF, PR) : A3; UXP(UNIT, PR) : B;
  UXI(UNIT, PLANT) : K;
  PXM(PLANT, MARKET) : RD1, MUF;
  PXI(PR, PLANT) : Z;
  CFXPXM(CF, PLANT, MARKET) : X;
  CRXI(CR, PLANT) : U; CFXI(CF, PLANT) : E;
  CFXM(CF, MARKET) : D, V;
 ENDSETS

 ! Demand equations;
   @FOR(CFXM(C, J):
    D(C,J) = 5.209 * (1 + 40/ 100) * DD(J)/100);

  ! Transport rate  equations;
   @FOR(PXM(I, J)| RD1(I, J) #GT# 0:
    MUF(I, J) =  2.48 + .0084 * RD1(I, J));
   @FOR(PXM(I, J)| RD1(I, J) #LE# 0:
    MUF(I, J) = 0);
   @FOR(PLANT(I)| RD2(I) #GT# 0:
    MUE(I) = 2.48 + .0084 * RD2(I));
   @FOR(PLANT(I)| RD2(I) #LE# 0: MUE(I) = 0);
   @FOR(MARKET(J)| RD3(J) #GT# 0:
    MUV(J) = 2.48 + .0084 * RD3(J));
   @FOR(MARKET(J)| RD3(J) #LE# 0: MUV(J) = 0);
```

```
! For each plant I1;
 @FOR(PLANT(I1):

  ! Sources >= uses, final products;
   @FOR(CF(CF1): [MBF]
    @SUM(PR(P1): A3(CF1, P1) * Z(P1, I1)) >=
     @SUM(MARKET(J1): X(CF1, I1, J1)) +
      E(CF1, I1));

  ! Intermediate products;
   @FOR(CI(CI1): [MBI]
    @SUM(PR(P1): A2(CI1, P1) *
     Z(P1, I1)) >= 0);

  ! Raw materials;
   @FOR(CR(CR1): [MBR]
    @SUM(PR(P1): A1(CR1, P1) *
     Z(P1, I1)) + U(CR1, I1) >= 0);

  ! Capacity of each productive unit M1;
   @FOR(UNIT(M1): [CC]
    @SUM(PR(P1): B(M1, P1) * Z(P1, I1)) <=
     K(M1, I1));
 );

! For each final product CF1;
 @FOR(CF(CF1):

  ! Demand requirements for each market J1;
   @FOR(MARKET(J1): [MR]
    @SUM(PLANT(I1): X(CF1, I1, J1)) + V(CF1, J1)
     >= D(CF1, J1));

  ! Upper limit on exports ;
   [ME] @SUM(PLANT(I1): E(CF1, I1)) <= EB(CF1);
 );

! Components of objective;
 PHIPSI = @SUM(CR(CR1):
  @SUM(PLANT(I1): PD(CR1) * U(CR1, I1)));
 PHILAM = @SUM(CF(CF1):
  @SUM(PLANT(I1): @SUM(MARKET(J1):MUF(I1,J1)*
   X(CF1, I1, J1))) + @SUM(MARKET(J1):
    MUV(J1) * V(CF1, J1)) + @SUM(PLANT(I1):
     MUE(I1) * E(CF1, I1)));
 PHIPI = @SUM(CFXM(CF1, I1):
  PV(CF1) * V(CF1, I1));
 PHIEPS = @SUM(CFXP(CF1, I1):
  PE(CF1) * E(CF1, I1));
[OBJROW] MIN = PHIPSI + PHILAM + PHIPI - PHIEPS;
```

```
DATA:
  A1= -1.58, -1.38, 0,    0,    0,
      -0.63,  0,    0,    0,    0,
       0,    -0.57, 0,    0,    0,
       0,     0,    0,   -0.58, 0,
       0,     0,   -0.33, 0,   -0.12;
  A2=  1,     0,   -0.77, 0,   -0.95,
       0,     1,    0,   -1.09, 0;
  A3=  0,     0,    1,    1,    1;
  B = 1 0 0 0 0
      0 0 1 0 0
      0 0 0 0 1
      0 1 0 0 0
      0 0 0 1 0;
  K = 3.25, 1.40, 1.10, 0,    0,
      1.50, 0.85, 0,    0,    0,
      2.07, 1.50, 1.30, 0,    0,
      0,     0,   0,    0.98, 1,
      0,     0,   0,    1.13, 0.56;
  RD1= 1204  218   1125
       1017  0     1030
       819  1305   704
       1017  0     1030
       185  1085   760;
  RD2 = 739, 521,  0, 521, 315;
  RD3 = 428, 521,  300;
  PD = 18.7, 52.17, 14,    24,  105;
  PV = 150;
  PE = 140;
  EB = 1;
  DD = 55, 30, 15;
 ENDDATA

END
```

Model: MEXICO

Multiprod. Capac. Lot Sizing Model: MPSCHD

The following model is a production-planning model where multiple products compete for scarce capacity. Each product has a setup time, setup cost, production time, production cost, and holding cost. The question is how much of each product should we produce in each period to minimize total production costs.

```
MODEL:
! Multiproduct Capacitated lot sizing;
 SETS:
  PROD/1..2/: ! Each product has a ...;
   ST,   ! Setup time;
   VT,   ! Production time per unit;
   SC,   ! Setup cost;
   VC,   ! Production cost per unit;
   HC;   ! Holding cost per unit per period;
  TIME/1..6/:;
  PXT(PROD, TIME):

! Each product in each period has...;
   DEM,  ! Demand;
   MAKE, ! Amount to produce;
   Y;    ! = 1 if anything is produced;
 ENDSETS

 DATA:
  CAP = 200;     ! Capacity per period;
  ST = 0  0;     ! Setup times;
  VT = 1  1;     ! Production time per unit;
  SC = 150  45;  ! Setup costs;
  VC =  7  4;    ! Cost per unit to produce;
  HC =  2  1;    ! Holding cost per unit;
  DEM =  40  60 130   0 100 200  ! Demands;
          0  45  50  35  20  35;
 ENDDATA

!——————————————————;
!  The Eppen/Martin model;
 SETS:
  PXTXT(PROD, TIME, TIME)| &3 #GE# &2:
   PCOF,
   CCOF,
   X;
 ENDSETS

! Compute cost and production amounts for
  various production runs;
 @FOR(PROD(I):
  @FOR(TIME(S):
   PCOF(I, S, S) = DEM(I, S);
   CCOF(I, S, S) = VC(I) * DEM(I, S);
   @FOR(TIME(T)| T #GT# S:
    PCOF(I, S, T) = PCOF(I, S, T - 1) +
```

```
      DEM(I, T);
     CCOF(I, S, T) = CCOF(I, S, T - 1) +
     (VC(I) + HC(I) * (T - S)) * DEM(I, T);
    )
   )
  );

! The objective;
  MIN = TCOST;
  TCOST = @SUM(PXTXT: CCOF * X) +
   @SUM(PXT(I, S): SC(I) * Y(I, S));
  @FOR(PROD(I):

! In period 1, some production run must be started;
!  Note, watch out for periods without demand;
   @SUM(PXTXT(I, S, T) | S #EQ# 1:
    X(I, S, T)) = 1;
   @FOR(TIME(K)| K #GT# 1:

!  If we ended a run in period K - 1...;
    @SUM(PXTXT(I, S, T)| T #EQ# K - 1:
     X(I, S, K - 1))

!  then we must start a run in period k;
       = @SUM(PXTXT(I, K, T): X(I, K, T));
   );

! Setup forcing;
   @FOR(TIME(S):
    Y(I, S) =  @SUM(PXTXT(I, S, T)
     : (PCOF(I, S, T) #GT# 0) * X(I, S, T));

! Calc amount made each period;
    MAKE(I, S) = @SUM(PXTXT(I, S, T):
     PCOF(I, S, T) * X(I, S, T))
   )
  );

! The capacity constraints;
  @FOR(TIME(S):
   @SUM(PROD(I): ST(I) * Y(I, S)) +
    @SUM(PXTXT(I, S, T):
     VT(I) * PCOF(I, S, T) * X(I, S, T)) <= CAP
  );

! Make the Y's integer;
  @FOR(PXT: @GIN(Y));
END
```

Model: MPSCHD

Machine Repair Model — Model: MREPAR

This model illustrates the tradeoff between the cost of service people and the costs of machine downtime. In our model, we have ten machines that have a tendency to break down randomly. It costs $350/hour in lost production whenever a machine is broken. The question we need to answer is how many service people should we hire to minimize the total cost of down time and salaries for service people.

```
MODEL:
 ! Machine repair model;
 SETS:
  NREP/1..5/: !Consider 5 possible repair persons;
       NDOWN, !Expected no. of down machines;
       CPERHR,Expected cost/hour of down machines;
       TCOST; !Total expected cost/hour;
 ENDSETS

! The input data;
  NMACH = 10;! No. machines subject to breakdown;
  RTIME = 1; ! Average repair time;
  UPTIME = 5;! Mean time between failures;
  CR = 30;   ! Hourly cost of a repair person;
  CM = 350;  ! Hourly cost of a down machine

 ! The machine repairman queuing model;
 ! For each case of 1 - 5 service people calculate
   expected number of machines down, cost per hour
   of down machines, and total cost per hour of
   operations. @PFS calculates the Probability in
   a Finite Source, in this case expected number
   of machines under repair. ;
  @FOR(NREP(I):
   NDOWN(I) =
    @PFS(NMACH * RTIME / UPTIME, I, NMACH);
   CPERHR(I) = CM * NDOWN(I);
   TCOST(I) = CPERHR(I) + CR * I
  );

END
```

Model: MREPAR

Material Requirements Planning Model: MRP

Material Requirements Planning, or MRP, is used to generate production schedules for the manufacture of complex products. MRP takes the demand schedule for a finished product and the lead times to produce the finished product and all the various subcomponents that go into the finished product, and then works backwards to come up with a detailed, just-in-time production schedule that meets the demand schedule. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Data for this model is read from MRP.LDT;
SETS:
! The set of parts;
 PART: LT;
 ! LT(i) = Lead time to produce part i;

! The set of time periods;
 TIME;

! A relationship called USES between pairs of parts;
 USES( PART, PART): NEEDS;
 ! Parent part i needs NEEDS(i, j) units of
   child part j;

! For each part and time period we're interested in;
 PXT( PART, TIME): ED, TD;
 ! ED(i, j) = External demand for part i at time j;
 ! TD(i, j) = Total demand for part i at time j;
ENDSETS

DATA:

! Load the data from an external file;

 ! Parts list;
 PART = @FILE( 'MRP.LDT');

 ! Time periods;
 TIME = @FILE( 'MRP.LDT');

 ! Get the parent child relations and the
   number of parts required;
 USES, NEEDS = @FILE( 'MRP.LDT');

! Get the lead times from the file;
 LT  = @FILE( 'MRP.LDT');

! Get the external demands
  over time for each part;
 ED  = @FILE( 'MRP.LDT');
ENDDATA
```

```
! Set NP = no. of time periods in the problem;
 NP = @SIZE( TIME);

! For each part P and period T, the total demand =
  external demand + demand generated by parents
  one lead time in the future;
 @FOR( PXT( P, T) | T + LT( P) #LE# NP :
  TD( P, T) = ED( P, T + LT( P)) +
   @SUM( USES( P2, P): TD( P2, T + LT( P)) *
    NEEDS( P2, P));
 );

DATA:

! Display a table showing the production schedule;
 @TEXT() = ' The production schedule:';
 @TEXT() = @TABLE( TD);
ENDDATA
END
```

Model: MRP

Minimal Spanning Tree Model: MSPAN

In the minimal spanning tree, we need to find a set of links (a tree) in a network that connects all cities. Furthermore, the sum of the distances over all the links in the tree should be minimized. Among other things, this application is useful in constructing communications networks at minimal cost.

It turns out that this becomes a very difficult problem to solve using optimization as the number of nodes grows. For large versions of this problem, the optimization techniques provided by LINGO are not the appropriate tool. One would be wise to pursue alternatives such as heuristics or dynamic programming.

```
MODEL:
!Given the number of nodes and the distance
 between them, finding the shortest total distance
 of links on the network to connect all the nodes
 is the classic problem called minimal spanning tree (MST).
 This model finds the (MST) connecting Atlanta,
 Chicago, Cincinnati, Houston, LA, and Montreal so
 that messages can be sent from Atlanta (base) to
 other cities through the network at minimum cost;
  SETS:
    CITY / 1.. 6/: U; ! U(I) = level of city I;
                      ! U(1) = 0;
    LINK(CITY, CITY):
         DIST,  ! The distance matrix;
           X;  ! X(I, J) = 1 if we use link I, J;
  ENDSETS

DATA:   ! Distance matrix need not be symmetric;
          ! However, city 1 is base of the tree;
     !to: Atl  Chi  Cin  Hou   LA  Mon ;
   DIST =   0  702  454  842 2396 1196  !from Atl;
          702    0  324 1093 2136  764  !from Chi;
          454  324    0 1137 2180  798  !from Cin;
          842 1093 1137    0 1616 1857  !from Hou;
         2396 2136 2180 1616    0 2900  !from LA;
         1196  764  798 1857 2900    0; !from Mon;
  ENDDATA

! The model size: Warning, may be slow for N >= 8;
   N = @SIZE(CITY);

  ! Minimize total distance of the links;
   MIN = @SUM(LINK: DIST * X);

  ! For city K, except the base, ... ;
   @FOR(CITY(K)| K #GT# 1:

    ! It must be entered;
      @SUM(CITY(I)| I #NE# K: X(I, K)) = 1;
```

```
    ! If there are 2 disjoint tours from 1 city to
      another, we can remove a link without
      breaking connections. Note: These are not
      very powerful for large problems;
      @FOR(CITY(J)| J #GT# 1 #AND# J #NE# K:
         U(J) >= U(K) + X (K, J) -
         (N - 2) * (1 - X(K, J)) +
         (N - 3) * X(J, K); );
   );

  ! There must be an arc out of city 1;
   @SUM(CITY(J)| J #GT# 1: X(1, J)) >= 1;

  ! Make the X's 0/1;
   @FOR(LINK: @BIN(X); );

  ! The level of a city except the base is at
    least 1 but no more than N-1, and is 1 if it
    links to the base;
    @FOR(CITY(K)| K #GT# 1:
       @BND(1, U(K), 999999);
       U(K) <= N - 1 - (N - 2) * X(1, K);   );
END
```

Model: MSPAN

Multilevel Distribution Model: MULLDC

In this model, we minimize shipping costs over a three tiered distribution system consisting of plants, distribution centers, and customers. Plants produce multiple products that are shipped to distribution centers. If a distribution center is used, it incurs a fixed cost. Customers are supplied by a single distribution center.

```
MODEL:
! MULLDC;
! Multilevel DC location model, based on
  Geoffrion/Graves, Man. Sci., Jan., 1974;
! Original LINGO model by Kamaryn Tanner;
 SETS:
! Two products;
  PRODUCT/ A, B/;
! Three plants;
  PLANT/ P1, P2, P3/;
! Each DC has an associated fixed cost, F,
  and an "open" indicator, Z.;
  DISTCTR/ DC1, DC2, DC3, DC4/: F, Z;
! Five customers;
  CUSTOMER/ C1, C2, C3, C4, C5/;
! D = Demand for a product by a customer.;
  DEMLINK( PRODUCT, CUSTOMER): D;
! S = Capacity for a product at a plant.;
  SUPLINK( PRODUCT, PLANT): S;
! Each customer is served by one DC,
  indicated by Y.;
  YLINK( DISTCTR, CUSTOMER): Y;
! C= Cost/ton of a product from a plant to a DC,
  X= tons shipped.;
  CLINK( PRODUCT, PLANT, DISTCTR): C, X;
! G= Cost/ton of a product from a DC to a customer.;
  GLINK( PRODUCT, DISTCTR, CUSTOMER): G;
ENDSETS
DATA:
! Plant Capacities;
 S = 80, 40, 75,
     20, 60, 75;
! Shipping costs, plant to DC;
 C = 1,     3,   3,   5,       ! Product A;
     4,   4.5, 1.5, 3.8,
     2,   3.3, 2.2, 3.2,
     1,     2,   2,   5,       ! Product B;
     4,   4.6, 1.3, 3.5,
     1.8,   3,   2, 3.5;
! DC fixed costs;
 F = 100, 150, 160, 139;
! Shipping costs, DC to customer;
 G =   5,   5,   3,   2,   4,  ! Product A;
     5.1, 4.9, 3.3, 2.5, 2.7,
     3.5,   2, 1.9,   4, 4.3,
```

```
     1,   1.8, 4.9, 4.8,   2,
     5,   4.9, 3.3, 2.5, 4.1,  ! Product B;
     5,   4.8,   3, 2.2, 2.5,
     3.2,   2, 1.7, 3.5,   4,
     1.5,   2,   5,   5, 2.3;
! Customer Demands;
 D =  25,  30,  50,  15,  35,
      25,   8,   0,  30,  30;
ENDDATA
!-----------------------------------------------------;
! Objective function minimizes costs.;
 [OBJ] MIN = SHIPDC + SHIPCUST + FXCOST;
 SHIPDC = @SUM( CLINK: C * X);
 SHIPCUST =
  @SUM( GLINK( I, K, L):
   G( I, K, L) * D( I, L) * Y( K, L));
 FXCOST = @SUM( DISTCTR: F * Z);

! Supply Constraints;
 @FOR( PRODUCT( I):
  @FOR( PLANT( J):
   @SUM( DISTCTR( K): X( I, J, K)) <= S( I, J))
 );

! DC balance constraints;
 @FOR( PRODUCT( I):
  @FOR( DISTCTR( K):
   @SUM( PLANT( J): X( I, J, K)) =
    @SUM( CUSTOMER( L): D( I, L)* Y( K, L)))
 );

! Demand;
 @FOR( CUSTOMER( L):
  @SUM( DISTCTR( K): Y( K, L)) = 1
 );

! Force DC K open if it serves customer L;
 @FOR( CUSTOMER( L):
  @FOR( DISTCTR( K): Y( K, L) <= Z( K))
 );

! Y binary;
 @FOR( DISTCTR( K):
  @FOR( CUSTOMER( L): @BIN( Y( K,L)))
 );

END
```

Model: MULLDC

Network Equilibrium Model: NETEQ1

In this example, we have a network of pipelines capable of transporting either fluid or gas. There are a total of eight nodes and 11 arcs between them. Two of the nodes are source nodes, while the remaining nodes are net demanders of product. The pressures at the source nodes are given. The model determines the flow of product down each arc and the pressures at each node.

```
MODEL:
! Network equilibrium NETEQ1:based on Hansen et.al., Math.
   Prog. vol. 52, no.1;
 SETS:
  NODE/A, B, C, D, E, F, G, H/:
   P;      ! Pressure at this node;
  ARC(NODE, NODE)/ B A, C A, C B, D C, E D,
                F D, G D, F E, H E, G F, H F/ :
   R,      ! Resistance on this arc;
   FLO;    ! Flow on this arc;
  SRC(NODE)/ G, H/:
   PFIXED; ! Fixed pressure at source nodes;
  DEST(NODE) | #NOT# @IN(SRC, &1):
   DEMAND; ! Given demand at destination nodes;
 ENDSETS
 DATA:
  PFIXED =  240, 240;
  DEMAND =  1, 2, 4, 6, 8, 7;
  R =  1, 25, 1, 3, 18, 45, 1, 12, 1, 30, 1;
! For incompressible fluids and electricity:
  PPAM = 1, for gases: PPAM = 2;
  PPAM = 1;
! For optimization networks: FPAM = 0
  (for arcs withflow >= 0)
  electrical networks:   FPAM = 1
  other fluids:  1.8 <= FPAM <= 2;
  FPAM = 1.852;
 ENDDATA
!Set the pressures for the source/reservoir nodes;
 @FOR(SRC(I): P(I) = PFIXED(I));
! Conservation of flow at non-source nodes;
  @FOR(DEST(J):
   @SUM(ARC(I, J): FLO(I, J)) = DEMAND(J) +
    @SUM(ARC(J, K): FLO(J, K)));
! Relate pressures at 2 ends of each arc;
 @FOR(ARC(I, J):
  P(I)^ PPAM - P(J)^ PPAM = R(I, J) *
   FLO(I, J) ^ FPAM;);
END
```

Model: NETEQ1

Minimize Traffic Congestion Model: NLTRAZ

In this model, we have a network with seven nodes. Three of the nodes are source nodes and four of the nodes are destination nodes. Each source node can ship to any of the destination nodes. The problem is the more of a product you send down an arc, the longer it takes for it to arrive. This might be the case if the underlying network were a railroad, for instance. We assume that shipping times obey the following relationship:

Time = *Rate* / (1 - *Flow* / *Limit*)

where,

Time	time to ship one unit of product down a route,
Rate	time required to transport one unit down a route with no congestion along the route,
Flow	amount of product shipped down a route, and
Limit	maximum limit of product flow down a route.

Based on this relationship, we see shipping times go to infinity as the capacity of an arc is approached. The goal of the model is to determine how much of a product to ship from each source to each destination so as to minimize total shipping times.

```
MODEL:
! Traffic congestion transportation problem.
  Cost/unit increases to infinity as traffic on
  link approaches its link capacity.
  Truncated variation of an AMPL example;
 SETS:
   ORIG/ CHIC CINC ERIE/: SUPPLY;
   DEST / HAM AKR COL DAY/ : DEMAND;
   OXD(ORIG, DEST): RATE, LIMIT, TRAF;
 ENDSETS
 DATA:
   SUPPLY = 1200 800 1400;
   DEMAND = 1000 1200 700 500 ;
   RATE =   39   14   11   14
            27    9   12    9
            24   14   17   13 ;
   LIMIT = 500 1000 1000 1000
           500  800  800  800
           800  600  600  600 ;
 ENDDATA
 [TOTCOST] MIN =
   @SUM(OXD: RATE * TRAF/(1 - TRAF/ LIMIT));
 @FOR(ORIG(I):
    @SUM(OXD(I, J): TRAF(I, J)) = SUPPLY(I));
 @FOR(DEST(J):
    @SUM(OXD(I, J): TRAF(I, J)) = DEMAND(J));
 @FOR(OXD: @BND(0, TRAF, LIMIT););
END
```

Model: NLTRAZ

Newsboy with Fixed Order Charge Model: NUSBOY

In the simple newsboy model (*EZNEWS*), presented earlier in this chapter, we did not have a fixed ordering charge to deal with. We add this minor complication to the model below. Assuming you decide to order, the fixed charge is a sunk cost and you should therefore order up to the same quantity, *S*, as in the standard newsboy model. However, there may be cases where preexisting inventory is of a level close enough to *S* that the expected gains of a minimal increase in inventory are not outweighed by the fixed order charge. The problem now is to not only determine *S* (or "big S"), but also to determine the additional parameter, *s* (or "little s"), where when inventory exceeds *s* the optimal decision is to not incur a fixed charge by foregoing an order for additional stock. Inventory strategies such as this are referred to as "little s-big S", or (*s*,*S*), policies.

One additional feature to note in this model is we now assume demand to be normally distributed. This is acceptable because the newsboy model does not demand any particular form of demand distribution. As with the *EZNEWS* model, we could have also used a Poisson distribution if we felt it was more appropriate.

```
MODEL:
! Newsboy inventory model;
! This model calculates the optimal stock levels
  for a product with normally distributed demand
  and a fixed ordering cost;

ATA:
 P = 11;      ! Penalty/unit for not having enough;
 H = 5;       ! Holding cost/unit for excess;
 MU = 144;    ! Mean demand;
 SIGMA = 25;  ! Standard deviation in demand;
 K = 15;      ! Fixed cost of placing an order;
ENDDATA

! Compute reorder point, SLIL, and order up to
  point, SBIG;
! Calculate the order up to point, SBIG, using
  standard newsboy formula;
 @PSN(ZBIG ) = P /(P + H);
 ZBIG = (SBIG - MU)/ SIGMA;
! and the expected cost of being there, CSBIG;
 CSBIG = SIGMA * @PSL(ZBIG) * (P+H) + H * (SBIG-MU);

! The expected cost at the reorder point should
  differ from the expected cost at SBIG by the
  fixed order cost, K;
 CSLIL = K + CSBIG;

! Solve for SLIL;
 CSLIL=SIGMA*@PSL(ZLIL)*(P+H)*H*(ZLIL*SIGMA);
 ZLIL = (SLIL - MU)/ SIGMA;

END
```

Model: NUSBOY

Optimal Airline Overbooking I — Model: OBOOKO

Closely related to the newsboy problem (in a mathematical sense) is the airline-overbooking problem. Given that a certain percentage of fliers with reservations will not show up for a flight, airlines that don't overbook will be sending most planes up with empty seats. Assuming the penalty cost for overbooking is not too high, an airline that hopes to maximize revenue should overbook its flights. The following model determines the optimal number of reservations to allow on a flight, and assumes the number of no-shows on a flight has a binomial distribution.

```
MODEL:
!
 This overbooking model determines the number of
 reservations, M, to allow on a flight if the
 no-show distribution is binomial;
! Some available data ;
   N = 16;   ! total seats available;
   V = 225;  !Revenue from a sold seat;
   P = 100;  !Penalty for a turned down customer;
   Q = .04;  !Probability a customer is a no-show;
! The probability to turn down customers is
  @PBN(Q, M, M - N), therefore the corresponding
  expected loss due to imperfect information is:
  (V + P) * @PBN(Q, M, M - N), and we want the
  loss to equal the revenue V on the margin.  So,
  the break-even equation is:;
   (V + P) * @PBN(Q, M, M - N) = V;
! Note, you should round up if M is fractional;
END
```

Model: OBOOKO

Optimal Airline Overbooking II Model: OBOOKT

For those of you uncomfortable with the previous overbooking example, we use a "brute force" method here to compute the expected profits from overbooking 1 to 6 seats. Solving this model, you will find the results agree with the previous—the overbooking level that maximizes expected revenue is 1 passenger.

```
MODEL:
! A strategy for airlines to minimize loss from
  no-shows is to overbook flights.  Too little
  overbooking results in lost revenue.  Too much
  overbooking results in excessive penalties.
  This  model computes expected profits for
  various levels of overbooking.;

  SETS:
    SEAT/1..16/;           ! seats available ;
    EXTRA/1..6/: EPROFIT;  ! expected profits from
                            overbooking 1-6 seats;
  ENDSETS

! Available data;
    V = 225; ! Revenue from a sold seat;
    P = 100; ! Penalty for a turned down customer;
    Q = .04; ! Probability customer is a no-show;
  ! No. of seats available;
    N = @SIZE(SEAT);
  ! Expected profit with no overbooking;
    EPROFIT0 = V * @SUM(SEAT(I):
     (1 - @PBN(1- Q, N, I - 1)));
  ! Expected profit if we overbook by 1 is:
    EPROFIT0 + Prob(he shows) * (V - (V + P) *
    Prob(we have no room));
    EPROFIT(1) = EPROFIT0 +
     (1 - Q) * (V - (V + P) * @PBN(Q, N, 0));
  ! In general;
    @FOR(EXTRA(I)| I #GT# 1:
    EPROFIT(I) = EPROFIT(I - 1) +
     (1 - Q) * (V - (V + P) *
      @PBN(Q, N + I - 1, I - 1));
    );
END
```

Model: OBOOKT

Black & Scholes Options Pricing Model: OPTION

A *call option* is a financial instrument that gives the holder the right to buy one share of a stock at a given price (the exercise price) on or before some specified expiration date. A frequent question is, "How much should one be willing to pay for such an option?" An exact answer to this question eluded researchers for many years until Fischer Black and Myron Scholes derived an option pricing formula in 1973. A Nobel Prize was subsequently awarded for their work in 1997. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Computing the value of an option using the Black
  Scholes formula (see "The Pricing of Options and
  Corporate Liabilities", Journal of Political
  Economy, May-June, 1973);
SETS:
! We have 27 weeks of prices P(t), LOGP(t) is log of prices;
   WEEK/1..27/: P, LOGP;
ENDSETS
DATA:
! Weekly prices of National Semiconductor;
   P = 26.375, 27.125, 28.875, 29.625, 32.250,
       35.000, 36.000, 38.625, 38.250, 40.250,
       36.250, 41.500, 38.250, 41.125, 42.250,
       41.500, 39.250, 37.500, 37.750, 42.000,
       44.000, 49.750, 42.750, 42.000, 38.625,
       41.000, 40.750;

! The current share price;
   S = 40.75;

! Time until expiration of the option, expressed
  in years;
   T = .3644;

! The exercise price at expiration;
   K = 40;

! The yearly interest rate;
   I = .163;
ENDDATA

SETS:
! We will have one less week of differences;
   WEEK1(WEEK)| &1 #LT# @SIZE(WEEK): LDIF;
ENDSETS

! Take log of each week's price;
   @FOR(WEEK: LOGP = @LOG(P));

! and the differences in the logs;
   @FOR(WEEK1(J): LDIF(J) =
    LOGP(J + 1) - LOGP(J));

! Compute the mean of the differences;
   MEAN = @SUM(WEEK1: LDIF)/ @SIZE(WEEK1);
```

```
! and the variance;
   WVAR = @SUM(WEEK1: (LDIF - MEAN)^2)/
    (@SIZE(WEEK1) - 1);

! Get the yearly variance and standard deviation;
   YVAR = 52 * WVAR;
   YSD = YVAR^.5;

! Here is the Black-Scholes option pricing formula;
   Z = ((I + YVAR/2) *
    T + @LOG(S/ K))/(YSD * T^.5);

! where VALUE is the expected value of the option;
   VALUE = S *@PSN(Z) - K *@EXP(- I * T) *
    @PSN(Z - YSD *T^.5);

! LDIF may take on negative values;
   @FOR(WEEK1: @FREE(LDIF));

! The price quoted in the Wall Street Journal for
  this option when there were 133 days left was
  $6.625;
 END
```

Model: OPTION

Binomial Options Pricing Model: OPTIONB

Compared to the Black & Scholes example above, we take a slightly different approach to options pricing in this example. We now assume a stock's return has a binomial distribution, and use dynamic programming to compute the option's value.

```
MODEL:
SETS:
! Binomial option pricing model: We assume that
a stock can either go up in value from one period
to the next with probability PUP, or down with
probability (1 - PUP).  Under this assumption,
a stock's return will be binomially distributed.
In addition, the symmetric probabilities allow
us to build a dynamic programming recursion to
determine the option's value;
! No. of periods, e.g., weeks;
   PERIOD /1..20/:;
ENDSETS

DATA:
! Current price of the stock;
   PNOW   = 40.75;
! Exercise price at option expiration;
   STRIKE = 40;
! Yearly interest rate;
   IRATE  = .163;
! Weekly variance in log of price;
   WVAR   = .005216191 ;
ENDDATA

SETS:
! Generate our state matrix for the DP.
  STATE(S, T) may be entered from STATE(S, T - 1)
  if stock lost value, or it may be entered from
  STATE(S - 1, T - 1) if stock gained;
   STATE(PERIOD, PERIOD)| &1 #LE# &2:
      PRICE,   ! There is a stock price, and...;
      VAL;     ! a value of the option;
ENDSETS

! Compute number of periods;
   LASTP = @SIZE(PERIOD);

! Get the weekly interest rate;
   (1 + WRATE) ^ 52 = (1 + IRATE);

! The weekly discount factor;
   DISF = 1/(1 + WRATE);

! Use the fact that if LOG(P) is normal with
  mean LOGM and variance WVAR, then P has
  mean EXP(LOGM + WVAR/2), solving for LOGM...;
   LOGM = @LOG(1 + WRATE) - WVAR/ 2;
```

```
! Get the log of the up factor;
   LUPF = (LOGM * LOGM + WVAR) ^ .5;

! The actual up move factor;
   UPF = @EXP(LUPF);

! and the down move factor;
   DNF = 1/ UPF;

! Probability of an up move;
   PUP =  .5 * (1 + LOGM/ LUPF);

! Initialize the price table;
   PRICE(1, 1) = PNOW;

! First the states where it goes down every period;
   @FOR(PERIOD(T) | T #GT# 1:
    PRICE(1, T) = PRICE(1, T - 1) * DNF);

! Now compute for all other states S, period T;
   @FOR(STATE(S, T)| T #GT# 1 #AND# S #GT# 1:
    PRICE(S, T) = PRICE(S - 1, T - 1) * UPF);

! Set values in the final period;
   @FOR(PERIOD(S):
    VAL(S, LASTP) =
     @SMAX(PRICE(S, LASTP) - STRIKE, 0));

! Do the dynamic programing;
   @FOR(STATE(S, T) | T #LT# LASTP:
    VAL(S, T) = DISF *
     (PUP * VAL(S + 1, T + 1) +
      (1 - PUP) * VAL(S, T + 1)));

! Finally, the value of the option now;
   VALUE = VAL(1, 1);
END
```

Model: OPTION

Bond Portfolio Optimization Model: PBOND

In certain situations, a business or individual may be faced with financial obligations over a future number of periods. In order to defease (i.e., eliminate) this future debt, the debtor can determine a minimal cost mix of current assets (e.g., cash and bonds) that can be used to cover the future stream of payments. This problem is sometimes referred to as the *cash flow matching* problem, or the *debt defeasance* problem. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
!Bond portfolio/cash matching problem. Given cash
 needs in a series of future periods, what
 collection of bonds should we buy to meet needs?;
SETS:
   BOND/A B/ :
     MATAT,   ! Maturity period;
      PRICE,   ! Price;
      CAMNT,   ! Coupon;
      BUY;     ! Amount to buy;
   PERIOD/1..15/:
      NEED,    ! Cash needed each period;
      SINVEST; !Short term investment each period;
ENDSETS
DATA:
   STRTE = .04;        ! Short term interest rate;
   MATAT = 6, 13;      ! Years to maturity;
   PRICE = .980, .965;  ! Bond purchase prices;
   CAMNT = .060, .065;  ! Bond coupon amounts;
   NEED = 10, 11, 12, 14, 15, 17, 19, 20, 22, 24,
          26, 29, 31, 33, 36; ! Cash needs;
ENDDATA
! Minimize the total investment required to
  generate the stream of future cash needs;
   MIN = LUMP;

! First period is slightly special;
   LUMP = NEED(1) + SINVEST(1) +
    @SUM(BOND: PRICE * BUY);

! For subsequent periods;
   @FOR(PERIOD(I)| I #GT# 1:
    @SUM(BOND(J)| MATAT(J) #GE# I:
     CAMNT(J) * BUY(J)) +
    @SUM(BOND(J)| MATAT(J) #EQ# I:
     BUY(J)) + (1 + STRTE) * SINVEST(I - 1) =
    NEED(I) + SINVEST(I);
   );
! Can only buy integer bonds;
   @FOR(BOND(J): @GIN(BUY(J)));
```

Model: PBOND

Simple Product-Mix Model: PC

In this example, we illustrate a simple product-mix model for deciding how many of two types of computers to produce. Note, LINGO allows you to use scalar variables and forgo the use of sets, thus allowing straightforward entry of simpler models.

```
MODEL:
  !  Total profit for the week;
      MAX = 200 * WS + 300 * NC;
  !  The total number of Wordsmiths produced is
     limited by the supply of graphics chips;
       WS <= 60;
  !  The total number of Numbercrunchers produced
      is limited by the supply of math
   coprocessors;
       NC <= 40;
  !  The total amount of memory used in all
      machines manufactured for the week can't
       exceed 120 Mb;
       WS + 2 * NC <= 120;
END
```

Model: PC

Project Management Model: PERT

In this example, we will set up a PERT model to determine the *critical path* of tasks in a project involving the roll out of a new product. For those not familiar, PERT stands for *Project Evaluation and Review Technique.* PERT is a simple, but powerful, technique developed in the 1950s to assist managers in tracking the progress of large projects. A detailed discussion of this model may be found in Chapter 2, *Using Sets*.

```
MODEL:
SETS:
   TASKS / DESIGN, FORECAST, SURVEY, PRICE,
    SCHEDULE, COSTOUT, TRAIN/: TIME, ES, LS, SLACK;

   PRED( TASKS, TASKS) /
    DESIGN,FORECAST,
    DESIGN,SURVEY,
    FORECAST,PRICE,
    FORECAST,SCHEDULE,
    SURVEY,PRICE,
    SCHEDULE,COSTOUT,
    PRICE,TRAIN,
    COSTOUT,TRAIN /;
ENDSETS

DATA:
   TIME = 10, 14, 3, 3, 7, 4, 10;
ENDDATA

@FOR( TASKS( J)| J #GT# 1:
 ES( J) = @MAX( PRED( I, J): ES( I) + TIME( I))
);

@FOR( TASKS( I)| I #LT# LTASK:
 LS( I) = @MIN( PRED( I, J): ES( J) - TIME( I));
);

@FOR( TASKS( I): SLACK( I) = LS( I) - ES( I));

ES( 1) = 0;
LTASK = @SIZE( TASKS);
LS( LTASK) = ES( LTASK);

DATA:
!Use @TABLE() to display the precedence relations set, PRED;
   @TEXT() = @TABLE( PRED);
END
```

Model: PERT

Proj. Management with Crashing Model: PERTC

In the previous example, *PERT*, it was assumed each task takes a fixed amount of time to complete. However, if we allocated additional resources to a task, it is reasonable to assume that we could complete that task in a shorter time period. This option of speeding up a task by spending more on it is referred to as *crashing*. In this next model, we incorporate crashing as an option. The goal is to meet the project's due date, while minimizing total crashing costs.

```
MODEL:
  ! A PERT/CPM model with crashing;
  ! The precedence diagram is:
  !       /FCAST\—SCHED——COSTOUT\
  !      /       \               \
  ! FIRST         \               \
  !      \         \               \
  !       \SURVEY-PRICE————————FINAL;

SETS:
  TASK/ FIRST, FCAST, SURVEY, PRICE,
        SCHED, COSTOUT, FINAL/:
              TIME,  ! Normal time for task;
              TMIN,  ! Min time at max crash;
              CCOST, ! Crash cost/unit time;
                 EF, ! Earliest finish;
              CRASH; ! Amount of crashing;

 ! Here are the precedence relations;
  PRED(TASK, TASK)/  FIRST,FCAST   FIRST,SURVEY,
   FCAST,PRICE   FCAST,SCHED   SURVEY,PRICE,
   SCHED,COSTOUT   PRICE,FINAL   COSTOUT,FINAL/;
 ENDSETS

DATA:
  TIME = 0 14 3 3 7 4 10; ! Normal times;
  TMIN = 0  8 2 1 6 3  8; ! Crash times;
  CCOST = 0 4 1 2 4 5  3; ! Cost/unit to crash;
  DUEDATE = 31;           ! Project due date;
ENDDATA

! The crashing LP model;
! Define earliest finish, each predecessor of a
  task constrains when the earliest time the task
  can be completed. The earliest the preceding
  task can be finished plus the time required for
  the task minus any time that could be reduced by
  crashing this task.;
  @FOR(PRED(I, J):
    EF(J) >= EF(I) + TIME(J) - CRASH(J)
  );
```

```
! For each task, the most it can be crashed is the
  regular time of that task minus minimum time for
  that task;
  @FOR(TASK(J):
    CRASH(J) <= TIME(J) - TMIN(J)
  );

! Meet the due date;
! This assumes that there is a single last task;
  EF(@SIZE(TASK)) <= DUEDATE;

! Minimize the sum of crash costs;
  MIN = @SUM(TASK: CCOST * CRASH);
END
```

Model: PERTC

Product-Mix with Setup Costs Model: PRODMIX

In a product-mix model, the decision is how much of a number of different products should be produced to maximize total revenue. Each product competes for a number of scarce resources. In this example, we produce six different flying machines from six different raw materials. This model also has the feature that, should we produce a given product, we incur a fixed setup cost.

```
MODEL:
SETS:
   PLANES/ ROCKET, METEOR, STREAK,
    COMET, JET, BIPLANE /:
     PROFIT, SETUP, QUANTITY, BUILD;
   RESOURCES /STEEL, COPPER, PLASTIC,
    RUBBER, GLASS, PAINT/: AVAILABLE;
   RXP(RESOURCES, PLANES): USAGE;
ENDSETS
DATA:
   PROFIT SETUP =
     30     35
     45     20
     24     60
     26     70
     24     75
     30     30;
   AVAILABLE =
    800 1160 1780 1050 1360 1240;
   USAGE =  1 4 0 4 2 0
            4 5 3 0 1 0
            0 3 8 0 1 0
            2 0 1 2 1 5
            2 4 2 2 2 4
            1 4 1 4 3 4;
ENDDATA
MAX = @SUM(PLANES: PROFIT*QUANTITY - SETUP*BUILD);

@FOR(RESOURCES(I):
 @SUM(PLANES(J):
  USAGE(I,J) * QUANTITY(J)) <= AVAILABLE(I)
);
@FOR(PLANES:
 QUANTITY <= 400 * BUILD;
  @BIN(BUILD)
);
@FOR(PLANES:
 @GIN(QUANTITY)
);
END
```

Model: PRODMIX

Scenario Portfolio Selection Model: PRTSCEN

Scenarios here refer to outcomes of events with an influence on the return of a portfolio. Examples might include an increase in interest rates, war in the Middle East, etc. In the scenario-based approach to portfolio selection, the modeler comes up with a set of scenarios, each with a certain probability of occurring over the next period. Given this set of scenarios and their probabilities, the goal is to select a portfolio that minimizes some measure of risk, while meeting a target return level. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Scenario portfolio model;
SETS:
   SCENE/1..12/: PRB, R, DVU, DVL;
   STOCKS/ ATT,  GMT,  USX/:  X;
   STXSC(SCENE, STOCKS): VE;
ENDSETS

DATA:
   TARGET = 1.15;

! Data based on original Markowitz example;
   VE =
    1.300    1.225    1.149
    1.103    1.290    1.260
    1.216    1.216    1.419
    0.954    0.728    0.922
    0.929    1.144    1.169
    1.056    1.107    0.965
    1.038    1.321    1.133
    1.089    1.305    1.732
    1.090    1.195    1.021
    1.083    1.390    1.131
    1.035    0.928    1.006
    1.176    1.715    1.908;

! All scenarios happen to be equally likely;
   PRB= .08333;
ENDDATA
! Compute expected value of ending position;
   AVG = @SUM(SCENE: PRB * R);
! Target ending value;
   AVG >= TARGET;
   @FOR(SCENE(S):
! Compute value under each scenario;
     R(S) = @SUM(STOCKS(J): VE(S, J) * X(J));
! Measure deviations from average;
      DVU(S) - DVL(S) = R(S) - AVG
   );
! Budget;
   @SUM(STOCKS: X) = 1;
```

```
! Our three measures of risk;
   [VARI] VAR = @SUM(SCENE: PRB * (DVU + DVL)^2);
   [SEMI] SEMIVAR = @SUM(SCENE: PRB * (DVL) ^2);
   [DOWN] DNRISK = @SUM(SCENE: PRB * DVL);
! Set objective to VAR, SEMIVAR, or DNRISK;
   [OBJ] MIN = VAR;
END
```

Model: PRTSCEN

Quadratic Assignment Model: QASGN

In this example, we need to assign airline flights to gates at a hub to minimize the distance traveled from gate to gate by passengers transferring between flights. This model is called the *quadratic assignment* model because we are assigning planes to gates, and a straightforward formulation would involve the use of quadratic terms in the objective. By complicating things slightly through the introduction of an additional variable (Y in this case), we are able to replace each quadratic objective term with one of the new variables. The result of this substitution is a linear model, allowing us to tackle much larger models.

```
MODEL:
!  A quadratic assignment problem:
   Given transfers between flights and distance between gates,
   assign flights to gates to minimize total transfer distance;
 SETS:
 FLIGHT/1..3/;   !  There are three flights;
 GATE/1..4/;     !  There are five gates;
 FXG(FLIGHT, GATE):   X; !Flight-gate assignment;
 GXG(GATE, GATE):     T; !Distance between gates;
 FXF(FLIGHT, FLIGHT): N; !Transfers btwn flights;
 ENDSETS
DATA:
 N =   0 30  5      ! No. transfers between flights;
      20  0  0
      30 40  0 ;
   T = 0  5 10 14   ! distance between gates;
       5  0  5 10
      10  4  0  6
      15 10  5  0 ;
 ENDDATA

 SETS:
   ! Transfer between 2 flights must be required and related to
    2 different gates. Warning: this set gets big fast.;
   TGTG(FLIGHT, GATE, FLIGHT, GATE)|
    &1 #LT# &3 #AND# ((N(&1, &3) #NE# 0) #AND#
    (T(&2, &4) #NE# 0) #OR# (N(&3, &1) #NE# 0)
      #AND# (T(&4, &2) #NE# 0)): Y;
 ENDSETS
   ! Each flight, B, must be assigned to a gate;
   @FOR(FLIGHT(B):
    @SUM(GATE(J): X(B, J)) = 1);
   ! Each gate, J, can receive at most one flight;
   @FOR(GATE(J):
    @SUM(FLIGHT(B): X(B, J)) <= 1);
   ! Force Y(B,J,C,K)=1 if B assigned to J and C
     assigned to K;
   ! Assumes the T and N matrices are nonnegative;
   @FOR(TGTG(B, J, C, K):
    Y(B, J, C, K) >= X(B, J) + X(C, K) - 1);
```

```
   ! Min the sum of transfers * distance;
   MIN = @SUM(TGTG(B, J, C, K): Y(B, J, C, K) *
    (N(B, C) * T(J, K) + N(C, B) * T(K, J)));
   ! Make the X's 0/1 (Y's will naturally be 0/1);
   @FOR(FXG: @BIN(X));
END
```

Model: QASGN

Economic Order Quantity Model: QDISCX

The classic EOQ (Economic Order Quantity) inventory model detailed in every introductory operations research text tells us the optimal order quantity of an item given its demand rate, unit holding cost, and fixed order cost. A common situation not handled by the standard EOQ model is quantity discounts. In this model, we extend the EOQ analysis by allowing for quantity discounts.

```
MODEL:
! Economic order quantity with quantity discounts;
! This model determines the optimal order quantity
  for a product that has quantity discounts;
SETS:
  ! Each order size range has;
  RANGE/1..4/:
   B,   ! An upper breakpoint;
   P,   ! A price/unit over this range;
   H,   ! A holding cost/unit over this range;
   EOQ, ! An EOQ using this ranges H and K;
   Q,   ! An optimal order qty within this range;
   AC;  ! Average cost/year using this range's Q;
ENDSETS
DATA:
  D = 40000;  ! The yearly demand;
  K = 90;     ! The fixed cost of an order;
  IRATE = .2; ! Yearly interest rate;
!The upper break points, B, and price per unit, P:
  Range:   1       2       3       4;
  B =  10000,  20000,  40000,  60000;
  P = .35225, .34525, .34175, .33825;
ENDDATA

  ! The model;
  ! Calculate holding cost, H, and EOQ for each
    range;
   @FOR(RANGE:
    H = IRATE * P;
     EOQ = (2 * K * D/ H) ^.5;
   );

  ! For the first range, the optimal order
    quantity is equal to the EOQ ...;
   Q(1) = EOQ(1)

  ! but, if the EOQ is over the first breakpoint,
    lower it;
     - (EOQ(1) - B(1) + 1) *
     (EOQ(1) #GE# B(1));
   @FOR(RANGE(J)| J #GT# 1:

 ! Similarly, for the rest of the ranges, Q = EOQ;
    Q(J) = EOQ(J) +
```

```
  ! but, if EOQ is below the lower breakpoint,
    raise it up;
     (B(J-1) - EOQ(J)) *
      (EOQ(J) #LT# B(J - 1))

  ! or if EOQ is above the upper breakpoint,
    lower it down;
       - (EOQ(J) - B(J) + 1) *
        (EOQ(J) #GE# B(J));
   );

  ! Calculate average cost per year, AC,
    for each stage;
   @FOR(RANGE: AC = P * D + H * Q/ 2 + K * D/ Q);

  ! Find the lowest average cost, ACMIN.;
   ACMIN = @MIN(RANGE: AC);

 ! Select the Q that gives the lowest AC per year;
  ! Note: TRUE = 1, FALSE = 0;
   QUSE = @SUM(RANGE: Q * (AC #EQ# ACMIN));
END
```

Model: QDISCX

An interesting feature to note in this model is the use of logical expressions as in the following:

```
QUSE = @SUM(RANGE: Q * (AC #EQ# ACMIN));
```

In this formula, we have the logical expression:

```
AC #EQ# ACMIN
```

Logical expressions will return the value 1 if they evaluate to TRUE, or 0 if they evaluate to FALSE. As you know, expressions of this nature are discontinuous, and will make it very difficult for the solver to find reliable answers to an optimization model. It turns out in this model, however, that all the variables and formulas are fixed. When a formula is fixed in value, logical expressions contained in the formula do not cause a problem.

Simple Queuing System Model: QMMC

In this simple queuing model, we use LINGO's runtime prompt feature to prompt the user for the arrival rate of service customers, the time required by a typical service call, and the number of available servers. Once you enter these values, LINGO computes various statistics about the system including using the Erlang busy function (*@PEB*) to compute the probability that a customer must wait for service.

```
MODEL:
 ! Compute statistics for a multi-server system
   with Poisson arrivals, exponential service time
   distribution.
 ! We prompt the user for he system parameters;

DATA:
   ARV_RATE = ?;
   SRV_TIME = ?;
   NO_SRVRS = ?;
 ENDDATA

! The model;
! Average no. of busy servers;
   LOAD = ARV_RATE * SRV_TIME;

! Probability a given call must wait;
   PWAIT = @PEB(LOAD, NO_SRVRS);

!Conditional expected wait, i.e., given must wait;
   WAITCND = SRV_TIME/(NO_SRVRS - LOAD);

! Unconditional expected wait;
   WAITUNC = PWAIT * WAITCND;
END
```

Model: QMMC

Minimal Cost Queuing Model: QUEUEL

The objective of this model is to choose the number of servers in a queuing system that minimizes total cost. If all servers are busy when a customer arrives, then the customer is lost. Total cost is composed of the cost of hiring our servers plus the expected cost of lost customers. The *@PEL* function is used to get the fraction of customers lost due to all servers being busy when they arrive.

```
MODEL:
! Model of a queuing system with N servers, each
  of which costs $17/hour. Arrivals occur at a
  rate of  70 per hour in a Poisson stream. Arrivals finding
  all servers busy are lost. A lost customer costs
  $35. The average time to process a customer is 5
  minutes;

! Minimize total cost =
  service costs + lost customer cost;
    [COST]  MIN = SCOST + LCOST ;

! Cost of servers;
    SCOST = 17 * N ;

! Cost of lost customers;
    LCOST = 35 * 70 * FLOST ;

! The fraction of customers lost;
    FLOST = @PEL(70 * 5 / 60 , N);
END
```

Model: QUEUEL

Steady State Queuing Model — Model: QUEUEM

A useful approach for tackling general queuing models is to use the *Rate In = Rate Out Principle* (RIRO) to derive a set of steady state equations for a queuing system. RIRO assumes a system can reach a state of equilibrium. In equilibrium, the tendency to move out of a certain state must equal the tendency to move towards that state. Given the steady state equations derived from this assumption, we can solve for the probability that a system is in a given state at any particular moment. A detailed discussion of this model may be found in Chapter 12, *Developing More Advanced Models*.

```
MODEL:
! Model of a queue with arrivals in batches.  In
  this particular example, arrivals may show up in
  batches of 1, 2, 3, or 4 units;

SETS:
! Look at enough states so that P(i) for large i
  is effectively zero, where P(i) is the steady
  state probability of i customers in the system;
   STATE/ 1..41/: P;

! Potential batch sizes are 1, 2, 3 or 4 customers,
  and A(i) = the probability that an arriving
  batch contains i customers;
   BSIZE/ 1..4/: A;
ENDSETS

DATA:
   ! Batch size distribution;
   A = .1, .2, .3, .4;
   ! Number of batches arriving per day;
   LMDA = 1.5;
   ! Number of servers;
   S    = 7;
   ! Number of customers a server can
     process per day;
   MU   = 2;
ENDDATA

! LAST = number of STATES;
   LAST = @SIZE(STATE);

! Balance equations for states where the number of
  customers in the system is less than or equal to
  the number of servers;
   @FOR(STATE(N)| N #LE# S:
    P(N) * ((N - 1)* MU + LMDA) =
     P(N + 1) * MU * N +
      LMDA * @SUM(BSIZE(I)| I #LT# N: A(I)
       * P(N - I))
   );

! Balance equations for states where number in system is
  greater than the number of servers, but less than the limit;
   @FOR(STATE(N)| N #GT# S #AND# N #LT# LAST:
```

```
    P(N) * (S * MU + LMDA) =
     P(N + 1) * MU * S +
      LMDA * @SUM(BSIZE(I)| I #LT# N: A(I) *
       P(N - I))
   );

! Probabilities must sum to 1;
   @SUM(STATE: P) = 1;
END
```

Model: QUEUEM

Designing a Computer Cabinet — Model: BOX

In this example, we create a nonlinear optimization model to design the case for a computer.

```
MODEL:
! Design a box at minimum cost that meets area,
  volume, marketing and aesthetic requirements;
 [COST] min = 2*(.05*(d*w + d*h) +.1*w*h);
 [SURFACE]  2*(h*d + h*w + d*w) >= 888;
 [VOLUME]   h*d*w >= 1512;

! These two enforce aesthetics;
 [NOTNARRO] h/w <= .718;
 [NOTHIGH]  h/w >= .518;

! Marketing requires a small footprint;
 [FOOTPRNT] d*w <= 252;
END
```

Model: BOX

Linear Regression Model: REGRES

Linear Regression is a forecasting technique used to predict the value of one variable (called the dependent variable) based upon the value of one or more other variables (the independent variables).

Our example is a simple linear regression model with one independent variable. The data is fit to a linear equation of the form:

$$Y(i) = CONS + SLOPE * X(i)$$

where Y is the dependent variable, X is the independent variable, $CONS$ is the value of Y when $X = 0$, and $SLOPE$ is the rate of change in Y with a unit change in X.

For our example, the dependent variable, Y, is the number of annual road casualties and the independent variable, X, is the number of licensed vehicles. We have 11 years of data.

```
MODEL:
! Linear Regression with one independent variable;
! Linear regression is a forecasting method that
  models the relationship between a dependent
  variable to one or more independent variable.
  For this model we wish to predict Y with the equation:
     Y(i) = CONS + SLOPE * X(i);

 SETS:
    ! The OBS set contains the data points for
      X and Y;
     OBS/1..11/:
      Y, ! The dependent variable (annual road
           casualties);
      X; ! The independent or explanatory variable
           (annual licensed vehicles;
    ! The OUT set contains model output.;
     OUT/ CONS, SLOPE, RSQRU, RSQRA/: R;
 ENDSETS

 ! Our data on yearly road casualties vs. licensed
   vehicles, was taken from Johnston, Econometric
   Methods;

 DATA:
  Y = 166 153 177 201 216 208 227 238 268 268 274;
  X = 352 373 411 441 462 490 529 577 641 692 743;
 ENDDATA

 SETS:
    ! The derived set OBS contains the mean
      shifted values of the independent and
      dependent  variables;
     OBSN(OBS): XS, YS;
 ENDSETS
```

```
! Number of observations;
 NK = @SIZE(OBS);

! Compute means;
 XBAR = @SUM(OBS: X)/ NK;
 YBAR = @SUM(OBS: Y)/ NK;

! Shift the observations by their means;
 @FOR(OBS(I):
 XS(I) = X(I) - XBAR;
 YS(I) = Y(I) - YBAR);

! Compute various sums of squares;
 XYBAR = @SUM(OBSN: XS * YS);
 XXBAR = @SUM(OBSN: XS * XS);
 YYBAR = @SUM(OBSN: YS * YS);

! Finally, the regression equation;
 R(@INDEX(SLOPE)) = XYBAR/ XXBAR;
 R(@INDEX(CONS)) = YBAR - R(@INDEX(SLOPE)) * XBAR;
 RESID= @SUM(OBSN:(YS - R(@INDEX(SLOPE)) * XS)^2);

! A measure of how well X can be used to predict Y
  - the unadjusted (RSQRU) and adjusted (RSQRA)
  fractions of variance explained;
 R(@INDEX(RSQRU)) = 1 - RESID/ YYBAR;
 R(@INDEX(RSQRA)) = 1 - (RESID/ YYBAR) *
  (NK - 1)/(NK - 2);

! XS and YS may take on negative values;
 @FOR(OBSN: @FREE(XS); @FREE(YS));
END
```

Model: REGRES

Acceptance Sampling I — Model: SAMPLE

In this example, we have a lot of 400 items. We take a sample of 100 items from the lot. We accept the entire lot as being good if the sample has 2 or less defective items.

We use the hypergeometric distribution (*@PHG*) to determine the exact producer risk (probability of rejecting a good lot), and the exact consumer risk (probability of accepting a bad lot). In the days before computers were widely available, statisticians had to rely on published tables of the probability distributions to compute probabilities such as these. Because the hypergeometric distribution is specified by *four* parameters, it would have been unrealistic to carry around hypergeometric tables that covered all possible scenarios. Instead, statisticians routinely used distributions of fewer parameters to approximate the hypergeometric. So, in deference to the good old days, we make use of the binomial, Poisson, and normal approximations to the hypergeometric to compute these same risk probabilities. The interested reader can compare the accuracy of the various approximations.

```
MODEL:
! Acceptance sampling: taking one or more samples at random
   from a lot, inspecting each of the items in the sample(s),
   and deciding on the basis of inspection results whether to
   accept or reject the entire lot. This Acceptance Sampling
   model illustrates the effect choice of distribution.;
 ! From a lot of 400 items;
  LOTSIZE = 400;

 ! We take a sample of size 100;
  SAMPSIZE = 100;

 ! Producer considers the lot good if
    the lot fraction defective is .0075 or less;
  FGOOD = .0075;

 ! Consumer considers the lot bad if
    the lot fraction defective is .025 or more;
  FBAD = .025;

 ! We accept the lot if sample contains 2 or less;
  ACCEPTAT = 2;

! The model;
 ! What is producer risk of rejecting a good lot;
   !Using the (exact) hypergeometric distribution;
  PGOODH = 1 - @PHG(LOTSIZE, LOTSIZE * FGOOD,
   SAMPSIZE, ACCEPTAT);

   ! Using binomial approx. to the hypergeometric;
  PGOODB = 1 - @PBN(FGOOD, SAMPSIZE, ACCEPTAT);

   ! Using the Poisson approx. to the binomial;
  PGOODP = 1 - @PPS(FGOOD * SAMPSIZE, ACCEPTAT);

   ! Using Normal approximation;
  PGOODN =
   1 - @PSN((ACCEPTAT + .5 - MUG) / SIGMAG);

   ! where;
```

```
  MUG = SAMPSIZE * FGOOD;
  SIGMAG = (MUG * (1 - FGOOD)) ^ .5;

!What is the consumer risk of accepting a bad lot;
! Using the hypergeometric;
  PBADH = @PHG(LOTSIZE, LOTSIZE * FBAD,
   SAMPSIZE, ACCEPTAT);

   ! Binomial;
  PBADB = @PBN(FBAD, SAMPSIZE, ACCEPTAT);

   ! Poisson;
  PBADP = @PPS(FBAD * SAMPSIZE, ACCEPTAT);

   ! Using Normal approximation;
  PBADN = @PSN((ACCEPTAT + .5 - MUB) / SIGMAB);

  ! where;
  MUB = SAMPSIZE * FBAD;
  SIGMAB = (MUB * (1 - FBAD)) ^ .5;
END
```

Model: SAMPLE

Stratified Sampling Design — Model: SAMPLE2

In this model, we want to come up with a sampling strategy that yields a variance within a specified target at minimal cost. We have four strata of a population that we will be querying on two topics. There is a maximum variance limit on the two questions. We know the variance in responses for each stratum on each question. How many respondents must you select from each stratum to meet your maximal variance requirements at minimal cost?

```
MODEL:
!  Stratified sampling plan design, taken from Bracken and McCormick.
   Minimize the cost of sampling from 4 strata, subject to
   constraints on the variances of the sample based estimates of two
   categories;

 SETS:
  STRATUM/1..4/: SIZE, POP, COST, WEIGHT;
  CATEGORY/1..2/: VARMAX, K2;
  SXC(STRATUM, CATEGORY): VAR, K1;
 ENDSETS

!  POP = population of each stratum. COST = cost of sampling in each.
   VARMAX = variance limits. VAR = variance for each category in each
   stratum. CFIX = a fixed cost;

 DATA:
  POP = 400000, 300000, 200000, 100000;
  COST =     1,      1,      1,      1;
  VARMAX = .043, .014;
  VAR  =    25    1
            25    4
            25    16
            25    64;
  CFIX = 1;
 ENDDATA

 [OBJ] MIN = CFIX + @SUM(STRATUM: SIZE * COST);
! Compute some parameters;
 TOTP =  @SUM(STRATUM(I): POP(I));
 @FOR(STRATUM(I):

! Weight given each stratum;
  WEIGHT(I) = POP(I)/TOTP;
  @GIN(SIZE(I));
 );

 @FOR(CATEGORY(J):
  K2(J) =
   @SUM(STRATUM(I): VAR(I, J)^2 *
    WEIGHT(I)/ POP(I));
 );

 @FOR(SXC(I, J):
  K1(I, J) =  VAR(I, J)^2* WEIGHT(I)^2;
 );
```

```
 @FOR(CATEGORY(J):
  @SUM(STRATUM(I): K1(I, J) / SIZE(I))
   - K2(J) <= VARMAX(J)
 );

@FOR(STRATUM(I):
  @BND(0.0001, SIZE(I), POP(I) -1);
 );
END
```

Model: SAMPLE2

Acceptance Sampling II — Model: SAMSIZ

We are sampling items from a large lot. If the number of defectives in the lot is 3% or less, the lot is considered "good". If the defects exceed 8%, the lot is considered "bad". We want a producer risk (probability of rejecting a good lot) below 9% and a consumer risk (probability of accepting a bad lot) below 5%. We need to determine *N* and *C*, where *N* is the minimal sample size, and *C* is the critical level of defects such that, if defects observed in the sample are less-than-or-equal-to *C,* we accept the lot.

```
MODEL:
!  Acceptance sampling design. From a large lot, take a sample
   of size N, accept if C or less are defective;
! Poisson approximation to number defective is used;

DATA:
  AQL  = .03;    ! "Good" lot fraction defective;
  LTFD = .08;    ! "Bad"  lot fraction defective;
  PRDRISK = .09; ! Tolerance for rejecting good lot;
  CONRISK = .05; ! Tolerance for accepting bad lot;
  MINSMP = 125;  ! Lower bound on sample size to help solver;
ENDDATA

  [OBJ] MIN = N;
! Tolerance for rejecting a good lot;
   1 - @PPS(N * AQL, C) <= PRDRISK;

! Tolerance for accepting a bad lot;
   @PPS(N * LTFD, C) <= CONRISK;

! Give solver some help in getting into range;
   N >= MINSMP; C>1;

! Make variables general integer;
   @GIN(N);  @GIN(C);
END
```

Model: SAMSIZ

Seasonal Sales Forecasting Model: SHADES

We have quarterly observations of sales for the last two years. We would like to estimate a base, trend, and seasonal factors to form a sales forecasting function that minimizes the sum of squared prediction errors when applied to the historical sales. A detailed discussion of this model may be found in Chapter 3, *Using Variable Domain Functions*.

```
MODEL:
SETS:
   PERIODS /1..8/: OBSERVED, PREDICT,
    ERROR;
   QUARTERS /1..4/: SEASFAC;
ENDSETS

DATA:
   OBSERVED = 10 14 12 19 14 21 19 26;
ENDDATA

MIN = @SUM(PERIODS: ERROR ^ 2);

@FOR(PERIODS: ERROR = PREDICT - OBSERVED);

@FOR(PERIODS(P): PREDICT(P) = SEASFAC(@WRAP(P, 4))
  * (BASE + P * TREND));

@SUM(QUARTERS: SEASFAC) = 4;

@FOR(PERIODS: @FREE(ERROR));

END
```

Model: SHADES

Exponential Smoothing Model: SIMXPO

Exponential smoothing is a technique that is relatively easy to implement, and yet has proven to be an effective tool at forecasting sales. In its simplest form, this technique assumes, on average, sales are constant, but include a random error term about the average. It is also assumed the underlying average can drift from period to period. These assumptions lead to the following smoothing equation:

$$S_t = \alpha X_t + (1 - \alpha) S_{t-1}$$

where,

S_t = predicted sales, or signal, in period t,
X_t = observed sales in period t, and
α = a constant term between 0 and 1.

From this equation, we can see the closer α is to 1, the more the current observation affects our signal and, subsequently, the less "memory" our equation has. Frequently, a value for α is chosen in the range of .01 to .3. In this example, we will solve for an α that minimizes the sum of the squared prediction errors.

For more information on exponential smoothing, see Winston (1995).

```
MODEL:
SETS:
   PERIODS /1..8/: OBSERVED, ERROR, PREDICT;
ENDSETS

DATA:
! The degree of the objective. N may be changed
   to 1 to minimize absolute deviation;
   N = 2;
! The observed values of the time series;
   OBSERVED = 10 14 12 19 14 21 19 26;
ENDDATA

! Force Period 1 prediction to 10;
  PREDICT(1) = 10;

! The objective function;
   [OBJ] MIN= @SUM(PERIODS: @ABS(ERROR) ^ N);

! Calculate the forecasts;
   @FOR(PERIODS(T) | T #GT# 1:
    PREDICT(T) = ALPHA * OBSERVED(T - 1) +
    (1 - ALPHA) * PREDICT(T - 1));

! Calculate forecast errors;
   @FOR(PERIODS: ERROR = PREDICT - OBSERVED);

! Error terms may be negative as well as positive;
   @FOR(PERIODS: @FREE(ERROR));
```

```
! Exclude meaningless Alphas of zero or one;
    @BND(.01, ALPHA,.9999);
END
```

Model: SIMXPO

Placing Songs on a Cassette Tape Model: SONGS

In this model, we have seven songs, each with a different length, that must be placed on a cassette tape. The goal is to maximize the number of songs on one side of the tape without exceeding half of the total time of the music on the other side.

```
MODEL:
SETS:
  SONG/1..7/: LENGTH, Y;
ENDSETS

  ! Maximize number of songs on short side;
   MAX = @SUM(SONG: Y);

  ! It must contain at most half the music;
   @SUM(SONG: LENGTH * Y) <= HALF;

  ! Compute half the length;
   HALF = @SUM(SONG: LENGTH)/ 2;

  ! We want the Y's to be 0/1;
   @FOR(SONG: @BIN(Y));

 DATA:
   LENGTH = 7, 5, 2, 2, 2, 2, 2;
 ENDDATA
END
```

Model: SONGS

Computing Sort Order Model: SORTIN

This simple model sorts cities according to their distance from the equator.

```
MODEL:
! Compute sort order ;
SETS:
CITY:      ! Some cities;
  LAT,     ! Their latitudes;
  RANKLT,  ! Compute rank in distance from equator;
  RDRLIST; ! Store in this ordered list;
ENDSETS
DATA:
 CITY = BEIJING LONDON   PARIS    NYC    LA MOSCOW  TOKYO;
  LAT =   39.6    51.3    48.5   40.4  34.1   55.5   35.4;
ENDDATA
CALC:
  ! Minimize output;
  @SET( 'TERSEO', 2);
  ! Compute rank of each city;
  RANKLT = @RANK( LAT);
  ! Put the original indices in order in a list;
  @FOR( CITY(i):
    RDRLIST( RANKLT(i)) = i;
  );
  !Display them;
  @WRITE(' The cities from closest to farthest from equator:',
   @NEWLINE(1));
  @WRITE(' Latitude    City', @NEWLINE(1));
  @FOR( CITY(i):
      @WRITE('   ', LAT(RDRLIST(i)),'       ', CITY(RDRLIST(i)),
   @NEWLINE(1));
      );

ENDCALC
END
```

Model: SORTIN

Traveling Salesman Problem — Model: TSP

In the traveling salesman problem (TSP), we have a network of cities connected by roads. We need to find a tour that visits each city exactly once, minimizing the total distance traveled.

As it turns, large TSP models are difficult to solve using optimization, and are best approached using some form of heuristic (see Lin and Kernighan, 1973). The problem lies in the fact that solutions to large models tend to contain *subtours*. A subtour is a tour of a subset of cities unconnected to the main tour. One can add constraints to break the subtours, but the number of constraints required grows dramatically as the number of cities increase.

```
MODEL:
 ! Traveling Salesman Problem for the cities of
 Atlanta, Chicago, Cincinnati, Houston, LA,
 Montreal;

 SETS:
  CITY / 1.. 6/: U; ! U(I) = sequence no. of city;
  LINK(CITY, CITY):
       DIST,  ! The distance matrix;
          X;  ! X(I, J) = 1 if we use link I, J;
 ENDSETS

DATA: !Distance matrix, it need not be symmetric;
  DIST =   0  702  454  842 2396 1196
         702    0  324 1093 2136  764
         454  324    0 1137 2180  798
         842 1093 1137    0 1616 1857
        2396 2136 2180 1616    0 2900
        1196  764  798 1857 2900    0;
 ENDDATA

 !The model:Ref. Desrochers & Laporte, OR Letters,
  Feb. 91;
  N = @SIZE(CITY);
  MIN = @SUM(LINK: DIST * X);
  @FOR(CITY(K):

  !  It must be entered;
   @SUM(CITY(I)| I #NE# K: X(I, K)) = 1;

  !  It must be departed;
   @SUM(CITY(J)| J #NE# K: X(K, J)) = 1;

  !Weak form of the subtour breaking constraints;
  !These are not very powerful for large problems;
   @FOR(CITY(J)| J #GT# 1 #AND# J #NE# K:
       U(J) >= U(K) + X (K, J) -
       (N - 2) * (1 - X(K, J)) +
       (N - 3) * X(J, K)
   );
  );

  ! Make the X's 0/1;
  @FOR(LINK: @BIN(X));
```

```
  ! For the first and last stop we know...;
  @FOR(CITY(K)| K #GT# 1:
   U(K) <= N - 1 - (N - 2) * X(1, K);
   U(K) >= 1  + (N - 2) * X(K, 1)
  );
END
```

Model: TSP

The Log Gamma Function Model: EZCOUNT

The factorial function is used in many probability computations. Unfortunately, the factorial function can generate some very large numbers that can exceed the fixed word size of most computers. A common way around this is to use the *Log Gamma function* (*@LGM*), which returns the logarithm of the factorial function. In the following model, we use *@LGM* to compute the number of possible poker hands.

```
MODEL:
! This model computes the number of ways of
  selecting 5 objects from a set of 52 objects;

! This is expressed by 52! / (5! * 47!).  The
  actual computation uses the log-gamma function;
       WAYS = @EXP(@LGM(53) - @LGM(6) - @LGM(48));

! Note that the arguments of the @LGM functions
  are one greater than the corresponding arguments
  of the factorial functions, due to the
  definition of the Gamma function;
END
```

Model: EZCOUNT

Vehicle Routing Problem Model: VROUTE

The *vehicle routing problem* occurs in many service systems such as delivery, customer pick-up, repair and maintenance. A fleet of vehicles, each with fixed capacity, starts at a common depot and returns to the depot after visiting locations where service is demanded. The objective is to minimize the total distance of all the routes.

In general, it takes much longer to find the best routes as the number of locations grow. Large versions of this model may have to be tackled using some form of heuristics.

In this particular example, we are delivering one product to seven cities with the depot at city 1.

```
MODEL:
! The Vehicle Routing Problem (VRP) occurs in many service systems
such as delivery, customer pick-up, repair and maintenance. A fleet
of vehicles, each with fixed capacity, starts at a common depot and
returns to the depot after visiting locations where service is
demanded. The objective is to minimize total routes.

In general it takes much longer time to find the best routes when the
number of locations become larger.

This model involves delivering the required amount of goods to seven
cities with depot at city 1 ;

SETS:
! Q(I) is the amount required at city I
    U(I) is the accumulated delivers at city I ;
   CITY/1..8/: Q, U;
! DIST(I,J) is the distance from city I to city J
    X(I,J) is 0-1 variable: It is 1 if some vehicle travels from
                            city I to J,  0 if none ;
   CXC(CITY, CITY): DIST, X;
ENDSETS

DATA:
! city 1 represent the common depot, i.e. Q(1) = 0;
   Q  =  0    6    3    7    7   18    4    5;

! distance from city I to city J is same from city J to city I
distance from city I to the depot is 0, since the vehicle has to
return to the depot ;
     DIST =  ! To City;
    ! Chi   Den  Frsn  Hous    KC    LA  Oakl  Anah   From;
        0   996  2162  1067   499  2054  2134  2050!   Chicago;
        0     0  1167  1019   596  1059  1227  1055!   Denver;
        0  1167     0  1747  1723   214   168   250!   Fresno;
        0  1019  1747     0   710  1538  1904  1528!   Houston;
        0   596  1723   710     0  1589  1827  1579!   K. City;
        0  1059   214  1538  1589     0   371    36!   L. A.;
        0  1227   168  1904  1827   371     0   407!   Oakland;
        0  1055   250  1528  1579    36   407     0;!   Anaheim;
! VCAP is the capacity of a vehicle ;
     VCAP = 18;
```

```
ENDDATA
! The objective is to minimize total travel distance;
   MIN = @SUM(CXC: DIST * X);
! for each city, except depot....;
   @FOR(CITY(K)| K #GT# 1:
! a vehicle does not traval inside itself,...;
        X(K, K) = 0;
! a vehicle must enter it,... ;
        @SUM(CITY(I)| I #NE# K #AND# (I #EQ# 1 #OR#
              Q(I) + Q(K) #LE# VCAP): X(I, K)) = 1;
! a vehicle must leave it after service ;
        @SUM(CITY(J)| J #NE# K #AND# (J #EQ# 1 #OR#
              Q(J) + Q(K) #LE# VCAP): X(K, J)) = 1;
! U(K) is at least amount needed at K but can't exceed capacity;
        @BND(Q(K), U(K), VCAP);
! If K follows I, then can bound U(K) - U(I);
        @FOR(CITY(I)| I #NE# K #AND# I #NE# 1: U(K) >=
         U(I) + Q(K) - VCAP + VCAP*(X(K, I) + X(I, K))
         - (Q(K) + Q(I)) * X(K, I);
        ) ;
! If K is 1st stop, then U(K) = Q(K);
         U(K) <= VCAP - (VCAP - Q(K)) * X(1, K);
! If K is not 1st stop...;
         U(K)>= Q(K)+ @SUM(CITY(I)| I #GT# 1: Q(I) * X(I, K));
   );
! Make the X's binary;
   @FOR(CXC(I, J): @BIN(X(I, J)) ;
   );
! Minimum no. vehicles required, fractional and rounded;
   VEHCLF = @SUM(CITY(I)| I #GT# 1: Q(I))/ VCAP;
   VEHCLR = VEHCLF + 1.999 - @WRAP(VEHCLF - .001, 1);

! Must send enough vehicles out of depot;
   @SUM(CITY(J)| J #GT# 1: X(1, J)) >= VEHCLR;
END
```

Model: VROUTE

Home Mortgage Calculation Model: WHATIF

This example models a home mortgage. The user is prompted for the monthly payment, the length of the mortgage, and the interest rate. The model then solves for the value of the home that can be purchased with the mortgage.

```
MODEL:
! A model of a home mortgage(WHATIF.LNG);
! The user is prompted for values for the
 payment, years, and interest rate. The
 face value of the mortgage (LUMP) is
 solved for.;

DATA:
! User is prompted for these:
   PAYMENT  = ?; ! Monthly payment;
   YEARS    = ?; ! No. of years;
   YRATE    = ?; ! Interest rate;
 ENDDATA

! Relate no. of months to no. of years;
   MONTHS = YEARS * 12;

! Relate monthly interest rate to yearly rate;
   (1 + MRATE) ^ 12  =  1 + YRATE;

! Relate lump sum to monthly payment, monthly
  interest rate, and no. of months;
   LUMP = PAYMENT * @FPA(MRATE, MONTHS);

END
```

Model: WHATIF

Transportation Problem — Model: WIDGETS

In this example, we want to ship a product from warehouses to vendors at minimal cost. An in-depth description of this model can be found in Chapter 1, *Getting Started with LINGO*.

```
MODEL:
! A 6 Warehouse 8 Vendor Transportation Problem;
SETS:
   WAREHOUSES / WH1 WH2 WH3 WH4 WH5 WH6/: CAPACITY;
   VENDORS / V1 V2 V3 V4 V5 V6 V7 V8/   : DEMAND;
   LINKS(WAREHOUSES, VENDORS): COST, VOLUME;
ENDSETS

! The objective;
   MIN = @SUM(LINKS(I, J):
    COST(I, J) * VOLUME(I, J));

! The demand constraints;
   @FOR(VENDORS(J):
    @SUM(WAREHOUSES(I): VOLUME(I, J)) =
     DEMAND(J));

! The capacity constraints;
   @FOR(WAREHOUSES(I):
    @SUM(VENDORS(J): VOLUME(I, J)) <=
     CAPACITY(I));

! Here is the data;
DATA:
   CAPACITY = 60 55 51 43 41 52;
   DEMAND = 35 37 22 32 41 32 43 38;
   COST = 6 2 6 7 4 2 5 9
          4 9 5 3 8 5 8 2
          5 2 1 9 7 4 3 3
          7 6 7 3 9 2 7 1
          2 3 9 5 7 2 6 5
          5 5 2 2 8 1 4 3;
ENDDATA
END
```

Model: WIDGETS

Appendix B: Error Messages

Listed below by code number are the error messages you may encounter when using LINGO. Suggestions for overcoming the errors are also included.

0. THE MODEL GENERATOR RAN OUT OF MEMORY.

LINGO's model generator ran out of working memory. The model generator converts the text of your LINGO model to an expanded form suitable for an appropriate solver engine. On most platforms, you can increase the amount of working memory allocated to the model generator. Note that memory set aside for LINGO's model generator will not be available to LINGO's various solver engines. Given this, you should not allocate an overly excessive amount of memory to the generator.

To increase the memory allocation for the model generator, select the *Solver|Options* command, then the *General* tab, and in the *Generator Memory Limit* box increase the amount of working memory. Press the *Save* button and then restart LINGO. You can verify the new memory allotment by issuing the *Help|About LINGO* command.

1. TOO MANY LINES OF TEXT IN THE MODEL.

There are too many lines in the model's text. For all practical purposes, the limit on total lines of text is large enough that this error message should never be encountered.

2. TOO MANY CHARACTERS IN THE MODEL TEXT.

There are too many characters in the model's text. For all practical purposes, the limit on the number of characters is large enough that this message should never be encountered.

3. OVERLENGTH LINE, CHARACTERS MAY HAVE BEEN LOST OFF END. USE CARRIAGE RETURN TO BREAK UP OVER SEVERAL INPUT LINES.

Input lines are limited to 200 characters. You will need to break up long input lines into shorter ones.

4. VALID LINES ARE 1 TO *N*. TYPE 'ALL' TO REFERENCE ALL LINES.

The *LOOK* command expects a range of row numbers. If the range is invalid, you will get this message. Enter a new range with valid numbers.

5. THERE IS NO CURRENT MODEL.

Any command that makes sense only if there is a current model in memory will print this message if invoked without the presence of a model. You need to load a model with the *File|Open* command in Windows or the *TAKE* command on other platforms, or enter a new model with the *File|New* command in Windows or the *MODEL* command on other platforms.

6. TOO MANY NESTED TAKE COMMANDS.
You have exceeded LINGO's limit of ten nested *TAKE* commands within a command script. If possible, try combining some commands into a single file.

7. UNABLE TO OPEN FILE: *FILENAME*.
The file you tried to read doesn't exist, or you misspelled its name. Try opening the file again.

8. TOO MANY CONSECUTIVE COMMAND ERRORS. REVERT TO TERMINAL INPUT.
LINGO prints this message after having encountered a number of consecutive errors in a command script. LINGO assumes that something has gone seriously awry, closes the script file, and returns you to command level.

9. NOT USED.

10. NOT USED.

11. INVALID INPUT. A SYNTAX ERROR HAS OCCURRED.
This is the generic error issued by the LINGO compiler when it detects a syntax error. In Windows, when you close the error box, the cursor will be on the line where the error occurred. Other versions of LINGO will try print out the general area where the error has occurred, but LINGO cannot always pinpoint the exact line. Examine this area for any obvious syntax errors. If you are unable to find any obvious errors, a useful technique is to comment out small sections of the model until the error goes away. This should give you a good idea of exactly where the error is occurring.

Syntax errors may also occur if you are not invoking the correct compiler in LINGO. Most users will choose to build models using the native LINGO syntax, however, some users may prefer building their models using LINDO syntax. LINGO can compile models written in either native LINGO syntax or LINDO syntax. LINGO chooses the compiler based on a model's file extension. LINGO models must have an extension of lg4 (the default) or lng. LINDO models must have an ltx extension. The default model extension may be set by clicking on: LINGO | Options | Interface | File Format. Each model window's title bar displays whether it is a LINGO or LINDO model.

12. MISSING RIGHT PARENTHESIS.
A LINGO expression is missing at least one closing right parenthesis. LINGO will point to the end of the expression where the error occurred. Count the number of parentheses in this expression to verify if you have input the correct number, or, in Window use the *Edit|Match Parenthesis* command to find the unmatched parenthesis.

13. A SPECIFIED SPREADSHEET RANGE WAS NOT FOUND. *RANGE_NAME*.
You specified a spreadsheet range titled *RANGE_NAME* that LINGO was unable to find. Check the spelling of the range name and be sure that the range is defined in the spreadsheet.

14. NOT ENOUGH TEMPORARY OPERATOR STACK SPACE.
LINGO uses a stack to temporarily hold operators and prefix functions during compilation of an expression. It is possible, though unlikely, that this stack will overflow. If so, try breaking up lengthy expressions, or adding parentheses to offending expressions.

15. NO RELATIONAL OPERATOR FOUND.
Each LINGO expression (with the exception of variable domain expressions) must contain one relational operator (e.g., =, <, >). Check to be sure that all expressions contain relational operators.

16. ALL MODEL OBJECTS MUST HAVE THE SAME PARENT SET FOR THIS OPERATION.
You have attempted to use an import or export function in the data section that involves two or more attributes from different parent sets. Break the function into multiple calls, so each instance refers to attributes belonging to the same parent set.

17. NOT ENOUGH INDEX STACK SPACE.
LINGO uses a stack to keep track of indices referenced by set operators. It is possible, though unlikely, that this stack will overflow. The only way to remedy this situation is to rewrite the given expression, so it uses fewer indices.

18. OUT OF SET STACK SPACE.
LINGO uses a stack to keep track of sets referenced by set operators. This stack may overflow, though it's unlikely. The only way to remedy this situation is to rewrite the given expression, so the maximum number of pending set operators is reduced.

19. INVALID USE OF @INDEX FUNCTION.
The *@INDEX* function expects an optional set name followed by a mandatory set number. Check to see that your arguments comply with these restrictions.

20. IMPROPER USE OF SET NAME.
A set name has been used in an improper manner. For instance, you may have attempted to set the name equal to a quantity. Check your model's syntax.

21. IMPROPER USE OF ATTRIBUTE NAME.
This message is printed if an attribute name is used incorrectly. For instance, you may have attempted to use it as a scalar (single value) variable. Check your model's syntax.

22. TOO MANY INEQUALITY OR EQUALITY RELATIONS.
A constraint may have only one relational operator (i.e., =, <, or >). A two-sided constraint such as 2 < X < 4 is not permitted. Instead, write it as two one sided constraints: X > 2; and X < 4;.

23. IMPROPER NUMBER OF ARGUMENTS.
LINGO's predefined functions generally expect a specific number of arguments. You'll get this message if you are passing an incorrect number of arguments. Check the syntax of the function in question.

24. INVALID SET NAME.
If LINGO was expecting a set name and didn't find it, you will get this message. Note that all sets must be defined in a sets section before they are referenced in a model expression.

25. NOT USED.

26. IMPROPER NUMBER OF INDEX VARIABLES.
When using indices in conjunction with a set in a set operator function, LINGO checks to be sure that you have specified the correct number of indices for the set. If not, it prints this message.

27. THE FOLLOWING SPREADSHEET RANGE IS DISCONTINUOUS: *RANGE_NAME*.
At present, LINGO only supports continuous ranges. Continuous ranges are simple, rectangular ranges. Discontinuous ranges are unions of two, or more, continuous ranges. You will need to break your discontinuous range up into a set of equivalent, continuous ranges.

28. INVALID USE OF A ROW NAME.
A row name may be input within brackets at the start of a constraint and may contain up to 32 characters. This error message indicates some improper use. Check your model's syntax.

29. INVALID NUMBER OF INITIALIZATION VALUES IN A DATA/INIT/CALC SECTION. THE REQUIRED NUMBER OF VALUES IS: *N*
When assigning values to an array, you must assign a value to every element in data, init and calc statements. LINGO keeps track of the number of values you specified and checks this against the length of each array being assigned. If the two numbers don't agree, LINGO prints this message along with the number of values that are required.

30. A GENERAL FAILURE OCCURRED WHILE ATTEMPTING A LINK TO EXCEL.
LINGO attempted to open an OLE link to Excel, but was unsuccessful. Be sure that Excel is installed on your machine (version 5, or later). Also, if your machine is busy, this error message can occur because it is taking an excessive amount of time to load Excel. In which case, simply retry the action when the machine is less busy.

31. INVALID ARGUMENT LIST FOR AN @TEXT FUNCTION CALL.
An instance of the *@TEXT* function has an invalid argument list. Check the documentation on this function for the correct syntax.

32. ATTEMPT TO IMPORT A BLANK SET MEMBER NAME.
You have attempted to import a set member name that is entirely blank. All set member names must conform to standard LINGO naming conventions and may not be blank. Assign a nonblank name to the set member and retry.

33. INVALID ARGUMENT LIST FOR AN @OLE FUNCTION CALL.
Check the documentation on the *@OLE* function to determine the correct syntax.

34. RANGE LENGTHS MUST BE IDENTICAL WHEN IMPORTING DERIVED SETS.
You are importing a derived set from multiple ranges of varying lengths. When using multiple ranges, each range must contain the same number of cells. Adjust the sizes of the ranges so they agree and retry.

35. UNRECOGNIZED NAME IN AN OUTPUT FUNCTION.
You have attempted to use an output function (e.g., *@OLE* or *@ODBC*) to export the value of a variable that does not exist. Check your output functions to see if all the variables exist and that they are spelled correctly.

36. ATTEMPT TO IMPORT INVALID PRIMITIVE SET ELEMENT NAME: *NAME*.
A primitive set name imported from an external source is invalid. Make sure the name conforms to normal LINGO naming standards.

37. NAME ALREADY IN USE: *NAME*. CHOOSE A DIFFERENT NAME.
A model can't use duplicate names. Select a new, unique name.

38. THE ODBC SERVER RETURNED THE FOLLOWING ERROR MESSAGE: *MESSAGE-TEXT*.
The ODBC server encountered an error. The text of the message will be displayed. In many cases, the text of the message should help clarify the problem. Unfortunately, some ODBC servers will not return an explanatory message for all error conditions, in which case, you may need to experiment with the format of your data to determine the cause of the problem.

39. THE FOLLOWING SET ELEMENT WAS NOT DERIVED FROM ITS PARENT PRIMITIVE SET: *SET-ELEMENT*.
When defining a sparse derived set, LINGO checks each set element against the parent set. If the element is not found in the parent set, you will get this message. Please check the spelling and ordering of the set elements in the sparse derived set you are defining.

40. INVALID NUMBER OF DERIVED SET INDEX ELEMENTS.
If a sparse derived set is formed from, say, three primitive sets, then there must be a multiple of three primitive set element names in the explicit definition of the derived set. LINGO checks this, and issues this message if there is not a match.

41. OUT OF ATTRIBUTE INDEX SPACE.
LINGO imposes a limit on the total number of primitive sets used in defining derived sets. The current limit is quite large and should not pose a problem.

42. EXPLICIT VARIABLE IN A SET CONDITION FOR SET: *SET_NAME*.
When using a conditional expression to define the members of a derived set, you cannot reference a variable that has not been fixed in a previous data statement. LINGO must be able to completely evaluate these conditional expressions during compilation so it knows the size of the set.

43. EXECUTOR ERROR IN SET CONDITION FOR SET: *SET_NAME*.
LINGO prints this message if an arithmetic error occurred when it was trying to evaluate a conditional expression used to define the members of a derived set. Check all arithmetic operations in the set definition in question.

44. UNTERMINATED CONDITION.
Each conditional expression placed on a set operator must be terminated with a colon (:). LINGO prints this message if you fail to add one.

45. INVALID ARGUMENT LIST FOR AN @ODBC FUNCTION CALL.
You have a syntax error in an argument list of an *@ODBC* function call. Check the documentation on the *@ODBC* function to determine the proper syntax.

46. INADMISSIBLE FILE NAME: *FILENAME*.
The file name is either too long or it contains characters not permitted on this platform. Use a different file name and try again.

47. TOO MANY FILES OPEN: *NAME_OF_LAST_FILE*.
LINGO imposes a limit on the total number of files that can be opened simultaneously through use of the *@FILE* command. You can try placing all the data in fewer files. Also, avoid using the LINGO end-of-record mark (~) at the end of files. This allows LINGO to "see" the end of the file, forcing it to close the file down, thus allowing for an additional open file.

48. UNABLE TO OPEN FILE: *FILENAME*.
LINGO prints this message when it is unable to open a file. Check the spelling of the filename. Be sure a copy of the file exists on your disk and that the file is not open in another application.

49. ERROR READING FILE: *FILENAME*.
LINGO prints this message in case an error occurred while reading a file with the *@FILE* function. Check the file to be sure it is not damaged. Another possibility is that you do not have read access to the disk or directory where the file resides.

50. IMPROPER USE OF @FOR FUNCTION.
LINGO prints this message if you've attempted to nest an *@FOR* function inside some other set operator. You can nest *@FOR* functions within other *@FOR*s, and other set operators within *@FOR*s, but nesting an *@FOR* inside any function other than another *@FOR* is not permitted.

51. RAN OUT OF GENERATOR MEMORY COMPILING MODEL.
LINGO exhausted available generator memory compiling a model and was forced to halt. See error message 0 for suggestions on increasing the model generator's allotment of working memory.

52. IMPROPER USE OF @IN FUNCTION.
You passed incorrect arguments to the *@IN* function. Check the documentation on the *@IN* function.

53. UNABLE TO LOCATE RANGE NAME: *RANGE_NAME*.
You specified a range name in a spreadsheet interface function, which is either inadmissible or was not found in the specified worksheet. Please check the worksheet file to be sure the range name exists as spelled.

54. ERROR (*N*) READING FROM FILE.
LINGO was unable to read from the file FILE_NAME. Possible causes include the file being locked by another user or the file being corrupted. Please check to see that the file is valid and available for reading.

55. UNABLE TO OPEN @TEXT DATA FILE: *FILENAME*.
You have specified a file as part of the *@TEXT* function that could not be opened. Check to see that the file exists and that you have spelled the name correctly.

56. ERROR READING FROM @TEXT DATA FILE: *FILENAME.*
A read error was encountered when LINGO attempted to read data from a file specified in an *@TEXT* function. Check to be sure that the file is not corrupted.

57. INVALID INPUT ENCOUNTERED IN @TEXT DATA FILE: *TEXT_STRING*.
The *@TEXT* function may be used in the data section to read and write numeric values. This error message results when nonnumeric data is encountered in a file being read with *@TEXT*. If you need to import nonnumeric data from a text file, use the *@FILE* function.

58. NOT ENOUGH VALUES FOUND IN @TEXT DATA FILE: *FILENAME*.
***N* VALUES NEEDED.**
***M* VALUES FOUND.**
The *@TEXT* function may be used in the data section to read and write numeric values. This error message results when an insufficient number of data points were found in a file being read with *@TEXT*. Add enough data to the file to fully initialize the attributes of interest.

59. TOO MANY VALUES ENCOUNTERED IN @TEXT DATA FILE: *FILENAME.*
The *@TEXT* function may be used in the data section to read and write numeric values. This error message results when too many data points were found in a file being read with *@TEXT*. Remove data from the file until you have the exact number required to initialize the attributes of interest.

60. FILE NAME REQUIRED IN AN @TEXT INPUT OPERATION.
The *@TEXT* function may be used in the data section to read and write numeric values. This error message results when an input file was not specified when attempting to read data points from a file using *@TEXT*.

61. COMMAND DISREGARDED.
LINGO prints this message when it was unable to interpret a command you typed at the colon prompt (:). Check the spelling of the command and that you are using correct syntax for the command.

62. RAN OUT OF WORKSPACE IN MODEL GENERATION (*N*).
LINGO ran out of working memory generating your model. Refer to error message 0 for strategies to increase the model generator's working memory allotment.

63. MODEL IS ILL DEFINED. CHECK FOR UNDEFINED INDICES AND/OR CONDITIONS IN EXPRESSION: *EXPRESSION*.

LINGO will print this message for one of two reasons: 1) a conditional expression used to qualify a set operator function cannot be evaluated, or 2) a subscript expression cannot be evaluated. When we say an expression cannot be evaluated, we mean that one or more variables in the expression are not fixed. LINGO will report the expression number, or row name (if used), where the fault occurred. Please be sure to add constraints or data statements to fix all the variables in the conditional or subscript expression in question. Also, if you are using primitive set member names in the model's equations, you must use the *@INDEX* function to get the index of the primitive set member. If you don't use the *@INDEX* function, LINGO will treat the primitive set member as if it is a new scalar variable. You can have LINGO check for primitive set names that are in use in both the data and the model equations by checking the *Check for duplicate names in data and model* box on the *General Solver* tab of the *Solver|Options* command dialog box (or the *SET CHKDUP 1* command from the LINGO command line). Finally, if you would like to use primitive set names directly in your model's equations (a practice we don't recommend), you can force LINGO to allow this by checking the *Allow unrestricted use of primitive set member names* checkbox on the *General Solver* tab (or the *SET USEPNM 1* from the LINGO command line).

64. TOO MANY NESTED @FOR OPERATORS.

LINGO maintains a stack to keep track of pending nested *@FOR* functions. Too many nested *@FOR*s could cause this stack to overflow, although it would be an unusual model that triggers this error. You can avoid nesting some *@FOR*s by forming derived sets, and looping over a single derived set as opposed to many primitive sets.

65. IMPROPER USE OF @WARN FUNCTION.

You have used the *@WARN* function incorrectly. Check the documentation on *@WARN* to determine where you went wrong.

66. WARNING: TOTAL FIXED ROWS WITH NONUNIQUE ROOTS: *N*

When LINGO generates a model, it determines if a variable can be solved for directly and substituted out of the model. These variables are referred to as being fixed, and the row in the model used to solve for a fixed variable is referred to as a fixed row. When LINGO solves for the value of a fixed variable in its fixed row, it will look around a neighborhood of the variable's value to determine if multiple roots exist, and display this message if any are found. An example of an equation with such multiple roots would be: *@SIGN(X) = 1*. All non-negative values of *X* would satisfy this relation. LINGO will display the names of up to three variables and their corresponding fixed rows with this symptom. LINGO will continue to process the model, however. You should examine the model closely when this error occurs, because it would be unusual to find a well formulated model displaying this characteristic. Keep in mind that there might be a better solution involving a different value for the fixed variable than was chosen by LINGO.

67. UNSUPPORTED STRING ARITHMETIC OPERATION.

You've attempted to perform an arithmetic operation on a text object.

68. MULTIPLE OBJECTIVE FUNCTIONS IN MODEL.
The model contains more than one objective function. Possible solutions are to delete some objectives, convert some objectives to constraints, or combine the multiple objectives into a single objective using a weighting scheme.

69. UNDEFINED ARITHMETIC OPERATION.
LINGO ran into an undefined arithmetic operation during execution (e.g., 1/0). Check the constraints to be sure all operations are defined.

70. SUBSCRIPT OUT OF RANGE ON ATTRIBUTE: *ATTRIB*.
While executing your model, LINGO found a subscript that was out of range. For example, if you defined the attribute *STAFF* with the set /1..5/, then referencing *STAFF(6)* would result in this error message. Nonintegral subscripts will also generate this message. Please be sure your sets are defined across the intended range and that any subscript computations are correctly specified.

71. IMPROPER USE OF A VARIABLE DOMAIN FUNCTION (E.G. , @GIN, @BIN, @FREE, @BND).
This error results when the syntax of a variable domain function has been violated. Check the model's use of these functions.

72. UNABLE TO SOLVE FOR FIXED VARIABLE *VAR_NAME* IN CONSTRAINT *CONSTRAINT_NAME*.
LINGO has determined that it should be able to solve for a particular variable in a given row. The root finder was not able to converge on a solution. Be sure a solution exists for the row (e.g., the expression: -1=*x*^.5, would have no real solution). If possible, rewrite the equation in the form *x* = *f*(.), where *x* appears only once and is on the left-hand side of the expression. LINGO is always able to solve expressions in this form, as long as the function is defined and evaluates to a real number (e.g., *x* = *@LOG*(-3) evaluates to an imaginary number; and *x*/0 is undefined). If this is not possible, then you might try solving the expression by some other method, and enter the variable's value as a constant in LINGO.

73. A USER INTERRUPT OCCURRED.
LINGO was interrupted before the solver completed its run.

74. MAGNITUDE OF BOUND EXCEEDS: 1.E+21 ON VARIABLE: *VARIABLE_NAME*.
You have entered a bound outside LINGO's allowable limit. The magnitude of bounds input using *@BND* may not exceed 1.E32.

75. CONFLICTING BOUNDS ON VARIABLE: *VAR_NAME*.
LINGO has detected a bound on the named variable that is outside of the range of another bound. For instance, *@BND(-6, X, 6)*, followed by *@BND(-5, X, 5)* would not yield this error. However, following it with *@BND(7, X, 9)* would.

76. ERROR GENERATING MODEL.
LINGO was unable to pass the model to the optimizer. If this error occurs, contact LINDO Systems technical support.

77. ARITHMETIC ERROR GENERATING MODEL.
LINGO was unable to generate the model due to an undefined arithmetic operation (e.g., division by zero). Remove all undefined arithmetic operations from your model.

78. SET SECTIONS NOT ALLOWED IN SUBMODELS.
Set sections can not be placed inside submodels. You can move the set section before the relevant submodel to correct the problem.

79. NOT USED.

80. NOT ENOUGH WORKSPACE GENERATING MODEL.
LINGO ran out of working memory while trying to generate the model. Please refer to message 0 for strategies to increase the model generator's working memory.

81. NO FEASIBLE SOLUTION FOUND.
LINGO was unable to find a solution that simultaneously satisfies all the constraints. Check the model's consistency. Try dropping constraints until the problem goes away to get an idea of where the trouble may lie. Also, check the solution report. Constraints contributing to the infeasibility will have a nonzero dual price.

82. UNBOUNDED SOLUTION.
LINGO was able to increase the objective function without bound. Be sure that you have added all constraints to your model and that they have been input correctly.

83. ATTEMPT TO USE A FUNCTION THAT IS NOT VALID IN A MODEL SECTION.
Not all functions may be used in a model's constraints. Some functions, for example *@WRITE*, only make sense in and may only be used in calc and data sections. You will receive this error message if you attempt to use such functions in a model's constraints or objective function.

84. @ODBC AND @POINTER MAY NOT BE USED WITH @WRITE/@WRITEFOR IN CALC SECTIONS.
Calc sections do not presently support the use of the *@ODBC* and *@POINTER* output operators in conjunction with @WRITE/@WRITEFOR. In order to use *@ODBC* and *@POINTER*, you must pass complete attributes as per the following example:

```
CALC:
   @POINTER( 1) = X;
   @POINTER( 'MYDATA','MYTABLE','MYCOLUMN') = X;
ENDCALC
```

85. STACK OVERFLOW. EXPRESSION TOO COMPLEX.
LINGO uses a stack to store temporary values while executing the expressions in a model. The default stack size is quite large. Thus, this message should not occur. Should you receive this message, contact LINDO Systems technical support.

86. ARITHMETIC ERROR IN CONSTRAINT: *CONSTRAINT*.
INSTRUCTION POINTER: *N*
An undefined arithmetic operation (e.g., 1/0 or *@LOG*(-1)) occurred while LINGO was generating the model. If you have specified a row name for the constraint, LINGO will print the name of the constraint. If you haven't specified row names in your model, you may want to add them to assist in tracking down this error. Check the referenced constraint for any undefined operations.

87. IMPROPER USE OF @IN FUNCTION.
You have specified improper arguments for the *@IN* function. Refer to the documentation on this function for more details.

88. SOLUTION IS CURRENTLY UNDEFINED.
LINGO was not able to solve the model to completion for some reason. In which case, any attempt to print out a solution will result in this message. Try re-solving the model and determining the reason LINGO was unable to solve the model.

89. RUNTIME ERROR IN SOLVER ROUTINES. CONTACT LINDO SYSTEMS.
An unexpected runtime error has occurred in LINGO's solver routines. Please contact LINDO Systems for assistance.

90. THE SOLVER TIME LIMIT WAS EXCEEDED.
The solver time limit was reached and the solution process was halted. If you believe that there should be no time limit, then you should check to see what the current time limit is. In Windows, check to see what the current setting is using the *Solver|Options* command to check the Time box on the *General Solver tab*. On other platforms, run the *HELP SET* command to see what the setting is for the *TIMLIM* parameter.

91. INVALID RUNTIME PARAMETER VALUE.
At runtime, LINGO will prompt for any variable set to equal a question mark in the data section. If LINGO receives an invalid value, you will get this error message. Correct your data and try again.

92. WARNING: THE CURRENT SOLUTION MAY BE NONOPTIMAL/INFEASIBLE FOR THE CURRENT MODEL.
If you've solved a model and brought another model window to the front, LINGO prints this message to remind you that the solution you've asked for may not belong to the frontmost model window. LINGO also prints this message whenever you attempt to examine a nonoptimal solution. For instance, when you've interrupted the solver, or when LINGO couldn't find a feasible answer. In the latter case, correct any errors in the model and re-solve.

93. INVALID SWITCH IN COMMAND LINE.
Some LINGO commands accept switches, or modifiers. If there was an error in one of these command-line modifiers, LINGO will print this message. Refer to the documentation on the specific command to learn the available modifiers.

94. MAXIMUM ARRAY LENGTH EXCEEDED.
LINGO attempted to allocate an array with more than 2^31 elements. At present, this is the largest array LINGO can handle. The model is most likely extremely large and will have to be reduced in size in order to solve.

95. NOT USED.

96. NOT USED.

97. NOT USED.

98. NOT USED.

99. NOT USED.

100. A SUBMODEL REFERENCE WAS EXPECTED.
A reference was made to an undefined submodel. Submodels are "models within models" that may be solved independently or combined with other submodels. You will need to either define the referenced submodel, or move it to an earlier part of the main model before any references to the submodel occur. Refer to the *SUBMODEL* statement for more information.

101. PASSWORDS DON'T MATCH -- MODEL NOT EXTENDED.
You've attempted to extend a password protected model using a model fragment with a password that differs from the main model's password. The passwords must be identical in order to add the new model fragment, so you will need to re-save either the main model or the model fragment with a matching password.

102. UNRECOGNIZED MODEL OBJECT NAME: OBJECT_NAME.
The model object name you have specified does not exist. Please check your spelling.

103. TEXT LITERALS NOT ALLOWED IN VECTOR-BASED, NON-TEXT OUTPUT OPERATIONS.
When outputting solutions in vector format (i.e., passing an entire attribute at a time), text literals may not be included in the list of output objects. For example, suppose you had the following output statement in your model:

```
DATA:
  @OLE( 'MYBOOK.XLS', 'HEADER', 'VOLUME') = 'SHIP=', VOLUME;
ENDDATA
```

In this case, *VOLUME* is an attribute containing multiple values, while 'SHIP=' is a single text literal. This is a vector-based output statement in that we aren't embedding output objects in *@WRITE* or *@WRITEFOR* statements. The use of text literals is not permitted in vector based output. You will need to break the statement up into two statements, one using *@WRITE* to output the text literal:

```
DATA:
  @OLE( 'MYBOOK.XLS', 'HEADER') = @WRITE( 'SHIP=');
  @OLE( 'MYBOOK.XLS', 'VOLUME') = VOLUME;
ENDDATA
```

104. MODEL NOT SOLVED YET, OR THE MODEL HAS BEEN SOLVED AND FOUND TO CONTAIN NO CONSTRAINTS OR VARIABLES.
If the model has not already been solved then issue the SolveLINGO_Solve command. If the model has been solved then it was found to be vacuous.

105. THE FOLLOWING TABLE IS TOO SMALL TO RECEIVE ALL REQUESTED OUTPUT.
An ODBC output statement to a database has more rows than is contained in the output statement. You will need to either increase the size of the table in the database or reduce the number of rows in the output table.

106. MINIMUM AND MAXIMUM VALUES FOR WIDTH ARE: 68 800.
When changing terminal width using the *WIDTH* command, you must enter an integer between 68 and 800.

107. INVALID @POINTER INDEX VALUE
Your model contains a reference to the *@POINTER* function with an invalid argument list. The *@POINTER* function requires one argument that must be integer valued and greater-than-or-equal-to 1.

108. THE MODEL'S DIMENSIONS EXCEED THE CAPACITY OF THIS VERSION.
Your model is too large for your version of LINGO. Some versions of LINGO limit one or more of the following model properties: total variables, integer variables, nonlinear variables, and constraints. This error message displays the dimensions of the model and the limits of your version. Refer to this information to determine the specific limit that is being exceeded.

You can also view the limits of your version of LINGO by issuing the *Help|About* command.

Nonlinear variables are allowed only if you have purchased the nonlinear option for your LINGO system. If you aren't sure which parts of yout model, you can run the *Generate | Display nonlinear rows* command to display the rows in the model containing nonlinearities. It's possible that some or all of the nonlinear rows can be rewritten so that they become linear.

In general, you must either make your model smaller by simplifying it or upgrade to a larger version of LINGO. If you are exceeding the limit on constraints and if you have simple bounds entered as constraints, you should enter them with the *@BND* function. Constraints entered using *@BND* don't count against the constraint limit.

The limits for the various versions of LINGO are:

Version	Total Variables	Integer Variables	Nonlinear Variables	Constraints
Demo/Web	300	30	30	150
Solver Suite	500	50	50	250
Super	2,000	200	200	1,000
Hyper	8,000	800	800	4,000
Industrial	32,000	3,200	3,200	16,000
Extended	*Unlimited*	*Unlimited*	*Unlimited*	*Unlimited*

If your version of LINGO doesn't include the nonlinear option, then your models can't contain any nonlinear variables.

109. THE SOLVER ITERATION LIMIT WAS EXCEEDED.
The solver iteration limit was reached and the solution process was halted. If you believe that there should be no iteration limit, then you should check to see what the current iteration limit is. In Windows, check to see what the current setting is using the *Solver|Options* command to check the Iterations box on the *General Solver tab* (or enter the *HELP SET* command from the LINGO command line to see what the setting is for the *ITRLIM* parameter).

110. PASSWORDS ARE LIMITED TO 56 CHARACTERS.
The password associated with your *HIDE* command is too long. It must be 56 characters, or less.

111. COMMAND NOT AVAILABLE WHEN MODEL IS HIDDEN.
Any command that reveals the content of a model is not permitted when the model is hidden. If you know the password to unhide the model, enter it with the *HIDE* command and try again.

112. INCORRECT PASSWORD... MODEL REMAINS HIDDEN.
You have entered an incorrect password. The model may not be viewed without the correct password.

113. LOOK/SAVE FAILED... THE FOLLOWING LINE IS TOO LONG: *N*. INCREASE THE TERMINAL WIDTH.
An attempt to view a given line, all lines, or to save the model, has failed because a line is too long. Lines can't exceed 800 characters in length. You will need to break each long line into two or more shorter lines.

114. PASSWORD VERIFICATION FAILED... MODEL WILL NOT BE HIDDEN.
You must enter the same password twice to verify it.

115. MODEL MUST BE AT LEAST 64 CHARACTERS LONG FOR ENCRYPTION.
In order to encrypt a model with the HIDE command it must be at least 64 characters long. If need be, you can extend the model's length using comments.

116. THE MAXIMUM NUMBER OF CHARACTERS IN A FILE NAME IS: *N*.
The maximum number of characters in a file name, including path, has been exceeded.

117. INVALID COMMAND. TYPE 'COM' TO SEE VALID COMMANDS.
You have entered a command that is not implemented in LINGO. Check the list of available commands with the *COMMANDS* command to see whether you've misspelled a command.

118. AMBIGUOUS COMMAND. TYPE 'COM' TO SEE VALID COMMANDS.
You have probably typed an abbreviated command that can be interpreted in more than one way. Spell out the command name completely.

119. TOO MANY CONSECUTIVE ERRORS IN BATCH MODE. LINGO WILL STOP.
When running a command script, LINGO will stop after the sixth consecutive error.

120. UNABLE TO WRITE CNF FILE TO STARTUP AND WORKING DIRECTORIES.
LINGO was unable to write its configuration file. You must have write permission for either the working directory or the startup directory. If you're connected to a network, check with your network administrator to determine your disk privileges.

121. RANGE ANALYSIS NOT ALLOWED ON INTEGER PROGRAMMING MODELS.
Range reports are not possible on IP models, given that they have no meaning in this context.

122. RANGE REPORTS NOT POSSIBLE WHEN RANGE ANALYSIS IS DISABLED.
Range computations are currently disabled. To enable range computations in Windows versions of LINGO, run the *Solver|Options* command, click the *General Solver* tab, and select the *Prices and Ranges* option from the *Dual Computations* list box. To enable range computations in command-line versions of LINGO, use the command: *SET DUALCO 2*. Be aware that range computations will increase solution times.

123. MODELS MUST BE EITHER INFEASIBLE OR UNBOUNDED IN ORDER TO BE DEBUGGED.
Debugging is permitted only on models that are either infeasible or unbounded.

124. ATTEMPT TO INITIALIZE AN ATTRIBUTE OF A NULL SET.
You are attempting to assign values to an attribute of a null set. Given that the set is null, each of its attributes also have no members, and, therefore, may not be assigned values. Check the initialization of the attribute's parent set to be sure it is being assigned at least one member.

125. THE @USER FUNCTION IS NOT CURRENTLY SUPPORTED ON THIS PLATFORM.
The *@USER* user-supplied function is not currently supported on this platform. You may wish to see if a newer release of Lingo supports this feature on your platform.

126. THE FOLLOWING VARIABLE NAME WAS NOT RECOGNIZED: *VAR_NAME*.
You have attempted to use an ODBC link to export an unknown variable to a database. Check the spelling of the variable's name and try again.

127. THE FOLLOWING VARIABLE IS OF A DIFFERENT DIMENSION: *VAR_NAME*.
You have attempted to use an ODBC link to export multiple variables of different dimensions to a database. All variables being simultaneously exported must be of the same dimension.

128. THE PARAMETER INDEX MUST BE BETWEEN 1 AND *N*.
You have input an invalid parameter index as part of the *SET* command. Use the *HELP SET* command to determine the valid indices.

129. THE PARAMETER VALUE IS NOT VALID.
You have input an invalid parameter value as part of the *SET* command. LINGO will print the valid range of values. You should adjust the value of the parameter, so it falls within the valid range, and reenter the command.

130. THE FOLLOWING PARAMETER NAME WAS NOT RECOGNIZED: *PARAMETER_NAME*.
You have input an invalid parameter name as part of the *SET* command. Use the *HELP SET* command to determine the valid parameter names.

131. UNABLE TO WRITE PARAMETERS TO CONFIGURATION FILE.
You have attempted to save LINGO's current configuration using the *FREEZE* command. LINGO was unable to write the parameter values to its configuration file (*LINGO.CNF*). Perhaps you don't have write access to the drive or the disk is full.

132. ONE OR MORE ERRORS OCCURRED WHILE READING THE CONFIGURATION FILE.
LINGO was unable to successfully read its configuration file (*LINGO.CNF*). Perhaps the file has become corrupted. Beware that some of the parameters you have set may revert back to their default values.

133. UNABLE TO INITIALIZE WORKBOOK FOR OLE TRANSFERS.
You have attempted to link to a spreadsheet that LINGO was unable to load into Excel. Try reading the file into Excel yourself, and check to be sure that some other user isn't accessing the file.

134. UNABLE TO COMPLETE ALL OUTPUT OPERATIONS.
You have attempted to use interface functions in the data section to export parts of the solution. One or more of these operations failed. You will receive additional messages from LINGO detailing the problem.

135. THE FOLLOWING FIXED VARIABLE VIOLATES ITS SEMICONTINUOUS BOUNDS: VARIABLE.
You have specified semicontinuous bounds for the specified variable using the *@SEMIC* function. The variable was determined to be fixed to a particular value that violates its semicontinuous property. For example, if *X* were specified to be semicontinuous using the statement *@SEMIC(5, X, 100)*, then *X* must be either 0 or lie within the range [5,100]. If the

model generator determines that *X* is fixed at some value other than this, you will receive this error message. The most likely cause of this error is incorrect semicontinuous bounds, i.e., in this example, the bounds of 5 and 100

136. ERROR PERFORMING @TEXT OUTPUT OPERATION: *N*.
You have attempted to use the *@TEXT* interface function to export data to a text file. For some reason, this operation has failed. Perhaps the disk is full or you don't have write access to the drive.

137. ENCRYPTED FILE NOT IN PROPER FORMAT.
An encrypted model (see the *HIDE* command) was found to not be in the proper format. Please check the file name to be sure you are referring to the correct file.

138. TOO MANY NESTED DIVERT FILES.
LINGO allows you to nest *DIVERT* commands up to 10 levels deep. If you attempt to go beyond 10 levels, you will get this error message.

139. DIVERT COMMAND FAILED. UNABLE TO OPEN FILE.
You have attempted to open a file for output using the *DIVERT* command. LINGO was unable to open the files. Perhaps the disk is full or you don't have write access to the drive.

140. DUAL VALUES WERE REQUESTED BUT DUAL COMPUTATIONS ARE DISABLED.
You have attempted to export dual values when the solver is in primals only mode. In primals only mode, dual values are not computed and, therefore, can't be exported. To enable dual computations, run the *Solver|Options* command, click the *General Solver* Tab, and select the *Prices* option from the *Dual Computations* list box. To enable range computations as well as dual computations, select the *Prices & Ranges* option from the *Dual Computations* list box. (Alternatively, in the LINGO command-line enter `SET DUALCO 1` to enable dual computations, or enter `SET DUALCO 2` to enable both range computations and dual computations.)

141. RANGE VALUES WERE REQUESTED ON A ROW THAT WAS SUBSTITUTED FROM THE MODEL.
You have attempted to export range values on a row that is fixed in value. LINGO substitutes these rows from the model. Therefore, range values are not computed on these rows.

142. AN UNEXPECTED ERROR OCCURRED. PLEASE CONTACT LINDO SYSTEMS TECHNICAL SUPPORT.
In general, this error message should never occur. Should you receive this message, please contact a technical support representative.

143. OUTPUT OPERATION FAILED. MODEL OBJECTS NOT OF SAME LENGTH.
You have attempted to use an interface function to export two or more model objects simultaneously. This interface function requires all objects to be of the same dimension. Break the output operation up into individual operations that all contain objects of the same dimension.

144. INVALID ARGUMENT LIST FOR @POINTER FUNCTION.
The *@POINTER* function only accepts a single positive integer as an argument. Review the documentation on the use of the *@POINTER* function.

145. ERROR *N* PERFORMING @POINTER OUTPUT OPERATION.
A *@POINTER* output function operation failed. Values for *N* and their interpretations are:

No.	Interpretation
2	Attempt to export an invalid variable
3	Ran out of working memory
4	Requested duals in primals only mode
5	Range values were requested on fixed rows
6	Unexpected error, call LINDO Systems Technical Support
7	Range values requested with range computations disabled

146. THE FOLLOWING NAMES APPEARED IN THE MODEL AND THE DATA: *NAME1 NAME2 NAME3*.
If you go to the *Model Generator* tab on the *Solver|Options* command's dialog box, you will see a *Check for duplicates names in data and model* checkbox. When this option is enabled, LINGO will compare primal set member names to all the variable names used in the model's equations. If any duplicates are found, LINGO will print the first three and print this error message. (To enable this option in the command, use the *SET CHKDUP 1* command.)

147. UNABLE TO EVALUATE ALL @WARN FUNCTIONS.
Conditional expressions contained in *@WARN* functions must contain fixed variables only. When this is not the case, LINGO can't evaluate the *@WARN* functions and you will receive this error message.

148. @OLE FUNCTION NOT SUPPORTED ON THIS PLATFORM.
At present, the *@OLE* function is supported only in Windows versions of LINGO. If you don't have a Windows version of LINGO, you can export the data from your spreadsheet to a file and use the *@FILE* function in your LINGO model to import the data.

149. TARGET RANGE NAMES AND ODBC COLUMN NAMES CAN'T TAKE DEFAULT VALUES FOR INDEX VARIABLE OUTPUT.
If you are outputting an index variable's value to a workbook or database, you must explicitly provide the name of the workbook range or the database column. LINGO cannot provide default names in this instance.

150. ODBC INTERFACE NOT SUPPORTED ON THIS PLATFORM.
LINGO's ODBC link to databases is supported only in Windows versions. If you don't have a Windows version of LINGO, you can use text files to move data in and out of LINGO. See Chapter 8 of the users manual, *Interfacing with External Files*, for more details.

151. @POINTER NOT SUPPORTED ON THIS PLATFORM.
LINGO's *@POINTER* function for interfacing with calling applications is supported only in Windows versions. If you don't have a Windows version of LINGO, you can use text files to move data in and out of LINGO. See Chapter 8 of the users manual for more details.

152. COMMAND NOT SUPPORTED ON THIS PLATFORM.
You have selected a command that is not supported on your platform.

153. SET DEFINITIONS NOT ALLOWED IN INIT SECTIONS.
Sets can't be initialized in an init section. You must change the model, so the set is initialized in either a sets section or in a data section.

154. ATTEMPT TO REDEFINE A PREVIOUSLY DEFINED SET.
You have attempted to define a set twice in the same model. A set name can only be used once. Choose a different name for the two sets and try again.

155. SET MEMBER NAMES MAY NOT BE OMITTED IN DATA STATEMENTS.
When initializing a set in a model's data section, you must explicitly list each member. You may not skip elements as you can with set attributes.

156. INCORRECT NUMBER OF ARGUMENTS IN A DATA SECTION.
ARGUMENT MUST BE A MULTIPLE OF: *N*
NUMBER OF ARGUMENTS FOUND: *M*
You have a data, init or calc statement in your model that doesn't have the correct number of values to initialize a specified list of attributes and/or sets. LINGO will let you know how many values it found and what the number of arguments should be a multiple of. Add or subtract values in the data statement and try again.

157. ATTEMPT TO USE @INDEX ON AN UNDEFINED SET.
LINGO can't compile an instance of the *@INDEX* function without knowing of the existence of the set that *@INDEX* is referencing. Move the expression with the *@INDEX* function after the set definition to correct this problem.

158. A SET MEMBER NAME WAS EXPECTED.
You have used a function that was expecting a set member name as an argument. Correct the arguments to the function and try again.

159. A DERIVED SET MEMBER IS NOT CONTAINED IN ITS PARENT SET.
You have specified a set member in a derived set that is not contained in the parent set. Each member in a derived set must be derived from a member of its parent set(s). You have probably misspelled the name of the derived set member. Check the spelling and try again.

160. ONLY ONE SET MAY BE DEFINED IN EACH DATA STATEMENT.
You have attempted to define more than one set in an individual data statement. Break the data statement into multiple statements with no more that one set per statement.

161. INDEX VARIABLES MAY NOT SHARE NAMES WITH OTHER VARIABLES.
The index variables used in set looping functions may not use the same names as those used by the structural variables in the model. For example, in the following model:

```
MODEL:
  SETS:
    S1/1..5/: X;
  ENDSETS
  MAX=I;
  @SUM( S1( I): X( I)) <= 100;
END
```

the variable name *i* is used in the objective row for a structural variable. In the next to the last statement in the model, the name *i* is also being used as an index variable, which will trigger the error. In the case, you would need to change the name of either the variable in the objective or the name of the index variable in the *@SUM* loop.

162. ATTEMPT TO INITIALIZE MIXED DATA TYPES (TEXT AND NUMERIC) FROM A SINGLE WORKBOOK RANGE.
When specifying a single range for multiple model objects, all the objects must be of the same data type—either text (set members) or numeric (set attributes). LINGO's spreadsheet interface can't handle ranges with mixed data types. Break the data statement up into two—one containing text model objects and the other containing numeric objects.

163. INVALID NUMBER OF DATA VALUES FOR OBJECT: *OBJECT_NAME*.
You have attempted to initialize a model object of known length with an incorrect number of values. Check the initialization statement to be sure that the values are specified correctly.

164. INVALID LINGO NAME: *NAME*.
You have used a symbol name that doesn't conform to LINGO's naming conventions. Please correct the name and try again.

165. SET EXPORTS NOT SUPPORTED WITH THIS FUNCTION.
Not all of LINGO's export functions can handle set members. Switch to a different export function and try again.

166. ATTEMPT TO OUTPUT OBJECTS OF VARYING LENGTHS TO A TABULAR DEVICE.
LINGO requires output to be in tabular form for certain output devices (databases and text files). You have constructed an output statement with two or more model objects of varying length. In which case, it is not obvious how to transform the data into tabular form. Break the output function call up into two or more function calls such that each call contains model objects of identical length.

167. INCORRECT NUMBER OF RANGES SPECIFIED: *N*. NUMBER OF RANGES REQUIRED: *M*.
You didn't specify the correct number of ranges in an import/export function. In general, you will need one range for each model object.

168. OUTPUT MODIFIERS NOT ALLOWED ON TEXT DATA.
When exporting set members, use of the *@DUAL*, *@RANGEU*, and *@RANGED* modifier functions are not allowed. Remove the modifier function and try again.

169. RUNTIME INPUT OF SET MEMBERS NOT ALLOWED.
When initializing attributes in a data section, you can initialize all or part of an attribute with a question mark. In which case, LINGO will prompt you for the values each time the model is run. This is not the case with sets—all set members must be explicitly listed when initializing a set in the data section.

170. LICENSE KEY WAS NOT FOUND OR IS INVALID.
LINGO did not recognize your license key. Check to make sure you have entered it correctly. If you received it as part of an email, then you may cut-and-paste it from the email into LINGO's license dialog box.

171. LICENSE KEY IS INVALID.
Your license key is not recognized by LINGO. LINGO will continue to operate, but only in demonstration mode.

172. INTERNAL SOLVER ERROR. CONTACT LINDO SYSTEMS.
LINGO's solver encountered an unexpected error condition and was unable to continue. Please contact LINDO Systems for assistance.

173. NUMERIC ERROR IN THE SOLVER.
LINGO's solver experienced numerical problems and was unable to continue. Scaling the model's coefficients so that they don't cover as large a range may be helpful in eliminating this error. Also, check for potentially undefined arithmetic operations in the model. If these remedies fail, please contact LINDO Systems for assistance.

174. OUT OF MEMORY IN PREPROCESSOR.
LINGO's solver ran out of memory during the preprocessor phase. Refer to error message 175 for possible remedies.

175. NOT ENOUGH VIRTUAL SYSTEM MEMORY.
LINGO's solver ran out of virtual system memory. LINGO's solver accesses memory from the system heap. Adding more memory to your machine may help. Note, also, that the memory allocated to LINGO's model generator is not available to the solver. Thus, if too much memory is allocated to the generator there may not be enough left over for the solver. If you are using a Windows version of LINGO, see the *Solver|Options* command for information on adjusting the generator memory level, otherwise, refer to the *SET* command.

176. OUT OF MEMORY DURING SOLVER POSTPROCESSING.
LINGO's solver ran out of memory during the post-processing phase. Refer to error message 175 for possible remedies.

177. OUT OF MEMORY ALLOCATING SET: *SET_NAME*.
LINGO failed to allocate the internal data structures for a set due to lack of generator memory. First, check to see that the set's dimensions are not unreasonably large, perhaps indicating a typo in the model. If all appears well, then you will need to boost the available generator memory, restart LINGO and try again.

178. OUT OF STACK SPACE ALLOCATING SET: *SET_NAME*.
When LINGO allocates the data structures for a set, it may find that the set is derived from another set. If this parent set is also unallocated, LINGO must back track and allocate it, too. In some rare instances, LINGO might have to backtrack many generations. As LINGO back tracks, it keeps information on a stack. This error message is triggered when this stack runs out of space. However, it would be a highly unusual model that causes this error.

179. THE MODEL TRANSLATOR HAD TO PATCH NAMES TO MAKE THEM COMPATIBLE
VAR NAMES PATCHED: *N*
ROW NAMES PATCHED: *M*
This message occurs when LINGO imports a non-native file converting it to a LINGO model. Non-native file formats (e.g., MPS, MPI and/or LP) allow for variable and row names that aren't entirely compatible with LINGO's syntax (e.g., spaces in names). In order to overcome this, LINGO will patch these names by replacing inadmissible characters in the names with underscores and truncating names longer than 64 characters. This can create problems because two or more unique MPS names may get mapped into one LINGO name. A technique for avoiding this problem is to use R/C naming conventions. Refer to the *Use R/C format names for MPS I/O* option for more information.

180. UNABLE TO CREATE MODEL DATA STRUCTURE.
LINGO was unable to create the internal data structures required for transferring an MPS file. The most likely cause of the problem is insufficient system memory. Try cutting back on LINGO's allotment of generator memory or using a machine with more memory.

181. ERROR EXTRACTING DATA FROM MODEL STRUCTURE.
LINGO was unable to extract model information from a data structure. The most likely cause of the problem is insufficient system memory. Try cutting back on LINGO's allotment of generator memory, or try using a machine with more memory.

182. ERROR IN MPS OR MPI FILE ON LINE NUMBER: *N*
TEXT OF INVALID LINE: *TEXT*
LINGO encountered an error reading an MPS or MPI format file. Go to the line number in the file, correct the error, and try again.

183. INVALID USE OF SP TABLE DECLARATION FUNCTION @SPDISTTABLE.
The *@SPDISTTABLE* function is a matrix oriented function that is uses in stochastic programming models to declare outcome tables and assign to assign random variables to those tables. Correct usage is:

```
@SPDISTTABLE( OUTCOME_TABLE_ATTR, RANDOM_VAR_VECTOR[,
PROBABILITY_VECTOR]);
```

The outcome table argument must be either one or two dimensions, with each row of the table corresponding to an outcome of the random variable(s) in the random variable vector. The number of elements in the random variable vector must equal the number of columns in the outcome table argument. The third argument is optional, is used to pass the probabilities of the outcomes, and must have one element for each row of the outcome table.

184. RANGE VALUES CAN'T BE REPORTED WHEN RANGE ANALYSIS IS DISABLED.
You have attempted to reference range values while the solver is not set to compute ranges. To enable range computations, run the *Solver|Options* command, click the *General Solver* Tab, and select the *Prices and Ranges* option from the *Dual Computations* list box. (To enable dual and range computations in LINGO's command-line, use the command: `SET DUALCO 2`.)

185. BARRIER SOLVER REQUESTED WITHOUT A LICENSE.
LINGO's barrier solver capability is an additional option. To enable the option, contact LINDO Systems for licensing information and fees.

186. ONLY ATTRIBUTES CAN BE INITIALIZED WITH @QRAND.
The *@QRAND* function may be used for initializing attributes of dense sets only. Other entities, such as set members, cannot be initialized with this function.

187. ONLY ONE LHS ATTRIBUTE MAY BE INITIALIZED IN A @QRAND STATEMENT.
You can initialize only one attribute at a time with the @QRAND function. For more information, refer to the *Probability Functions* section in Chapter 7, *Operators & Functions*.

188. ATTRIBUTES INITIALIZED WITH @QRAND MUST BELONG TO DENSE SETS.
You can initialize only one attribute of dense sets with the @QRAND function. For more information, refer to the *Probability Functions* section in Chapter 7, *Operators & Functions*.

189. INVALID SEED.
You have attempted to use an invalid seed value for a random number generating function. Seed values must have non-negative, integer values.

190. INVALID IMPLICIT SET DEFINITION.
LINGO allows you to express sets implicitly. An example would be *Jan..Dec*, which would give you a set of 12 elements consisting of the months of the year. There are a number of syntax rules required of implicit set definitions. For information on the specific rules, refer to Chapter 2, *Using Sets*.

191. THE LINDO API RETURNED THE FOLLOWING ERROR CODE: *N ERROR_TEXT*.
The LINDO API is the core solver engine used by LINGO. On rare occasions, the LINDO API may raise an unexpected error condition. LINGO will display a text message from the API regarding the error. In most cases, this message should clarify the situation. If not, please contact LINDO Systems technical support.

192. @WKX NO LONGER SUPPORTED…USE @OLE INSTEAD.
The *@WKX* function is no longer supported. You must now use the *@OLE* function to interface with spreadsheets.

193. A SOLUTION IS NOT AVAILABLE FOR THIS MODEL.
There is no solution associated with the model. Either an error occurred while attempting to solve the model, or the model has not been solved yet.

194. UNABLE TO CREATE ENVIRONMENT DATA STRUCTURE.
LINGO was not able to allocate some internal data structures. This may due to insufficient memory in your system's dynamic memory pool or that your system date is not set correctly. You can try running on a machine with more memory, increasing the machine's allotment of virtual memory, or reducing the amount of generator memory allocated to LINGO's model generator. You should also check that your system date is correctly set.

195. AN ERROR OCCURRED WHILE ATTEMPTING TO WRITE TO A FILE.
LINGO experienced problems writing to a file. Be sure your disk is not full and that you have write access to the target file.

196. A DUAL SOLUTION DOES NOT EXIST FOR THIS MODEL.
The solver was unable to successfully compute the dual solution for the model. Given this, the solution report will only display primal values. If you don't require dual information, you can avoid the error by disabling *Dual Computations* on the *General Solver Tab*.

197. THE MODEL CONTAINS ONE OR MORE VACUOUS @MAX OR @MIN FUNCTIONS.
The model contains an @MAX or @MIN function that has no arguments. An example of this situation is:

```
MODEL:
SETS:
S1/1..5/: X;
ENDSETS
THEMAX = @MAX( S1( I) | I #LT# 0: X( I));
END
```

Note that the index variable I will take on values 1, 2, 3, 4 and 5. In no case will I ever be less than 0. This means that the condition (I #LT# 0) on the @MAX function will never be satisfied, meaning that there are no explicit arguments to the @MAX function. Please check that all the @MAX and @MIN functions in the model are correctly specified.

198. QUADRATIC MODEL IS NOT CONVEX.
Quadratic models must be convex in order for them to be solved by the quadratic solver. You must disable the quadratic solver by turning off *Quadratic Recognition* and then re-solve. You can disable *Quadratic Recognition* with the command:

LINGO | Options | Nonlinear Solver | Quadratic Recognition

With command-line versions of LINGO, use the command:

SET USEQPR 0

This will cause the general purpose, nonlinear solver to be invoked. Please see the section titled *Convexity* for more information.

199. A BARRIER SOLVER LICENSE IS REQUIRED FOR THE QUADRATIC SOLVER.
You will need a license for the barrier option to run the quadratic solver. You can proceed by disabling quadratic recognition and re-solving. This will cause the general purpose, nonlinear solver to be invoked.

200. UNABLE TO COMPUTE DUAL SOLUTION.
LINGO was unable to compute the dual values. You can proceed by turning off dual calculations and re-solving.

201. THE MODEL IS LOCALLY INFEASIBLE.
The solver was unable to find a feasible solution within a local region, however, a feasible solution may exist elsewhere. The global or multistart solvers may have more success.

202. THE NUMBER OF NONLINEAR VARIABLES IN THE MODEL: *N1* EXCEEDS THE GLOBAL SOLVER LIMIT IN THIS VERSION: *N2*
Some versions of LINGO impose a limit on the total number of nonlinear variables when running the global solver. You will either need to reduce the number of nonlinear variables, turn off the global solver, or upgrade to a larger version of LINGO.

203. THE GLOBAL SOLVER OPTION WAS REQUESTED WITHOUT A LICENSE. LINGO WILL REVERT TO USING THE DEFAULT NONLINEAR SOLVER.
Your installation of LINGO does not have the global solver option enabled. The global solver is an add-on option to LINGO. LINGO will use the standard nonlinear solver in upgrading your license in order to enable this option.

204. THE MULTISTART OPTION WAS REQUESTED WITHOUT A LICENSE. LINGO WILL REVERT TO USING THE DEFAULT NONLINEAR SOLVER.
Your installation of LINGO does not have the global solver option enabled. The multistart solver is a component of the global solver add-on option. LINGO will use the standard nonlinear solver in place of the multistart solver. You can contact LINDO Systems for information on upgrading your license in order to enable this option.

205. THE MODEL IS POORLY SCALED AND MAY YIELD ERRATIC RESULTS. THE UNITS OF THE ROWS AND VARIABLES SHOULD BE RESCALED SO THE COEFFICIENTS COVER A MUCH SMALLER RANGE.
After LINGO generates a model, it checks all the nonzero coefficients in the model and computes the ratio of the largest to smallest coefficients. This ratio is an indicator of how well the model is scaled. When the ratio gets to be too high, scaling is considered to be poor, and numerical difficulties may result during the solution phase. If the scaling ratio exceeds the value of the *SCALEW* parameter, LINGO will display this error message. The default value for *SCALEW* is 1e9. Instead of simply increasing the *SCALEW* setting to eliminate error 205, we strongly suggest that you attempt to rescale the units of your model so as to reduce the largest-to-smallest coefficient ratio.

In some instances, changing the units of measure can be an easy way to improve a model's scaling. For instance, suppose we have a model with the following budget constraint in dollar units:

1000000 * X + 2200000 * Y + 2900000 * Z <= 5000000;

This constraint introduces several large coefficients into the model. If we rewrote the constraint so that it is in units of millions of dollars, then we would have:

X + 2.2 * Y + 2.9 * Z <= 5;

The coefficients in this new constraint are much less likely to present a problem.

As part of this error message, LINGO reports the values of the largest and smallest coefficients, as well as where they appear in the model. This information should help in tracking down the problem. You may also run the *Solver|Generate* command to track down other extreme coefficients. This *Generate* command displays the full, generated model and specifically lists all the coefficients.

206. NOT USED.

207. MISSING LEFT PARENTHESIS.
A unmatched right parenthesis was encountered in the model. Use the *Edit|Match Parenthesis* command to help pair up parentheses.

208. @WRITEFOR MAY ONLY APPEAR IN A DATA AND CALC SECTIONS.
The *@WRITEFOR* function is permitted only in the data and calc section of a model. You will need to move the expression to a data or calc section.

209. RELATIONAL OPERATORS NOT ALLOWED IN @WRITEFOR STATEMENTS.
The *@WRITEFOR* function is used to display output. Constraint relational operators may not be used inside the *@WRITEFOR* function.

210. INVALID USAGE OF @WRITEFOR FUNCTION.
The *@WRITEFOR* function is being used incorrectly. *@WRITEFOR* may only be used in the data and calc sections for the purpose of creating reports. Refer to the documentation for the correct usage of *@WRITEFOR.*

211. ARITHMETIC ERROR IN OUTPUT OPERATION.
An arithmetic error (e.g., 1/0) occurred inan output statement. You will need to correct the error to continue.

212. SUBSCRIPT OUT OF RANGE ON SET NAME: *SET_NAME*
A subscript was found to be out of range while attempting to output a set member. Check all output operations that refer to the set *SET_NAME* for correctness.

213. TEXT OPERAND NOT PERMITTED HERE.
A text argument to a function was encountered where something other than text was expected. Please check all function arguments to be sure they are correct.

214. DUPLICATE INITIALIZATION OF A VARIABLE.
A variable has been initialized more than one time in the model's data sections. Eliminate all duplicate variable initializations.

215. OUTPUT MODIFIERS NOT ALLOWED HERE (E.G., @DUAL)
Output modifiers (e.g., *@DUAL, @RANGEU, @RANGED*) are not allowed here. Remove them to continue.

216. PREFIX FUNCTION EXPECTED A TEXT ARGUMENT.
Output modifiers (e.g., *@DUAL, @RANGEU, @RANGED*) are not allowed here. Remove them to continue.

217. PREFIX FUNCTION EXPECTED A NUMERIC ARGUMENT.
A function was expecting a numeric argument, but found something other than a numeric value. Check all function arguments for correctness.

218. PREFIX FUNCTION EXPECTED A ROW OR VARIABLE INDEX ARGUMENT.
A function was expecting a variable or row name as an arguments but found something different. Check all function arguments for correctness.

219. UNABLE TO FIND A SPECIFIED INSTANCE OF A ROW NAME.
You've requested the value of a row that does not exist in the model.

220. UNSUPPORTED OPERATION ON TEXT OPERAND.
You've attempted to perform an undefined operation on a text operand.

221. ARGUMENT OVERFLOW IN @WRITE() OR @WRITEFOR().
This error occurs when there are too many text objects being output within a single *@WRITE* or *@WRITEFOR* function call. Reduce the size of any large arguments lists to these functions.

222. A VARIABLE OR ROW REFERENCE WAS EXPECTED.
A variable or row reference was expected in this location. You will need to correct this to continue.

223. A DYNAMIC RANGE EXCEEDED BOUNDARY LIMITS WRITING TO RANGE: *RANGE_NAME*
You attempted to write more values to a spreadsheet than it can actually hold. In general, this means the dynamic range created by LINGO to receive all the values has overflowed the row limit of the spreadsheet. An option is to specify the range yourself so that it stays within the limits of the workbook.

224. THE FOLLOWING RANGE IS TOO SMALL TO RECEIVE ALL REQUESTED OUTPUT: *RANGE_NAME*
You've specified a workbook range that is too small to receive all the exported values. You'll need to increase the size of the range.

225. INVALID FORMAT FOR A CALC EXPRESSION.
A calc section may not contain simultaneous expressions that are to be optimized. Calc sections can only contain explicit statements that are to be executed in sequential order. Because of this, various functions related to model/simultaneous sections are not permitted in the calc section (e.g., *@GIN* and *@FREE*). You must remove any disallowed functions to continue.

226. DEFAULT WORKBOOK NOT OPEN.
If all arguments are omitted to the *@OLE* spreadsheet interface function, then Excel must be open with a workbook in memory (*@OLE* will default to using this open workbook). Open Excel and then load the workbook containing the data for your model.

227. THE FOLLOWING RANGE: *RANGE_NAME* MUST HAVE A COLUMN COUNT EQUAL TO THE NUMBER OF MODEL OBJECTS BEING IMPORTED/EXPORTED.
When importing values from or exporting values to a workbook you need to consider the number of model objects involved. When multiple objects are being either sent to or received from a single range in a workbook, the number of columns in the range must coincide with the number of model objects. Keep in mind that a derived set counts as more than one object depending upon its dimension. You must either adjust the size of the workbook range to the correct number of columns or edit the set of model objects involved in the interface statement.

228. INVALID API PARAMETER INDEX.
The parameter index you specified as part of the *APISET* command is not valid. Refer to the LINDO API documentation for a list of valid parameter indices.

229. INVALID API PARAMETER TYPE.
The parameter type you specified as part of the *APISET* command is not valid. The parameter type must be specified as either "int" for integer or "double" for double precision floating point. Refer to the LINDO API documentation for a list of valid parameter indices and their types.

230. INVALID API PARAMETER VALUE.
The parameter value you specified as part of the *APISET* command is not valid. Refer to the LINDO API documentation for a list of valid parameter indices and their permissible values.

231. UNABLE TO SET LINDO API PARAMETER.
The parameter value you specified as part of the *APISET* command is not valid. Refer to the LINDO API documentation for a list of valid parameter indices and their permissible values. To clear all parameter values set with *APISET* enter the command: `APISET DEFAULT`.

232. RANGE ANALYSIS NOT AVAILABLE ON QUADRATIC PROGRAMS.
LINGO cannot currently perform range analysis on quadratic programs. You'll need to disable range analysis by running the *Solver|Options* command, selecting the *General Solver* tab, and set the Dual Computations option to disable range analysis. On platforms other than Windows, you'll need to set the DUALCO option to disable range analysis.

233. THE DEBUGGING PROCESS WAS INTERRUPTED.
You interrupted the model debugger before it could complete its analysis. Given this, the resulting report will be incomplete or empty.

234. *N* LARGE NUMBER(S) WERE TRUNCATED TO A VALUE OF: *<MACHINE_INFINITY>*.
You attempted to input one or more numbers that are too large to be handled on your hardware. LINGO reduced the numbers to your machine's infinity. In general, you should scale your models so that very large numbers are not required.

235. THE MODEL IS NONLINEAR AND MAY NOT BE GENERATED AS A LINEAR PROGRAM
You have attempted to generate a nonlinear model with the *Assume model is linear* option enabled. You will need to run *the Solver|Options* command, select the *Model Generator* tab, and disable this option. (Alternatively, from LINGO's command line enter the SET LINEAR 0 command.)

236. INCORRECT @FORMAT USAGE IN AN OUTPUT STATEMENT.
The arguments to an *@FORMAT* reference are not valid. *@FORMAT* requires one numeric argument (the numeric value to be formatted) and one text argument (the format template). For example,

```
@FORMAT( X,  '14.5g')
```

will cause X to be displayed using 5 significant digits in a field of 14 characters.

237. RANGE ANALYSIS NOT CURRENTLY AVAILABLE ON THIS MODEL.
LINGO does not have range analysis data available for the model. You'll need to disable range analysis by running the *Solver|Options* command, selecting the *General Solver* tab, and set the *Dual Computations* option to enable range analysis. You should then re-solve the model.

238. OBJECTS DISPLAYED BY @TABLE ARE LIMITED TO A DIMENSION OF: 16.
The *@TABLE* function can be used to display sets and attributes in tabular format. At the moment, these objects are limited to a maximum dimension of 16, which should be more than adequate for most models.

239. A REPORT IS TOO WIDE TO DISPLAY. INCREASE THE PAGE WIDTH LIMIT AND TRY AGAIN.
You have requested a report that is too wide for the current page width setting. You will need to increase this setting by running the *Solver|Options* command, selecting the *Interface* tab and then increase the *Width* parameter in the *Page Size Limits* box. (Alternatively, in LINGO's comman line you can adjust the line width parameter with the *SET LINLEN* command.)

240. INVALID USE OF @TABLE OUTPUT FUNCTION.
The model contains an invalid reference to the *@TABLE* function. The valid forms of arguments lists for *@TABLE* are:

- @TABLE(AttrName|SetName)
- @TABLE(AttrName|SetName, HzPrimSet)
- @TABLE(AttrName|SetName, HzPrimSet1,…, HzPrimSetI)
- @TABLE(AttrName|SetName, HzPrimSet1,…, HzPrimSetK)
- @TABLE(AttrName|SetName, HzPrimSet1,…, HzPrimSetN, NHz)

Currently, *@TABLE* output can only be routed to either the standard output device or text files.

241. @IFC STRUCTURE IS TOO DEEPLY NESTED.
Calc sections can use *@IFC/@ELSE* statements to implement conditional branching. Presently, you may nest *@IF* statements up to 64 levels deep. You will receive this error message if your model exceeds this limit.

242. INVALID USE OF @IFC CONSTRUCT.
Calc sections can use *@IFC/@ELSE* statements to implement conditional branching. You will receive this error message if an *@IFC* is note being used correctly. An example would be an *@ELSE* statement appearing before a corresponding *@IFC*. Refer to the documentation on the use of *@IFC* for more information.

243. MULTIPLE @ELSE BLOCKS WERE FOUND.
Calc sections can use the *@IFC/@ELSE* statements to implement conditional branching. You will receive this error message if a compound *@IFC* statement contained more than one *@ELSE* branch.

244. @IFC CONTROL STRUCTURE ONLY ALLOWED IN CALC SECTIONS.
Calc sections can use the *@IFC/@ELSE* statements to implement conditional branching. You have attempted to use these statements outside of a calc section. You will need to either move them to a calc section or delete them.

245. AN UNTERMINATED @IFC STRUCTURE WAS FOUND.
Calc sections can use the *@IFC/@ELSE* statements to implement conditional branching. An *@IFC/@ELSE* compound statement is terminated with a right parenthesis, which, in this instance, is missing. You will need to add the terminating right parenthesis at the appropriate place.

246. THE @SOLVE FUNCTION IS ONLY VALID IN CALC SECTIONS.
A number of command functions are allowed only in calc sections, one of which is the *@SOLVE* function for solving a model. You will need to either delete the function reference or move it to a calc section.

247. INVALID USE OF A SUBMODEL NAME.
LINGO allows you to define submodels within a main model. These submodels can be solved using the *@SOLVE* function in calc sections. Your model has an invalid reference to a submodel. You must remove this reference in order to proceed.

248. SUBMODEL NAMES MUST BE UNIQUE.
LINGO allows you to define submodels within a main model using the *SUBMODEL* statement. These submodels can be solved using the *@SOLVE* function in calc sections. You

have attempted to use the same submodel name more that once. You must change the names of the submodels so that all the names are unique.

249. SUBMODEL NOT VALID HERE.
LINGO allows you to define submodels within a main model using the *SUBMODEL* statement. These submodels can be solved using the *@SOLVE* function in calc sections. You have attempted to locate a submodel in an invalid section of the model. Please move the submodel to the main part of the model, outside of all data, calc and init sections.

250. SUBMODEL NOT DEFINED YET.
LINGO allows you to define submodels within a main model using the *SUBMODEL* statement. These submodels can be solved using the *@SOLVE* function in calc sections. You have referenced a submodel that has not been defined. Submodels must be defined in the model before any occurrences of references to them. You may be able to solve this error by moving the submodel in question to an earlier position in the model text.

251. MODEL CONTAINS AN UNTERMINATED SUBMODEL STATEMENT.
LINGO allows you to define submodels within a main model using the *SUBMODEL* statement. These submodels can be solved using the *@SOLVE* function in calc sections. Your model contains a submodel without a terminating *ENDSUBMODEL* statement.

252. A VARIABLE OR ROW REFERENCE WAS EXPECTED.
LINGO was expecting a reference to a row or variable name. You will need to correct the problem to continue.

253. OPERATION ALLOWED ONLY IN CALC SECTIONS.
A number of operators are allowed only in calc sections. An example would be the *@SOLVE* function. You will need to either delete the function reference or move it to a calc section.

254. INVALID PARAMETER ARGUMENT FOR @SET(PARAM, VALUE).
The *@SET* function can be used in a calc section to set any of LINGO's parameters . You must provide the parameter's name as text, and a permissible value for the parameter. For instance, to turn set the iteration limit to one million, you would use *@SET('ITRLIM', 1.e6).* You can refer to documentation on the *SET* command to learn all the parameters and their names.

255. IMPROPER USE OF A CALC COMMAND FUNCTION.
You may include a calc section in your model that contains a series of commands for LINGO to execute. Examples would include the *@SOLVE* and *@WRITE* functions. This error is triggered when one of these command functions is used incorrectly. You should refer to the documentation to learn the correct usage and syntax for the function.

256. @WHILE() ALLOWED ONLY IN CALC SECTION.
At present, the *@WHILE* statement is allowed only in calc sections and may not appear in the model section. You may be able to use the *@FOR* statement instead.

257. INVALID ARGUMENT FOR @SET('PARAM', VALUE).
In order to set a parameter via the *@SET* command you must specify a correct parameter name in quotes and a correct value. Please be sure you have spelled the parameter name

correctly and that you have placed it in quotes. Also, check that the specified value is permissible for the parameter.

258. MODEL EXECUTION HALTED. STOP STATEMENT ENCOUNTERED.
LINGO encountered an *@STOP* statement while processing a calc section. This terminates execution of the current model. If a text string was specified in the reference to *@STOP,* then it will also be displayed as part of this message. *@STOP* is typically used in response to some unexpected condition.

259. FILE NAME LENGTH EXCEEDS: N
You've specified a file name whose length exceeds the maximum length allowed of *N* characters. You will need to shorten the length of the file name to continue.

260. @BREAK MAY ONLY APPEAR IN @WHILE AND @FOR LOOPS INSIDE CALC SECTIONS
The *@BREAK* statement is used for unconditional breaks out of *@FOR* and *@WHILE* loops. It may also only be used inside calc sections, and is not valid in the model section. You will need to delete the reference to *@BREAK* to continue.

261. INVALID ARGUMENT FOR @RELEASE. ARGUMENTS MUST BE A SINGLE VARIABLE REFERENCE, E.G., @RELEASE(X).
The *@RELEASE* statement is used in calc sections to release a variable that was previously fixed so it may once again become optimizable. To release an entire attribute, place the *@RELEASE* reference in an *@FOR* loop, e.g., *@FOR(SET(I): @RELEASE(X(I))).*

262. INVALID ARGUMENT FOR @APISET(PARAM-ID, 'INT|DOUBLE', VALUE).
The *@APISET* statement is used in calc sections to set options in the LINDO API (LINGO's solver engine) that aren't available through the standard option set in LINGO. The argument list consists of a parameter-id (an integer value), a string specifying if the parameter is an integer or a double precision value, and the parameter value. More information on the parameters available in the LINDO API may be found in the LINDO API documentation and the Lindo.h header file included with your LINGO installation.

263. MODEL WAS FOUND TO BE FEASIBLE AND CANNOT BE DEBUGGED.
You have run the *Solver|Debug* to find the infeasibilities in a model that was found to be feasible. In which case the model debugger will stop its search for an infeasible subset of constraints.

264. LINEARIZED MODEL TOO LARGE FOR THIS VERSION.
You have enabled LINGO's linearization option, which attempts to rewrite nonlinear models in an equivalent, but linear, manner. This requires the addition of variables and constraints, whose increased counts have exceeded the limits of your version. You will need to either disable linearization or upgrade to a larger capacity version of LINGO.

265. DUPLICATE VARIABLES WERE FOUND IN THE SEMI-CONTINUOUS SET.
LINGO supports semi-continuous (SC) variables. SC variables are restricted to being either 0, or to lie within some non-negative range. Semi-continuous variables are declared using the *@SEMIC* statement. A variable may only be declared semi-continuous once in a model. You

will receive this error message if one or more variables are declared semi-continuous more than one time.

266. BOUNDS OUT OF RANGE ON A SEMI-CONTINUOUS VARIABLE.
LINGO supports *semi-continuous* (SC) variables. SC variables are restricted to being either 0, or to lie within some non-negative range. Semi-continuous variables are declared using the *@SEMIC* statement. The syntax for *@SEMIC* is:

@SEMIC(*lower_bound, variable, upper_bound*)

You will receive this error whenever the variable bounds fall outside the range permitted by LINGO.

267. @SPCHANCE, @SOS OR @CARD SET LABEL EXCEEDS LIMIT OF: <N>
LINGO supports *chance-constrained constraint sets, special ordered sets* (SOS) of variables as well as *cardinality* sets of variables. Each set is denoted by a unique name/label, which is limited to *<N>* characters in length. You will need to choose a shorter label name for the variable set.

268. VARIABLE SET LABEL USED TO REPRESENT MORE THAN ONE SET TYPE.
LINGO supports *chance-constrained constraint sets, special ordered sets* (SOS) of variables as well as *cardinality* sets of variables. Each set is denoted by a unique label/name. You have attempted to use the same set label for different set types (e.g., SOS1 *and* SOS2). Check to be sure you've specified the correct set name and/or set type.

269. THE FIRST ARGUMENT TO @SOS/@CARD MUST BE A STRING.
LINGO supports special ordered sets (SOS) of variables as well as cardinality sets of variables. Each set is denoted by a unique label/name. The first argument to the *@SOS* and *@CARD* functions must be a string representing the label name for the particular variable set. Furthermore, the label name must be contained in single or double quotation marks.

270. THE SOS/CARD SET: *<SET_NAME>* CONTAINS THE FIXED VARIABLE: *<VAR_NAME>*
LINGO supports special ordered sets (SOS) of variables as well as cardinality sets of variables. All variables belonging to these sets must not be fixed, i.e., they must be optimizable decision variables. You will need to remove the fixed variable from the designatged set.

271. NO CARDINALITY WAS SPECIFIED FOR @CARD SET: *<SET_NAME>*
LINGO supports *cardinality* sets of variables. All variables belonging to a particular cardinality set must sum to a specified integer value. To form a cardinbality set for variable *X, Y* and *Z* that sums to 2, you would specify the following in your model:

```
@CARD( 'MYSET', X);
@CARD( 'MYSET', Y);
@CARD( 'MYSET', Z);
@CARD( 'MYSET', 2);
```

You will receive this error message when the cardinality value, @CARD('MYSET', 2) in this case, is omitted. You will need to add a cardinality value reference for the set.

272. DUAL FORMULATIONS MAY ONLY BE GENERATED FOR LINEAR PROGRAMS.

LINGO can generate the dual formulation for linear programming models. You have attempted to generate the dual for a model that is not a linear program, i.e., the model is either nonlinear or it contains integer variables. If it is possible to rewrite the model so that it becomes linear without any integer variables, then you should be able to generate the dual formulation. Otherwise, it will not be possible to generate the dual formulation.

273. THE LICENSE MANAGER RETURNED THE FOLLOWING ERROR: *<LICENSE_ERROR>*

LINGO's license manager was unable to find a valid license file. The license file is normally located in your main LINGO subdirectory under the name: lndlng<version>.lic, where <version> is the version number of your LINGO installation. You may want to examine this file for any obvious errors or problems. The license manager will also return a brief string with more specific information (e.g., "No dongle found"), which will be displayed as part of this message. This additional information should help you narrow down the problem. If you are still unable to resolve this problem, please contact LINDO Systems Technical Support for assistance.

274. THE MODEL CONTAINS FUNCTIONS NOT SUPPORTED BY THE GLOBAL SOLVER.

The model contains functions not supported by the global solver. You will need to either remove these functions or disable the global solver options.

The global solver supports most of the functions of the LINGO language. However, there are few that aren't currently, or can't be supported. You may refer the *Use Global Solver* section of Chapter 5 for a listing of functions not supported by the global solver.

275. A BARRIER LICENSE IS REQUIRED TO RUN BARRIER IN A CORE.

LINGO can run multiple linear programming solvers in parallel on multi-core machines. You have requested to run the barrier solver in parallel, but your installation does not have a license for the barrier solver. You will need to run the *Solver*|Options command, go to the Linear Solver tab, and disable the request to run the barrier solver in parallel.

276. PRIMAL2 MAY NOT BE RUN WITHOUT BARRIER RUNNING IN PARALLEL.

LINGO can run multiple linear programming solvers in parallel on multi-core machines. You have requested to run the Primal2 solver in parallel without also requesting the barrier solver. You will need to run the *Solver*|Options command, go to the Linear Solver tab, and either enable the barrier solver to run parallel or disable Primal2 from running in parallel.

277. INCONSISTENT BOUNDS ON VARIABLE: *<NAME>* IN THE SOS/CARD SET: *<SET>*

LINGO supports special ordered sets (SOS) of variables as well as cardinality sets of variables. You have specified bounds on variables in the designated set that are inconsistent

with variables belonging to such a set. Check to be sure you've specified the bounds correctly.

278. ERROR IN DYNAMIC SET DEFINITION FOR SET: SET_NAME.
In some instances, you may not need to explicitly define a set. Instead, you can have LINGO infer the dimensions of the set from the number data points assigned to an attribute of the set in the model's Data section. This is referred to as a *dynamic set definition*. It's not always possible for LINGO to accurately deduce a set's dimension from a data statement, in which case, you will receive this error message. A valid solution is to explicitly define the set in question in the model's Sets section.

279. AN UNALLOCATED SET WAS REFERENCED: <*SET_NAME*>
You've referenced members of a set that has not been allocated. You must allocate sets in a model before they can be referenced. Many times, all that is required is to move the set's allocation to a point in the model that precedes any references to it. Set allocation refers to the declaration of the set's members. Allocation is generally done in a sets section or in a data section.

280. INVALID SYNTAX FOR A CALC SECTION I/O OPERATION.
You have incorrectly specified an I/O function (*@TEXT*, *@OLE*, *@ODBC*, *@POINTER*) in a calc section. Please refer to the documentation for the correct usage of these functions.

281. I/O OPERATIONS MAY ONLY APPEAR IN CALC, DATA AND INIT SECTIONS.
You have attempted to us an I/O function (*@TEXT*, *@OLE*, *@ODBC*, *@POINTER*) in the model section. These functions are available only in calc, data and init sections of a model. Please refer to the documentation for the correct usage

282. SETS MAY ONLY BE INITIALIZED IN SETS AND DATA SECTIONS.
You have attempted to initialize a set outside of a set and data section. You will need to move the initiation statement to the correct model section.

283. TEXT AND NUMERIC OBJECTS CAN'T BE COMBINED IN A SINGLE @POINTER OPERATION.
You have attempted to reference both text and numeric objects in a single *@POINTER* statement. A given reference to the *@POINTER* function can only import/export items of one type, i.e., text or numeric. You will need to split the statement up into two or more statements, such that each new statement only references either text or only references numeric objects.

284. INVALID @POINTER SET I/O OPERATION. @POINTER CAN ONLY REFERENCE ONE SET AT A TIME.
You have attempted to reference both more than one set in a single *@POINTER* statement. A given reference to the *@POINTER* function can only import/export one set at a time. You will need to split the statement up into two or more statements, such that each new statement only references no more than one set.

285. INCORRECT USAGE OF @RANK OR @SORT.
You used one of the *@RANK* or *@SORT* sorting functions incorrectly. Proper usage is either:

ATTR_RANK = @RANK(ATTR1[,...,ATTRN);
or,
ATTR_SORT = @SORT(ATTR1[,...,ATTRN);

where *ATTR_RANK (ATTR_SORT)* receives the ranking (sorting) of attribute *ATTR1*, with ties being broken by the optionally supplied attributes *ATTR2* through *ATTRN*. In addition, *@RANK* and *@SORT* are only valid in calc sections. Please refer to the documentation for more details.

286. INCORRECT USAGE OF @SOLU.
You used the *@SOLU* sorting solution report function incorrectly. This function displays the standard LINGO solution report and may be called in calc sections. Proper usage is:

@SOLU([0|1[, OBJECT_NAME[, "HEADER_TEXT"]]]);

where the first argument is set to 1 if only nonzero variable values are desired (and binding rows), *OBJECT_NAME* is the optional name of an object to report on (the entire report will be generated otherwise), and the optional *HEADER_TEXT* is a header line to display in the report. Please refer to the documentation on *@SOLU* for more details.

287. INCORRECT USAGE OF @SPSTGVAR/@SPSTGRNDV/@SPSTGROW.
You used either the @SPSTGVAR, @SPSTGRNDV or @SPSTGROW function incorrectly. Proper usage is:

@SPSTG[VAR|RNDV|ROW](STAGE, VARIABLE_OR_ROW_NAME);

where the first argument is the stage of the variable or row and the second argument the name of the variable or row.

288. INVALID STAGE VALUE.
An invalid stage value was specified. Stage values must be non-negative, integer values. In addition, random variables must have stages of 1, or higher. Please correct the indicated stage value and retry.

289. STOCHASTIC PROPERTIES MAY ONLY BE ASSIGNED ONCE.
You may assign stochastic properties to variable and rows one time only. Please remove multiple declarations and retry.

290. INVALID DECLARATION OF A STOCHASTIC DISTRIBUTION.
A stochastic distribution declaration was incorrectly specified. Please refer to the *Declaring Distributions* section of Chapter 14 for clarification.

291. NAME IS TOO LONG.
The indicated name in the model exceeds the maximum name length of 64 characters. You must shorten the name to continue.

292. NAME ALREADY IN USE.
The indicated name is already in use. To continue, you will need to change the.name so that it is unique.

293. INVALID DISTRIBUTION PARAMETER.
The indicated parameter is invalid for the chosen probability distribution. Please refer to the *Declaring Distributions* section of Chapter 14 for clarification.

294. INVALID USE OF @SPTABLEOUTC.
The *@SPTABLEOUTC* function for declaring discrete distribution outcomes was used incorrectly. The correct usage is:

@SPTABLEOUTC('DIST_NAME', OUTC_PROB, OUTC_VAL1[,..., OUTC_VALN])

For more information, refer to the Declaring Distributions Using Outcome Tables in Chapter 14.

295. OUTCOME TABLE NAME NOT PREVIOUSLY DEFINED.
You have attempted to use the designated outcome table name before defining it. For more information on defining outcome tables, please refer to the *Declaring Outcome Tables* section of Chapter 14. Note that the declaration of an outcome table must occur before the name is used for other purposes.

296. INCORRECT NUMBER OF OUTCOMES FOR OUTCOME TABLE.
When declaring an outcome table using *@SPDISTTABLE* you declare its total number of outcomes. The number of outcomes loaded via *@SPTABLEOUTC* does not match this total number for the specified table. Please check the declaration of the table and the calls to *@SPTABLEOUTC*. For more information on defining outcome tables, please refer to the *Declaring Outcome Tables* section of Chapter 14.

297. INVALID USE OF @SPTABLERNDV.
The *@SPTABLERNDV* is used to associate random variables with an instance of a discrete outcome table. The format of *@SPTABLERNDV* is:

@SPTABLERNDV('DIST_INSTANCE_NAME', RANDOM_VAR_1[, ... , RANDOM_VAR_N]);

Please check to see that the arguments conform to this syntax. For more information on defining outcome tables and using *@SPTABLERNDV*, please refer to the *Declaring Outcome Tables* section of Chapter 14.

298. INCORRECT NUMBER OF RANDOM VARIABLES ASSIGNED TO DISTRIBUTION.
The *@SPTABLERNDV* is used to associate random variables with an instance of a discrete outcome table. The format of *@SPTABLERNDV* is:

@SPTABLERNDV('DIST_INSTANCE_NAME', RANDOM_VAR_1[, ... , RANDOM_VAR_N]);

When the parent outcome table was defined using *@SPDISTTABLE*, the number of associated random variables was also declared. The number of variables associated with the specified outcome table instance does not match this declared number. For more information on defining outcome tables and using *@SPTABLERNDV*, please refer to the *Declaring Outcome Tables* section of Chapter 14.

299. VARIABLES ASSIGNED TO DISTRIBUTIONS MUST BE DESIGNATED AS RANDOM VARIABLES.
All variables associated with a probability distribution must first be declared as random variables using the *@SPSTGRNDV* function. For more information on identifying random variables and associating them with distributions, please refer to the *Language Features for SP Models* section in Chapter 14.

300. RANDOM VARIABLES CAN ONLY BE ASSIGNED TO ONE DISTRIBUTION.
Only one random variable may be associated with a probability distribution. For more information on identifying random variables and associating them with distributions, please refer to the *Language Features for SP Models* section in Chapter 14.

301. INVALID USE @SPTABLEINST.
The *@SPTABLEINST* function is used to declare a particular instance of an outcome table. The syntax for *@SPTABLEINST* is as follows:

@SPTABLEINST('PARENT_TABLE_NAME', 'TABLE_INSTANCE_NAME')

The parent table must have already been declared using *@SPDISTTABLE*. For more information on declaring outcome tables and using *@SPTABLEINST*, please refer to the *Declaring Outcome Tables* section of Chapter 14.

302. INVALID USE @SPSAMPSIZE.
The *@SPSAMPSIZE* function is used to declare a sample size for a particular stage of a stochastic programming model. The syntax for *@SPSAMPSIZE* is as follows:

@SPSAMPSIZE(STAGE, SAMPLE_SIZE)

For more information on using *@SPSAMPSIZE*, please refer to the *Declaring Parametric Distributions* section of Chapter 14.

303. PROBABILITIES DO NOT SUM TO 1 FOR OUTCOME TABLE.
Outcome tables are loaded with outcomes and their respective probabilities using the *@SPDISTOUTC* function. You will receive this error message if the sum of the probabilities for all outcomes for the specified outcome table do not sum to 1. You should check the probability values for the specified distribution. For more information on defining outcome tables and using *@SPTABLEOUTC*, please refer to the *Declaring Outcome Tables* section of Chapter 14.

304. NO OUTCOMES DECLARED FOR OUTCOME TABLE.
Outcome tables are loaded with outcomes and their respective probabilities using the *@SPDISTOUTC* function. You will receive this error message if no outcomes were declared for the specified outcome table. You will need to load at least one or more outcomes into the

specified table instance. For more information on defining outcome tables and using *@SPTABLEOUTC*, please refer to the *Declaring Outcome Tables* section of Chapter 14.

305. NO RANDOM VARIABLES ASSIGNED TO STAGE.
Each stage other than stage 0 of a stochastic programming model must have at least one random variable assigned to it. Random variables are assigned to stages using the *@SPSTGRNDV* function, which has the following syntax:

@SPSTGRNDV(STAGE, VARIABLE_NAME);

You will need to assign a random variable to the specified stage or reduce the number of stages in the model. For more information on identifying random variables and associating them with distributions, please refer to the *Language Features for SP Models* section in Chapter 14.

306. SP CORRELATION TYPES CAN'T BE MIXED IN THE SAME MODEL.
There are three methods available for inducing correlations between random variables - Pearson, Kendall and Spearman. You may use only one method throughout an entire model. Trying to use more than one particular method will trigger this error message.

307. STOCHASTIC PROGRAMS MUST HAVE AT LEAST TWO STAGES.
Stochastic programs must have two, or more, stages. The first stage, stage 0, is the initial decision stage. The second stage, stage 1, is a recourse decision stage which occurs after the stage 1 random variable values have been revealed. Subsequent rounds of random variable revelations and recourse decisions are permitted but not required. For more information on adding random variables and stages to an SP model, please refer to the *Language Features for SP Models* section in Chapter 14.

308. LINEARIZATION OPTION NOT PERMITTED ON STOCHASTIC PROGRAMS.
LINGO has the capability to reformulate certain nonlinear expressions so that they become linear, while maintaining mathematical equivalency to the original model. This linearization option is only available on deterministic models. If you enable the linearization option on SP models, you will receive this error message. An option is to enable the global solver instead (assuming your license allows for this optional feature). If the SP model can be completely linearized, the global solver will realize this and perform the linearization, thereby allowing for a globally optimal solution.

309. THE DETEQ MODEL GENERATED BY THE SP SOLVER EXCEEDS THE CAPACITY OF THIS VERSION.
LINGO's SP solver generates a deterministic model to solve the original SP model. This model is called the deterministic equivalent (deteq). Depending on the number of stages, random variables and samples sizes, the deteq may get to be very large. In which case, there is a possibility that it will exceed the capacity of your LINGO license. This can occur even when the core model lies well within the capacity of your license. You will need to either reduce the size of the model (by using fewer stages, less random variables or smaller sample sizes), or upgrade to a license with more capacity.

310. INVALID SP CORRELATION MATRIX.
The correlation matrix entered for the random variables using *@SPCORREL* is not valid. Please check the correlations to see that they lie within a range of [-1,+1]. Also check to see that explicit correlation values lie within any implied range, e.g., CORR(X, Y) = .9 and CORR(Y, Z) = .9 implies that CORR(X, Z) is at least .62.

311. ATTEMPT TO CORRELATE NON-RANDOM VARIABLES.
In a stochastic programming (SP) model, only random variable may be correlated via the *@SPCORREL* function. In addition, the correlated variables must be flagged as random variables using the *@SPSTGRNDV* function prior to inputting their correlation coefficient using *@SPCORREL*.

312. INVALID CORRELATION COEFFICIENT.
You've input an invalid correlation coefficient that lies outside [-1,+1]. Please correct the coefficient's value and try again.

313. SCENARIO INDICES MUST BE INTEGRAL.
In referring to a scenario, you've attempted to use an out-of-range index. In other words, the index is either less than 1 or greater than the total number of scenarios. Please correct the index and retry.

314. SCENARIO INDEX OUT OF RANGE.
The scenario index must be an integer greater-than-or-equal-to 1 and may not exceed the total number of scenarios in the model.

315. THE FOLLOWING SET MEMBER: *SET-MEMBER* APPEARS MORE THAN ONCE IN SET: *SET-NAME*.
The indicated set member, *SET-MEMBER*, appears more than one in its set *SET-NAME*. Please correct the duplicated set members to continue.

316. UNABLE TO GENERATE DETERMINISTIC EQUIVALENT.
LINGO's SP solver generates a deterministic model to solve the original SP model. This model is called the deterministic equivalent (deteq). Depending on the number of stages, random variables and samples sizes, the deteq may get to be very large. There is a remote chance that LINGO will not be able to generate the deteq. This would be a very unusual situation. If there is no obvious reason for this problem, you may wish to contact LINDO Systems Technical Support for assistance.

317. THE MODEL IS NOT A STOCHASTIC PROGRAM.
You've attempted to request information relevant only to stochastic programs while running a deterministic model.

318. THIS OPERATION REQUIRES THE STOCHASTIC PROGRAMMING OPTION.
You're attempting to solve a stochastic program (SP) on a LINGO installation that does not support the SP option. You will need to upgrade your license to include the option.

319. ONE OR MORE SP SCENARIO SOLUTIONS DO NOT EXIST.
LINGO was unable to generate solutions for all the models scenarios. This would be a very unusual situation. If there is no obvious reason for this problem, you may wish to contact LINDO Systems Technical Support for assistance.

320. @SPLOADSCEN FAILED FOR SCENARIO: *N*
LINGO was unable to load the solution for scenario N via the @SPLOADSCEN function. This would be an unusual situation. If there is no obvious reason for this problem, you may wish to contact LINDO Systems Technical Support desk for assistance.

321. UNABLE TO GENERATE INDIVIDUAL SCENARIO MODEL.
LINGO was unable to generate a requested individual scenario for an SP mode. This would be an unusual situation. If there is no obvious reason for this problem, you may wish to contact LINDO Systems Technical Support for assistance.

322. NO CONSTRAINTS WERE ASSIGNED TO STAGE: *N*
One or more stages in the stochastic program do not have any constraints. If this is an error, then correct it and continue. If this is not an error, then you should fold the random variables from the offending stage into the subsequent stage. This will reduce the total number of stages in the model by one.

323. THE FOLLOWING RANDOM VARIABLE IS NOT ASSIGNED TO A STAGE.
Each random variable declared with *@SPSTGRNDV* must also be assigned to a stochastic distribution. LINGO will display the name of the first unassigned random variable. You will need to use either *@SPDISTTABLE* or *@SPTABLERNDV* to assign the variable to a distribution.

324. THE FOLLOWING RANDOM VARIABLE ATTRIBUTE HAS ELEMENTS BELONGING TO DIFFERENT STAGES.
The attribute in question was assigned to an outcome table using *@SPDISTTABLE*. All elements of the attribute must be random variables and must all be assigned to the same stage using the *@SPSTGRNDV* function. Please check to see that all the elements of the attribute have been assigned to the correct and identical stage.

325. THE FOLLOWING PROBABILITY OR OUTCOME ATTRIBUTE CONTAINS NON-CONSTANT ELEMENTS.
The attribute in question was used to construct an outcome table distribution using *@SPDISTTABLE*. All elements of outcome and probability attributes must be constants and may not be optimizable. Please assign appropriate values to each element of the attribute.

326. THE SP SAMPLE SIZES ARE NOT LARGE ENOUGH TO GENERATE A MEANINGFUL SAMPLE.
The sample sizes in your SP model are not large enough to allow for generation of a meaningful sample of the random variables. Note that models that induce correlation between random variable will require larger sample sizes. You will need to use the *@SPSAMPSIZE* function to increase the sample sizes.

327. UNSUPPORTED MPI INSTRUCTION CODE: *N*

You are attempting to read an MPI format file that contains an instruction code not currently supported by LINGO. LINGO supports most of the frequently used MPI instruction codes, however, it does not support 100% of the codes offered by the MPI format. In order to continue, you will need to either remove the unsupported function code or rewrite the expression using supported instructions codes.

328. MULTIPLE WILDCARD CHARACTERS (*) NOT SUPPORTED.

You are attempting to use the multi-character wildcard (*) more than once in a field. You will need to either eliminate all but one asterisk or use multiple single-character wildcards (%).

329. ERROR EXECUTING AN @CHART FUNCTION.

An *@CHART* function failed to draw the requested chart. This is typically due to lack of memory. Try closing some windows and /or other applications and then run the model again.

330. INVALID CHART TYPE.

You've requested an unsupported chart type. Currently supported types are: Bar, Bubble, Contour, Curve, Histogram, Line, Pie, Radar, Scatter, or Surface.

331. INVALID ARGUMENT FOR AN @CHART FUNCTION.

You've specified an invalid argument for an @CHART function. Please refer to the @CHART documentation for information on valid argument syntax.

332. @CHART NOT SUPPORTED IN LINGO API.

The *@CHART* class of functions for generating charts are only supported in interactive versions of LINGO. If you need charting support in an application that calls the LINGO API, then you will need to find a third-party charting tool.

333. NOT USED

Error code not currently in use.

334. NO RECOURSE VARIABLES ASSIGNED TO STAGE: *N*

You have a stochastic program without recourse variables in stage *N*. You will need to use the *@SPSTGVAR* function to assign one or more recourse variables to stage *N*.

335. INVALID @SPCHANCE USAGE.

Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. You identify these constraint sets with the *@SPCHANCE* function. Refer to the documentation on *@SPCHANCE* for information on proper syntax.

336. CHANCE-CONTRAINED PROGRAMS CAN CURRENTLY HAVE ONLY ONE STAGE.

Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. At present, the CCP solver can handle only single-stage models.

337. THE FOLLOWING CHANCE-CONSTRAINED SET IS EMPTY: *SET-NAME*
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. You've established a CCP set with the *@SPCHANCE* function but have assigned any constraints to the set. Refer to the documentation on *@SPCHANCE* for information on proper syntax.

338. THE CHANCE-CONSTRAINED SET NAME IS ALREADY IN USE.
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. You may establish CCP sets with the *@SPCHANCE* function but set names must be unique and may not be reused. Refer to the documentation on *@SPCHANCE* for information on establishing CCP sets.

339. THE CHANCE-CONSTRAINED SET NAME WAS NOT PREVIOUSLY DEFINED.
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. Refer to the documentation on *@SPCHANCE* for information on defining a CCP set.

340. THE CHANCE-CONSTRAINED ROW: ROW-NAME DOES NOT CONTAIN ANY RANDOM VARIABLES.
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. Each constraint in a CCP set must contain at least one random variable. You will need to use the *@SPRNDVAR* function to identify at least one random variable in the row.

341. THE OBJECTIVE ROW MAY NOT BE ASSIGNED TO A CHANCE-CONSTRAINED SET.
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. Note that a model's objective function may not be assinged to a CCP set – only constraints may be assigned to a CCP set.

342. THE 'ASSUME MODEL IS LINEAR' OPTION MAY NOT BE APPLIED TO STOCHASTIC PROGRAMS.
The *Assume Model is Linear* option is an option available for linear models that reduces peak memory usage. This option is currently not available for stochastic and chance-constrained programs.

343. STOCHASTIC MODELS MUST HAVE AT LEAST TWO SCENARIOS.
A stochastic program must have at least two scenarios. Without at least two scenarios, the model would have no stochastic properties. You will need to increase the sample size using the *@SPSAMPSIZE* function.

344. A NON-CHANCE-CONSTRAINED ROW CONTAINS RANDOM VARIABLES: ROW-NAME
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. Each random variable in a CCP must appear in a CCP constraint set. TYou will get this error message

when a random variable is found in a row not contained in any CCP set. Refer to the documentation on *@SPCHANCE* for information on how to add a constraint to a CCP set.

345. THE FOLLOWING ROW HAS MORE THAN ONE CCP ASSIGNMENT.
Chance-constrained programs (CCPs) are stochastic models that allow one or more sets of constraints to be violated up to a specified percentage of total scenarios. You may establish CCP sets with the *@SPCHANCE* function, but each constraint may appear in no more that one CCP set. Refer to the documentation on *@SPCHANCE* for more information on establishing CCP sets.

346. AN INVALID SET REFERENCE WAS FOUND INSIDE THE MODEL'S CONSTRAINTS.
There is a reference to a set or set member in a constraint. You will need to remove this reference in order to continue.

347. AN INVALID/UNSUPPORTED FUNCTION CODE WAS ENCOUNTERED.
The model was found to contain an unsupported function. In general, this error message indicates a problem in processing the model. You should contact technical support if you encounter this error.

348. UNABLE TO COMPLETE ALTER COMMAND -- PROCESSING TERMINATED.
An *ALTER* command to edit the model was too complicated for execution. You should simplify the command by breaking it up into multiple, more compact commands.

349. THE FOLLOWING VARIABLE VIOLATES ITS BOUNDS: <VAR_NAME>
This error message indicates that a variable whose value was set in either a calc or data section was found to violate its bounds. You can either change the variable's value or modify its bounds to include the variable's value. Another solution is to disable the *Enforce bounds in CALC and DATA* option on the *Model Generator* tab of the *Solver|Options* command.

350. THE FOLLOWING VARIABLE VIOLATES ITS INTEGRALITY CONDITION: <VAR_NAME>
This error message indicates that a variable whose value was set in either a CALC or DATA section was found to violate an integrality condition set with either the *@BIN* or *@GIN* functions. You can either change the variable's value to an integer or disable the *Enforce bounds in CALC and DATA* option on the *Model Generator* tab of the *Solver|Options* command.

351. THE MULTISTART OPTION MAY NOT BE USED IN CONJUNCTION WITH SLP.
You have selected both the Multistart Solver and the Successive Linear Programming (SLP) Solver options. These two solvers may not be used simultaneously. In order to continue, LINGO will disable the Multistart option. You will need to disable at least one of these two options in order to eliminate this error message.

352. NESTED CALC AND/OR PROCEDURE SECTIONS NOT ALLOWED.
Calc sections and/or model procedures may not be nested inside one another. You will need to separate them into individual sections.

353. THE LIMIT ON RECURSIVE PROCEDURE CALLS IS: *N*

There is a limit on the number of times a procedure may call itself. You will need to reduce the number of recursive calls.

354. DEBUGGING CURRENTLY NOT SUPPORTED FOR STOCHASTIC MODELS.

The model debugger may not currently be used on stochastic models. As an alternative, you may generate the stochastic equivalent of the model and debug it. One common problem that makes stochastic models infeasible is having continuous random variables that can take on negative values. These negative values may get assigned to other variables with default lower bounds of zero, making the stochastic model infeasible.

355. INVALID SET MEMBER FOR INSERTION INTO SET: *SET-NAME*

An attempt has been made to insert an invalid set member into set *SET-NAME* using the *@INSERT* function. Set members may only be inserted into *derived sets*, and the member being inserted must be a member of the derived set's parent *primitive set(s).* The following example illustrates:

```
MODEL:

SETS:
S1 /M1, M2, M3/;
S2( S1);
ENDSETS

CALC:
 !The following is valid because M2 is a member
  of S2's parent primitive set, S1;
 @INSERT( S2, @INDEX( M2));

 !The following is not valid because M4 is not a
  member of S1;
 @INSERT( S2, @INDEX( M4));

 !The following is not valid because S1 is not a
  derived set;
 @INSERT( S1, @INDEX( M4));

ENDCALC

END
```

356. INSERTING MEMBERS INTO PRIMITIVE SETS IS NOT CURRENTLY SUPPORTED.

An attempt has been made to insert a set member into a *primitive set* using the *@INSERT* function. Set members may only be inserted into *derived sets*, and the member being inserted must be a member of the derived set's parent primitive sets. The following example illustrates:

```
MODEL:

SETS:
S1 /M1, M2, M3/;
S2( S1);
ENDSETS

CALC:
 !The following is valid because M2 is a member
  of S2's parent primitive set, S1;
 @INSERT( S2, @INDEX( M2));

 !The following is not valid because M4 is not a
  member of S1;
 @INSERT( S2, @INDEX( M4));

 !The following is not valid because S1 is not a
  derived set;
 @INSERT( S1, @INDEX( M4));

ENDCALC

END
```

357. THE ENTIRE MODEL IS FIXED. YOU MAY NEED TO DISABLE FIXED VARIABLE REDUCTION.

When LINGO exports a model to either MPS or MPI format, it first substitutes out any fixed variables in the model. The purpose is to export a model that exactly matches the one LINGO is solving internally. In this instance, all the variables were found to be fixed in value, leaving no model to export. You may disable fixed variable reduction with the *LINGO | Options | Model Generator | Fixed Var Reduction* option. Keep in mind, however, that turning off variable reduction may result in a nonlinear model, which may not be represented in MPS format.

358. IMPROPER USE OF @BLOCKROW(BLOCKID, ROWNAME) FUNCTION.

The *@BLOCKROW* function is used in conjunction with the branch-and-price (BNP) solver to assign rows to blocks. The BNP solver requires that a model be partitioned into two or more blocks of mostly independent rows. The *BLOCKID* argument is a non-negative integer, indicating the block index, while the ROWNAME argument is the name of a row, perhaps indexed, that appears in the model. For more information on *@BLOCKROW*, please refer to the BNP solver documentation.

359. UNABLE TO FIND A BLOCK PARTITIONING.
The branch-and-price (BNP) solver requires that a model be partitioned into two or more blocks of mostly independent rows. This error typically occurs if you are explicitly providing the block structure (as opposed to letting LINGO find the block structure) and an input error was found in the block assignments. For more information on *@BLOCKROW*, please refer to the BNP solver documentation.

360. INVALID PROCEDURE REFERENCE.
Procedures may only be called from within calc sections and may not be embedded in other statements. Refer to the documentation on procedure calls for more information.

361. INVALID USE OF @INTEGRAL.
You have not correctly set up a call to LINGO's numeric integration function, *@INTEGRAL*. The correct usage is:

@INTEGRAL(PROCEDURE, X, LB_X, UB_X, Y);

where *PROCEDURE* refers to the name of the procedure containing the function to integrate, *X* is the variable to integreate over, *LB_X* and *UB_X* are the lower and upper bounds on *X*, and *Y* is the variable that stores the value of the function being integrated. Refer to the *@INTEGRAL* documentation for more information.

362. INVALID USE OF @POSD
The model contains an improperly formulated call to the *@POSD* function for restricting matrices to be positive semi-definite. Correct usage is:

@POSD(X);

where *X* is a matrix of optimizable variables. The matrix must be lower triangular, upper triangular or square. If the matrix is sparse, then it must be lower or upper triangular. *@POSD* statements may not appear in calc sections.

363. BARRIER AND CONIC REQUIRED FOR @POSD
The *@POSD* function for requiring a matrix to be positive semi-definite requires licenses for the optional barrier and conic solvers. You will need to upgrade your license to proceed.

364. @CHART NOT SUPPORTED ON THIS PLATFORM
LINGO's charting functions are not supported on all platforms. You may want to go to www.lindo.com to see if there is a newer release of LINGO that supports these functions.

365. INVALID @EIGEN USAGE.

The *@EIGEN* function returns eigenvectors and eigenvalues for the matrix *A*. The syntax for *@EIGEN* is:

LAMDAR, VR, LAMBDAI, VI, err = @EIGEN(A);

where,

- **A** an *n*x*n* matrix,
- **LAMBDAR** a vector of length *n* for returning the real parts of the eigenvalues,
- **VR** an *n*x*n* matrix for returning the real parts of the eigenvectors (one vector per column),
- **LAMBDAI** a vector of length *n* for returning the imaginary parts of the eigenvalues,
- **VI** an nxn matrix for returning the imaginary parts of the eigenvectors, and
- **err** an error flag that will be 1 if a numeric problem occurs, otherwise 0.

All matrices must be dense and cannot be defined on sparse sets, and *@EIGEN* may only be used in a model's calc section.

As long as at least one left-hand side argument is present, all other arguments may be omitted. If *err* is present, then LINGO will not halt the run if a numeric error occurs; it will be up to your model to handle the error. If *err* is not present and a numeric error occurs, then LINGO will halt model execution.

For more information, refer to the section *Matrix Functions.*

366. THE MATRIX IS NOT POSITIVE DEFINITE.

Some of LINGO's matrix operations require that the matrix be positive definite (e.g., *@CHOLESKY*). You will receive this error message if this requirement is not met. You may wish to investigate the use of LINGO's *@POSD* function to force a matrix to be positive definite.

367. THE SET'S DIMENSION EXCEEDS THE LIMIT OF: N.

There's a limit to the number of dimensions in a set. For practical purposes, this limit is quite large, and reasonably formulated models should never hit the limit.

368. INVALID @REGRESS USAGE.

The *@REGRESS* function is used to perform *multiple linear regression*, a technique that models the linear relationship, *Y = b0 + B*X*, between one dependent variable and one or more independent variables The syntax for *@REGRESS* is:

B, b0, RES, rsq, f, p, var = @REGRESS(Y, X);

where,

- ***B*** The vector of coefficient terms for the regression. If there are p independent variables, then B is a vector of length p.
- ***b0*** The constant term of the regression. If a constant term is not desired, then you can omit this argument and LINGO will force the constant term to 0.
- ***RES*** The residuals, or error terms. These are the differences between the predicted and actual observed values for the dependent variable.
- ***rsq*** The R-squared statistic, a measure of strength of the relationship between the model and dependent variable. The fraction of the original variance removed by the forecast formula.
- ***f*** The F-value, a measure of the overall significance of the model.
- ***p*** The p-value for the regression, a measure of the significance of the F-test.
- ***var*** Estimate of variance of the residual terms.
- ***Y*** The observed values for the dependent variable. If there are n observations, then Y should be a vector of length n.
- ***X*** The independent variable values. If there are n observations and p independent variables, then X should be an nxp matrix.

All matrices must be dense and cannot be defined on sparse sets, and *@REGRESS* may only be used in a model's calc section.

As long as at least one left-hand side argument is present, all other arguments may be omitted. If *err* is present, then LINGO will not halt the run if a numeric error occurs; it will be up to your model to handle the error. If *err* is not present and a numeric error occurs, then LINGO will halt model execution.

For more information, refer to the section *Matrix Functions.*

369. INVALID @CHOLESKY USAGE.

The *@CHOLESKY* function performs a Cholesky factorization of the symmetric, positive-definite matrix *A,* where *LL' = A*, and *L* is a lower-triangular matrix with real and positive diagonal elements. One can think of *L* as the "square root" of matrix *A*. Both *A* and *L* must be square and of the same dimension. The upper triangle of *L* will be filled with zeros. Syntax for *@CHOLESKY* is:

L[, err] = @CHOLESKY(A);

Both matrices must be dense and cannot be defined on sparse sets, and *@CHOLESKY* may only be used in a model's calc section.

The optional argument, *err*, is an error flag that will be 0 if the factorization was successful, otherwise it will be 1. If *err* is present, any error will be ignored by LINGO and will be left for your model to handle. If *err* is not present, then, on any error, LINGO will display an error message and terminate the run.

For more information, refer to the section *Matrix Functions.*

370. INVALID @INVERSE USAGE.
The *@INVERSE* function returns the inverse of matrix *A*. The syntax for *@INVERSE:*

AINV[, err] = @INVERSE(A);

where both *AINV* and *A* must be square and dense *n*x*n* matrices.

The optional argument, *err*, is an error flag that will be 0 if the invert operation was successful, otherwise it will be 1. A situation where *err* would be nonzero is when *A* is a singular matrix. If *err* is present, any error will be ignored by LINGO and will be left for your model to handle. If *err* is not present, then, on any error, LINGO will display an error message and terminate the run.

For more information, refer to the section *Matrix Functions.*

371. INVALID @TRANSPOSE USAGE.
The *@TRANSPOSE* function returns the transpose of matrix *A*. The syntax for *@TRANSPOSE* is:

T = @TRANSPOSE(A);

Matrix *A* does not have to be square, but if *A* is an *m*x*n* matrix, then *T* must be of dimension *n*x*m*. In addition, both matrices must be dense and cannot be defined on sparse sets, and *@TRANSPOSE* may only be used in a model's calc section.

For more information, refer to the section *Matrix Functions.*

372. INVALID @DETERMINANT USAGE.
The *@DETERMINANT* function returns the determinant value for matrix *A*. The syntax for *@DETERMINANT* is:

@DETERMINANT(A);

The *A* matrix must be dense and cannot be defined on a sparse set, and *@DETERMINANT* may only be used in a model's calc section.

For more information, refer to the section *Matrix Functions*

373. INVALID @MTXMUL USAGE.
The *@MTXMUL* function multiplies matrix *B* by matrix *C* and places the result in matrix *A*. Syntax for *@MTXMUL* is:

A = @MTXMUL(B, C);

The matrices must all be defined on dense sets. The dimensions of the matrices must also agree, for example, if *B* is an *m* x *n* matrix, then *C* must be an *n* x *p*, and *A* must be an *m* x *p* matrix. Furthermore, *m*, *n* and *p* must be greater than 1, i.e., inner and outer products are not currently allowed. *@MTXMUL* may only be used in calc sections, and may not appear in the optimizable model sections.

374. INVALID DATA FRAME USAGE.

The data frame style of input allows us to enter the first dimension of a two-dimensional set along with an attribute of the same two-dimensional set in a single data statement. In the example below, *ROUTES* is a two-dimensional set and one of its attributes is *COST*. The first dimension of *ROUTES* is the primitive set *WAREHOUSE*. Both the *WAREHOUSE* set and the *COST* attribute are initialized in the data section with a data frame:

```
! A 3 Warehouse, 4 Customer
  Transportation Problem;
SETS:
  WAREHOUSE    : CAPACITY;
  CUSTOMER     : DEMAND;
  ROUTES( WAREHOUSE, CUSTOMER) : COST, VOLUME;
ENDSETS

DATA:
  CUSTOMER = C1 C2 C3 C4;

  ! Here we use a data frame to input
    both WAREHOUSE and COST;
  WAREHOUSE, COST =
         W1   6  2  6  7
         W2   4  9  5  3
         W3   8  8  1  5
  ;

  CAPACITY =   30, 25, 21;
  DEMAND   = 15 17 22 12;
ENDDATA
```

375. INVALID @READRM/@READLN USAGE.

@READLN and @READRM may be used to programmatically read data from text files. These functions are only valid in calc sections. Fore more information, refer to the documentation for these functions.

376. K-BEST SOLVES REQUIRE AT LEAST ONE BINARY VARIABLE.

Models must have one or more binary variables to use the *@NEXTKBEST* function to find additional feasible solutions. You may add binary variables to a model with the *@BIN* function. *@NEXTKBEST* can then be used in a calc section to iterate through feasible solutions ranked by objective value.

377. INVALID @QRFACTOR USAGE.

The *@QRFACTOR* function returns a QR factorization of a matrix. The syntax for *@QRFACTOR* is:

Q, R = @QRFACTOR(A);

where $A = QR$, with R being upper triangular and Q being orthogonal (i.e., $Q'Q$ is the identity matrix). The matrices must all be defined on dense sets, with A and Q being $m \times n$ matrices and R an $n \times n$ matrix. *@QRFACTOR* may only be used in calc sections.

378. THE NUMBER OF SCENARIOS EXCEEDS THE MAXIMUM LIMIT.
In a stochastic programming model, the total number of scenarios is currently limited to approximately 2^31. Your model has exceeded this limit. You will need to reduce the number of time periods and/or reduce the sample sizes of the random variables in some of the time periods.

379. UNABLE TO ESTABLISH A LINK TO R.
LINGO was not able to establish a link to R. A link to the R package is required to use any of the R-interface functions, e.g, @RCMD. To establish a link, you will need to do the following

- Install a version of R that has the same bit level (32 or 64 bits) as the copy of LINGO you are using.
- The R bin folder must be on the system path. Typically, the R bin folder will have a name like C:\R\R-<version number>\bin\x64 when running a 64-bit version of R on Windows. R most likely installed this folder name in your system path during installation, but, if not, you will need to modify the path to include this folder.
- The R_HOME environment must point to the main R folder. For instance, on Windows, if you enter the SET command in a command window, you should see something like the following: R_HOME=C:/R/R-<version>, where <version> is the version number of R.
- Finally, you must install the RInside library in your copy of R. You may do this by running R and then entering the command: install.packages("RInside"). You can verify the installation of RInside by entering the R command: library("RInside").

380. A CALL TO THE R PACKAGE FAILED.
LINGO has a connection to R in this case, but a call to R failed. LINGO will display any error message that it retrieved from R, which may give you more insight into the problem. Typically, though, this error results from R commands that don't have valid syntax.

381. THE R SYSTEM FILE R.DLL WAS NOT FOUND ON THE SYSTEM PATH.
In order to interface with R, LINGO must be able to load the R.DLL system file from somewhere on the system path. You should check your PATH environment variable to see that it contains something like the following: C:\R\R-<version number>\bin\x64, where <version number> is the version number of R you have installed. If you open this folder, you should find a copy of R.DLL.

382. THE R_HOME ENVIRONMENT VARIABLE IS NOT DEFINED.
In order for LINGO to interface with R, there must be an environment variable call R_HOME that points to the main R folder. If you run the SET command from a command window, you

should see something like the following: R_HOME=C:/R/R-<version>, where <version> is the version number of R. If this environment variable is not present, you will need to add it.

383. IMPROPER SYNTAX FOR R INTERFACE.
There is a syntax error in a call to the R package. The current syntax is fairly simple in that you may transfer only one object at a time. For example, reading a set and an attribute from R in one statement is not possible. You will need to split the call up into two statements—one to read the set and then a second to read the attribute values.

384. R OBJECT IS NOT NUMERIC.
LINGO is expecting to read an object from R that is numeric. As an example, trying to read a text object into and attribute will cause this error.

385. INVALID @NULLSET(SET_NAME) USAGE.
This function deletes all members of a specified derived set set_name, and is only available for use in calc sections. Note that *@NULLSET* does not currently work on primitive sets. But you can workaround this by creating a derived set directly from a primitive set, and then null out this derived set. For example:

```
SETS:
  MYPRIMITIVESET1 /1..3/;
  MYDERIVEDSET( MYSET1);
ENDSETS
CALC:
  ! The following nulling of a primitive set is not allowed;
  @NULLSET( MYPRIMITIVESET);

  ! The next statement is allowed, given that we are nulling a
   derived set;
  @NULLSET( MYDERIVEDSET);
ENDCALC
```

Once a set is nulled, you will typically add selected members back using the @INSERT function..

386. ATTEMPT TO ALLOCATE TOO LARGE OF A MEMORY BLOCK.
Working memory blocks have a maximum size of 2^31-1 (2,147,483,647) bytes. LINGO will point to the location in the model where the error occurred. You will need to modify the model to reduce the size of the memory block.

387-1000. UNUSED.

1001. NO MATCHING PARENTHESIS WAS FOUND.
LINGO did not find a closing parenthesis for the selected parenthesis. Add a closing parenthesis and try again.

1002. UNABLE TO SOLVE ... NO MODEL CURRENTLY IN MEMORY.
LINGO does not have the text of a model available to attempt to solve. Select a model window and try solving again.

1003. LINGO IS BUSY AND CAN'T COMPLETE YOUR REQUEST RIGHT NOW. TRY AGAIN LATER.
You have attempted an operation that is not possible because LINGO is currently solving a model. To halt the solver, press the *Interrupt Solver* button on the solver status window.

1004. ERROR WRITING TO LOG FILE. PERHAPS THE DISK IS FULL?
LINGO can't write any additional output from the command window to an open log file. Check to see if there is any free space remaining on the disk.

1005. UNABLE TO OPEN LOG FILE.
LINGO could not open the log file that you requested. Does the path exist? Do you have write access to the selected path?

1006. COULD NOT OPEN FILE: *FILENAME*.
LINGO could not open a requested file. Did you spell the file's name correctly? Does the file exist?

1007. NOT ENOUGH MEMORY TO COMPLETE COMMAND.
LINGO was unable to allocate additional system memory while attempting to execute a command. Exit any other applications and try again. If the error still occurs, you may have set the model generator's working memory area to be too large. You can adjust the generator's memory allocation downward on the *General* tab of the *Solver|Options* command's dialog box.

1008. UNABLE TO CREATE A NEW WINDOW.
LINGO was unable to create a new window. This is probably caused by insufficient system resources.

1009. UNABLE TO ALLOCATE ENOUGH MEMORY FOR SOLUTION REPORT.
LINGO could not allocate enough memory to store the solution report to the model. System resources must be very low in order to receive this message. Exit any other applications and try again. If the error still occurs, you may have set the model generator's working memory area to be too large. You can adjust the generator's memory allocation downward on the *General* tab of the *Solver|Options* command's dialog box. If all else fails, you can use the *Solver|Solution* command to request smaller solution reports for individual variables.

1010. NOT USED.

1011. UNABLE TO COMPLETE GRAPHICS REQUEST.
LINGO was unable to generate a requested graph. Either LINGO is incorrectly installed or there is not enough free memory to perform the operation. Try cutting back on the amount of memory allocated to LINGO's model generator using the *Solver|Options* command. If this doesn't help, try reinstalling LINGO.

1012. NOT USED.

1013. NOT USED.

1014. NOT USED.

1015. EDIT COMMAND NOT SUPPORTED UNDER WINDOWS. USE THE FILE|OPEN COMMAND INSTEAD.
The *EDIT* command is not supported in Windows versions of LINGO. If you need to edit the file, use the *File|Open* command to load the file into a window for editing.

1016. COULD NOT SAVE FILE.
LINGO was unable to save the file. Check for a valid path name and sufficient disk space.

9999. OPTIMIZER FAILED DUE TO A SERIOUS ERROR. PLEASE REFER TO YOUR DOCUMENTATION UNDER APPENDIX B, "ERROR MESSAGES".
The optimizer was unable to continue due to a serious error. The most likely cause of this error stems from a problem in evaluating the functions within your model. Not all functions are defined for all values of their arguments. For example, *@LOG(X – 1)* is undefined for values of *X* less-than-or-equal-to 1. You should check your model for any such functions, and use the *@BND* function to place bounds on your variables to keep the optimizer from straying into any regions where functions become undefined.

Another complicating factor can be nonlinear relations in a model. Linear models can be solved much more reliably and quickly than nonlinear models of comparable size. If possible, try to linearize your model by approximating nonlinear functions with linear ones, or by eliminating nonlinear equations.

Another possible remedy when dealing with nonlinear models is to attempt different starting points. A starting point may be input in the model's *INIT* section. Refer to Chapter 5, *Windows Commands*, for more details.

If this problem persists after bounding any variables that could potentially lead to a problem, removing any unnecessary nonlinearities, and attempting new starting points, please contact LINDO systems for assistance.

Appendix C: Bibliography and Suggested Reading

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